

Our situation in IPTW ATT Robust variance

$$A(\xi) = -E \begin{bmatrix} \frac{\partial \psi_{d0}}{\partial \alpha_0} & \frac{\partial \psi_{d0}}{\partial \alpha_1} & \frac{\partial \psi_{d0}}{\partial \alpha_2} & \frac{\partial \psi_{d0}}{\partial \mu_1} & \frac{\partial \psi_{d0}}{\partial \mu_0} \\ \frac{\partial \psi_{d1}}{\partial \alpha_0} & \frac{\partial \psi_{d1}}{\partial \alpha_1} & \frac{\partial \psi_{d1}}{\partial \alpha_2} & \frac{\partial \psi_{d1}}{\partial \mu_1} & \frac{\partial \psi_{d1}}{\partial \mu_0} \\ \frac{\partial \psi_{d2}}{\partial \alpha_0} & \frac{\partial \psi_{d2}}{\partial \alpha_1} & \frac{\partial \psi_{d2}}{\partial \alpha_2} & \frac{\partial \psi_{d2}}{\partial \mu_1} & \frac{\partial \psi_{d2}}{\partial \mu_0} \\ \frac{\partial \psi_1}{\partial \alpha_0} & \frac{\partial \psi_1}{\partial \alpha_1} & \frac{\partial \psi_1}{\partial \alpha_2} & \frac{\partial \psi_1}{\partial \mu_1} & \frac{\partial \psi_1}{\partial \mu_0} \\ \frac{\partial \psi_0}{\partial \alpha_0} & \frac{\partial \psi_0}{\partial \alpha_1} & \frac{\partial \psi_0}{\partial \alpha_2} & \frac{\partial \psi_0}{\partial \mu_1} & \frac{\partial \psi_0}{\partial \mu_0} \end{bmatrix}$$

Annotations: a_{11} (top row), a_{21} (bottom row), $\frac{\partial \psi_1}{\partial \alpha_0} = 0$, $\frac{\partial \psi_1}{\partial \alpha_1} = 0$, $\frac{\partial \psi_1}{\partial \alpha_2} = 0$, $\frac{\partial \psi_1}{\partial \mu_1} = 0$, $\frac{\partial \psi_1}{\partial \mu_0} = 0$, $P(A=1)$, 0 , 0 , $P(A=1)$

우리의 상황은 $P(A_T=1 | L_T) = \exp(\alpha_0 + \alpha_1 B + \alpha_2 C) / 1 + \exp(\alpha_0 + \alpha_1 B + \alpha_2 C)$ $\rightarrow h(L_T; \alpha)$ 이 define.

1) $\psi_{d0} = A_T - \{h(L_T; \alpha) / (1 + h(L_T; \alpha))\}$

① $\frac{\partial \psi_{d0}}{\partial \alpha_0} = - \frac{h(L_T; \alpha)}{(1 + h(L_T; \alpha))^2}$ ② $\frac{\partial \psi_{d0}}{\partial \alpha_1} = - \frac{h(L_T; \alpha) \times B_T}{(1 + h(L_T; \alpha))^2}$ ③ $\frac{\partial \psi_{d0}}{\partial \alpha_2} = - \frac{h(L_T; \alpha) \times C_T}{(1 + h(L_T; \alpha))^2}$

2) $\psi_{d1} = \{A_T - (h(L_T; \alpha) / (1 + h(L_T; \alpha)))\} B_T$

① $\frac{\partial \psi_{d1}}{\partial \alpha_0} = - \frac{h(L_T; \alpha) \times B_T}{(1 + h(L_T; \alpha))^2}$ ② $\frac{\partial \psi_{d1}}{\partial \alpha_1} = - \frac{h(L_T; \alpha) \times B_T^2}{(1 + h(L_T; \alpha))^2}$ ③ $\frac{\partial \psi_{d1}}{\partial \alpha_2} = - \frac{h(L_T; \alpha) \times B_T \times C_T}{(1 + h(L_T; \alpha))^2}$

3) $\psi_{d2} = \{A_T - (h(L_T; \alpha) / (1 + h(L_T; \alpha)))\} C_T$

① $\frac{\partial \psi_{d2}}{\partial \alpha_0} = - \frac{h(L_T; \alpha) \times C_T}{(1 + h(L_T; \alpha))^2}$ ② $\frac{\partial \psi_{d2}}{\partial \alpha_1} = - \frac{h(L_T; \alpha) \times B_T \times C_T}{(1 + h(L_T; \alpha))^2}$ ③ $\frac{\partial \psi_{d2}}{\partial \alpha_2} = - \frac{h(L_T; \alpha) \times C_T^2}{(1 + h(L_T; \alpha))^2}$

\Rightarrow 이때, $h(L_T; \alpha) / (1 + h(L_T; \alpha))^2 = \frac{h(L_T; \alpha)}{1 + h(L_T; \alpha)} \times \frac{1}{1 + h(L_T; \alpha)} = P(A_T=1 | L_T) \times (1 - P(A_T=1 | L_T))$

그러면, $a_{11} = \begin{bmatrix} - \frac{h(L_T; \alpha)}{(1 + h(L_T; \alpha))^2} (1, B_T, C_T)^T \\ - \frac{h(L_T; \alpha)}{(1 + h(L_T; \alpha))^2} (B_T, B_T^2, B_T C_T)^T \\ - \frac{h(L_T; \alpha)}{(1 + h(L_T; \alpha))^2} (C_T, B_T C_T, C_T^2)^T \end{bmatrix}$ $\xrightarrow{\text{code}} W = \text{diag}(- \frac{h(L_T; \alpha)}{(1 + h(L_T; \alpha))^2})$

$X = \text{Design matrix} \begin{pmatrix} 1 & L_1 \\ 1 & L_2 \\ \vdots & \vdots \\ 1 & L_n \end{pmatrix}$

$\Rightarrow \text{crossprod}(\text{sqrt}(W), X)$

4) $\psi_0 = (Y_T - \mu_0) (1 - A_T) h(L_T; \alpha)$

① $\frac{\partial \psi_0}{\partial \alpha_0} = (Y_T - \mu_0) (1 - A_T) h(L_T; \alpha)$ ② $\frac{\partial \psi_0}{\partial \alpha_1} = (Y_T - \mu_0) (1 - A_T) h(L_T; \alpha) \times B_T$

③ $\frac{\partial \psi_0}{\partial \alpha_2} = (Y_T - \mu_0) (1 - A_T) h(L_T; \alpha) \times C_T$

그러면, $a_{21} = (Y_T - \mu_0) (1 - A_T) h(L_T; \alpha) (1, B_T, C_T)^T$

$$B\left(\frac{2}{3}\right) = E \left[\begin{array}{ccc|cc} \psi_{\alpha_0}^2 & \psi_{\alpha_0}\psi_{\alpha_1} & \psi_{\alpha_0}\psi_{\alpha_2} & \psi_{\alpha_0}\psi_1 & \psi_{\alpha_0}\psi_0 \\ \psi_{\alpha_0}\psi_{\alpha_1} & \psi_{\alpha_1}^2 & \psi_{\alpha_1}\psi_{\alpha_2} & \psi_{\alpha_1}\psi_1 & \psi_{\alpha_1}\psi_0 \\ \psi_{\alpha_0}\psi_{\alpha_2} & \psi_{\alpha_1}\psi_{\alpha_2} & \psi_{\alpha_2}^2 & \psi_{\alpha_2}\psi_1 & \psi_{\alpha_2}\psi_0 \\ \hline \psi_1\psi_{\alpha_0} & \psi_1\psi_{\alpha_1} & \psi_1\psi_{\alpha_2} & \psi_1^2 & \psi_1\psi_0 \\ \hline \psi_0\psi_{\alpha_0} & \psi_0\psi_{\alpha_1} & \psi_0\psi_{\alpha_2} & \psi_0\psi_1 & \psi_0^2 \end{array} \right]$$