Debugging Sandwich Variance Estimator of DR ATE

--- 10월 14일 Version

〈What To Do〉

1) $E[\psi(\theta^*)\psi(\theta^*)^T]$ 계산 때 # of obs = 100000 로 하여 값 얻음.

⇒ 그리고 # of obs = 20000인 Data 1000 set 생성하여 $\bar{\psi}(\theta^*) = \frac{1}{N}\sum_{i=1}^{N}\psi_i(\theta^*)$ 의 Sample Variance

* N 얻은 다음, 둘의 Elementwise absolute 차이 계산

2) $J(\theta) = E[\frac{\partial}{\partial\theta}\psi(\theta)]$ 에 대하여 # of obs = 100000 으로 하여 $J(\hat{\theta})$, $J(\theta^*)$ 구함

⇒ $\left|\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\psi_i(\theta^*) - \left(-\sqrt{N}\cdot J(\hat{\theta})(\hat{\theta}-\theta^*)\right)\right|$, $\left|\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\psi_i(\theta^*) - \left(-\sqrt{N}\cdot J(\theta^*)\cdot(\hat{\theta}-\theta^*)\right)\right|$

# of obs 바꾸어가며 계산! / 값 차이가 0에 근사하는지 확인!

〈Result〉

1) # of obs = 20000 일 때

```
> abs(nu_true_mean-nu_theta)
             [,1]        [,2]       [,3]        [,4]        [,5]        [,6]        [,7]        [,8]        [,9]
[1,] 0.47687463 0.028313816 0.05616488 0.059948048 0.062043434 0.098521826 0.039318202 0.043460282 0.059645777
[2,] 0.02831382 0.041980061 0.02486491 0.004421565 0.023803051 0.038776739 0.005709347 0.006314372 0.017887111
[3,] 0.05616488 0.024864907 0.05959470 0.010462546 0.041721687 0.043129297 0.010397515 0.010552535 0.014047166
[4,] 0.05994805 0.004421565 0.01046255 0.007105912 0.009100360 0.011432163 0.004820058 0.005200631 0.006071791
[5,] 0.06204343 0.023803051 0.04172169 0.009100360 0.035750391 0.033430542 0.005751727 0.006222270 0.002040694
[6,] 0.09852183 0.038776739 0.04312930 0.011432163 0.033430542 0.007329277 0.009284695 0.003093051 0.014118616
[7,] 0.03931820 0.005709347 0.01039751 0.004820058 0.005751727 0.009284695 0.002300704 0.001883389 0.002314726
[8,] 0.04346028 0.006314372 0.01055254 0.005200631 0.006222270 0.003093051 0.001883389 0.002953975 0.002473348
[9,] 0.05964578 0.017887111 0.01404717 0.006071791 0.002040694 0.014118616 0.002314726 0.002473348 0.009339369
```

v_1 v_0 α β

⇒ "v_1" term을 다시 살펴볼 필요가 있어 보인다.

2)-① : # of obs = 1000 일 때

```
> abs(one-two)
              [,1]
[1,] 1.574870e-03
[2,] 1.551027e+01
[3,] 5.703173e-02
[4,] 9.834696e-03
[5,] 1.132673e-01
[6,] 1.432825e-03
[7,] 8.679565e-05
[8,] 7.835005e-04
[9,] 1.757062e-03
> abs(one-three)
              [,1]
[1,] 1.574870e-03
[2,] 1.552728e+01
[3,] 5.703173e-02
[4,] 9.834696e-03
[5,] 1.132673e-01
[6,] 1.432825e-03
[7,] 1.408398e-04
[8,] 7.709809e-04
[9,] 1.831650e-03
```

$\left| \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \psi_T(\theta^*) - \sqrt{N} \cdot J(\hat{\theta})(\hat{\theta}-\theta^*) \right|$ 계산

$\left| \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \psi_T(\theta^*) - \sqrt{N} \cdot J(\theta^*)(\hat{\theta}-\theta^*) \right|$

$\tilde{\beta}$

$\tilde{\beta}$

∴ "v_0" term 다시 볼 필요 있어보임

2)-② : # of obs = 10000 일 때

```
> abs(one-two)
              [,1]
[1,]  0.091926262
[2,] 49.051893061
[3,]  0.015637277
[4,]  0.004149759
[5,]  0.010563418
[6,]  0.036708680
[7,]  0.012055775
[8,]  0.009985468
[9,]  0.008084158
> abs(one-three)
              [,1]
[1,]  0.091926262
[2,] 49.018512652
[3,]  0.015637277
[4,]  0.004149759
[5,]  0.010563418
[6,]  0.036708680
[7,]  0.014127568
[8,]  0.011822266
[9,]  0.005553918
```

$\left| \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \psi_T(\theta^*) - \sqrt{N} \cdot J(\hat{\theta})(\hat{\theta}-\theta^*) \right|$ 계산

$\left| \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \psi_T(\theta^*) - \sqrt{N} \cdot J(\theta^*)(\hat{\theta}-\theta^*) \right|$

∴ # of obs 가 커질수록 "v_0" term 이 두드러지게 보임.