

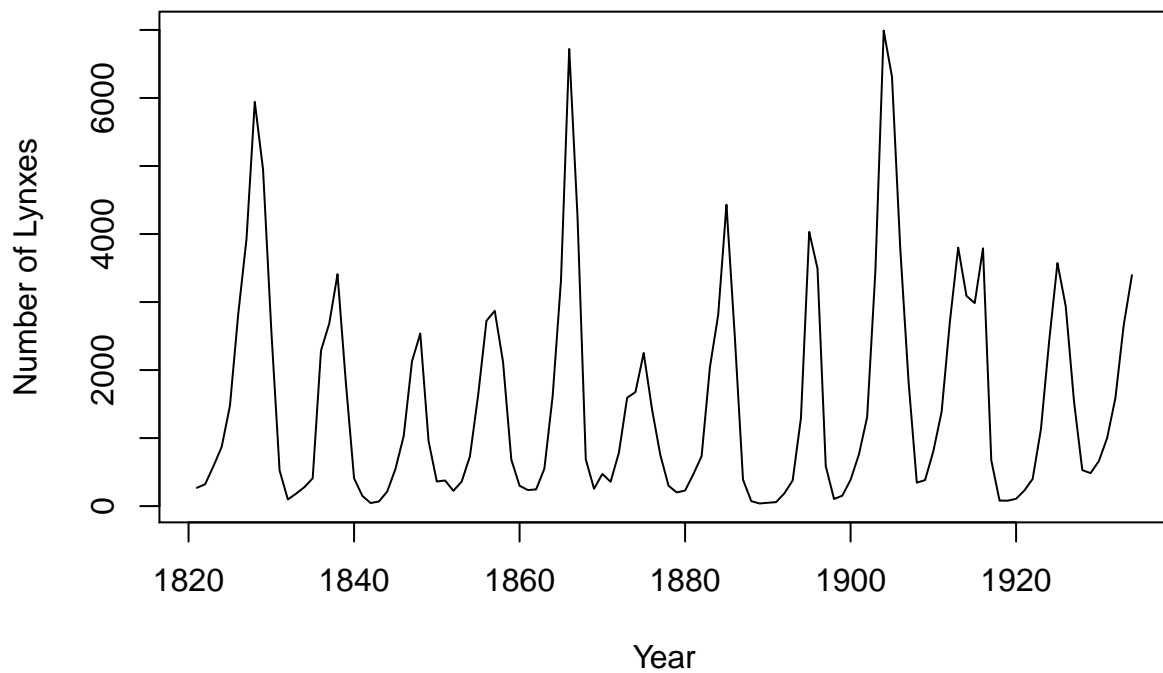
# 36-467 Homework 10

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## Question 1

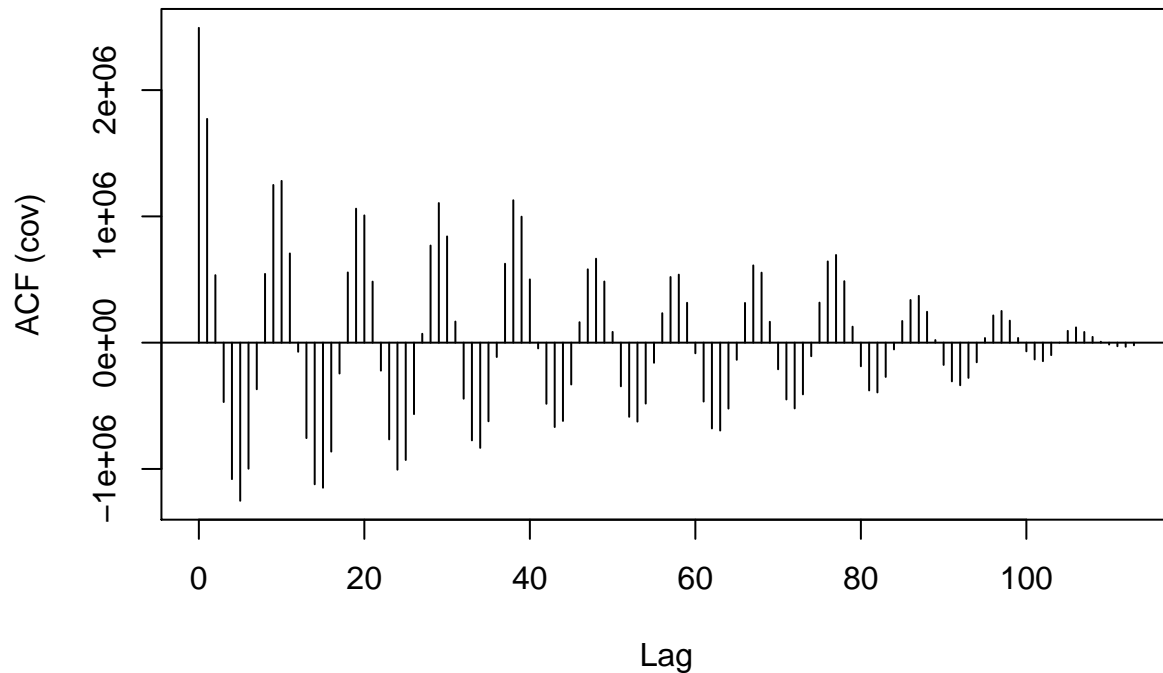
**Plot of Number of Lynxes against Year**



The peaks are roughly 10 years apart.

## Question 2

### ACF of Lynx Counts over Time



The peak of the second positive region is roughly at lag 10 years. The autocovariance at lag 10 is  $1.28 \times 10^6$ .

## Question 3

Table 1: Coefficients of AR(p) models

	Intercept	X(t-1)	X(t-2)	X(t-3)
AR(1)	454.152	0.720	NA	NA
AR(2)	710.106	1.152	-0.606	NA
AR(3)	733.630	1.136	-0.577	-0.026

The coefficient of  $X(t-1)$  does not really stay the same, and we should not expect it to. As we increase  $p$ , the degrees of freedom for fitting increases, allowing for other lags to play a role in fitting a model with less in-sample error. This will most probably change the coefficient of  $X(t-1)$ , as the data appears to be periodic and an AR(1) model will underfit the periodic trend.

## Question 4

In the previous `sim.ar1` function, we assumed the distribution of the innovation was Gaussian with  $\mu = 0$  and constant  $\sigma^2$  and thus sampled such a Gaussian distribution for the simulated innovations.

However, this new simulator resamples the estimated innovations, or residuals, from the input fitted AR model, which acts as an empirical distribution for the innovations.

The input `mdl` would have to a fitted `ar` model with intercept,  $X(t - 1)$  coefficient, and residuals.

## Question 5

Q5 a)

Table 2: Standard Error of Coefficients for AR(1) model

	Coefficient
Intercept	168.001
$X(t - 1)$	0.068

Q5 b)

Table 3: Standard Error of Coefficients for AR(2) model

	Coefficient
Intercept	104.007
$X(t - 1)$	0.072
$X(t - 2)$	0.071

Q5 c)

Table 4: Standard Error of Coefficients for AR(3) model

	Coefficient
Intercept	148.828
$X(t - 1)$	0.108
$X(t - 2)$	0.146
$X(t - 3)$	0.114

## Question 6

Q6 a)

Table 5: Summary statistics of `b3.star`

Statistic	Value
Min.	-0.265
1st Qu.	-0.083
Median	-0.012
Mean	-0.016
3rd Qu.	0.060
Max.	0.287

Table 6: Summary statistics of abs(b3.star)

Statistic	Value
Min.	0.004
1st Qu.	0.034
Median	0.071
Mean	0.079
3rd Qu.	0.105
Max.	0.287

**Q6 b)**

## [1] 0.2

The quantity has a value of 0.2, and is a p-value. The quantity is the proportion of  $|\hat{b}_3| \geq |b_3^*|$  in 100 runs, which approximates  $P(|\hat{b}_3| \geq |b_3^*|)$ . This is a two-tailed test of the following:

$$H_0 : \hat{b}_3 = b_3^* \quad (1)$$

$$H_a : \hat{b}_3 \neq b_3^* \quad (2)$$

This essentially is an approximate test of whether the AR(3) model is significantly differently from the AR(2) model.

## Question 7

Table 7: 95% Confidence Interval for b3

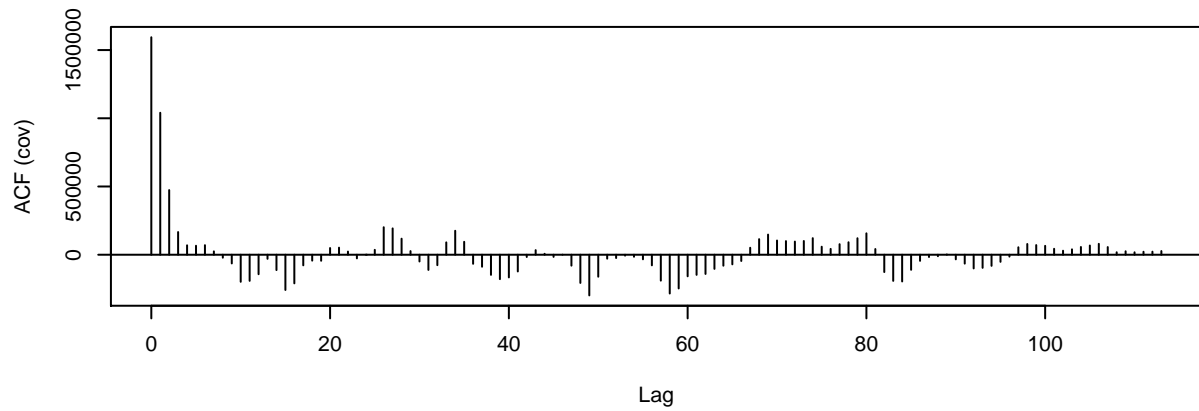
	Coefficient
Lower	-0.2373416
Upper	0.2006773

The confidence interval contains 0. This does form a test for whether the coefficient of  $X(t-3)$  is 0, which has a also positive rate of 5%.

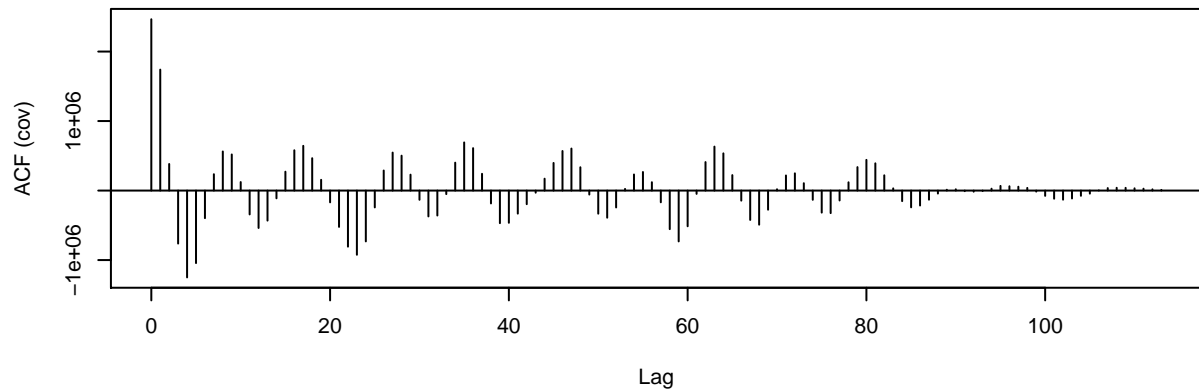
## Question 8

**Q8 a)**

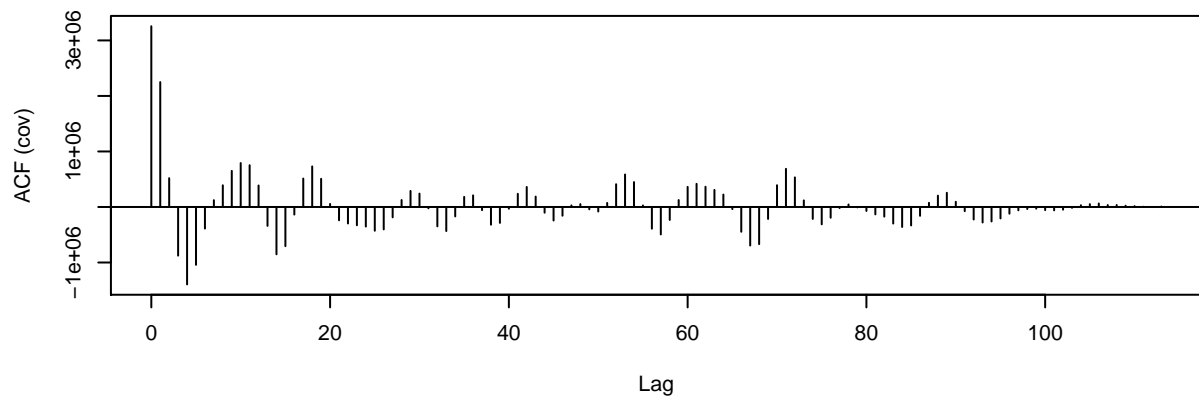
**ACF of AR(1) simulation**



**ACF of AR(2) simulation**



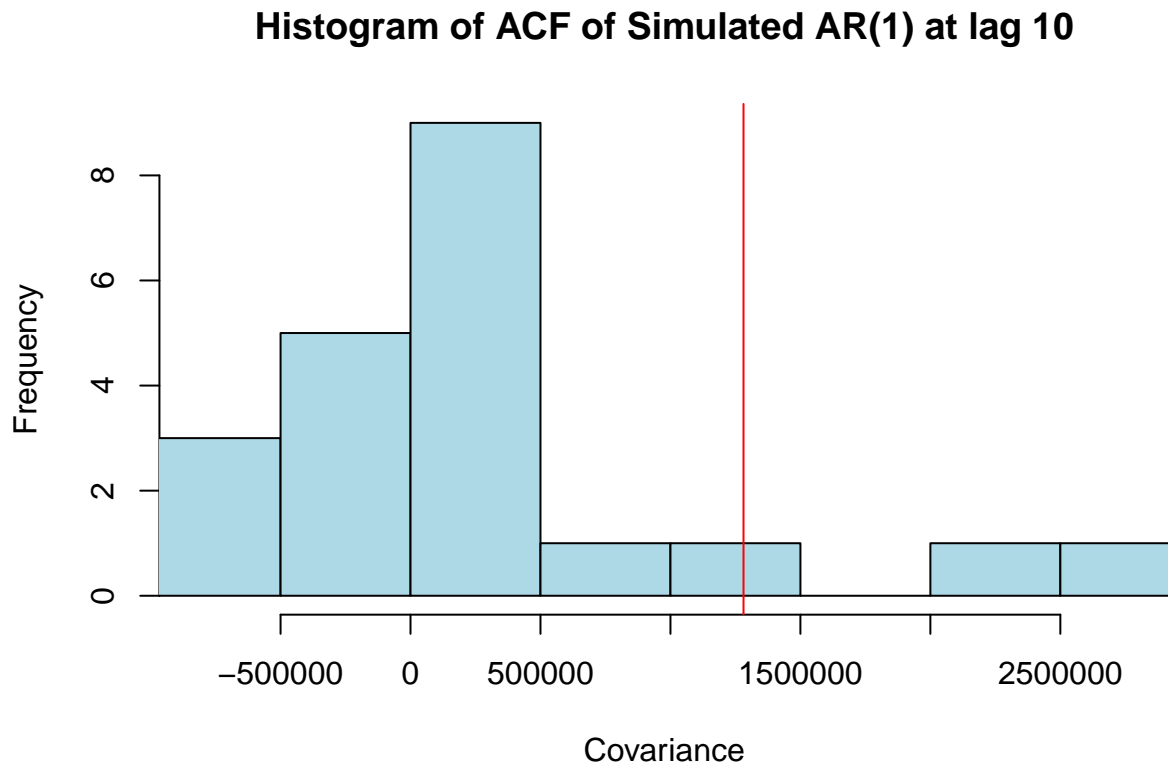
**ACF of AR(3) simulation**



In the ACF of the AR(1) plot, there is only a little resemblance of a periodic pattern, with largely varying periods and amplitudes of each crest and trough. In the ACF of the AR(2) plot, a more steady oscillatory pattern can be observed, with more similar periods that appear to be closer to the inter-peak interval of the original ACF. The oscillatory pattern also appears to be damped, with the amplitudes in general decreasing, such as in the original ACF. In the ACF of the AR(3) plot, we see that it is quite similar to that of the ACF

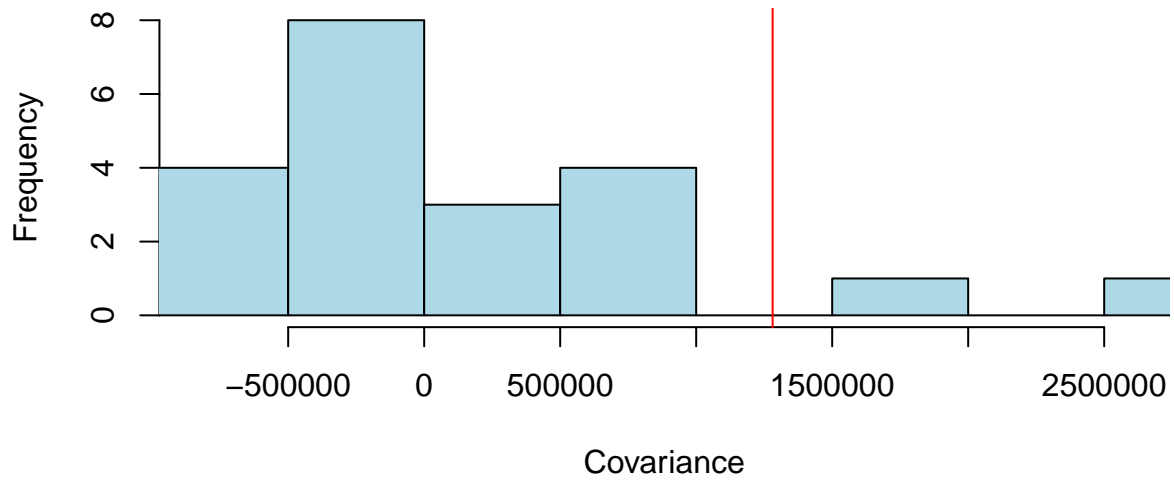
of the AR(2) plot. However, it also has a rough period of 10 and the oscillatory pattern here also appears to be damped.

Q8 b)



Q8 c)

**Histogram of ACF of Simulated AR(2) at lag 10**



Q8 d)

**Histogram of ACF of Simulated AR(3) at lag 10**

