Algebraic Topology

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1 Chapter One

Definition 1.1. *A* ***topology*** *on a set X is a collection $\mathcal{T}$ of subsets of X having the following properties:*

1. *$\varnothing$ and X are in $\mathcal{T}$*
2. *The union of the elements of any finite subcollection of $\mathcal{T}$ is in $\mathcal{T}$. Closed under arbitrary unions.*
3. *The intersection of the elements of any finite subcollection of $\mathcal{T}$ is in $\mathcal{T}$. Closed under intersection.*

A set X for a which a topology $\mathcal{T}$ has been specified is called a ***topological space*** *and $\mathcal{T}$ is called a topology on X.*

The sets in $\mathcal{T}$ are called open sets. If $U \in \mathcal{T}$ is in open set then $X \setminus U$ is closed.

Example 1.2. *Let $X \in \mathbb{R}$. Define a topology $\mathcal{T}$ on X. A set $U \in \mathbb{R}$ is in $\mathcal{T}$ if, given $x \in U$, there is an $\epsilon > 0$ such that $(x - \epsilon, x + \epsilon) \subset U$. This is called an open set.*

Theorem 1.3. *Let $(a, b) \subset \mathbb{R}$. Then (a, b) is an open set.*

Proof. Let $x \in (a, b)$. We need to find an $\epsilon > 0$ such that $(x - \epsilon, x + \epsilon) \subset (a, b)$. Let $\epsilon = \min\{|x - a|, |x - b|\}$. Thus, $(x - \epsilon, x + \epsilon) \subset (a, b)$.

□

Definition 1.4. *Let $X = \mathbb{R}^n$. The* ***open ball*** *of radius ϵ around x is defined as:*

$$B_\epsilon(x) = \{y \in \mathbb{R}^n \mid |x - y| < \epsilon\}$$

This is called the ***usual topology*** *on $\mathbb{R}^n$.*

Definition 1.5. *Let $(X, \mathcal{T})$ be a topological space. If $A \subset X$, then the collection*

$$\mathcal{T}_A = \{A \cap U \mid U \in \mathcal{T}\}$$

is a topology on A, called the ***subspace topology****.*