36-402 Homework 2

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January 29, 2019

Question 1

Table 1: Coefficients & Std. Error of linear model

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.03525	0.00665	-5.30037	0.00000
underval	0.00476	0.00218	2.18614	0.02898
log(gdp)	0.00630	0.00079	7.96591	0.00000

Since the coeficient of $log(gdp)$ is significantly positive, this model does not seem to support the idea of "catching-up" as countries with higher GDP have a higher economic growth rate. However, it does support the idea that under-valuing a currency boosts economic growth as the coefficient of *underval* is quite significantly positive (assuming 5% significance level), indicating a positive *underval* index, which represents undervaluing, leads to higher economic growth.

Question 2

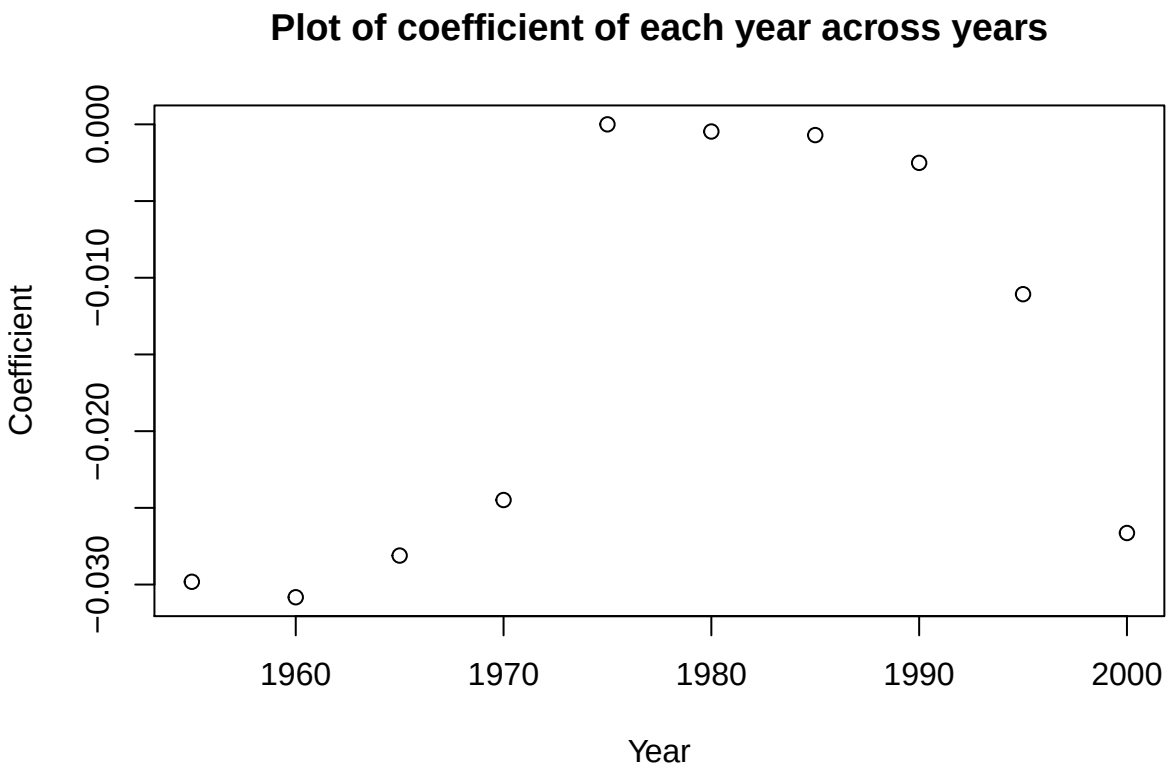
Q2 a)

Table 2: Coefficients & Std. Error of linear model

	Estimate	Std. Error	t value	Pr(>|t|)
underval	0.01361	0.00290	4.69667	0
log(gdp)	0.02892	0.00317	9.13254	0

Q2 b) It is more appropriate to use `factor(year)` as there are only 10 unique years that are 5 years apart. As such, it might be more appropriate to model year as a discrete variable rather than a continuous one. Modelling this way, we will have a slope for each 5-year interval rather than a single slope for each increment of year.

Q2 c)



Q2 d)

Since the coeficient of $log(gdp)$ is positive, this model does not seem to support the idea of "catching-up" as countries with higher GDP have a higher economic growth rate. However, it does support the idea that under-valuing a currency boosts economic growth as the coefficient of *underval* is positive, indicating a positive *underval* index, which represents undervaluing, leads to higher economic growth.

Question 3

Q3 a)

Table 3: R^2 values for each linear model

	Model 1	Model 2
R^2	0.04855	0.42924
Adj. R^2	0.04709	0.33214

Q3 b)

Table 4: $M\hat{S}E$ of linear models by LOOCV

	x
Model 1	0.001030348892

	x
Model 2	0.000952766927

The second linear model seems to perform better on predictions, having an estimated MSE that is 7.53% lower than the first model.

Q3 c)

Since the lowest count of any country is 2, doing 5-fold cross validation has a high chance of resulting in the training 4-folds missing some countries because the 2 rows are in the testing 1-fold that is left out. This would create models that leave out some countries by chance and fail to predict with a new category it has not seen in training.

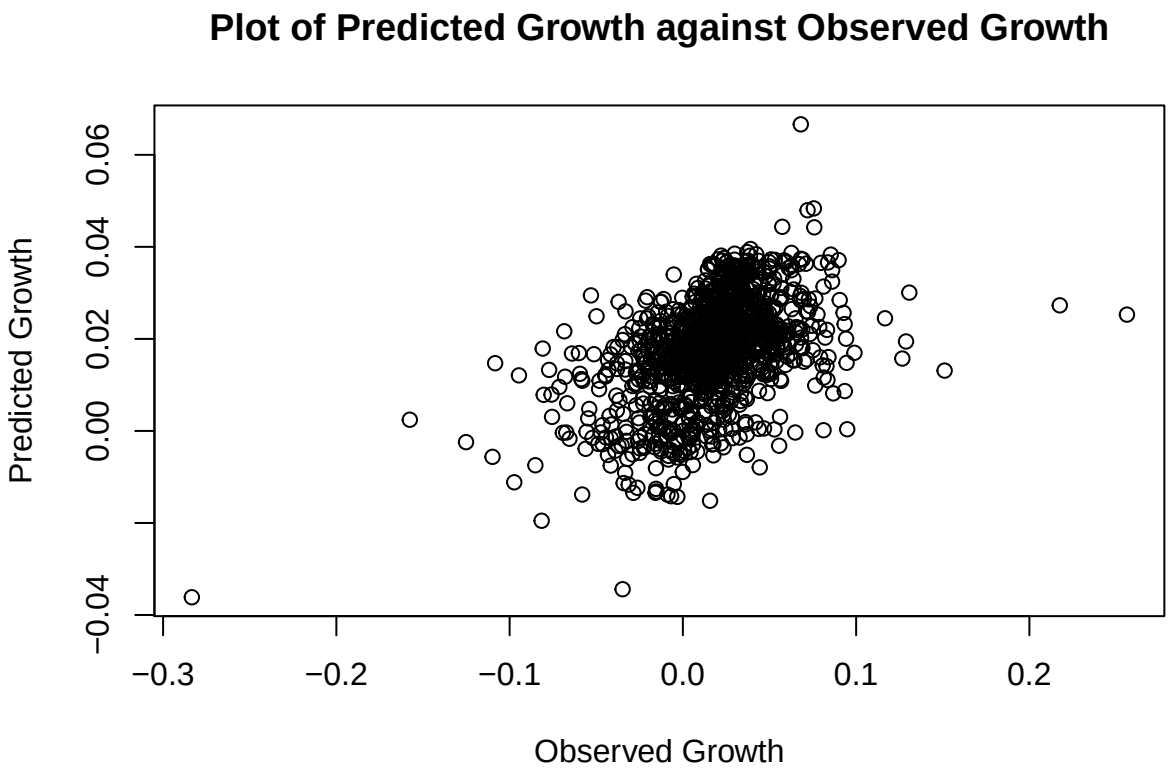
Question 4

```
## Nonparametric Kernel Methods for Mixed Datatypes (version 0.60-9)
## [vignette("np_faq",package="np") provides answers to frequently asked questions]
## [vignette("np",package="np") an overview]
## [vignette("entropy_np",package="np") an overview of entropy-based methods]
```

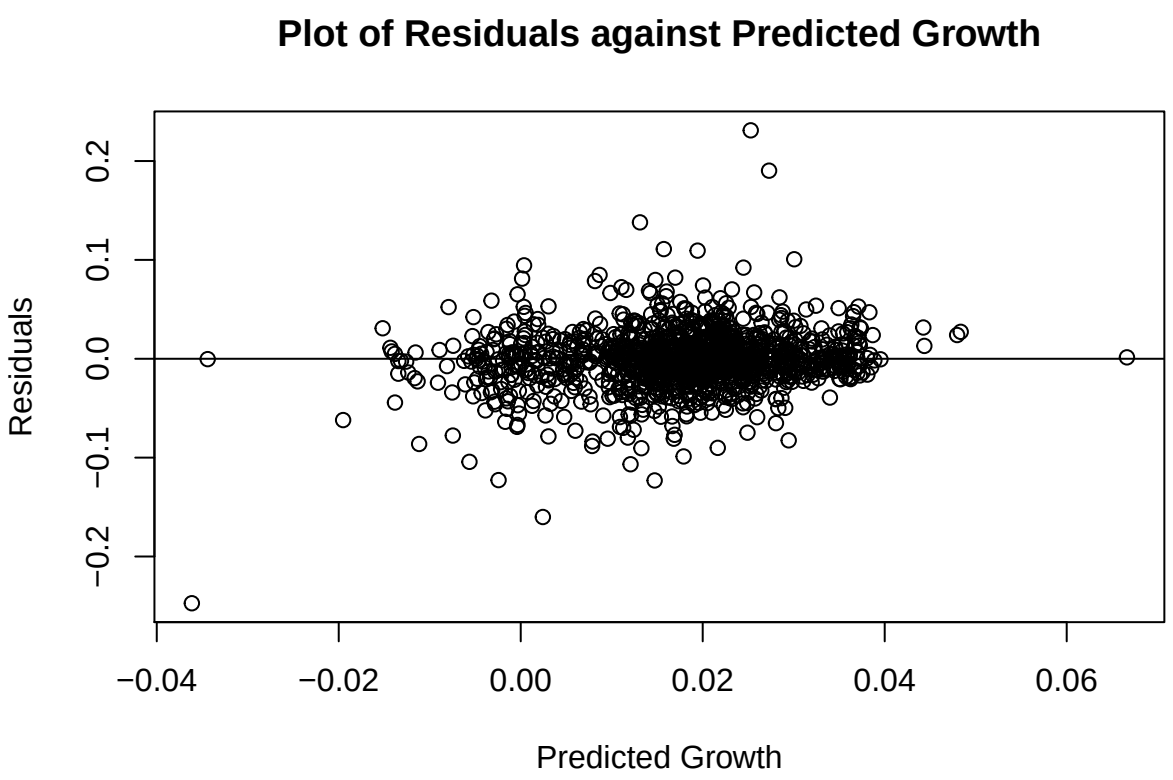
Q4 a)

There are no coefficients to report for kernel regression, as it is a non-parametric smoothing method that smooths the data with a kernel. The smoothing is only controlled by the choice of the kernel and the related bandwidth for the kernel, without any coefficients on the predictors.

Q4 b)


Plot of Predicted Growth against Observed Growth
Predicted Growth
0.06
0.04
0.02
0.00
−0.04
−0.3
−0.2
−0.1
0.0
0.1
0.2
Observed Growth

Q4 c)



The points should be scattered around a flat line at 0 if the model was right, as we assume $\mathbb{E}[\epsilon] = 0$. In this case, it would seem that the residuals indeed are roughly scattered around a flat line at 0.

Q4 d)

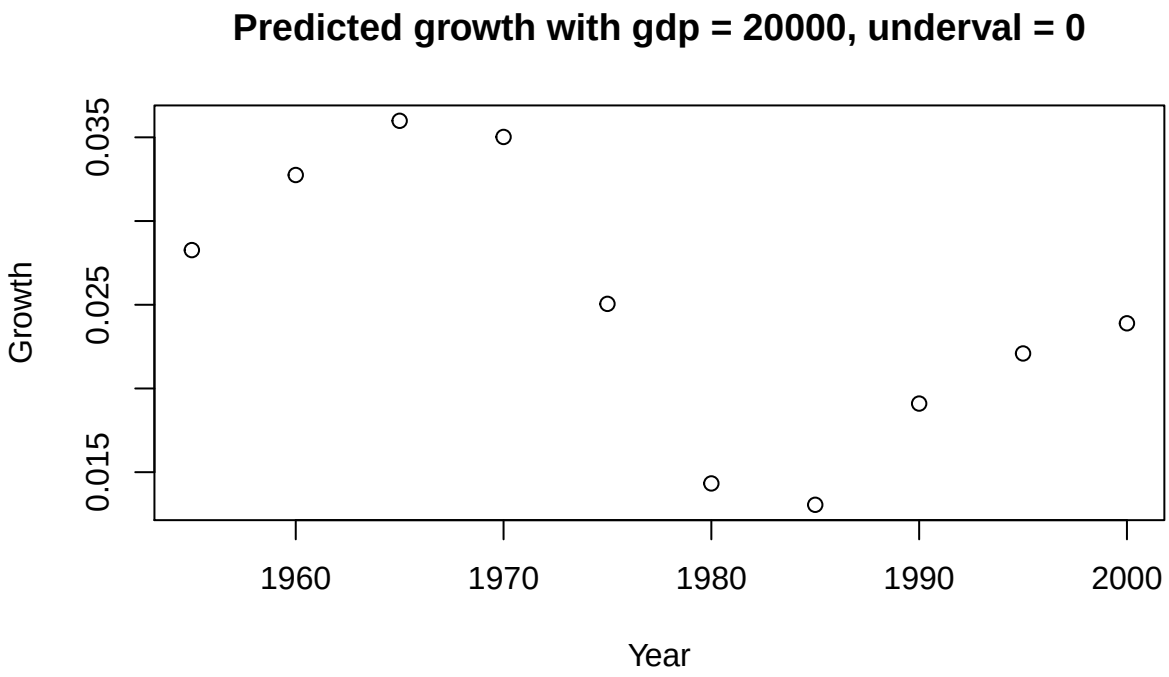
Table 5: $\hat{MSE}$ of the regressions via cross validation

	x
Linear Regression	0.0009688
Kernel Regression	0.0009481

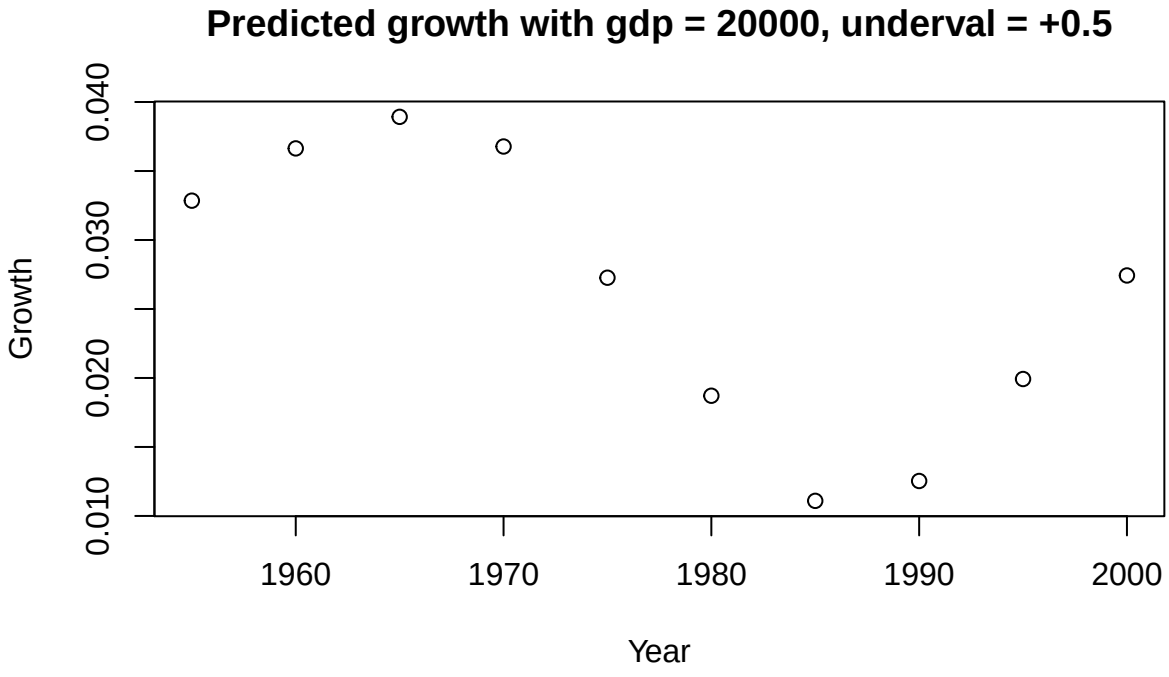
As seen from above, kernel regression has a lower estimated MSE than a linear model with the same covariates. Hence, the kernel regression is better in generalizing and predicting better than the linear model.

Question 5

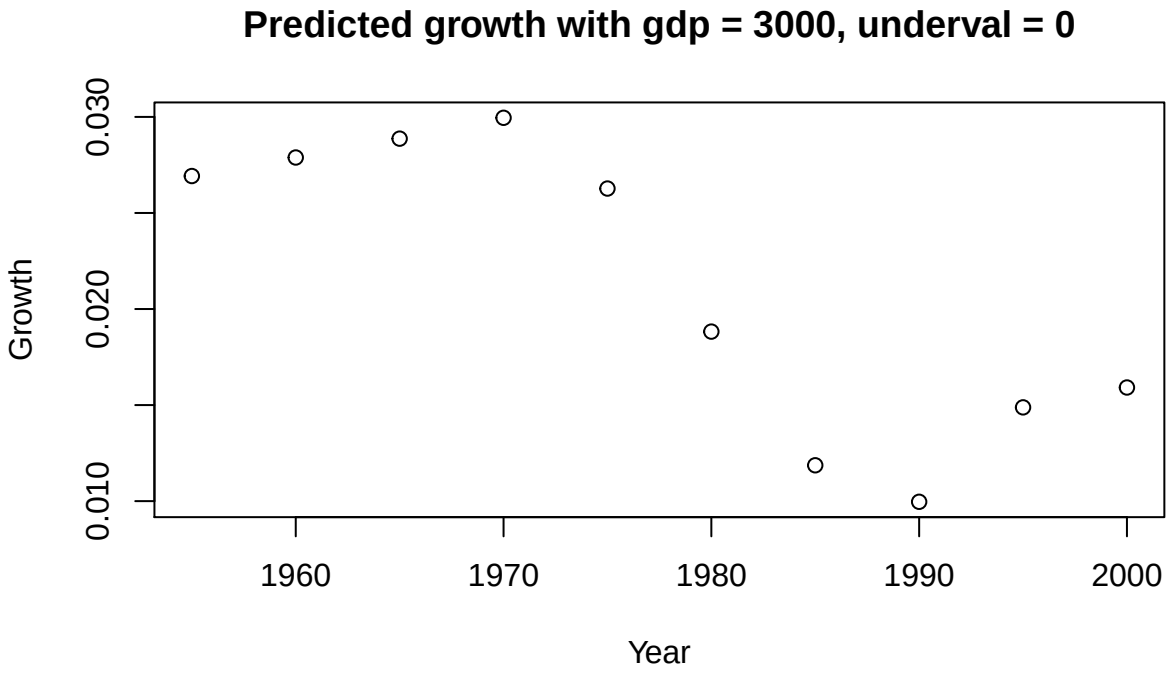
Q5 a)


Predicted growth with gdp = 20000, underval = 0
Growth
0.015
0.025
0.035
1960
1970
1980
1990
2000
Year

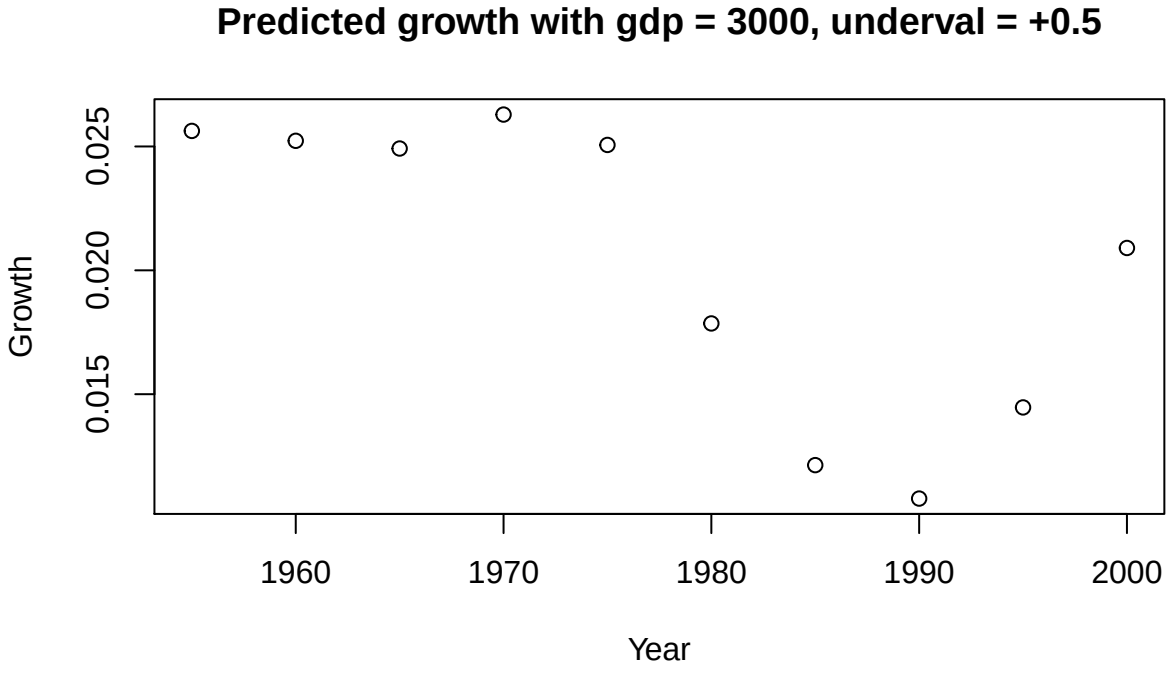
Q5 b)


Predicted growth with gdp = 20000, underval = +0.5
Growth
0.010
0.020
0.030
0.040
1960
1970
1980
1990
2000
Year

Q5 c)



Q5 d)

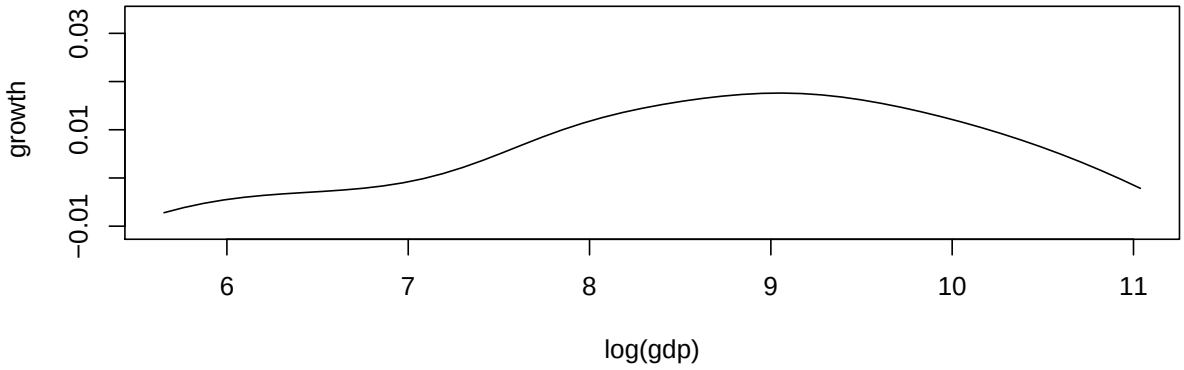


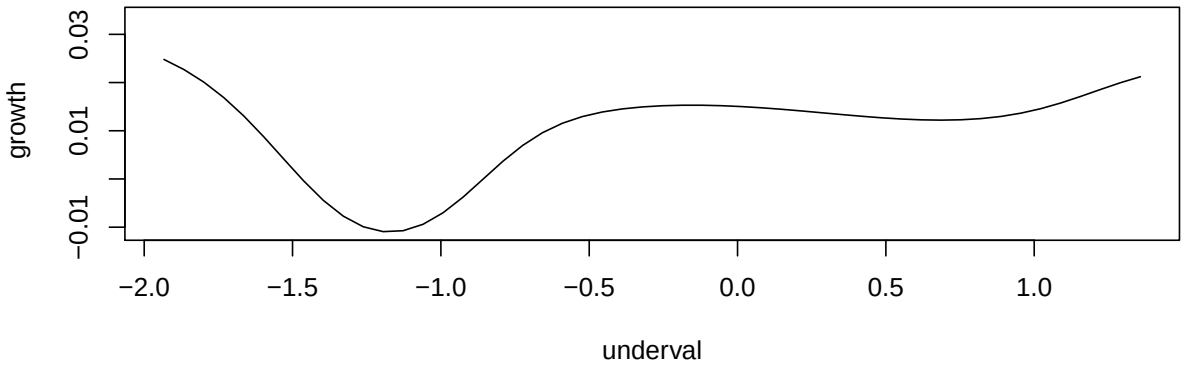
Q5 e)

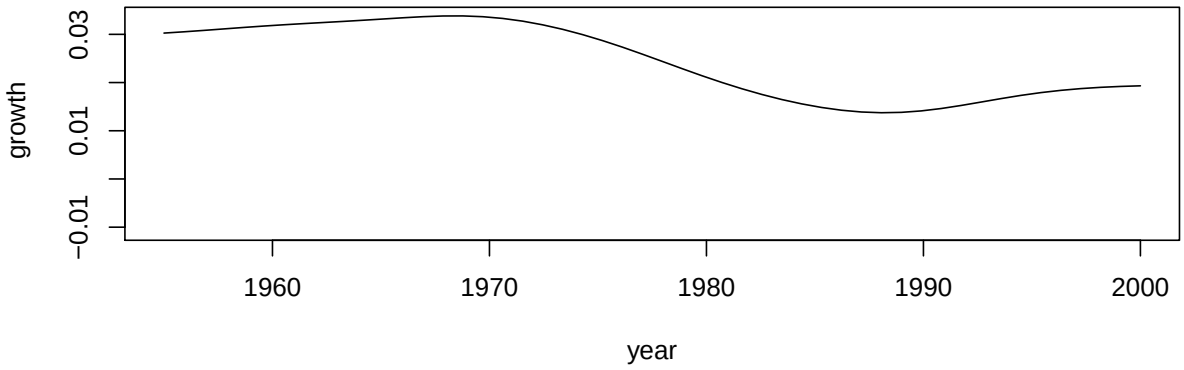
By comparing the graphs with initial $GDP = 20000$, we can see that by increasing under-valuation from 0 to 0.5, the predicted values increased from 1955 to 1970 but for $GDP = 3000$, increasing under-valuation

from 0 to 0.5 decreased the predicted values from 1955 to 1970. The graphs are not parallel across and the effects not similar when we change intial GDP and under-valuation, hence there should be some interaction between these two variables.

Q5 f)







From the plots of growth against each variable, we can see that both GDP and under-valuation have strong relationshipswith growth. For the plot of growth against $log(GDP)$, there seems to be significant variation from varying $log(GDP)$ from 7 to 11. For the plot of growth against under-valuation, there also seems to be significant variation from varying under-valuation from -2.0 to -0.5. Finally, there is also some relationship, a weaker one than the previous two, between growth and year. For the plot of growth against year, there seems to be a slightly significant variation from avarying year from 1970 to 1985.

Question 6

$$\begin{aligned}
\text{Optimism} &= \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n}(Y_i' - \hat{\mu}(x_i))^2\right] - \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{\mu}(x_i))^2\right] && (1)\\
&= \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}\left[(Y_i' - \mu(\hat{x_i}))^2\right] - \mathbb{E}\left[(Y_i - \mu(\hat{x_i}))^2\right] && (2)\\
&= \frac{1}{n}\sum_{i=1}^{n}\text{Var}\left[Y_i' - \hat{\mu}(x_i)\right] + \mathbb{E}\left[Y_i' - \mu(\hat{x_i})\right]^2 - \text{Var}\left[Y_i - \mu(\hat{x_i})\right] - \mathbb{E}\left[Y_i - \mu(\hat{x_i})\right]^2 && (3)\\
&= \frac{1}{n}\sum_{i=1}^{n}\text{Var}\left[Y_i'\right] + \text{Var}\left[\hat{\mu}(x_i)\right] - 2\text{Cov}\left[Y_i', \hat{\mu}(x_i)\right] - \text{Var}\left[Y_i\right] - \text{Var}\left[\hat{\mu}(x_i)\right] + 2\text{Cov}\left[Y_i, \hat{\mu}(x_i)\right] && (4)\\
&+ \left(\mathbb{E}\left[Y_i'\right] - \mathbb{E}\left[\hat{\mu}(x_i)\right]\right)^2 - \left(\mathbb{E}\left[Y_i\right] - \mathbb{E}\left[\hat{\mu}(x_i)\right]\right)^2 && (5)\\
&= \frac{1}{n}\sum_{i=1}^{n}2\text{Cov}\left[Y_i, \hat{\mu}(x_i)\right], \text{ as } \text{Var}\left[Y_i'\right] = \text{Var}\left[Y_i\right], \mathbb{E}\left[Y_i'\right] = \mathbb{E}\left[Y_i\right], \text{and } \text{Cov}\left[Y_i', \hat{\mu}(x_i)\right] = 0 && (6)\\
&= \frac{2}{n}\sum_{i=1}^{n}\text{Cov}\left[Y_i, \sum_{j=1}^{n}w(x_j, x_i)Y_j\right] && (7)\\
&= \frac{2}{n}\sum_{i=1}^{n}w(x_i, x_i)\text{Var}\left[Y_i\right], \text{ as } \text{Cov}\left[Y_i, Y_j\right] = 0 \forall i \neq j && (8)\\
&= \frac{2\sigma^2}{n}\text{tr}(\mathbf{w}) = \frac{2\sigma^2}{n}df(\hat{\mu}) && (9)
\end{aligned}$$

Question 7

$$\begin{aligned}
h\hat{_{CV}} &= h_{opt}O(n^{-1/10}) + 2h_{opt} && (10)\\
&= O(n^{-1/5})O(n^{-1/10}) + O(n^{-1/5}) && (11)\\
&= O(n^{-3/10}) + O(n^{-1/5}) && (12)\\
&= O(n^{-1/5}) && (13)\\
MSE(h\hat{_{CV}}) - \sigma^2(x) &= O(h\hat{_{CV}}^4) + O((nh\hat{_{CV}})^{-1}) && (14)\\
&= O((n^{-1/5})^4) + O(((n^{1-1/5}))^{-1}) && (15)\\
&= O(n^{-4/5}) + O(n^{-4/5}) && (16)\\
&= O(n^{-4/5}) && (17)
\end{aligned}$$