Introduction to Calculus

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1 Limits and Continuity

Calculus begins with the concept of limits and continuity.

1.1 Limit

The limit of a function $f(x)$ at a point $x = a$ is defined as:

$$\lim_{x \to a} f(x) = L$$

if for every $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$.

1.2 Continuity

A function $f(x)$ is continuous at a point $x = a$ if the following three conditions hold:

1. $f(a)$ is defined.
2. $\lim_{x \to a} f(x)$ exists.
3. $\lim_{x \to a} f(x) = f(a)$.

2 Derivatives

The derivative of a function measures its rate of change.

2.1 Derivative Definition

The derivative of a function $f(x)$ at a point $x = a$ is defined as:

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

2.2 Differentiability

A function $f(x)$ is differentiable at a point $x = a$ if the derivative $f'(a)$ exists.

3 Integrals

Integrals are used to calculate the accumulated area under a curve.

3.1 Definite Integral

The definite integral of a function $f(x)$ over the interval $[a, b]$ is denoted as:

$$\int_a^b f(x)\,dx$$

3.2 Indefinite Integral

The indefinite integral of a function $f(x)$ is denoted as:

$$\int f(x)\,dx$$

4 Conclusion

Calculus is a fundamental branch of mathematics that deals with limits, continuity, derivatives, and integrals. These concepts form the basis for understanding changes and accumulation in various mathematical and scientific fields.