Times Series Econometrics Project

In [1]:
```
# Import Librabries
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
sns.set(style="whitegrid")

from pandas_datareader import data as web
import datetime

from arch.unitroot import ADF, KPSS
from statsmodels.tsa.vector_ar.vecm import coint_johansen, VECM
from statsmodels.tsa.api import VAR

import warnings
warnings.filterwarnings("ignore")
```

Data gathering and collection

In [2]:
```
# Data gathering and collection

start = datetime.datetime(1959, 1, 1)
end = datetime.datetime(2025, 10, 1)

# Download data from FRED
pce = web.DataReader('PCECC96', 'fred', start, end)
dpi = web.DataReader('DPIC96', 'fred', start, end)
gdp = web.DataReader('GDP', 'fred', start, end)  # optional variable for advanced work

# Merge and clean
data = pd.concat([pce, dpi, gdp], axis=1)
data.columns = ['PCE', 'DPI', 'GDP']
data = data.dropna()
data.tail()
```

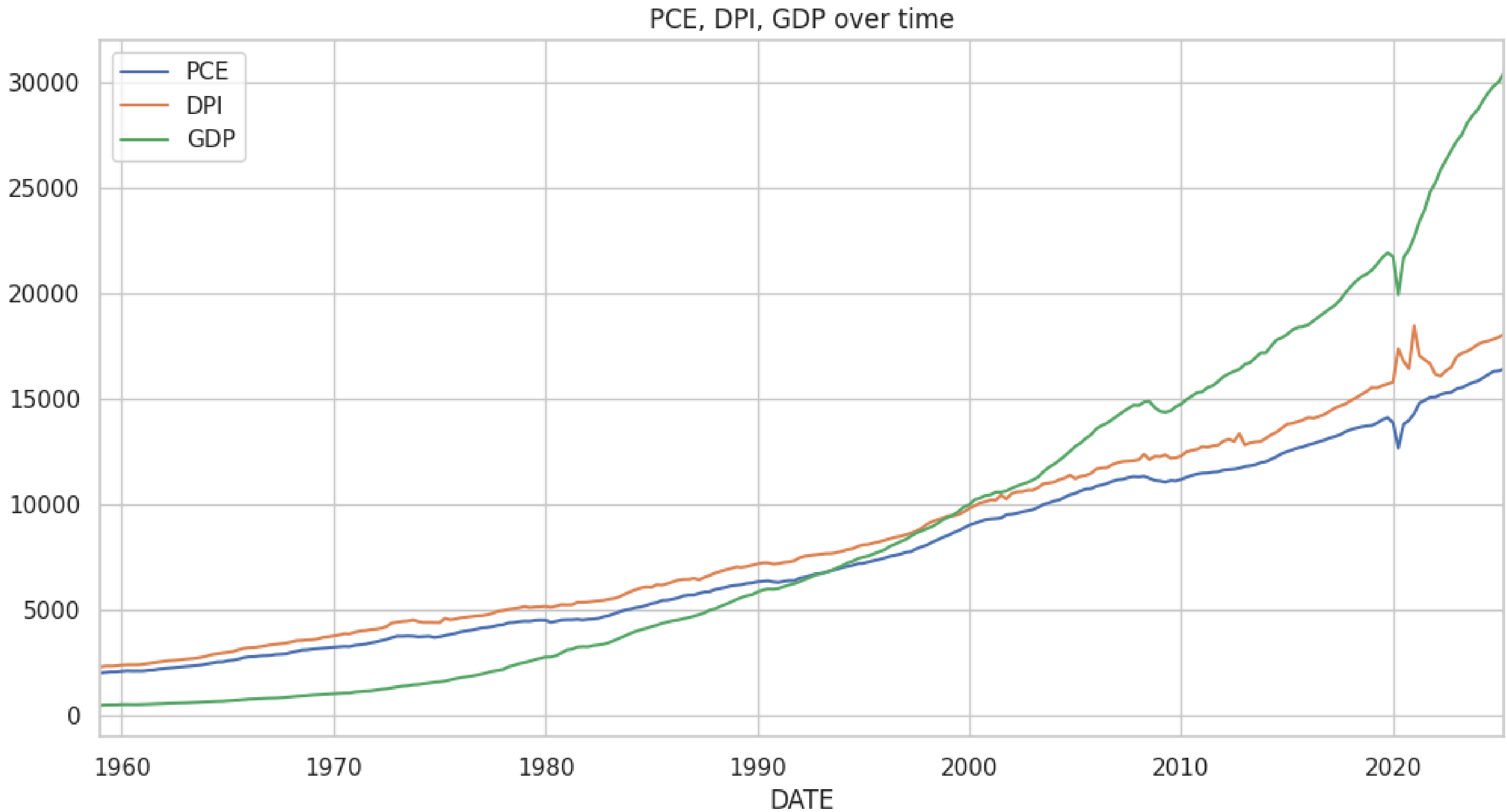
Out[2]:

DATE	PCE	DPI	GDP
2024-04-01	16009.637	17700.963	29147.044
2024-07-01	16165.768	17755.291	29511.664
2024-10-01	16320.890	17843.165	29825.182
2025-01-01	16345.793	17943.157	30042.113
2025-04-01	16445.685	18082.022	30485.729

Pre-Estimation Analysis

In [3]:
```
# Pre-Estimation Analysis

# Visual inspection
data.plot(figsize=(12,6), title="PCE, DPI, GDP over time")
plt.show()

# Log transformation for variance stabilization
data['ln_pce'] = np.log(data['PCE'])
data['ln_dpi'] = np.log(data['DPI'])
data['ln_gdp'] = np.log(data['GDP'])

# Stationarity tests
def adf_test(series, title):
    print(f'ADF Test: {title}')
    print(ADF(series, trend='ct').summary())
    print("\n")

def kpss_test(series, title):
    print(f'KPSS Test: {title}')
    print(KPSS(series, trend='ct').summary())
    print("\n")

print("=== Stationarity Tests ===")
adf_test(data['ln_pce'], 'ln_pce level')
```

```
adf_test(data['ln_dpi'], 'ln_dpi level')
adf_test(data['ln_gdp'], 'ln_gdp level')
adf_test(data['ln_pce'].diff().dropna(), 'Δln_pce')
adf_test(data['ln_dpi'].diff().dropna(), 'Δln_dpi')
adf_test(data['ln_gdp'].diff().dropna(), 'Δln_gdp')
```



```
=== Stationarity Tests ===
ADF Test: ln_pce level
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -1.564
P-value                         0.806
Lags                                0
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: -3.99 (1%), -3.43 (5%), -3.14 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.


ADF Test: ln_dpi level
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -2.302
P-value                         0.433
Lags                                6
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: -3.99 (1%), -3.43 (5%), -3.14 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.


ADF Test: ln_gdp level
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -0.399
P-value                         0.987
Lags                                2
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: -3.99 (1%), -3.43 (5%), -3.14 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

```
ADF Test: Δln_pce
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                -17.101
P-value                         0.000
Lags                                0
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: -3.99 (1%), -3.43 (5%), -3.14 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.


ADF Test: Δln_dpi
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -8.423
P-value                         0.000
Lags                                5
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: -3.99 (1%), -3.43 (5%), -3.14 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.


ADF Test: Δln_gdp
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -9.169
P-value                         0.000
Lags                                1
-------------------------------------

Trend: Constant and Linear Time Trend
Critical Values: -3.99 (1%), -3.43 (5%), -3.14 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

Cointegration Analysis (Johansen)

In [4]:
```
# Cointegration Analysis (Johansen)

johansen_test = coint_johansen(data[['ln_pce', 'ln_dpi']], det_order=1, k_ar_diff=2)
print("=== Johansen Test ===")
print("Trace statistic:", johansen_test.trace_stat)
print("Critical values:", johansen_test.cvt)
```

```
=== Johansen Test ===
Trace statistic: [19.4704735   3.65278798]
Critical values: [[16.1619 18.3985 23.1485]
 [ 2.7055  3.8415  6.6349]]
```

VECM Estimation

In [5]:
```
# 4. VECM Estimation

vecm_model = VECM(
    data[['ln_pce', 'ln_dpi']],
    k_ar_diff=4,
    coint_rank=1,
    deterministic="ci"
)
vecm_fit = vecm_model.fit()
print("=== VECM Summary ===")
print(vecm_fit.summary())
```

=== VECM Summary ===

Det. terms outside the coint. relation & lagged endog. parameters for equation ln_pce

	coef	std err	z	P>|z|	[0.025	0.975]
L1.ln_pce	-0.0261	0.065	-0.402	0.688	-0.153	0.101
L1.ln_dpi	0.3358	0.050	6.774	0.000	0.239	0.433
L2.ln_pce	0.0471	0.064	0.741	0.459	-0.077	0.172
L2.ln_dpi	0.1633	0.050	3.273	0.001	0.065	0.261
L3.ln_pce	0.0067	0.065	0.104	0.917	-0.120	0.134
L3.ln_dpi	0.1225	0.049	2.517	0.012	0.027	0.218
L4.ln_pce	0.0653	0.067	0.977	0.329	-0.066	0.196
L4.ln_dpi	0.0149	0.048	0.307	0.759	-0.080	0.110

Det. terms outside the coint. relation & lagged endog. parameters for equation ln_dpi

	coef	std err	z	P>|z|	[0.025	0.975]
L1.ln_pce	-0.0009	0.077	-0.011	0.991	-0.153	0.151
L1.ln_dpi	-0.1280	0.059	-2.167	0.030	-0.244	-0.012
L2.ln_pce	0.3102	0.076	4.100	0.000	0.162	0.458
L2.ln_dpi	0.0338	0.059	0.569	0.570	-0.083	0.150
L3.ln_pce	-0.4265	0.077	-5.528	0.000	-0.578	-0.275
L3.ln_dpi	0.1596	0.058	2.752	0.006	0.046	0.273
L4.ln_pce	-0.0341	0.080	-0.428	0.668	-0.190	0.122
L4.ln_dpi	-0.0357	0.058	-0.618	0.537	-0.149	0.078

Loading coefficients (alpha) for equation ln_pce

	coef	std err	z	P>|z|	[0.025	0.975]
ec1	0.0137	0.009	1.572	0.116	-0.003	0.031

Loading coefficients (alpha) for equation ln_dpi

	coef	std err	z	P>|z|	[0.025	0.975]
ec1	0.0672	0.010	6.473	0.000	0.047	0.088

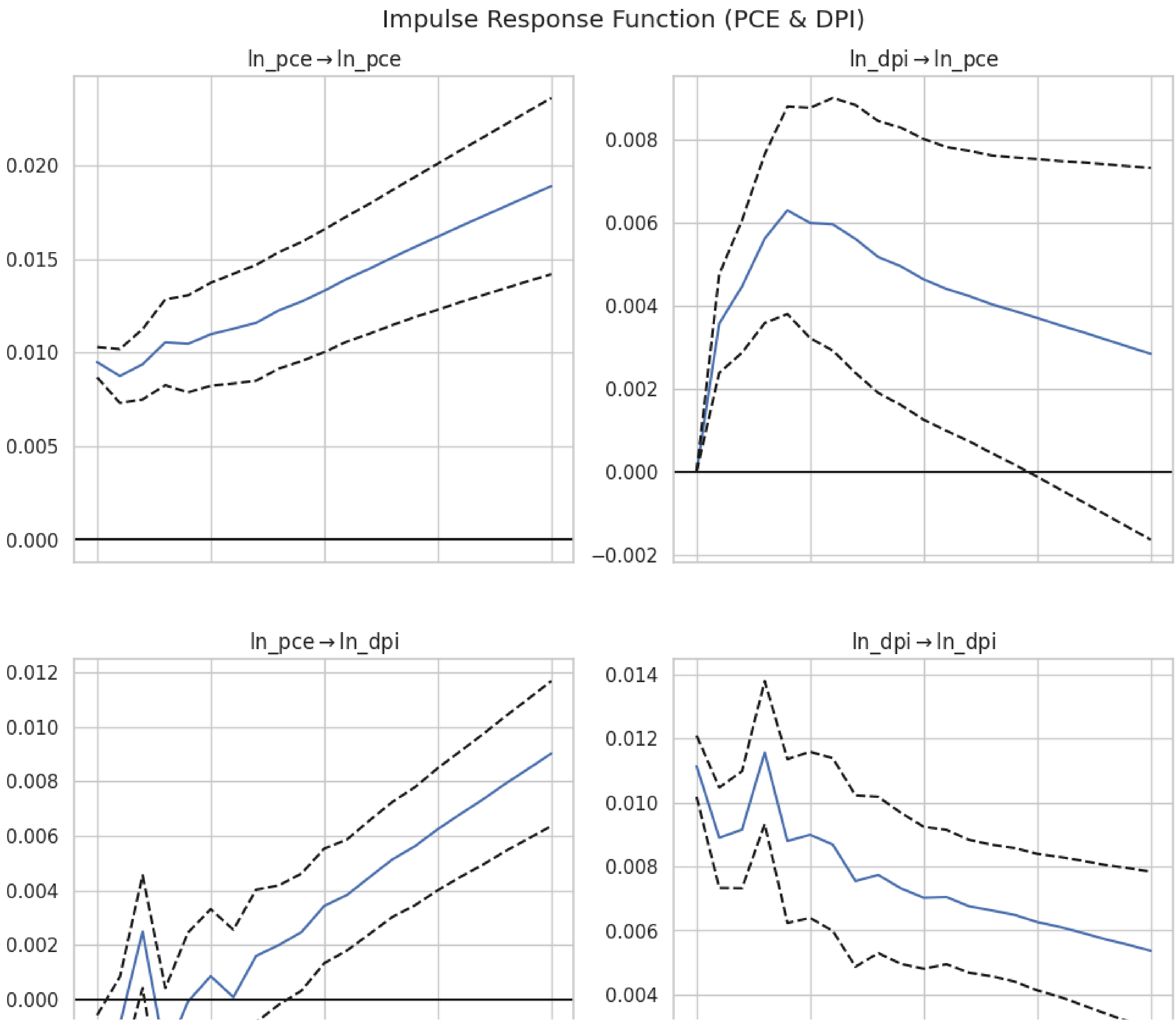
Cointegration relations for loading-coefficients-column 1

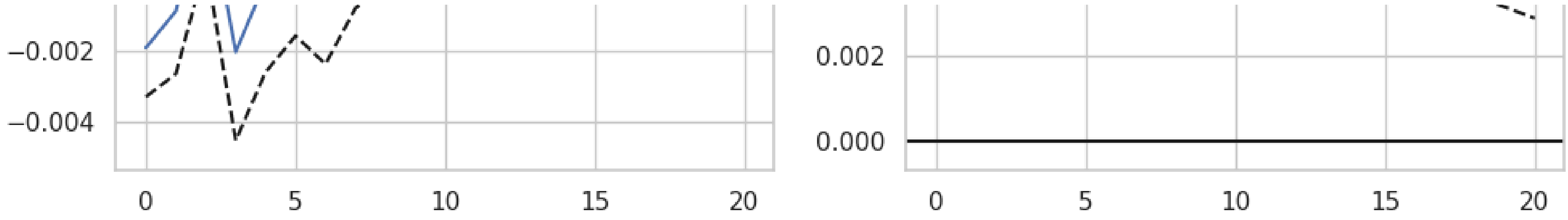
	coef	std err	z	P>|z|	[0.025	0.975]
beta.1	1.0000	0	0	0.000	1.000	1.000
beta.2	-1.0827	0.018	-60.776	0.000	-1.118	-1.048

```
const          0.9904      0.166      5.982      0.000       0.666       1.315
==============================================================================
```

Post-Estimation Analysis (IRF)

In [6]:
```
# Post-Estimation Analysis (IRF)

irf = vecm_fit.irf(20)
irf.plot(orth=True, signif=0.05)
plt.suptitle("Impulse Response Function (PCE & DPI)")
plt.show()
```


Impulse Response Function (PCE & DPI)
ln_pce → ln_pce
ln_dpi → ln_pce
ln_pce → ln_dpi
ln_dpi → ln_dpi
0.020
0.015
0.010
0.005
0.000
0.008
0.006
0.004
0.002
0.000
−0.002
0.012
0.010
0.008
0.006
0.004
0.002
0.000
0.014
0.012
0.010
0.008
0.006
0.004



Forecasting with Confidence Intervals

In [7]:
```
# 6. Forecasting with Confidence Intervals

steps = 12  # forecast horizon
forecast_vals, lower_vals, upper_vals = vecm_fit.predict(steps=steps, alpha=0.05)

forecast_df = pd.DataFrame(np.exp(forecast_vals), columns=['PCE_forecast', 'DPI_forecast'])
lower_df = pd.DataFrame(np.exp(lower_vals), columns=['PCE_lower', 'DPI_lower'])
upper_df = pd.DataFrame(np.exp(upper_vals), columns=['PCE_upper', 'DPI_upper'])

forecast_ci = pd.concat([forecast_df, lower_df, upper_df], axis=1)
print("VECM Forecast with 95% CI: \n")
forecast_ci
```

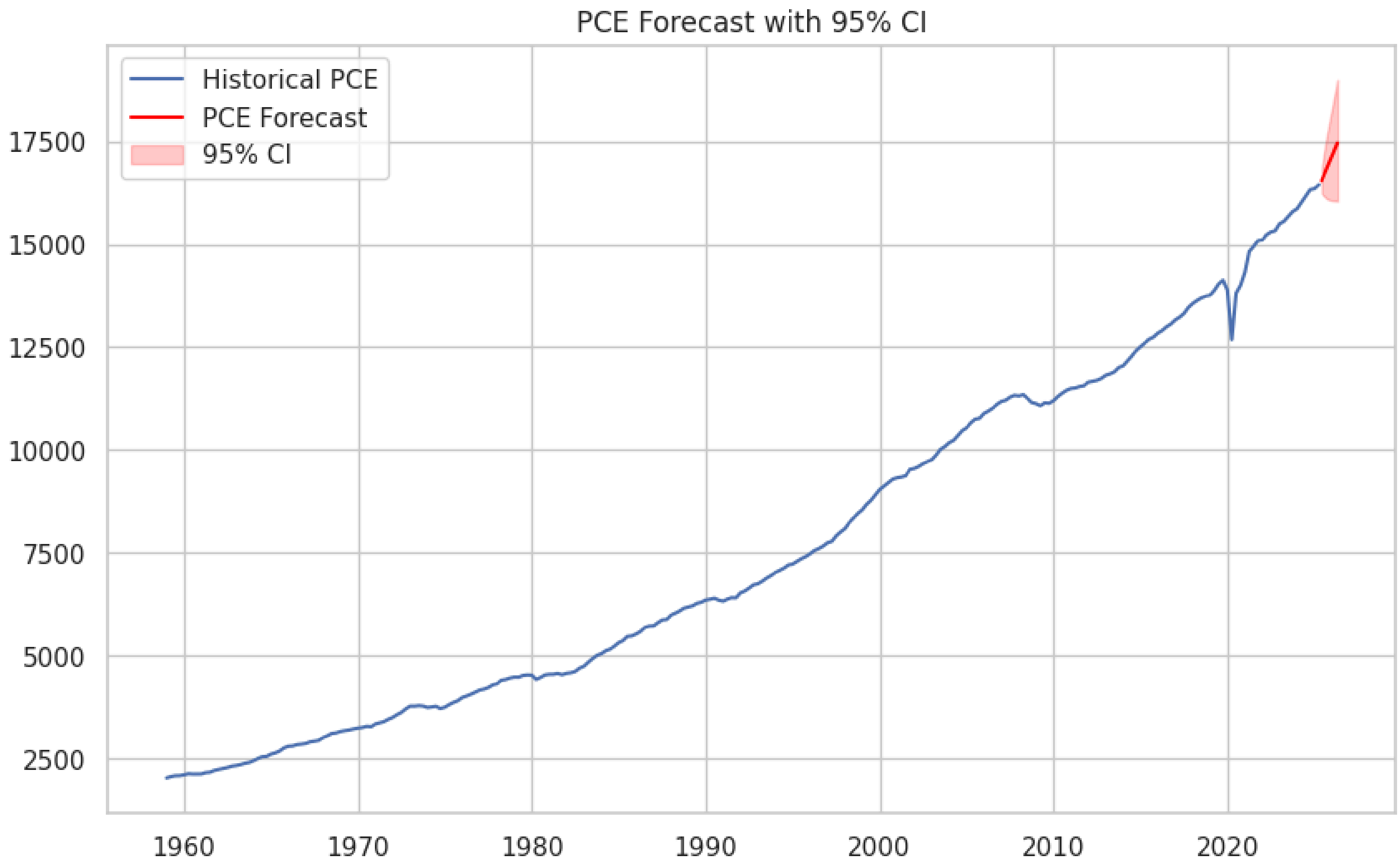
```
VECM Forecast with 95% CI:
```

Out[7]:

	PCE_forecast	DPI_forecast	PCE_lower	DPI_lower	PCE_upper	DPI_upper
0	16543.509330	18112.374014	16238.812727	17715.737180	16853.923100	18517.891131
1	16619.298163	18252.004211	16188.903973	17743.779790	17061.134706	18774.785399
2	16708.520962	18344.364848	16162.878490	17734.628591	17272.583772	18975.064515
3	16791.501662	18418.482711	16122.917170	17680.559691	17487.810989	19187.203985
4	16873.389677	18531.452256	16096.125862	17727.149409	17688.186687	19372.247325
5	16958.663320	18617.822729	16080.875221	17749.370501	17884.366220	19528.767128
6	17039.671355	18710.325189	16067.704249	17785.179251	18070.434668	19683.595185
7	17122.509174	18809.794830	16062.234787	17840.367128	18252.772686	19831.900264
8	17205.502602	18899.216071	16057.813410	17884.586199	18435.219804	19971.408011
9	17287.070353	18994.797173	16053.403249	17938.453279	18615.541936	20113.346118
10	17369.573818	19089.078886	16049.905333	17991.212318	18797.749167	20253.939883
11	17451.527181	19180.963538	16044.696639	18040.810430	18981.711391	20393.172673

Plot forecast

In [8]:
```
# Plot forecast
last_date = data.index[-1]
future_index = pd.date_range(start=last_date, periods=steps+1, freq="M")[1:]

plt.figure(figsize=(10,6))
plt.plot(data.index, data['PCE'], label='Historical PCE')
plt.plot(future_index, forecast_df['PCE_forecast'], color='red', label='PCE Forecast')
plt.fill_between(future_index, lower_df['PCE_lower'], upper_df['PCE_upper'], color='red', alpha=0.2, label='95% CI')
plt.title('PCE Forecast with 95% CI')
plt.legend()
plt.show()
```



Compare VECM to VAR

In [9]:

```
# Compare VECM to VAR
var_model = VAR(data[['ln_pce', 'ln_dpi']])
var_fit = var_model.fit(maxlags=4)
var_forecast = var_fit.forecast(y=data[['ln_pce', 'ln_dpi']].values[-var_fit.k_ar:], steps=12)
var_forecast_df = pd.DataFrame(np.exp(var_forecast), columns=['PCE_forecast_VAR', 'DPI_forecast_VAR'])
```

```
print("VAR Forecast (Next 12 Periods): \n")
var_forecast_df
```

```
VAR Forecast (Next 12 Periods):
```

Out[9]:

	PCE_forecast_VAR	DPI_forecast_VAR
0	16548.662420	18108.083644
1	16630.219789	18246.313860
2	16732.309389	18334.339971
3	16824.305692	18405.481622
4	16914.942806	18517.257608
5	17010.024206	18599.808467
6	17100.371113	18690.859354
7	17192.581364	18790.163547
8	17285.406027	18879.888526
9	17377.210400	18976.696839
10	17470.364657	19073.445060
11	17563.525934	19168.533887

Include GDP as additional variable in VECM

In [10]:

```
# Include GDP as additional variable in VECM
vecm_model_gdp = VECM(
    data[['ln_pce', 'ln_dpi', 'ln_gdp']],
    k_ar_diff=4,
    coint_rank=1,
    deterministic="ci"
)
vecm_fit_gdp = vecm_model_gdp.fit()
```

```
print("=== VECM with GDP Summary ===")
print(vecm_fit_gdp.summary())
```

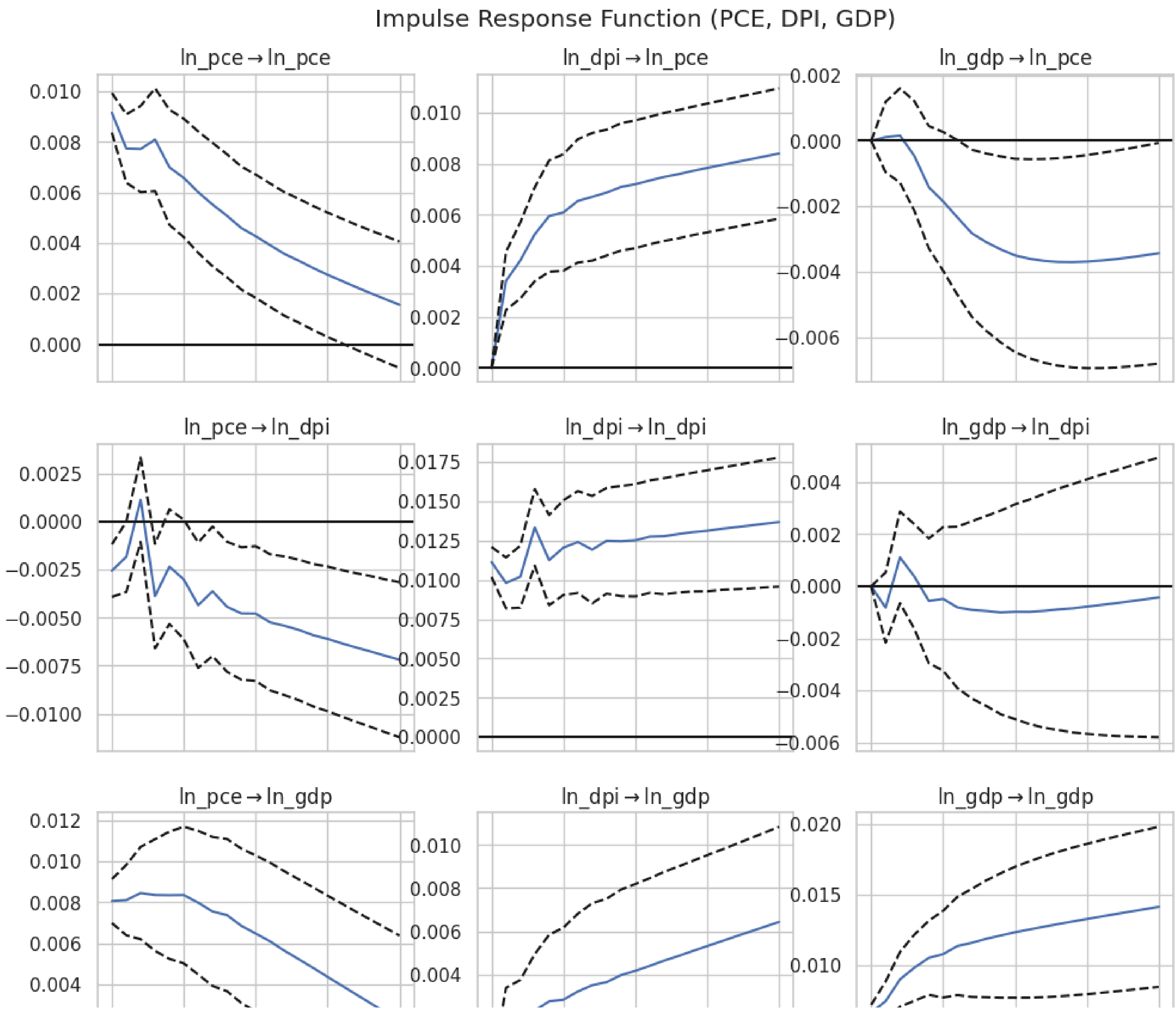
```
=== VECM with GDP Summary ===
Det. terms outside the coint. relation & lagged endog. parameters for equation ln_pce
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
L1.ln_pce     -0.0609      0.094     -0.649      0.516      -0.245       0.123
L1.ln_dpi      0.2871      0.053      5.392      0.000       0.183       0.391
L1.ln_gdp      0.0173      0.083      0.209      0.835      -0.145       0.180
L2.ln_pce     -0.0177      0.091     -0.193      0.847      -0.197       0.161
L2.ln_dpi      0.1140      0.052      2.202      0.028       0.013       0.215
L2.ln_gdp      0.0445      0.081      0.550      0.582      -0.114       0.203
L3.ln_pce      0.1235      0.089      1.391      0.164      -0.050       0.297
L3.ln_dpi      0.0614      0.050      1.232      0.218      -0.036       0.159
L3.ln_gdp     -0.1757      0.081     -2.158      0.031      -0.335      -0.016
L4.ln_pce      0.1614      0.090      1.795      0.073      -0.015       0.338
L4.ln_dpi     -0.0263      0.048     -0.545      0.586      -0.121       0.068
L4.ln_gdp     -0.1388      0.081     -1.712      0.087      -0.298       0.020
Det. terms outside the coint. relation & lagged endog. parameters for equation ln_dpi
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
L1.ln_pce      0.1779      0.117      1.518      0.129      -0.052       0.408
L1.ln_dpi     -0.1596      0.066     -2.401      0.016      -0.290      -0.029
L1.ln_gdp     -0.1232      0.103     -1.191      0.234      -0.326       0.080
L2.ln_pce      0.1461      0.114      1.282      0.200      -0.077       0.369
L2.ln_dpi      0.0163      0.065      0.252      0.801      -0.110       0.143
L2.ln_gdp      0.2862      0.101      2.832      0.005       0.088       0.484
L3.ln_pce     -0.3477      0.111     -3.138      0.002      -0.565      -0.131
L3.ln_dpi      0.1263      0.062      2.030      0.042       0.004       0.248
L3.ln_gdp     -0.0770      0.102     -0.757      0.449      -0.276       0.122
L4.ln_pce      0.1646      0.112      1.466      0.143      -0.055       0.385
L4.ln_dpi     -0.0518      0.060     -0.860      0.390      -0.170       0.066
L4.ln_gdp     -0.1712      0.101     -1.691      0.091      -0.370       0.027
Det. terms outside the coint. relation & lagged endog. parameters for equation ln_gdp
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
L1.ln_pce      0.0047      0.108      0.044      0.965      -0.207       0.216
L1.ln_dpi      0.2705      0.061      4.418      0.000       0.151       0.391
L1.ln_gdp      0.1247      0.095      1.308      0.191      -0.062       0.312
L2.ln_pce     -0.1807      0.105     -1.721      0.085      -0.387       0.025
L2.ln_dpi     -0.0039      0.060     -0.065      0.948      -0.121       0.113
```

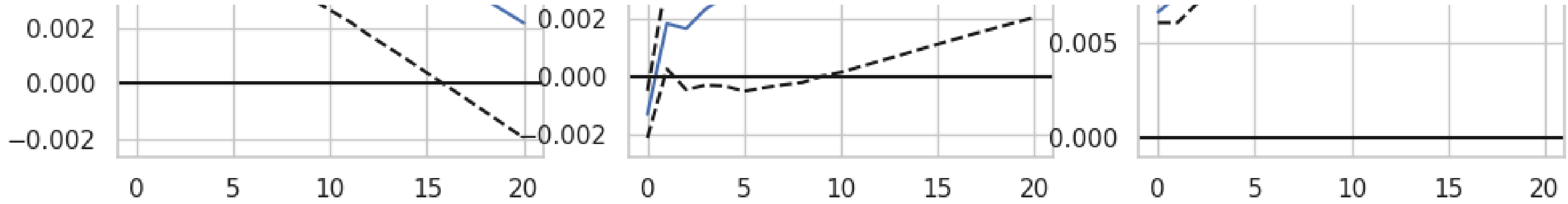
```
L2.ln_gdp       0.2541      0.093      2.727      0.006       0.071       0.437
L3.ln_pce      -0.0920      0.102     -0.902      0.367      -0.292       0.108
L3.ln_dpi       0.0216      0.057      0.377      0.706      -0.091       0.134
L3.ln_gdp      -0.0162      0.094     -0.173      0.863      -0.200       0.167
L4.ln_pce       0.0990      0.103      0.957      0.339      -0.104       0.302
L4.ln_dpi      -0.0018      0.055     -0.032      0.974      -0.110       0.107
L4.ln_gdp       0.0667      0.093      0.715      0.475      -0.116       0.250
               Loading coefficients (alpha) for equation ln_pce
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ec1           -0.0232      0.006     -4.030      0.000      -0.035      -0.012
               Loading coefficients (alpha) for equation ln_dpi
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ec1           -0.0280      0.007     -3.891      0.000      -0.042      -0.014
               Loading coefficients (alpha) for equation ln_gdp
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ec1           -0.0273      0.007     -4.122      0.000      -0.040      -0.014
          Cointegration relations for loading-coefficients-column 1
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
beta.1         1.0000          0          0      0.000       1.000       1.000
beta.2        -0.9194      0.221     -4.166      0.000      -1.352      -0.487
beta.3        -0.0051      0.106     -0.048      0.962      -0.213       0.202
const         -0.8480      1.074     -0.790      0.430      -2.953       1.257
==============================================================================
```

IRF with GDP

In [11]:

```
# IRF with GDP
irf_gdp = vecm_fit_gdp.irf(20)
irf_gdp.plot(orth=True, signif=0.05)
plt.suptitle("Impulse Response Function (PCE, DPI, GDP)")
plt.show()
```


Impulse Response Function (PCE, DPI, GDP)
ln_pce → ln_pce
ln_dpi → ln_pce
ln_gdp → ln_pce
ln_pce → ln_dpi
ln_dpi → ln_dpi
ln_gdp → ln_dpi
ln_pce → ln_gdp
ln_dpi → ln_gdp
ln_gdp → ln_gdp



Forecast with GDP

In [12]:

```python
# Forecast with GDP
forecast_gdp_vals, lower_gdp_vals, upper_gdp_vals = vecm_fit_gdp.predict(steps=12, alpha=0.05)
forecast_gdp_df = pd.DataFrame(np.exp(forecast_gdp_vals), columns=['PCE_forecast', 'DPI_forecast', 'GDP_forecast'])
print("VECM Forecast with GDP (Next 12 Periods): \n")
forecast_gdp_df
```

```
VECM Forecast with GDP (Next 12 Periods):
```

Out[12]:

	PCE_forecast	DPI_forecast	GDP_forecast
0	16564.106694	18117.732846	30851.581843
1	16657.127985	18289.149455	31211.855568
2	16751.435271	18374.981808	31566.031606
3	16835.808606	18443.685209	31910.858139
4	16923.823736	18556.024800	32253.629708
5	17010.903171	18637.144105	32611.197661
6	17096.139823	18730.557103	32956.103865
7	17184.857530	18829.080822	33309.847955
8	17272.784003	18919.387898	33665.370745
9	17360.869605	19015.968361	34021.558607
10	17450.569808	19111.185668	34381.515074
11	17540.054615	19205.932884	34744.023063

In [12]: