# Laplacian Edge Detection

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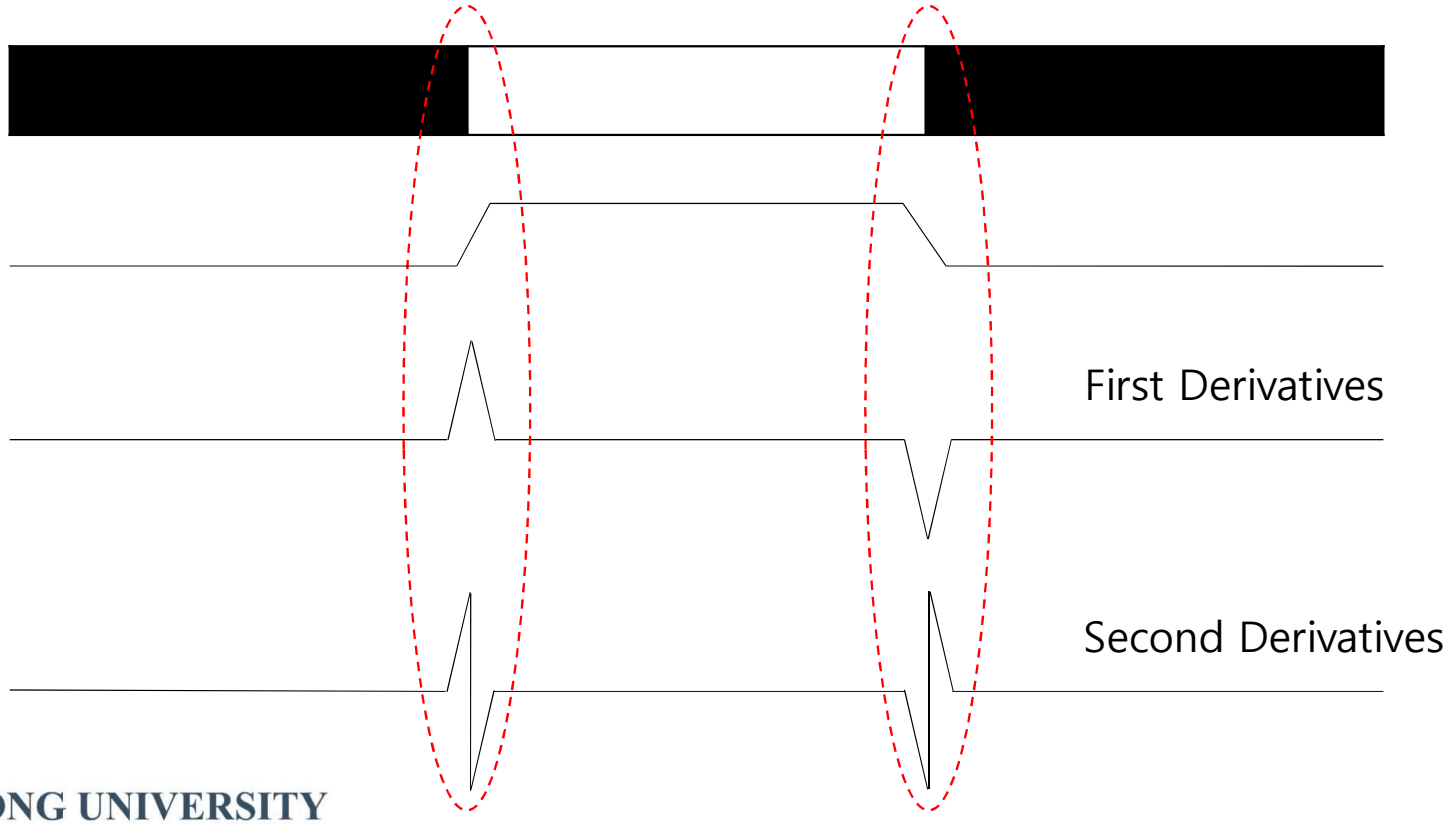
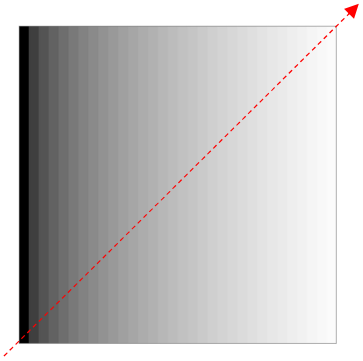
# Laplacian Filter

- Zero crossing edge detection using the second derivatives of an image



Gradual Change

Edge

No Edge

# Second Derivative of Discrete Signal

- Assuming that the first derivative is represented by difference between two consecutive pixel values

$$f'(x) = \frac{df}{dx} = \frac{f(x+1) - f(x)}{x+1-x} = f(x+1) - f(x) \approx f(x) - f(x-1)$$

- Second derivative calculating the rate at which the first derivative changes

$$f''(x) = \frac{d^2 f}{dx^2} = \frac{df(x+1)}{dx} - \frac{df(x)}{dx} = \big(f(x+1) - f(x)\big) - \big(f(x) - f(x-1)\big)$$

$$= f(x+1) - 2 \cdot f(x) + f(x-1)$$

# Second Derivatives of an Image

$$f''(x,y) = \frac{d^2f}{dx^2} + \frac{d^2f}{dy^2}$$

$$\frac{d^2f}{dx^2} = f(x+1,y) - 2\cdot f(x,y) + f(x-1,y)$$

$$\frac{d^2f}{dy^2} = f(x,y+1) - 2\cdot f(x,y) + f(x,y-1)$$

$$\rightarrow f(x+1,y) - 2\cdot f(x,y) + f(x-1,y) + f(x,y+1) - 2\cdot f(x,y) + f(x,y-1)$$

$$= f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4\cdot f(x,y)$$

# Laplacian Filter Coefficients

$$f''(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4 \cdot f(x,y)$$

Only One Kernel

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | -4 | 1 |
| 0 | 1 | 0 |

Second Derivatives in Single Pass

| | | |
|---|---|---|
| 0 | -1 | 0 |
| -1 | 4 | -1 |
| 0 | -1 | 0 |

| | | |
|---|---|---|
| -1 | -1 | -1 |
| -1 | 8 | -1 |
| -1 | -1 | -1 |

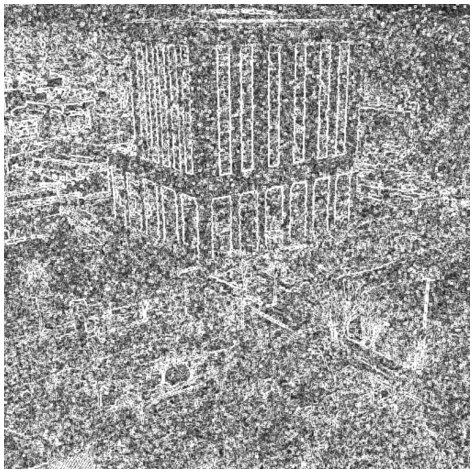
| | | |
|---|---|---|
| 1 | -2 | 1 |
| -2 | 4 | -2 |
| 1 | -2 | 1 |

| | | | | |
|---|---|---|---|---|
| 0 | 0 | -1 | 0 | 0 |
| 0 | -1 | -2 | -1 | 0 |
| -1 | -2 | 16 | -2 | -1 |
| 0 | -1 | -2 | -1 | 0 |
| 0 | 0 | -1 | 0 | 0 |

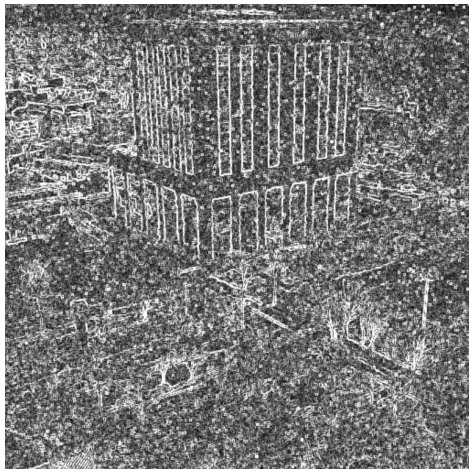
# Noise

- Sudden changes of intensity in an image, which is same as edge
- Points that image intensity has discontinuities, which is same as edge

AICenterY_CombinedNoise.bmp



Sobel

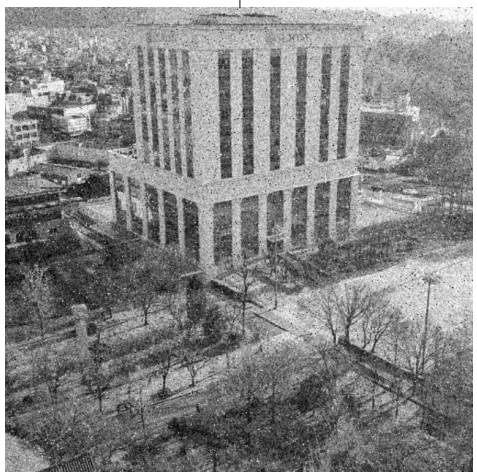


Prewitt

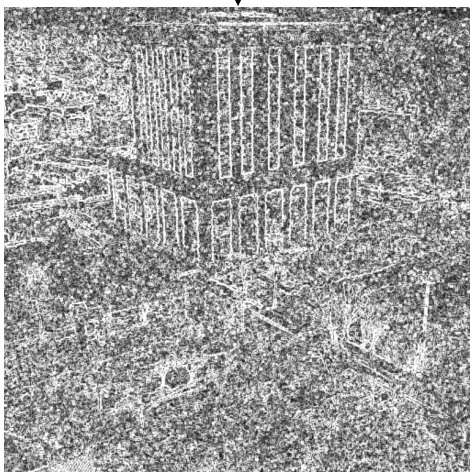
Roberts

# Noise Removal

- Image restoration, and then edge detection
- Median filter, mean filter, Gaussian filter, weighted average filter… for noise removal

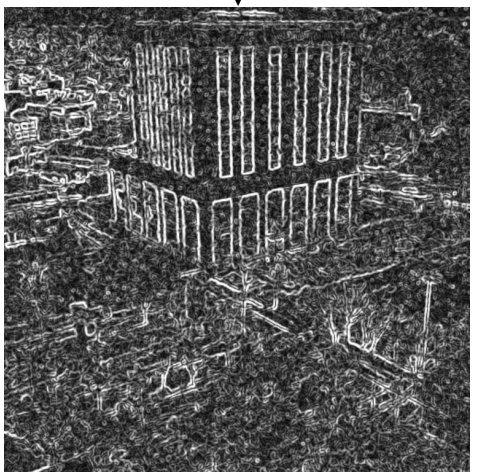


AICenterY_CombinedNoise.bmp



Sobel

Filtered Image



Sobel

Mean Filtering

# Canny Edge Detection

- Accurate edge detection with low error rate
- Three steps
  - Noise removal with Gaussian filter
  - Calculation of gradient magnitude with Sobel filter
  - Edge detection with thresholding

| | | |
|---|---|---|
| -1 | 0 | 1 |
| -2 | 0 | 2 |
| -1 | 0 | 1 |

| | | |
|---|---|---|
| -1 | -2 | -1 |
| 0 | 0 | 0 |
| 1 | 2 | 1 |

Sobel Filter

| | | | | |
|---|---|---|---|---|
| 2/159 | 4/159 | 5/159 | 4/159 | 2/159 |
| 4/159 | 9/159 | 12/159 | 9/159 | 4/159 |
| 5/159 | 12/159 | 15/159 | 12/159 | 5/159 |
| 4/159 | 9/159 | 12/159 | 9/159 | 4/159 |
| 2/159 | 4/159 | 5/159 | 4/159 | 2/159 |

Gaussian Filter