IPTW ATE & ATT Robust variance formula

1) IPTW ATE Robust (Sandwich) Variance ; Liang & Zegar Sandwich Variance 이용!

$\rightarrow \sum_{i=1}^{K} D_i' V_i^{-1} S_i S_i' D_i / K$ ($D_i = \partial \mu_i / \partial \beta$, $S_i = Y_i - \mu_i$, $V_i = Var(y_i | x_i) = \nu(\mu_i)\phi$)

R code) Variance = bread %*% meat %*% bread / v-scale

lm version 이용!

/ glm version 도 이후 추가로 필요

여기서 bread = $n \times (X^T w X)^{-1}$, v-scale = n, meat = $\underbrace{psi' psi}_{= X'W^T S^T S W X} / n$ (이때, psi = $\underbrace{S}_{잔차}$ * Weight * X)

Question) $V_i^{-1} \overset{?}{=}$ Weight

2) IPTW ATT Robust Sandwich Variance ; Saul and Hudgens, 2020 이용!

$V(\xi) = A(\xi)^{-1} B(\xi) \{A(\xi)^{-1}\}^T$, $\nabla g(\xi)^T = (0, \underbrace{0_{J+1}}_{회귀계수의 개수}, 1, -1) \Rightarrow \Sigma = \nabla g(\xi)^T V(\xi) \nabla g(\xi)$

$\rightarrow e(L_i ; \alpha) = P(A_i = 1 | L_i) = \exp(\alpha_0 + \alpha_1^T L_i) / 1 + \exp(\alpha_0 + \alpha_1^T L_i)$ → $h(L_i ; \hat{\alpha})$라 하자.

L_i를 i^{th} confounder set 이라 하면, $\psi_\alpha = \{A_i - e(L_i ; \alpha)\} \begin{pmatrix} 1 \\ L_i \end{pmatrix}$ → ψ_{α_0}, ψ_{α_1}

$\psi_1 = A_i + (1-A_i) h(L_i ; \alpha) A_i (Y_i - \mu_1) = A_i^2 (Y_i - \mu_1) + \underbrace{A_i (Y_i - \mu_1)(1-A_i)}_{A_i(1-A_i) = 0} h(L_i ; \alpha)$ (∵ A_i가 1 or 0)

check $= A_i^2 (Y_i - \mu_1) = A_i (Y_i - \underbrace{\mu_1}_{= E[Y^1 | A=1]})$ (∵ A_i는 Binary)

$\psi_0 = (A_i + (1-A_i) h(L_i ; \alpha))(1-A_i)(Y_i - \mu_0) = A_i \cancel{(1-A_i)}^{0} (Y_i - \underbrace{\mu_0}_{= E[Y^0 | A=1]}) + \underbrace{(1-A_i)^2}_{= 1-A_i} (Y_i - \mu_0) h(L_i ; \alpha)$

$= (1-A_i)(Y_i - \mu_0) h(L_i ; \alpha)$

$$A(\xi) = -E \left[\begin{array}{cc|cc} \partial \psi_{\alpha_0} / \partial \alpha_0 & \partial \psi_{\alpha_0} / \partial \alpha_1 & \partial \psi_{\alpha_0} / \partial \mu_1 & \partial \psi_{\alpha_0} / \partial \mu_0 \\ \partial \psi_{\alpha_1} / \partial \alpha_0 & \partial \psi_{\alpha_1} / \partial \alpha_1 & \partial \psi_{\alpha_1} / \partial \mu_1 & \partial \psi_{\alpha_1} / \partial \mu_0 \\ \hline \partial \psi_1 / \partial \alpha_0 & \partial \psi_1 / \partial \alpha_1 & \partial \psi_1 / \partial \mu_1 & \partial \psi_1 / \partial \mu_0 \\ \partial \psi_0 / \partial \alpha_0 & \partial \psi_0 / \partial \alpha_1 & \partial \psi_0 / \partial \mu_1 & \partial \psi_0 / \partial \mu_0 \end{array} \right] = -E \begin{bmatrix} a_{11} & O_{(J+1) \times 2} \\ a_{21} & P_1 I_2 \end{bmatrix}$$

$$B(\xi) = E \left[\begin{array}{cc|cc} \psi_{\alpha_0}^2 & \psi_{\alpha_0} \psi_{\alpha_1} & \psi_{\alpha_0} \psi_1 & \psi_{\alpha_0} \psi_0 \\ \psi_{\alpha_1} \psi_{\alpha_0} & \psi_{\alpha_1}^2 & \psi_{\alpha_1} \psi_1 & \psi_{\alpha_1} \psi_0 \\ \hline \psi_1 \psi_{\alpha_0} & \psi_1 \psi_{\alpha_1} & \psi_1^2 & \psi_1 \psi_0 \\ \psi_0 \psi_{\alpha_0} & \psi_0 \psi_{\alpha_1} & \psi_0 \psi_1 & \psi_0^2 \end{array} \right] = E \begin{bmatrix} b_{11} & b_{12} \\ b_{12}^T & b_{22} \end{bmatrix}$$

cf) 실제 분산 계산 때 필요한 부분은 [ ] 로 칠한 부분!

Meeting 때 교수님의 thinking)

1) $E[\psi_1] = 0$ / 2) $E[\psi_0] \neq 0$