

# # Our situation in IPTW ATT Robust variance #

$$A(\hat{\alpha}) = -E \begin{bmatrix} \frac{\partial \psi_{d0}}{\partial \alpha_0} & \frac{\partial \psi_{d0}}{\partial \alpha_1} & \frac{\partial \psi_{d0}}{\partial \alpha_2} & \frac{\partial \psi_{d0}}{\partial \mu_1} & \frac{\partial \psi_{d0}}{\partial \mu_0} \\ \frac{\partial \psi_{d1}}{\partial \alpha_0} & \frac{\partial \psi_{d1}}{\partial \alpha_1} & \frac{\partial \psi_{d1}}{\partial \alpha_2} & \frac{\partial \psi_{d1}}{\partial \mu_1} & \frac{\partial \psi_{d1}}{\partial \mu_0} \\ \frac{\partial \psi_{d2}}{\partial \alpha_0} & \frac{\partial \psi_{d2}}{\partial \alpha_1} & \frac{\partial \psi_{d2}}{\partial \alpha_2} & \frac{\partial \psi_{d2}}{\partial \mu_1} & \frac{\partial \psi_{d2}}{\partial \mu_0} \\ \frac{\partial \psi_1}{\partial \alpha_0} & \frac{\partial \psi_1}{\partial \alpha_1} & \frac{\partial \psi_1}{\partial \alpha_2} & \frac{\partial \psi_1}{\partial \mu_1} & \frac{\partial \psi_1}{\partial \mu_0} \\ \frac{\partial \psi_0}{\partial \alpha_0} & \frac{\partial \psi_0}{\partial \alpha_1} & \frac{\partial \psi_0}{\partial \alpha_2} & \frac{\partial \psi_0}{\partial \mu_1} & \frac{\partial \psi_0}{\partial \mu_0} \end{bmatrix}$$

Annotations:  $a_{11}$  (top row),  $a_{21}$  (bottom row),  $\frac{\partial \psi_1}{\partial \alpha_0} = 0$ ,  $\frac{\partial \psi_1}{\partial \alpha_1} = 0$ ,  $\frac{\partial \psi_1}{\partial \alpha_2} = 0$ ,  $\frac{\partial \psi_1}{\partial \mu_1} = 0$ ,  $\frac{\partial \psi_1}{\partial \mu_0} = 0$ . A green box contains  $\begin{bmatrix} P(A=1) & 0 \\ 0 & P(A=1) \end{bmatrix}$ .

우리의 상황은  $P(A_T=1 | L_T) = \exp(d_0 + d_1 B + d_2 C) / 1 + \exp(d_0 + d_1 B + d_2 C)$   $\rightarrow h(L_T; \alpha)$  이 define.

1)  $\psi_{d0} = A_T - \{h(L_T; \alpha) / (1 + h(L_T; \alpha))\}$

①  $\frac{\partial \psi_{d0}}{\partial \alpha_0} = - \frac{h(L_T; \alpha)}{(1 + h(L_T; \alpha))^2}$     ②  $\frac{\partial \psi_{d0}}{\partial \alpha_1} = - \frac{h(L_T; \alpha) \times B_T}{(1 + h(L_T; \alpha))^2}$     ③  $\frac{\partial \psi_{d0}}{\partial \alpha_2} = - \frac{h(L_T; \alpha) \times C_T}{(1 + h(L_T; \alpha))^2}$

2)  $\psi_{d1} = \{A_T - (h(L_T; \alpha) / (1 + h(L_T; \alpha)))\} B_T$

①  $\frac{\partial \psi_{d1}}{\partial \alpha_0} = - \frac{h(L_T; \alpha) \times B_T}{(1 + h(L_T; \alpha))^2}$     ②  $\frac{\partial \psi_{d1}}{\partial \alpha_1} = - \frac{h(L_T; \alpha) \times B_T^2}{(1 + h(L_T; \alpha))^2}$     ③  $\frac{\partial \psi_{d1}}{\partial \alpha_2} = - \frac{h(L_T; \alpha) \times B_T \times C_T}{(1 + h(L_T; \alpha))^2}$

3)  $\psi_{d2} = \{A_T - (h(L_T; \alpha) / (1 + h(L_T; \alpha)))\} C_T$

①  $\frac{\partial \psi_{d2}}{\partial \alpha_0} = - \frac{h(L_T; \alpha) \times C_T}{(1 + h(L_T; \alpha))^2}$     ②  $\frac{\partial \psi_{d2}}{\partial \alpha_1} = - \frac{h(L_T; \alpha) \times B_T \times C_T}{(1 + h(L_T; \alpha))^2}$     ③  $\frac{\partial \psi_{d2}}{\partial \alpha_2} = - \frac{h(L_T; \alpha) \times C_T^2}{(1 + h(L_T; \alpha))^2}$

$\Rightarrow$  이때,  $h(L_T; \hat{\alpha}) / (1 + h(L_T; \hat{\alpha}))^2 = \frac{h(L_T; \hat{\alpha})}{1 + h(L_T; \hat{\alpha})} \times \frac{1}{1 + h(L_T; \hat{\alpha})} = P(A_T=1 | L_T) \times (1 - P(A_T=1 | L_T))$

그러면,  $a_{11} = \begin{bmatrix} - \frac{h(L_T; \hat{\alpha})}{(1 + h(L_T; \hat{\alpha}))^2} (1, B_T, C_T)^T \\ - \frac{h(L_T; \hat{\alpha})}{(1 + h(L_T; \hat{\alpha}))^2} (B_T, B_T^2, B_T C_T)^T \\ - \frac{h(L_T; \hat{\alpha})}{(1 + h(L_T; \hat{\alpha}))^2} (C_T, B_T C_T, C_T^2)^T \end{bmatrix}$

4)  $\psi_0 = (Y_T - \mu_0) (1 - A_T) h(L_T; \alpha)$

①  $\frac{\partial \psi_0}{\partial \alpha_0} = (Y_T - \mu_0) (1 - A_T) h(L_T; \alpha)$     ②  $\frac{\partial \psi_0}{\partial \alpha_1} = (Y_T - \mu_0) (1 - A_T) h(L_T; \alpha) \times B_T$

③  $\frac{\partial \psi_0}{\partial \alpha_2} = (Y_T - \mu_0) (1 - A_T) h(L_T; \alpha) \times C_T$

그러면,  $a_{21} = (Y_T - \mu_0) (1 - A_T) h(L_T; \alpha) (1, B_T, C_T)^T$

$$B\left(\frac{2}{3}\right) = E \left[ \begin{array}{ccc|cc} \psi_{\alpha_0}^2 & \psi_{\alpha_0}\psi_{\alpha_1} & \psi_{\alpha_0}\psi_{\alpha_2} & \psi_{\alpha_0}\psi_1 & \psi_{\alpha_0}\psi_0 \\ \psi_{\alpha_0}\psi_{\alpha_1} & \psi_{\alpha_1}^2 & \psi_{\alpha_1}\psi_{\alpha_2} & \psi_{\alpha_1}\psi_1 & \psi_{\alpha_1}\psi_0 \\ \psi_{\alpha_0}\psi_{\alpha_2} & \psi_{\alpha_1}\psi_{\alpha_2} & \psi_{\alpha_2}^2 & \psi_{\alpha_2}\psi_1 & \psi_{\alpha_2}\psi_0 \\ \hline \psi_1\psi_{\alpha_0} & \psi_1\psi_{\alpha_1} & \psi_1\psi_{\alpha_2} & \psi_1^2 & \psi_1\psi_0 \\ \hline \psi_0\psi_{\alpha_0} & \psi_0\psi_{\alpha_1} & \psi_0\psi_{\alpha_2} & \psi_0\psi_1 & \psi_0^2 \end{array} \right]$$