PyData Delhi 2018

Introduction to Quantum Computing

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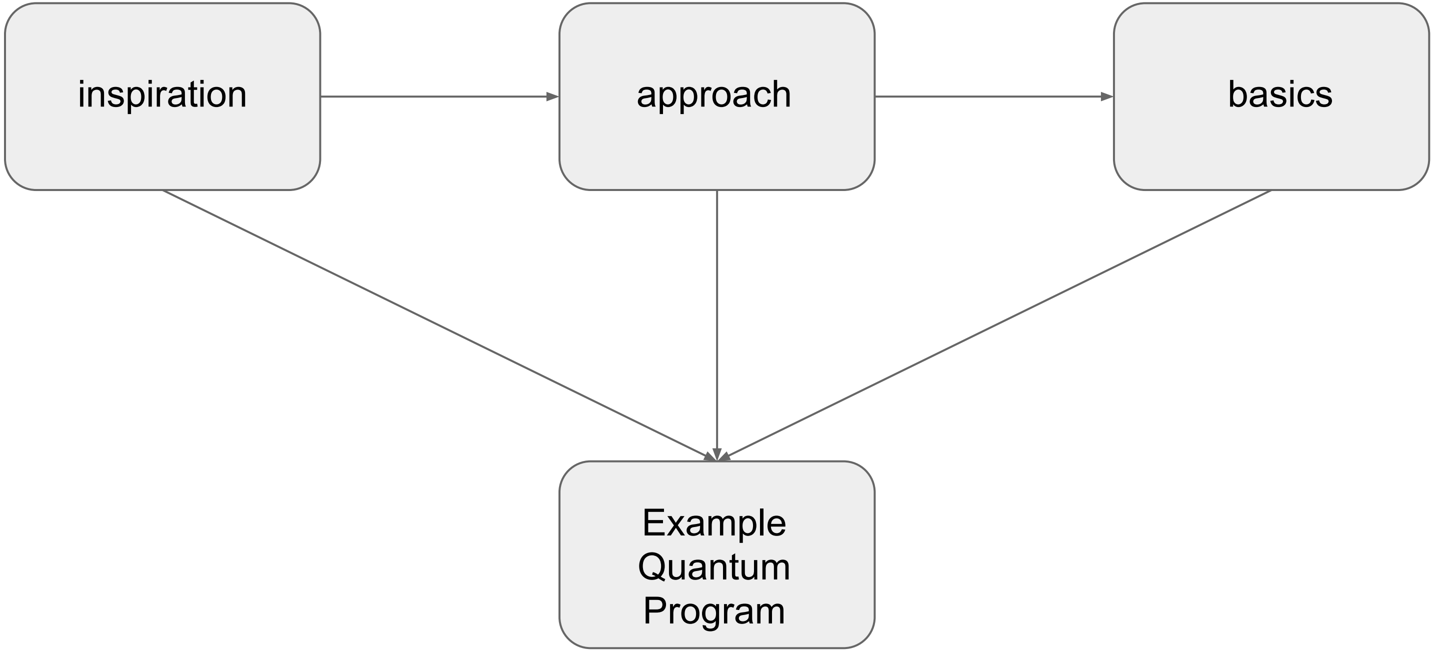
Structure and Aim of the Talk.



Inspiration behind Quantum computing

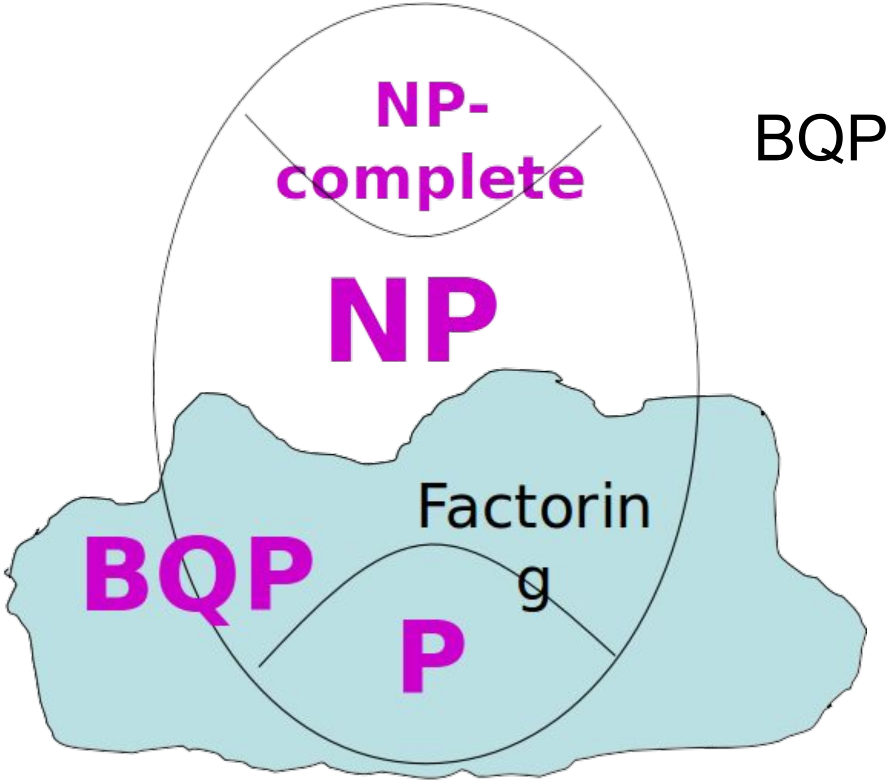
- ‘Physics based programming’.
- Nature itself computes solutions to problems that are seemingly impossible (read **NP hard**) for classical machine.

  - Example: Soap Bubble Computer (Steiner Tree Problem).

  - DNA programming, protein programming

- Issue:- nature doesn’t always settles for global optima.

What can a Quantum Computer 'supposedly Do Better?



BQP =Bounded error Quantum Polynomial time

But the Talk isn't about that!!!

0%

Is the probability that I am going to sing glories about Quantum Computing.

This Talk is about the Beauty of a System of Logic which has 'something' to do with Putting Cats in boxes.

How should you think about quantum computing- nOT LIKE A BUZZWORD

QUANTUM DEEP LEARNING

QUANTUM COGNITION

QUANTUM FINANCE

QUANTU
M
BITCOIN

Think of it as A Computing Model

...Having rules for calculating things.

As a system of Logic

Quantum Computation is very much like Euclid’s Geometry

- Trust its ‘laws’ just like you believed “All right angles are equal to one another” when you were in high school.

Don't think about cats

Don't think about this guy…

"Seriously…Quantum Systems are easy, they are just probabilities with negative signs"

Some Blah..Blah..Blah

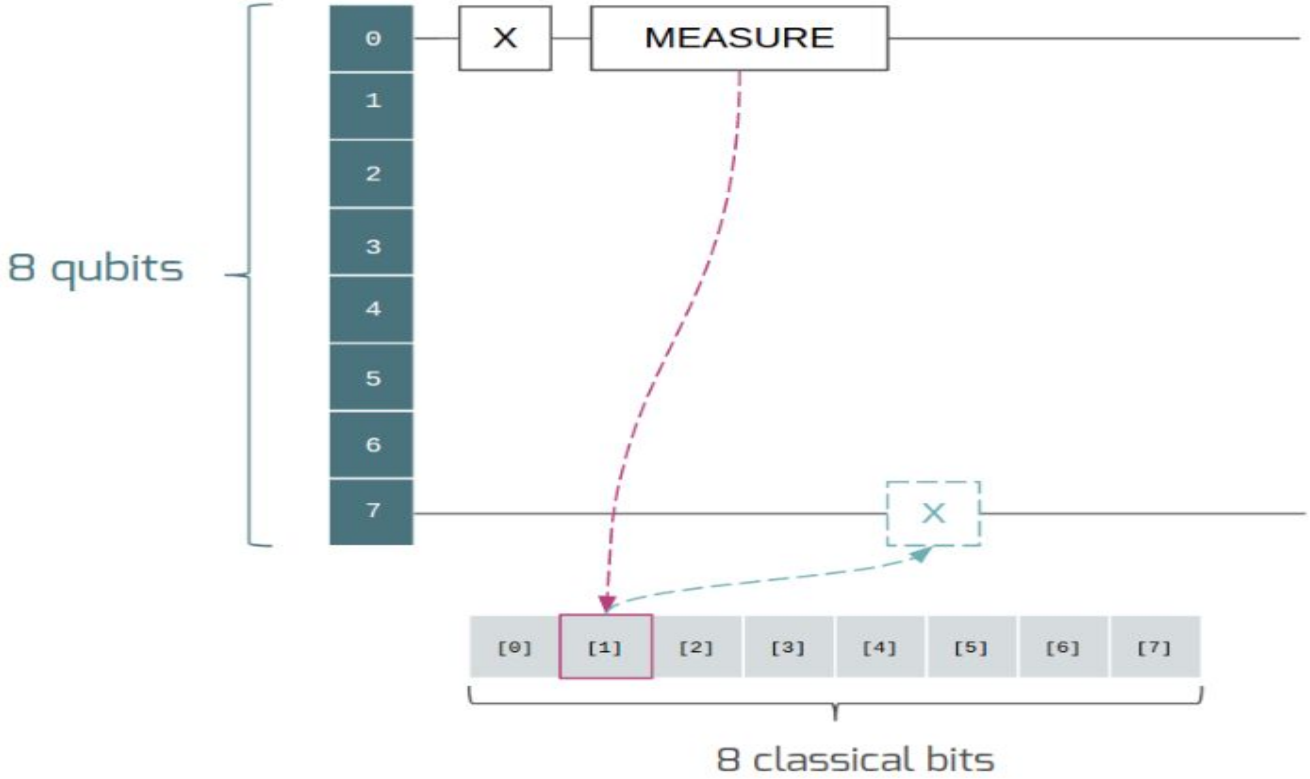
Things to watch out for in this talk

1. Quantum Interference
2. Quantum Entanglement
3. Measurement

BASICS..

Theoretical Model of Quantum Machine

Yeah classical and quantum systems are friends :)



Hilbert Space

- Fancy term for space of vectors.
- have complex numbers as components.
- These vectors can be added, subtracted, multiplied by a scalar or a vector
- Represented by a column matrix and have a special name in Quantum Mechanics...**KETS**
- Every ket can be expressed as a linear combination of orthonormal unit vectors called **basis vectors.**
- Symbol of ket

Hilbert Space continued

bra

$$\langle\psi| = (\psi_1^* \ \psi_2^* \ \cdots \ \psi_n^*)$$

$$|\psi\rangle \equiv \vec{\psi}$$

complex

Ket $\quad |\psi\rangle = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{pmatrix}$

$$\langle\phi|\psi\rangle = \underbrace{(\phi_1^* \ \phi_2^* \ \phi_3^* \cdots n)\begin{pmatrix} \phi_1 \\ \psi_2 \\ \psi_3 \\ \vdots \\ \psi_n \end{pmatrix}}_{\text{a scalar.}}$$

bra- ket

<u>In computing</u>

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

State

- Description of the system
- Is described completely by a ket having unit length. (**Important**).
- Unlike classical mechanics, in Quantum mechanics states can be added and multiplied with other states (kets).
- If a system can exist in two states, it can also exist in any linear combination of those two states. (**Quantum Interference**).

So what's all the Fuss about 'Qubits'?

Classical Bit

It can only be in two ‘classical’ states, a **0** or a **1.**
NOTHING ELSE.

CLASSICAL BITS ARE INDEPENDENT OF EACH OTHER.

Qubit

A Qubit can be in two states **|0>** and **|1>** (analogous to classical bits). These states are called ‘computational basis states’ (hi-fi term which means perpendicular axes).

BUT A Qubit CAN ALSO BE IN A LINEAR COMBINATION OF |0> AND |1>.

This means a*|0> + b*|1> is also a valid qubit state. (**no classical analog**).

What if there are more than 1 Qubit?

The computational basis for such a 2 Qubit system is **|00>, |01>, |10>, |11>.**

A general 2 Qubit system is **a*|00> + b*|01> + c*|10> + d*|11>.**

Is it possible to represent two single qubit system as a single multi-Qubit system?

- **YES**

'Classical' Example

2 coin system can have four states if 1 coin has two states H and T.

{**(HH),(HT),(TH),(TT)**}

$$\{0_u 0_v,\ 0_u 1_v,\ 1_u 0_v,\ 1_u 1_v\}$$

Combining single Qubit Systems

This operation is called the **Tensor Product.**

$$\underbrace{\alpha|0\rangle + \beta|1\rangle}^{\text{State 1}}) \; (\overbrace{\gamma|0\rangle + \delta|1\rangle}^{\text{state 2}}) = \underbrace{\alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle}_{\text{combined state}}$$

Concrete example

$$|\,u\rangle = \frac{1}{2}|\,0_u\rangle + \frac{1}{2}|\,1_u\rangle$$
$$|\,v\rangle = \frac{1}{2}|\,0_v\rangle + \frac{1}{2}|\,1_v\rangle,$$

$$|\,u\rangle \otimes |\,v\rangle = \frac{1}{4}|\,0_u 0_v\rangle + \frac{1}{4}|\,0_u 1_v\rangle + \frac{1}{4}|\,1_u 0_v\rangle + \frac{1}{4}|\,1_u 1_v\rangle.$$

'Basis' states of 2 qubit systems

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$|10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

4 Basis states fore 2 Qubit systems

Can all 2 Qubit Systems be written as a combination of Single Qubit System?

No...And such systems are said to be 'Entangled'

This phenomena really is the heart of Quantum Computing.

Classically one can treat a 2-bit system as a combination of single bit systems.

But entangled states in Quantum Mechanics cannot be interpreted in the same way.

Ex. **Bell State**

Operators

- These are linear operators represented by matrices.
- In the language of Quantum Computation, these are called **GATES** (analogous to gates in digital circuits).
- 1 ket into another

Identity gate

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$I|0\rangle \longrightarrow |0\rangle$$

$$I|1\rangle \longrightarrow |1\rangle$$

NOT Gate

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$X|0\rangle \longrightarrow |1\rangle$$

$$X|1\rangle \longrightarrow |0\rangle$$

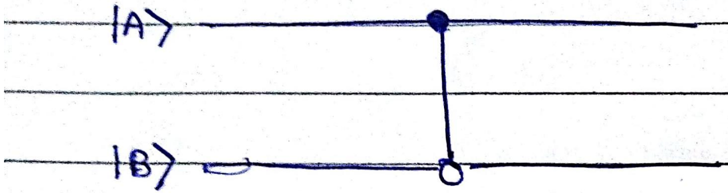
Hadamard Gate

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad -\boxed{H}-$$

$$H|0\rangle \longrightarrow \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$H|1\rangle \longrightarrow \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

CNOT Gate .. a binary gate



$|B\rangle$ is flipped only if $|A\rangle \equiv |1\rangle$

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

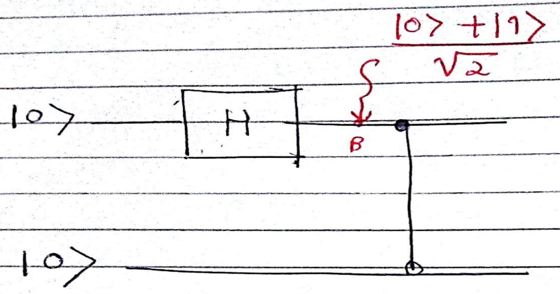
Pauli Gates

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Making an Entangled State

Bell State Preparation



Combined state at B

$$\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes \left(|0\rangle\right)$$

$$|\phi\rangle = \frac{|00\rangle + |10\rangle}{\sqrt{2}}$$

Bell state

$$\text{CNOT}\,|\phi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

OOff....Anything Else?

Perhaps the Most Unsettling Principle of Quantum Mechanics

Measurement of Quantum Systems

Measurement...A destructive phenomena

Though quantum states can be a linear combination of basis vectors, the act of ‘measurement’ **collapses**, to one of the basis vectors. After measurement the state remains equal to that basis vector.

wave function/state collapsed to a basis

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \xrightarrow{\text{measure}} \begin{cases} |0\rangle & \text{Probability} = |\alpha|^2 \\ |1\rangle & \text{Probability} = |\beta|^2 \end{cases}$$

$$|\psi\rangle = |0\rangle \xrightarrow{\text{measure}} |0\rangle \quad \text{prob} = 1$$

$$|\psi\rangle = |1\rangle \xrightarrow{\text{measure}} |1\rangle \quad \text{prob} = 1$$

Okay… So The Bare Minimum basics are Over, How much is all of this equivalent to classical Computation?

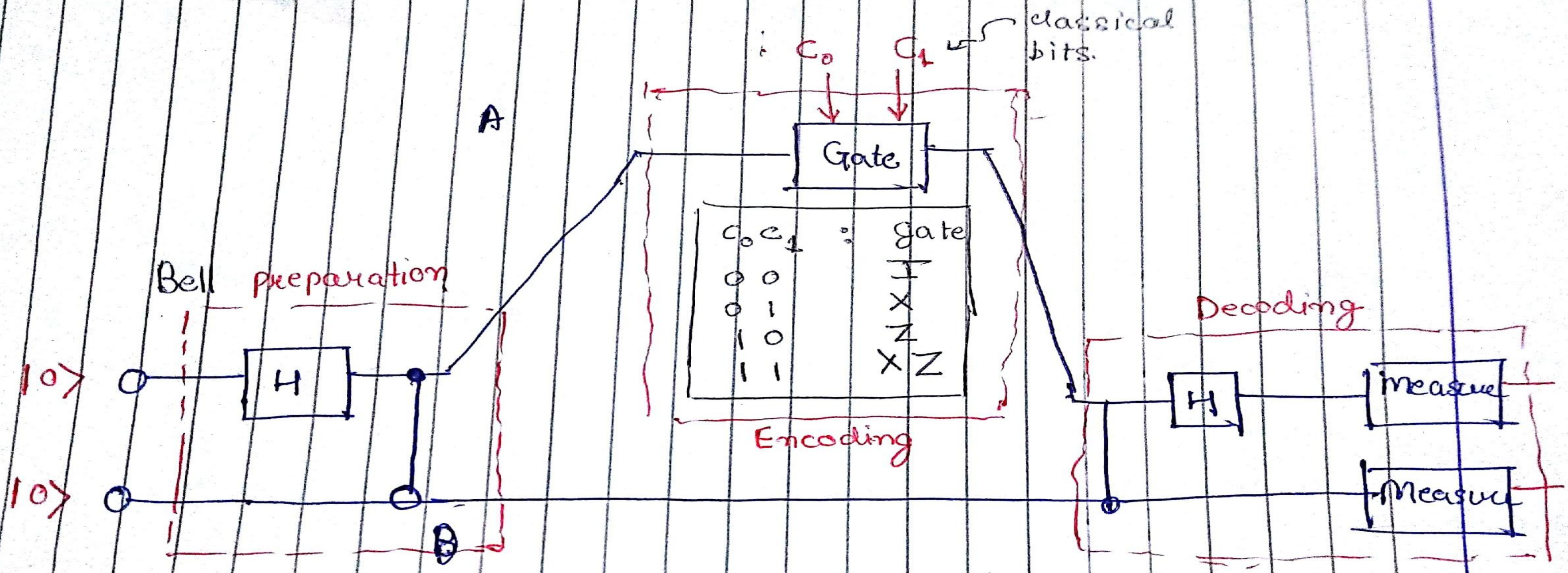
Equivalent to where you were when you were using AND, OR gates!!!

OMG!!! So Much for so Little?

Not Quite…There is a simple program that can demonstrate the power of Quantum Logic

SuperDense Coding

Sending 2 classical bits of information using a single Qubit.



Note: $Z|0\rangle = \cancel{-}|0\rangle$

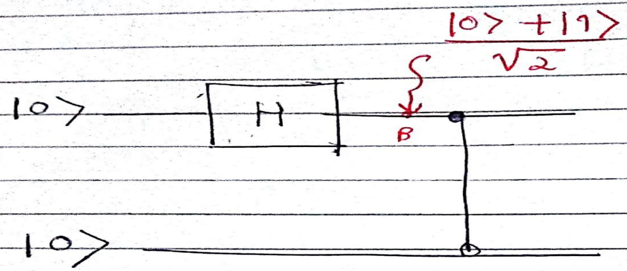
$Z|1\rangle = -|1\rangle$

Procedure Outline

- First entangle the states of the 2 Qubits
- Give one to the sender other to receiver.
- The sender would apply a gate to its Qubit depending on the classical bits it has.
- The sender would send its Qubit to the receiver.
- The receiver would then decode the classical bits from its Qubit and the Qubit it receives.

Calculations-1

Bell State Preparation



Combined state at B

$$|\phi\rangle = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes \left(|0\rangle\right) = \frac{|00\rangle + |10\rangle}{\sqrt{2}}$$

$$\text{CNOT}\,|\phi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Bell state

Calculations-2

Encoding

if $c_0 c_1 = 00 \Longrightarrow I|\phi\rangle$

$= 01 \Longrightarrow X|\phi\rangle$

$= 10 \Longrightarrow Z|\phi\rangle$

$= 11 \Longrightarrow XZ|\phi\rangle$

case 1: $|\phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

acts only on 1st part since gate applied on 1st Qubit

case 2: $X|\phi\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$

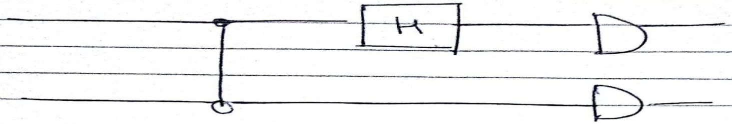
case 3: $Z|\phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + (-|11\rangle))$

case 4: $XZ|\phi\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)$

all these are the 4 bell states

Calculations-3

Decoding



Case 1: $\frac{|00\rangle + |11\rangle}{\sqrt{2}} \xrightarrow{CNOT} \frac{|00\rangle + |10\rangle}{\sqrt{2}} \xrightarrow{H} |00\rangle$

Case 2: $\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|11\rangle + |01\rangle) \xrightarrow{H} |01\rangle$

Case 3: $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle) \xrightarrow{H} |10\rangle$

Case 4: $\frac{1}{\sqrt{2}}(|10\rangle - |01\rangle) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|11\rangle - |01\rangle) \xrightarrow{H} |1$

all are product states

case 1: $\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)|0\rangle$ case 3: $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)|0\rangle$

Case 2: $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|1\rangle$ case 4: $\frac{-1}{\sqrt{2}}(|0\rangle - |1\rangle)|1\rangle$

Where is the Code?

What Next? How to go about studying more?

Brush up Your Linear Algebra

And Complex Numbers

Books

- Quantum Computation and Quantum Information, Michael Nielson
- Quantum Computing since Democritus.
- Michael Nielson’s Quantum Computing videos.

Speculation

Is Computational thinking another approach towards understanding Physics

- Information Theory and Statistical Mechanics -By E.T Jaynes

Questions ?

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