36-467 Homework 7

Eu Jing Chua

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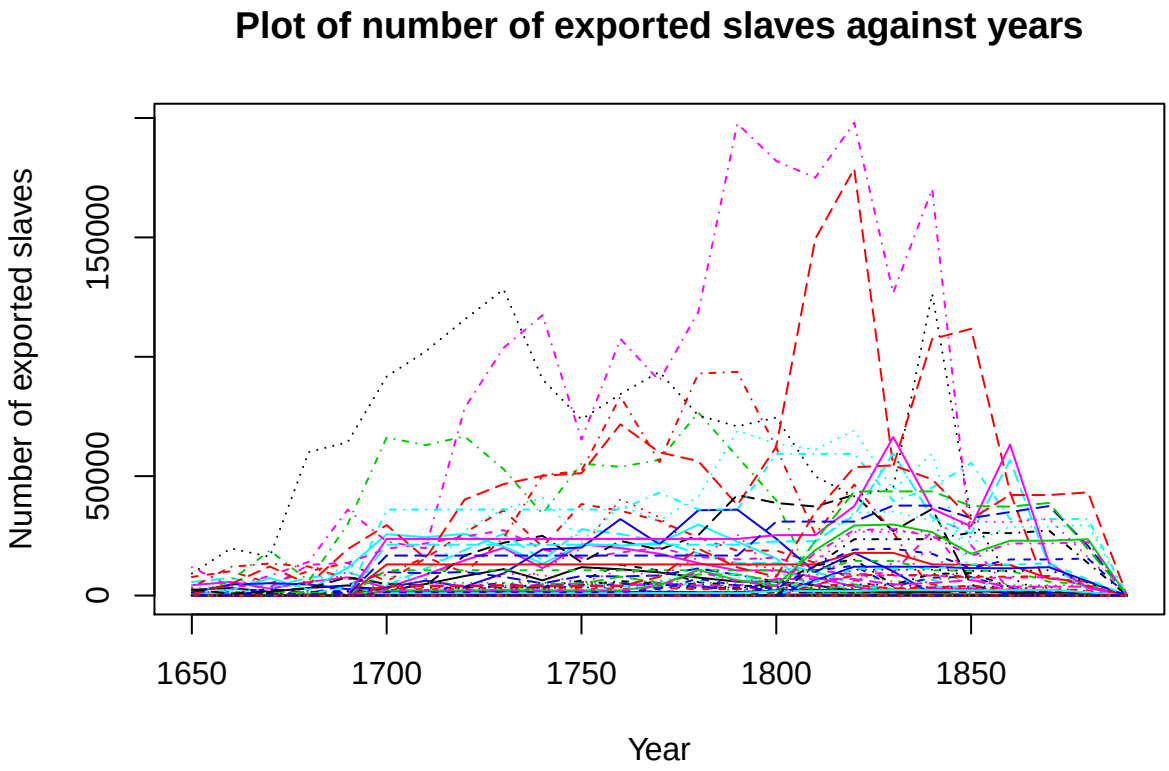
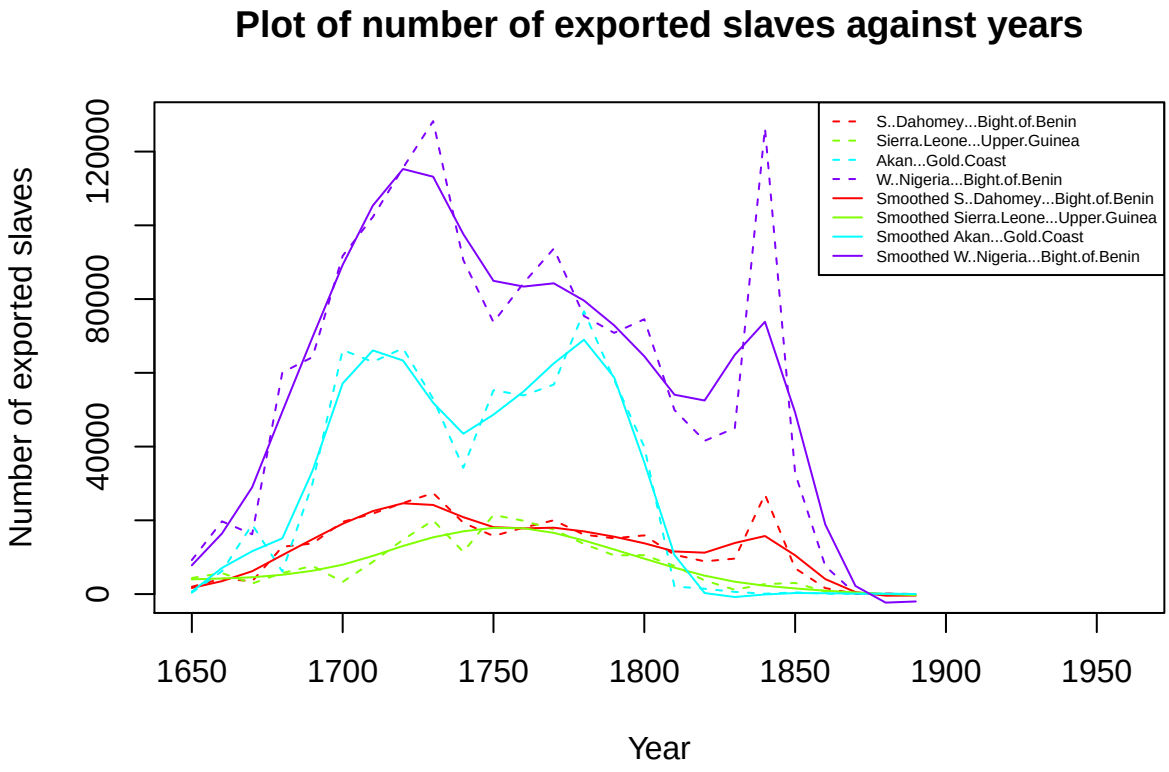
Question 1

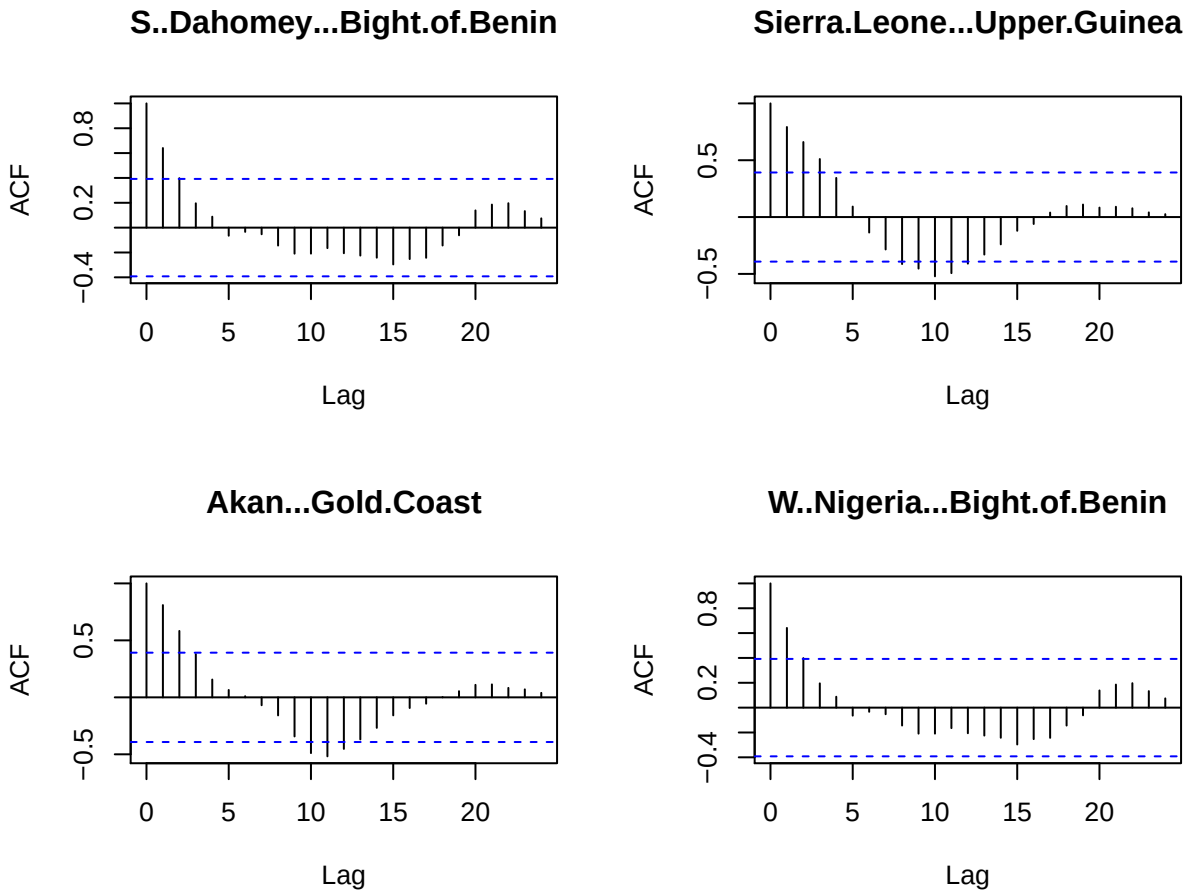
Q1 a)



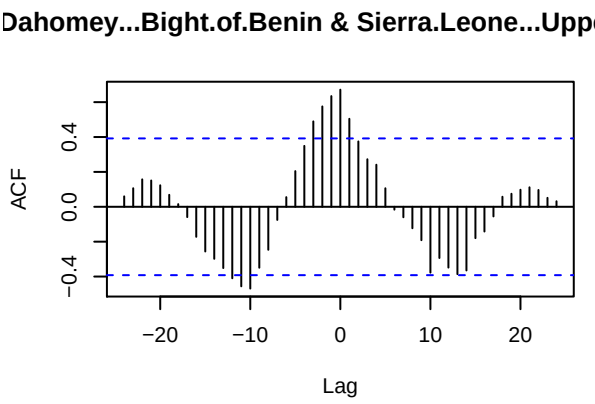
Q1 b)

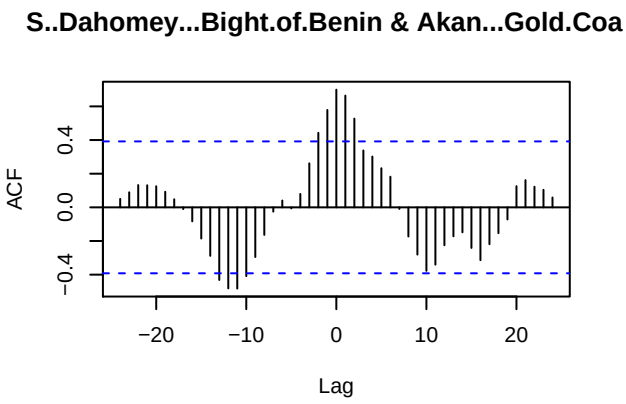

Plot of number of exported slaves against years
Number of exported slaves
0
40000
80000
120000
1650
1700
1750
1800
1850
1900
1950
Year
S..Dahomey...Bight.of.Benin
Sierra.Leone...Upper.Guinea
Akan...Gold.Coast
W..Nigeria...Bight.of.Benin
Smoothed S..Dahomey...Bight.of.Benin
Smoothed Sierra.Leone...Upper.Guinea
Smoothed Akan...Gold.Coast
Smoothed W..Nigeria...Bight.of.Benin

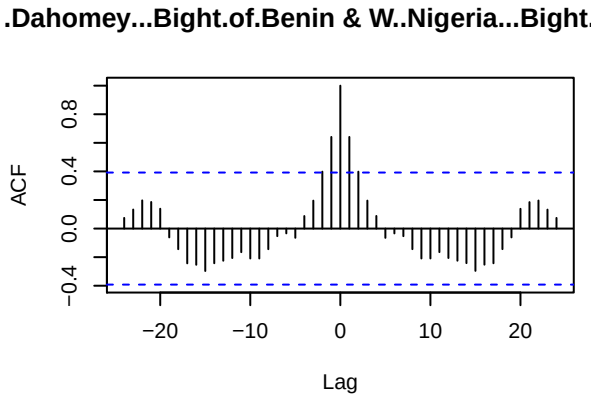
Q1 c)

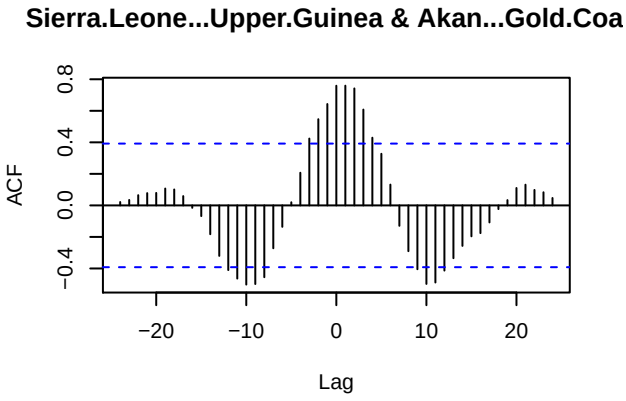

S..Dahomey...Bight.of.Benin
ACF
0.8
0.2
−0.4
0
5
10
15
20
Lag
Sierra.Leone...Upper.Guinea
ACF
0.5
−0.5
0
5
10
15
20
Lag
Akan...Gold.Coast
ACF
0.5
−0.5
0
5
10
15
20
Lag
W..Nigeria...Bight.of.Benin
ACF
0.8
0.2
−0.4
0
5
10
15
20
Lag

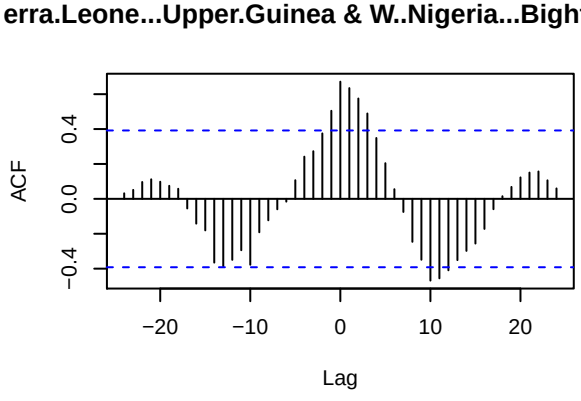
Q1 d)

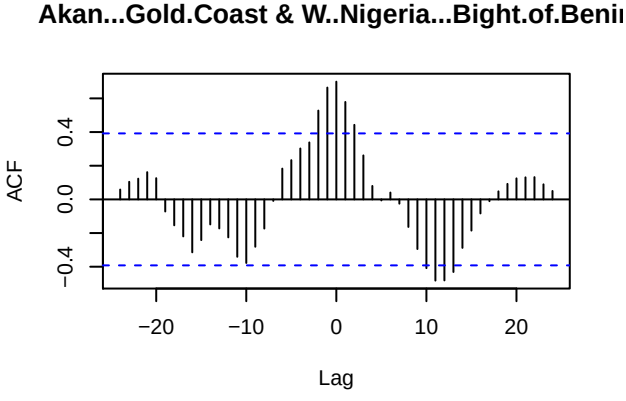












Question 2

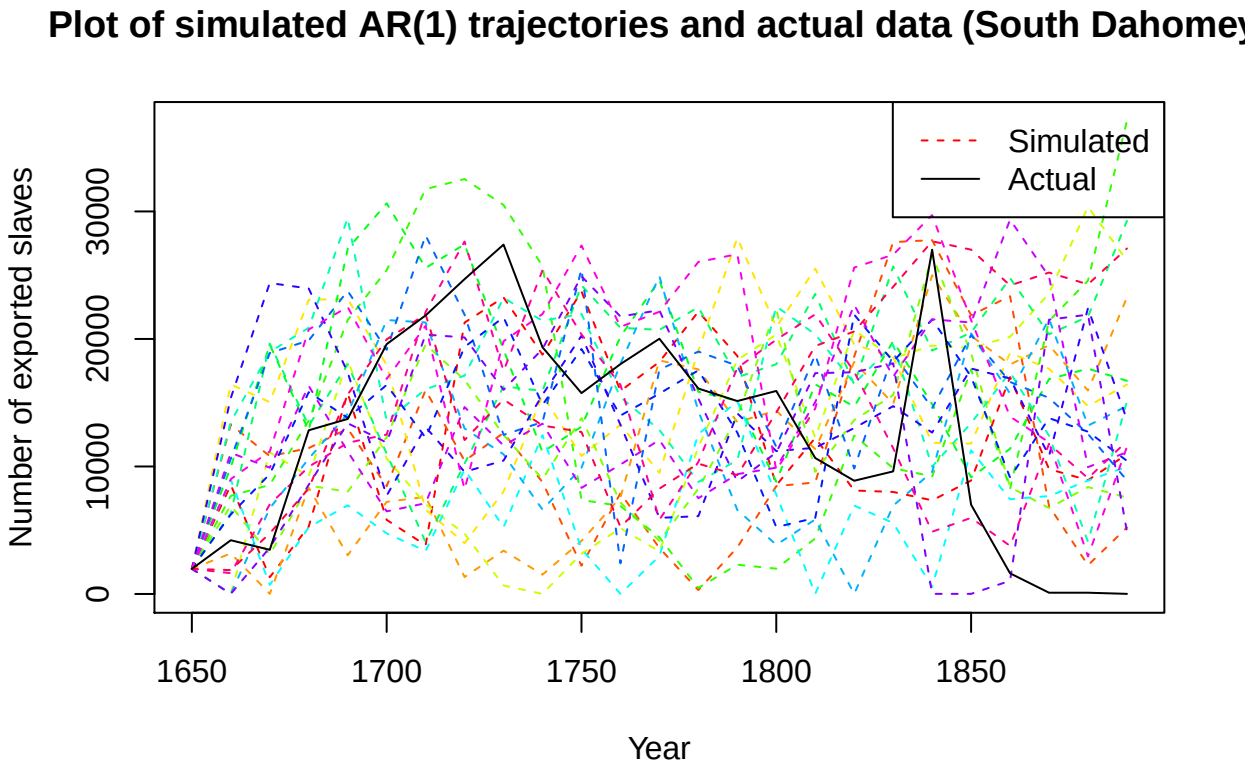
Q2 a)

	Coefficients
Slope	0.7022664
Intercept	3826.2165283

Q2 b) The stationary mean implied by this process is 12851 while the implied stationary variance is 36368859. The sample mean is 12601 and variance is 74742770. The means are very similar, but the variance is about half that of the sample variance.

Q2 c)

Assuming the innovations are distributed normally with mean 0 and variance as above, and the minimum value that can be generated is 10 as in the data:



The data does look like a run of the simulation.

Q2 d)

	Coefficients
X(t-1)	0.6775193
X(t-2)	0.0263159
Intercept	3847.0198526

Q2 e)

The RMS of the AR(1) model is 16629 while that of AR(2) is 13080.
Thus it seems AR(2) is performing better.

Question 3

Q3 a)

Table 3: VAR(1) Intercepts

	Coefficients
S. Dahomey	6085.068
Sierra Leone	370.789
Akan	2956.646
W. Nigeria	28412.853

Table 4: VAR(1) Slopes

	S. Dahomey	Sierra Leone	Akan	W. Nigeria
S. Dahomey	-77.338	-0.179	0.154	16.594
Sierra Leone	-5.413	0.510	0.078	1.177
Akan	-39.495	0.335	0.745	8.467
W. Nigeria	-363.706	-0.835	0.722	78.039

Q3 b)

Table 5: Eigenvalues of est. slope matrix of VAR(1)

Eigenvalues
0.892
0.495
0.398
0.171

The eigenvalues are all less than 1 in magnitude, which indicate that each step of the autoregression shrinks in each component of the eigenvectors, which acts to counter the addition of variance from innovation to form an approximately stationary process.

Q3 c)

Table 6: VAR(1) Slopes for S. Dahomey

	x
S. Dahomey	-77.338
Sierra Leone	-0.179
Akan	0.154
W. Nigeria	16.594

For each increase in number of slaves in each region at $t-1$, take Akan for example, holding all the other variables constant at $t-1$, the expected increase in number of slaves in South Dahomey at t is 0.154, the slope value. The same reasoning then carries on for the other variables of South Dahomey, Sierra Leone, Akan and West Nigeria at $t-1$.

Q3 d) The expected change in number of slaves exported in 1710 from Akan would be 0.335.
The expected change in number of slaves exported in 1720 from Sierra Leone would be $0.510^2 = 0.260$.

Question 4

Q4 a)

	Mauritania	Senegal	Gambia	Guine.Bissau	Guinee	Sierra.Leone
Slope	0.846	0.847	0.845	0.840	0.841	0.841
Intercept	586.388	1880.299	231.354	296.241	1452.784	1123.438

	Liberia	Ivory.Coast	Akan	TVT	Togo	S..Dahomey
Slope	0.819	0.820	0.843	0.842	0.843	0.702
Intercept	403.802	1244.176	4494.716	673.584	1748.853	3826.233

	W..Nigeria	E..Nigeria	Br..Cameroon	SW.Fr..Cam	Rio.Muni	Gabon
Slope	0.703	0.818	0.816	0.817	0.368	0.373
Intercept	17855.720	5158.509	497.835	1986.963	1685.634	4581.352

	Congo.Brazzaville	Low	Cabinda	W..Ubangi.Chari	Katanga	Angola.Other
Slope	0.639	0.639	0.623	0.638	0.798	0.798
Intercept	1533.262	18129.918	98.010	1272.516	5326.913	15240.731

	W..N..Rhodesia	Upper.Volta	Mali	N..Gold.Coast	N..Nigeria	Niger
Slope	0.798	0.820	0.820	0.811	0.759	0.758
Intercept	3251.289	1827.186	1481.383	509.019	7851.055	919.234

	N..Dahomey	N..Cam	Chad	E..Ubangi.Chari	S..Sudan	N..Sudan
Slope	0.754	0.758	0.774	0.773	0.556	0.556
Intercept	296.158	588.275	1067.070	296.373	8926.992	9943.512

	Br..Somalia	Ital..Somalia	Djibouti	Ethiopia	Eritrea	N..Nyasaland
Slope	0.832	0.831	0.822	0.830	0.832	0.884
Intercept	139.044	284.550	12.251	3262.076	199.390	276.001

	Kivu	E..N..Rhodesia	Tanganyika	Rwanda	Burundi	Kenya
Slope	0.884	0.884	0.883	0.885	0.882	0.882
Intercept	362.936	285.901	1398.959	404.319	425.943	1004.423

	Uganda	S..Mozambique	N..Mozambique	S..Nyasaland	Zimbabwe	Madagascar
Slope	0.883	0.801	0.801	0.801	0.801	0.801

	Uganda	S..Mozambique	N..Mozambique	S..Nyasaland	Zimbabwe	Madagascar
Intercept	861.141	3538.629	1931.761	1272.260	2337.633	1816.305

	Namibia	Botswana	South.Africa	Swaziland	Lesotho	Morocco
Slope	NA	NA	NA	NA	NA	NA
Intercept	NA	NA	NA	NA	NA	NA

	Algeria	Tunisia	Libya	Egypt
Slope	NA	NA	NA	NA
Intercept	NA	NA	NA	NA

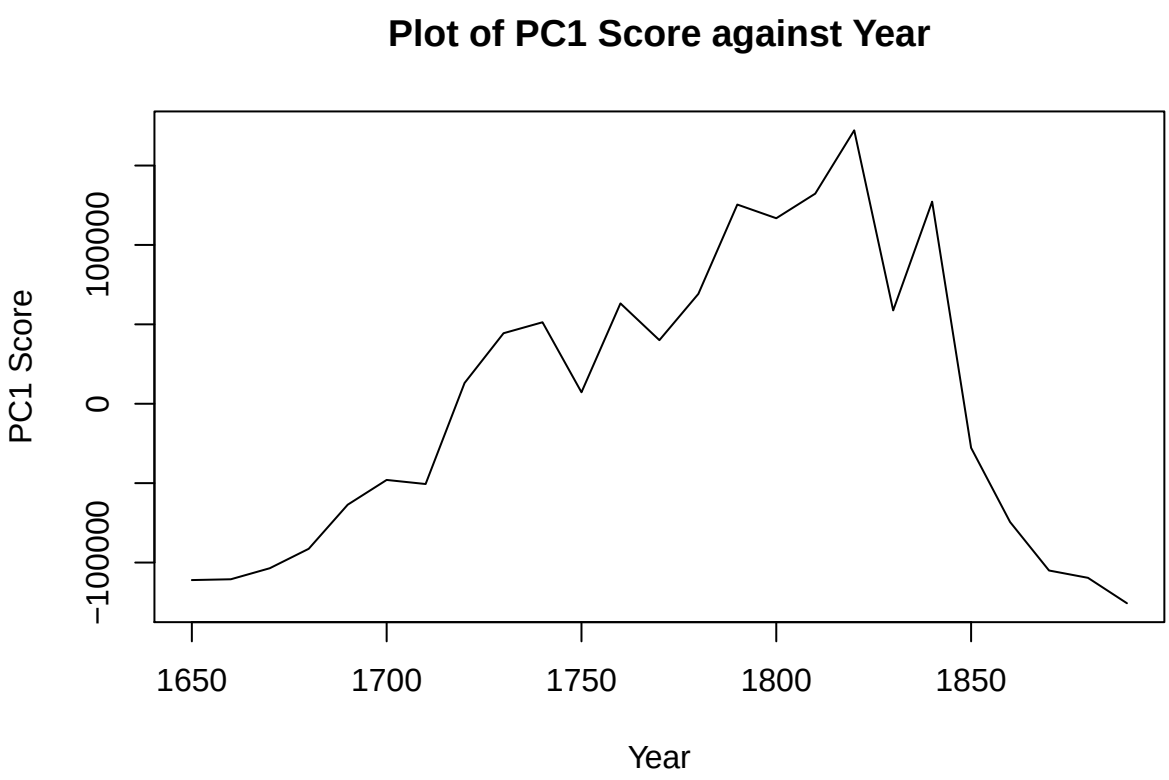
The last 10 areas do not have estimated AR(1) models as their historical data is constant (10 for a very small value), and thus no valuable analysis can be performed on them.

Q4 b) As all the slopes have magnitudes less than 1, we can see that all the 44 AR(1) models with estimates are probably stationary. As for the 10 that had no models, we cannot make any reasonable conclusion about their stationarity.

Question 5

Q5 a) PC1 captures 0.58356 of the variance.

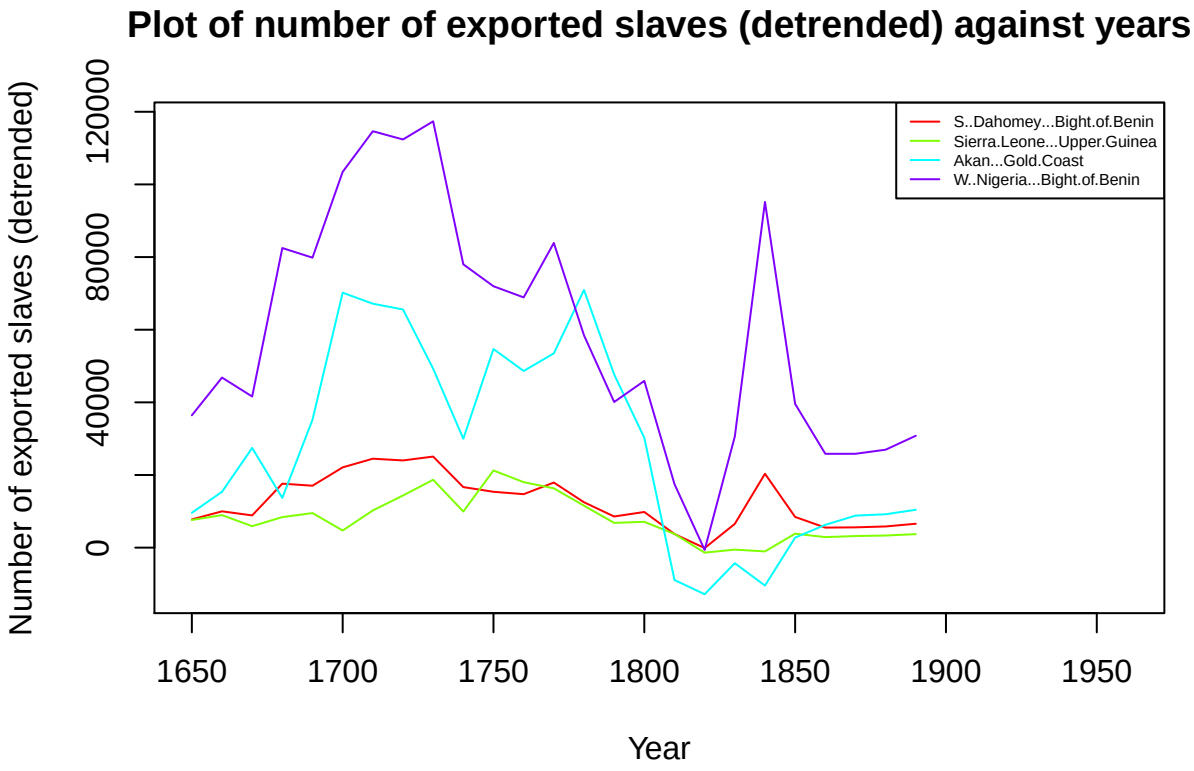
Q5 b)



The shape of the plot seems to be a peaked function, which increased over time up to a peak and then decreased over time after.

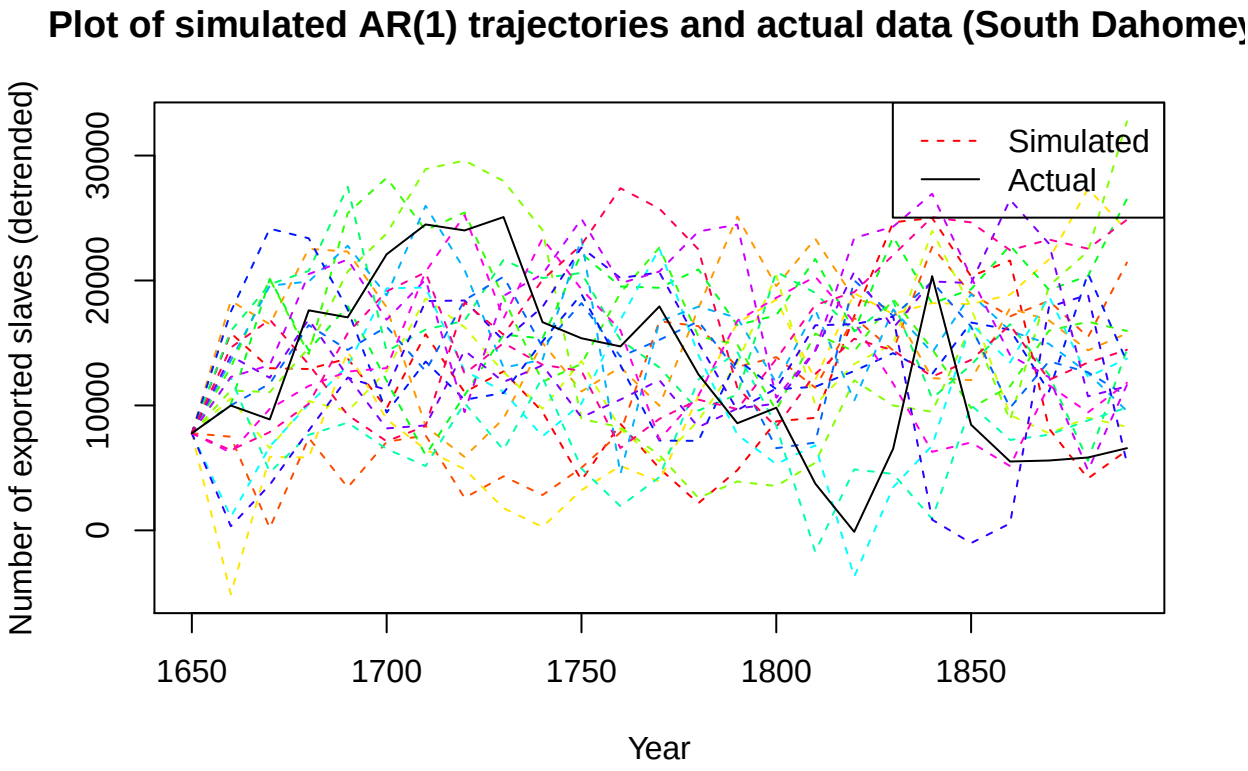
Q5 c) Since we know the values of PC1 over time, and we know that PC1 captures a large proportion of the variance, PC1 can be treated as a good estimator of the variation over time. By subtracting the value of PC1 at each point in time, weighed by the weighing factor of each location, from the actual observed values at each location, we are essentially removing a large proportion of the trend over time, which is akin to detrending.

Q5 d)



The new series look plausibly more detrended than before as the peaks are slighly reduced while the other areas are slightly increased.

Q5 e)



The simulations match the data slightly better after detrending, as before the data was consistently higher than most of the simulations or lower than most of the simulations, but now it is less so and more centered, except in the later years where is is consistently lower than most of the simulations.