Review: When do dynamic DeFi rate curves reduce capital efficiency?

Michael Bentley

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Original paper here: https://gauntlet.network/reports/pid.

1 Summary

Broadly, I agree with one of the papers main conclusions, which is that dynamic rates do not work if there is not sufficient interest rate arbitrage. That is kind of definitional, since dynamic interest rates depend on interest rate arbitrage to maintain their soft peg to the wider market interest rate. So if that assumption is violated, we should expect them to fail (this is partly why Euler has not implemented these models yet; I discuss reasons for the absence of interest rate arbitrage below.) However, I think a formal outline describing explicitly the risks involved and ways in which dynamic rates can be attacked is a very worthy exercise. Unfortunately, I was unable to fully grasp the steps of the attack outline in the paper in its current form. I've given feedback in the form of specific questions/comments below. Would love to follow up and try to understand it better. Indeed, it could be useful to incorporate this kind of approach into future risk assessments for determining when dynamic rates should/should not be integrated into lending protocols such as Euler in the future.

1.1 Questions/comments

Utilisation missing from borrow rate
I think a utilisation factor is missing from the definition of the borrow rate r_t^S on pg 5. (Tarun and I discussed this on Twitter a little.) Does this also change conclusions about dynamically adjusting spreads as a solution, on pg 9?

Interval of target utilisation, U^*, is unclear and might create issues
The target utilisation is defined in the paper as U^*. It should, as a minimum, take values $U^* \in [0, 1]$, but, more practically, should probably be defined in the interval $0.5 < U^* \leq 1$, because targeting high (but not too high) utilisation is usually the purpose of controlled interest rates. However, in the model setup on pg 7, we have $U_s = 2U^*$. This only works if $U^* \leq 0.5$, which is in contrast to some of the later assumptions/conclusions in the paper. On pg 10, for example, we ponder what happens when U^* is near 1. How much of the theory in this paper depends on this particular definition of U^* is unclear to me at the moment, but it would be good to clear this up. Presumably one can replace the '2' with a parameter in some way. However, intuitively, I think it might constrain the attack surface quite a bit.

The amount of supply required to move from S_{s-1} to S_s in main text is unclear and Fig. 1 does not marry well with the main text.
On pg 7, we are told that 'the current supply is fixed at S_s.' However, in Fig. 1 we see that both supply and demand (total borrowing) are increased between time $s-1$ and s. Indeed, it seems that $S_s > S_{s-1}$, but the amount of additional supply required to move the total from S_{s-1} to S_s is not defined explicitly anywhere in the text (except trivially as $S_s = S_{s-1} + \Delta S_{s-1}$). Given the relationship $D^*/S_s = U^*$, clearly we can choose D^* if S_s and U^* are known, but if S_s is unknown, then things are underdetermined. Overall, this makes it hard for me to reconcile the described steps of the attack with the theory (see below).

Unclear steps in the attack. Perhaps a worked example would help?
Here is my intuition about how such an attack might be carried out:

- Raise utilisation above the target to drive a spike in interest rates above the going market rate. E.g. make the rate 10% on Euler, whilst the rest of the market is at 1%.
- Immediately put utilisation back to the target, by spiking in supply, and/or reducing borrowing. In the absence of arbitrage, this prevents further interest rate changes since the rate of change of the interest rate is zero when utilisation is at the target. If the user has sufficient supply in the protocol, they can profit from increased rates at the expense of the protocol's borrowers. E.g. if the target utilisation is 0.5, and the pool currently has supply and borrows at 100 after step 1 (utilisation of 1), then the user can just add in supply of 100 to bring the utilisation back to 0.5, and earn a high rate of interest on that supply until arbitrage kicks in.

The devil is in the detail here, and this is what the paper attempts to address, but I'm unclear on how my intuition maps to the paper's model. In the model description on pg 7, we have the strategic user both increasing supply *and* reducing borrowing in a stepwise manner, over time. This process, of increasing supply and reducing borrowing at the same time, will *always* increase utilisation. However, in the paper a condition for keeping utilisation constant whilst carrying out this process is arrived at, which is contradictory.

Should strategic user share of lending/borrowing be incorporated, or is it just unimportant?

As an aside, I was also a bit surprised that none of the conditions arrived at in this part of the paper depend on the strategic user's share of the total lending/borrowing, both before or after the attack. Perhaps my intuition is wrong here, but it feels like the cost/benefit of the attack should depend on the user's share of lending/borrowing. If they make up a large % of the lending pool, then they stand to benefit more from the higher rates, for example.

Worked example would help bring clarity
Overall, I think it would not be too much trouble to outline a worked example. This would help bring clarity to the theory. Suppose a pool has $S_{s-1} = 100$ units of supply, and total borrows $D_{s-1} = 20$ units of demand, at some time $t = s - 1$, so that utilisation at that time is $U_{s-1} = 20/100 = 0.2$. If the target utilisation is $U^* = 0.7$, what does the strategic user do next? Can we follow it for a couple more steps to confirm the theory?

Models of control
I should point out that simple proportional or proportional-integral control is only one way to help shape the rate of change of interest rates. I will share the version(s) I worked on for Euler in due course, but they are quite different to the general cases outlined here. One can do interesting things with these kinds of models, like have a hybrid model that is partially controlled and partially purely a function of utilisation. Or create a model in which the rate of change depends on the distance of the utilisation from its target. Or a piecewise definition that depends on whether or not the utilisation is increasing or decreasing. Or a model that depends on derivatives (which may or may not be easy to manipulate). And so on. I think this is worth exploring further in the future. By defining the interest rate recursively, as $r_t = r_{t-1} + \Delta r$, it can be viewed as a sort of difference equation that gets integrated/resolved by the lenders and borrowers over time. One can get quite creative as a mechanism designer in what Δr is. Anyway, my point is that I would maybe caution against overly-broad claims/conclusions from this work and statements like 'As such, whenever DeFi enthusiasts talk about PID controlled interest rates, they usually only mean a P or PI controller.'

1.2 Dynamic rates on Euler and sufficient 'demand elasticity'

I designed a mechanism for controlled or 'dynamic' interest rates on Euler back in early 2020. A lot of people—including Tarun, in his latest Tweet thread—have pondered about why these rates have not yet gone live on Euler, given that the protocol has now been live on Ethereum for nearly a year. The answer to that is related to this latest piece of work, so is worth briefly mentioning here.

Dynamic interest rates using control theory are designed to work in an environment in which there is aggressive arbitrage of interest rates—what is called 'demand elasticity' in the paper discussed here. In that paper, it is pointed out that many suppliers and borrowers in the real-world are actually relatively insensitive to interest rate changes, meaning arbitrage is often weak. It is this phenomenon—weak arbitrage—that is claimed to enable a kind of just-in-time liquidity attack. I broadly agree with this, and it is partly why dynamic rates are not yet live on Euler.

Why is arbitrage weak? There are many explanations for this. In an efficient market, the theory goes that rational actors should move lending supply and loans between protocols in a way that maximises their economic output. In practice, however, there are lots of reasons why this does not happen. Moving positions costs time and money and may not be worth the effort. A 1% difference in trading fees between different exchange protocols could probably justify an immediate switch in user preferences, but is hardly likely to excite lenders/borrowers choosing between different lending protocols unless it has been shown to be sustained over the long-term. After all, a lower/higher interest rate elsewhere might only be transient. Or it might be offset by other factors. Other protocols may have different risk profiles, meaning the risk-adjusted interest rates are lower, even if the up-front rate appears to be significantly higher. Users are also sometimes 'loyal' or 'sticky' or apathetic. And so on.

In the environment in which I was thinking about dynamic interest rates in early 2020, there was reason to believe a new type of protocol would enable more efficient markets for interest rates. Specifically, the emergence of yield aggregators like Yearn suggested interest rate arbitrage would become the norm. Yield aggregators reduce many of the costs of arbitraging interest rates. Users pool their funds together, which can then be automatically and efficiently re-balanced across different protocols. For what it is worth, I still think this is the future for DeFi in the long-term, because it is orders of magnitude more efficient than what we have in tranditional finance. But it is clear that a major deterioration in market conditions in DeFi has tempered demand for using yield aggregators and lending protocols at the present time. This means interest rate arbitrage between Euler and other protocols remains relatively inefficient, and the conditions that favour the use of dynamic interest rate models are probably not met for the majority of assets today.