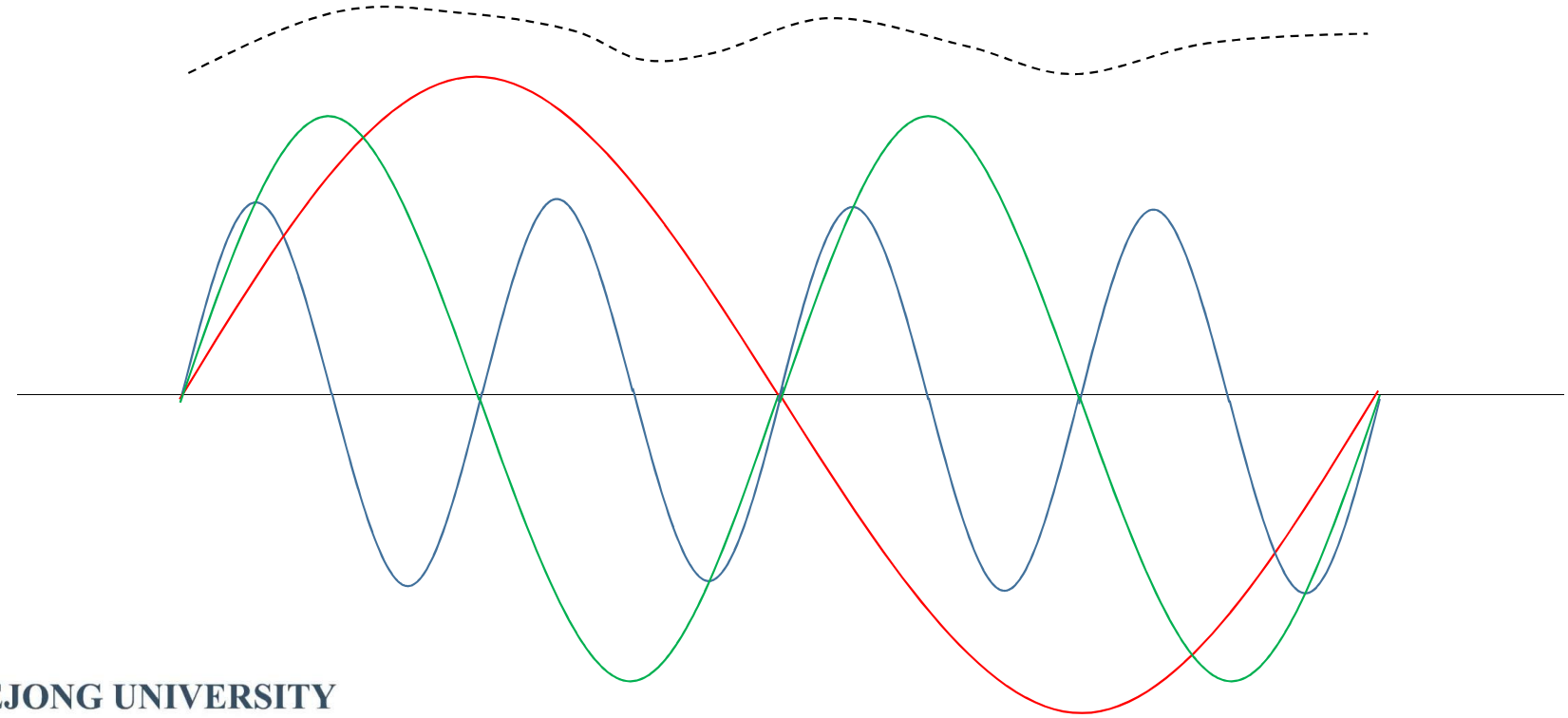
Transform

이진영

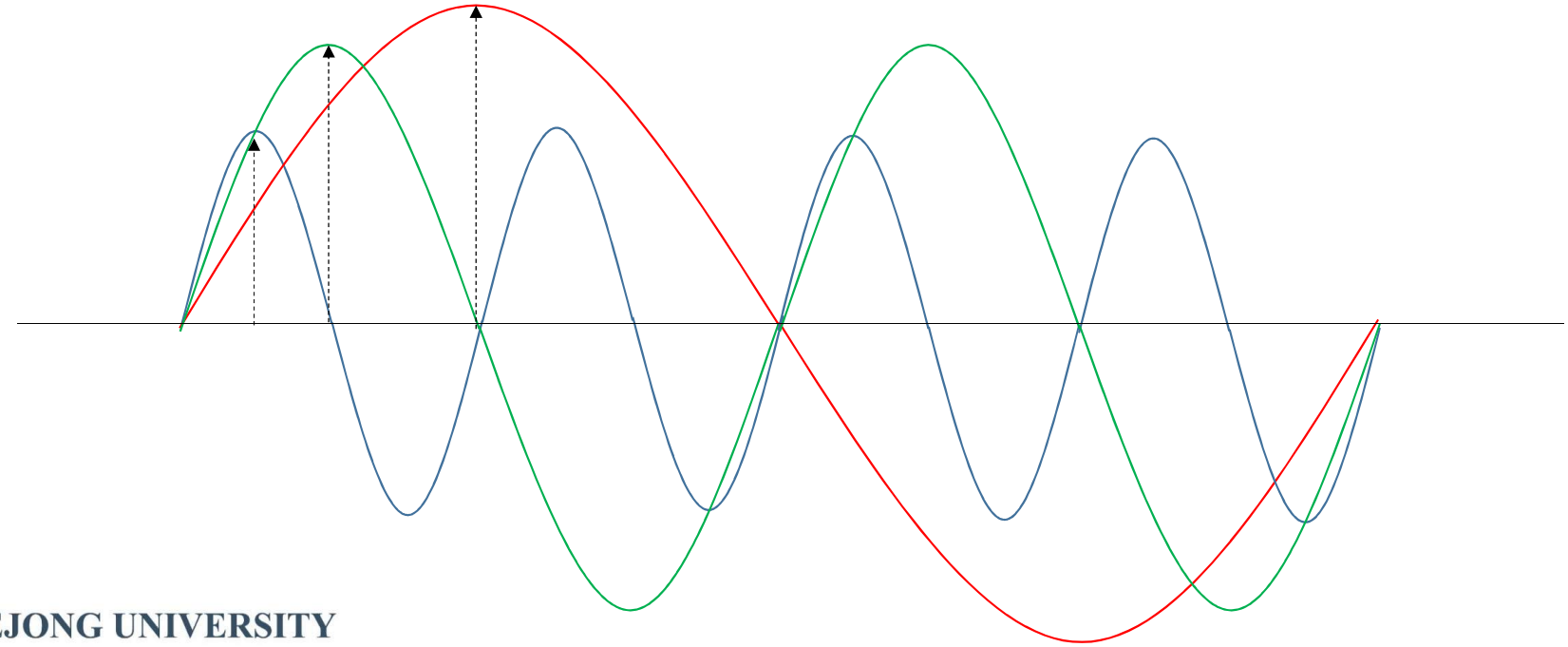
Signal

- Sum of sine waves of different frequencies



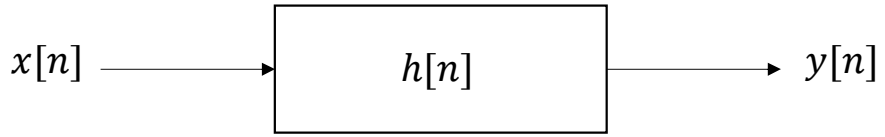
Frequency Domain

- Analysis of signals with respect to frequency, rather than time
- Representation of how much of the signal lies within each frequency

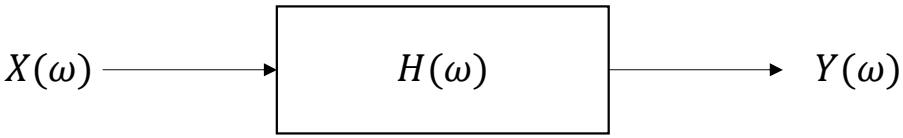


Digital Filter

- System that performs numerical operations on a discrete-time signal
- System that reduce, enhance, or restore certain aspects of a signal



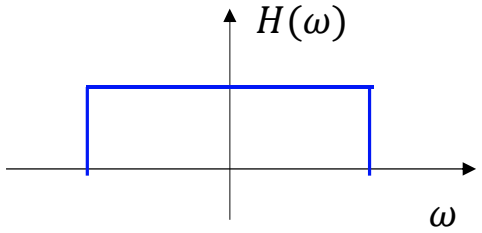
$$y[n] = x[n] * h[n]$$



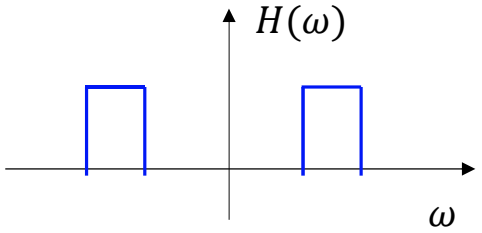
$$Y(\omega) = X(\omega)H(\omega)$$

Frequency Selective Filter

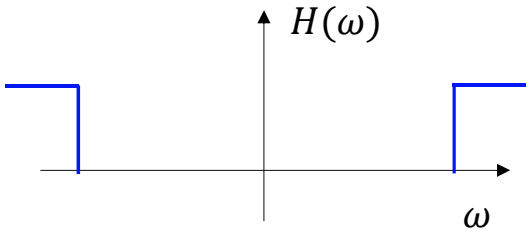
- Digital filter that passes or rejects desired frequency components in a signal
- Digital filter that separates a signal, based on frequencies



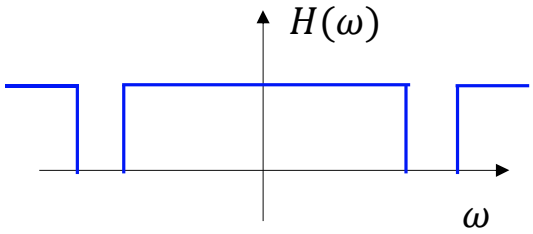
Low-Pass Filter (LPF)



Band-Pass Filter (BPF)



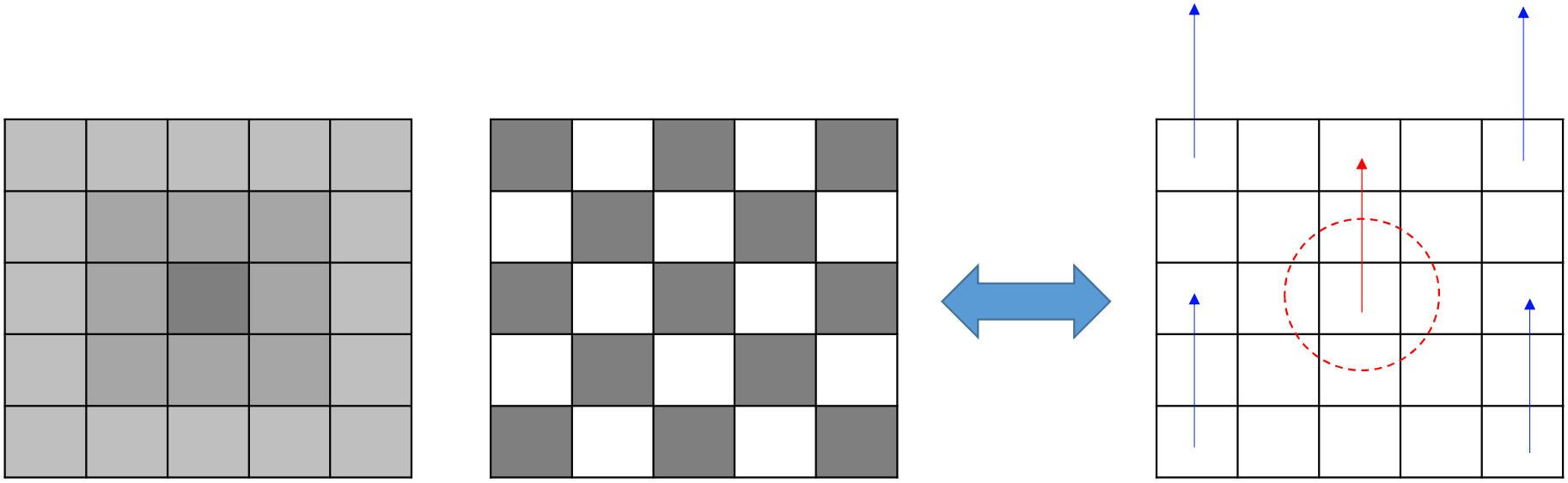
High-Pass Filter (HPF)



Band-Stop Filter (BSF)

Fourier Transform

- Decomposition of an image function into a frequency function
- Pixel values in an image domain, but frequency magnitudes in a frequency domain

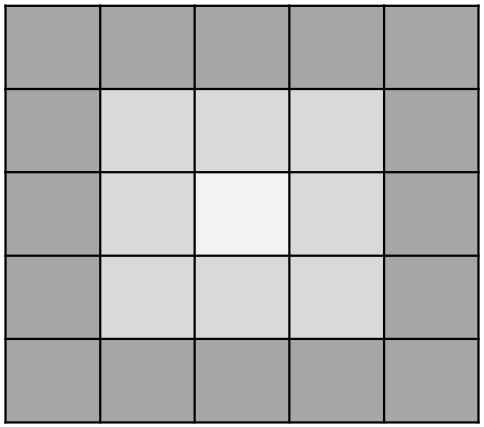


Image

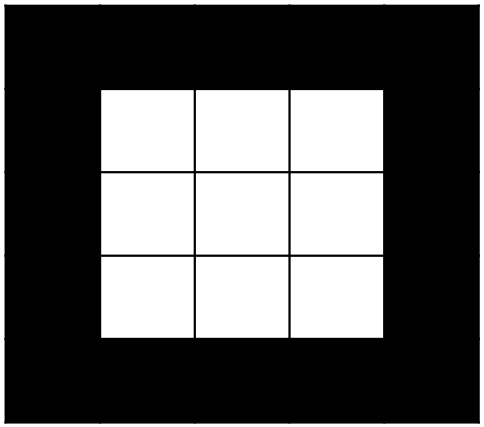
Frequency

Loss-Pass Filter

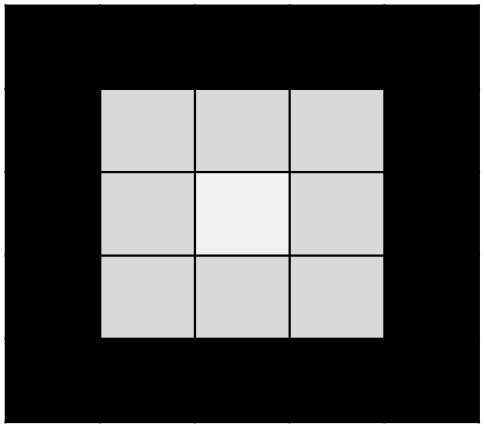
- Low-pass filter passing signals with frequencies lower than a cutoff frequency and blocking signals with frequencies higher than the cutoff frequency



Frequency (A)



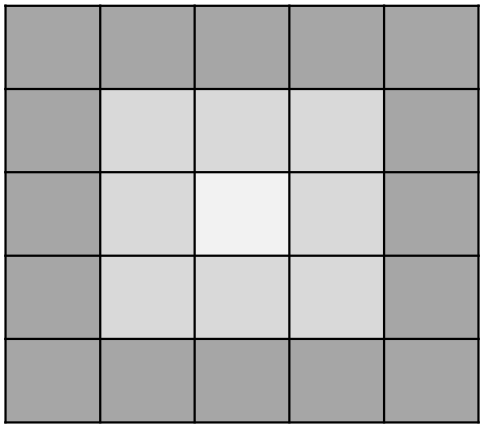
Low-Pass Filter (B)



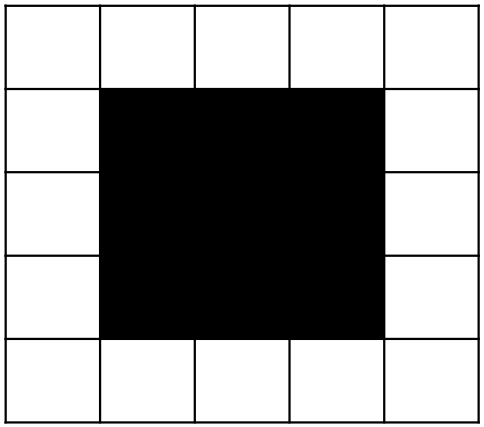
A∩B

High-Pass Filter

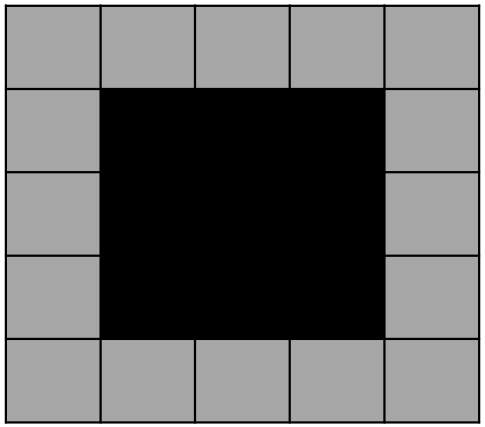
- High-pass filter passing signals with frequencies higher than a cutoff frequency and blocking signals with frequencies lower than the cutoff frequency



Frequency (A)



High-Pass Filter (B)

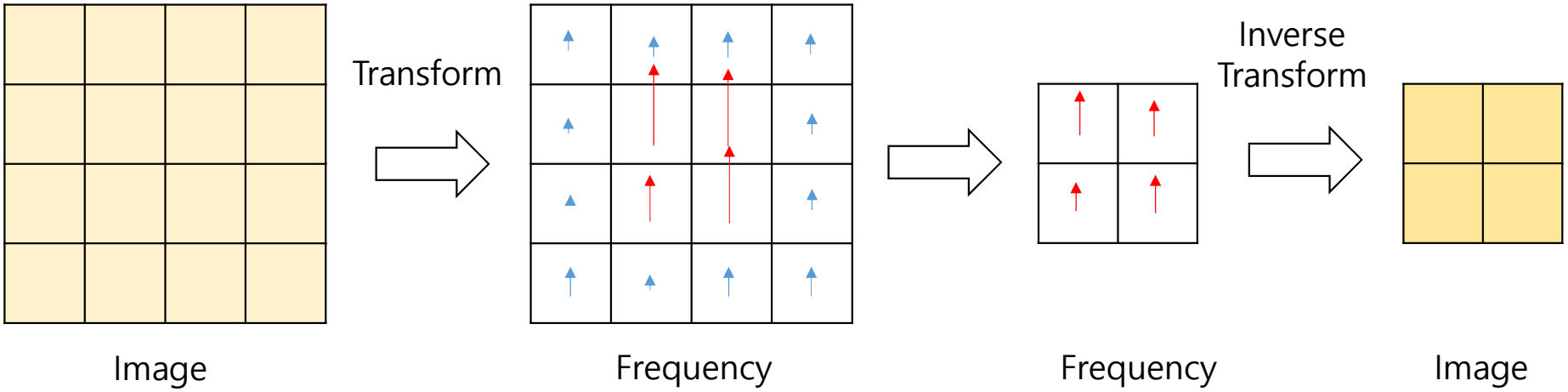


A∩B

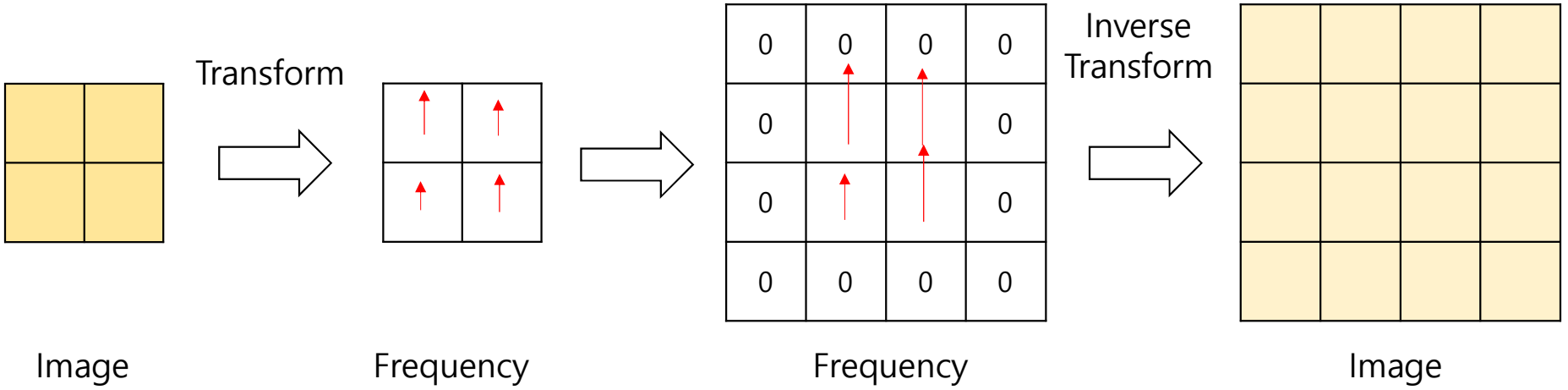
Application

- Frequency analysis for efficient image processing
- For example,
  - Downsampling and upsampling in image resizing
  - Noise reduction in image restoration
  - Edge detection in image enhancement
  - Quantization in image compression (Original, prediction, transform, quantization, inverse quantization, inverse transform, and then reconstruction)

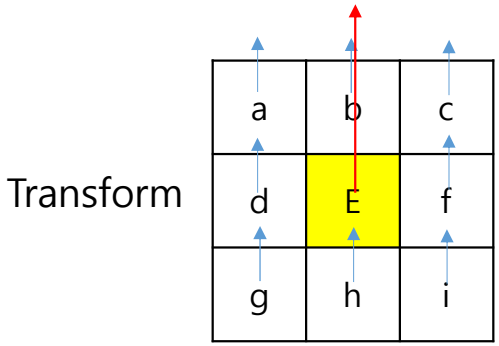
Downsampling

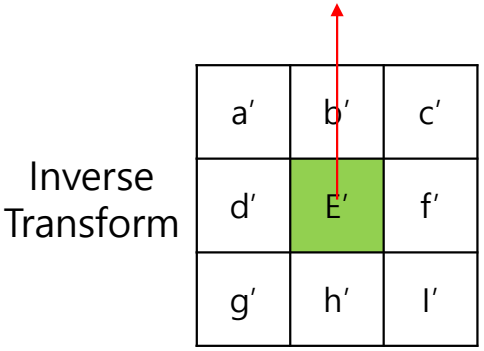


Upsampling


Transform
0 0 0 0
0 0
0 0
0 0 0 0
Inverse
Transform
Image
Frequency
Frequency
Image

Transform in Compression





Final

100	100	99
99	99	96
97	95	85

Original

-28	-3	-4
-29	-4	-7
-31	-3	-13

Prediction Error

0	0	0
0	E/5	0
0	0	0

Quantization ($e \div 5$)

0	0	0
0	×5	0
0	0	0

Inverse Quantization ($e \times 5$)

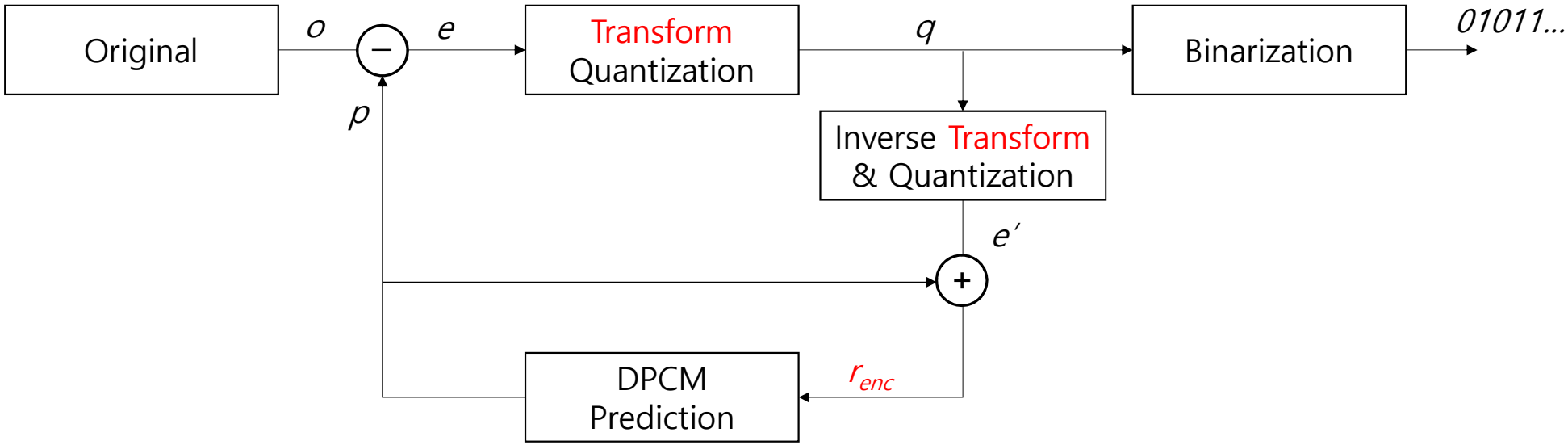
103	103	103
103	103	98
98	98	88

Reconstruction

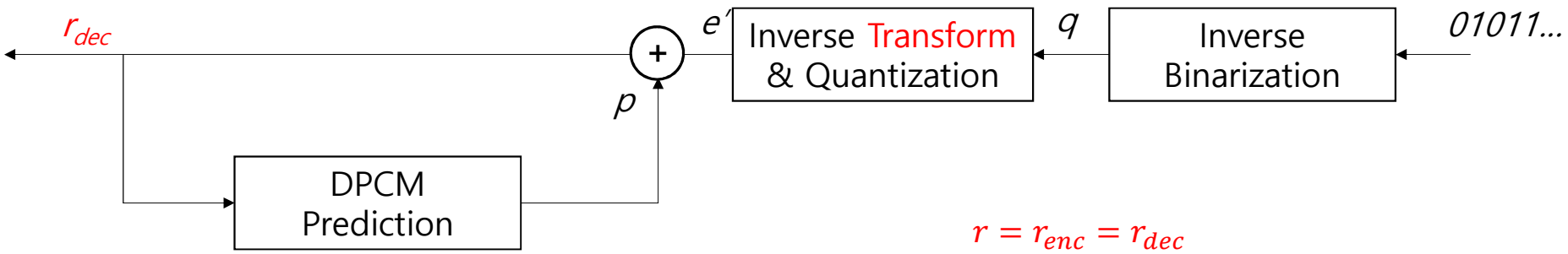
Encoder

Decoder

DPCM Based Encoder



DPCM Based Decoder



$r = r_{enc} = r_{dec}$

$$MSE = \frac{(o-r)^2}{\#Pixels}$$

$$PSNR = 10 \cdot \log(\frac{MAX^2}{MSE})$$