Our situation in IPTW ATT Robust variance

$$A(\xi) = -E\left[\begin{array}{ccc:cc} \partial\psi_{\alpha_0}/\partial\alpha_0 & \partial\psi_{\alpha_0}/\partial\alpha_1 & \partial\psi_{\alpha_0}/\partial\alpha_2 & \partial\psi_{\alpha_0}/\partial\mu_1 & \partial\psi_{\alpha_0}/\partial\mu_0 \\ \partial\psi_{\alpha_1}/\partial\alpha_0 & \partial\psi_{\alpha_1}/\partial\alpha_1 & \partial\psi_{\alpha_1}/\partial\alpha_2 & \partial\psi_{\alpha_1}/\partial\mu_1 & \partial\psi_{\alpha_1}/\partial\mu_0 \\ \partial\psi_{\alpha_2}/\partial\alpha_0 & \partial\psi_{\alpha_2}/\partial\alpha_1 & \partial\psi_{\alpha_2}/\partial\alpha_2 & \partial\psi_{\alpha_2}/\partial\mu_1 & \partial\psi_{\alpha_2}/\partial\mu_0 \\ \hdashline \partial\psi_1/\partial\alpha_0 = 0 & \partial\psi_1/\partial\alpha_1 = 0 & \partial\psi_1/\partial\alpha_2 = 0 & \partial\psi_1/\partial\mu_1 & \partial\psi_0/\partial\mu_0 \\ \partial\psi_0/\partial\alpha_0 & \partial\psi_0/\partial\alpha_1 & \partial\psi_0/\partial\alpha_2 & \partial\psi_0/\partial\mu_1 & \partial\psi_0/\partial\mu_0 \end{array}\right]$$

a_{11}

모두 0

a_{21}

$$\begin{bmatrix} P(A=1) & 0 \\ 0 & P(A=1) \end{bmatrix}$$

우리의 상황은 $P(A_T=1 \mid \underset{\sim}{L_T}) = \exp(\alpha_0+\alpha_1 B+\alpha_2 C) \,/\, 1+\underline{\exp(\alpha_0+\alpha_1 B+\alpha_2 C)}$ → $h(L_T;\alpha)$ 라 define.

1) $\psi_{\alpha_0} = A_T - \{h(\underset{\sim}{L_T};\alpha) / (1+h(\underset{\sim}{L_T};\alpha))\}$

① $\dfrac{\partial\psi_{\alpha_0}}{\partial\alpha_0} = -\dfrac{h(L_T;\alpha)}{(1+h(L_T;\alpha))^2}$ ② $\dfrac{\partial\psi_{\alpha_0}}{\partial\alpha_1} = -\dfrac{h(L_T;\alpha)\times B_T}{(1+h(L_T;\alpha))^2}$ ③ $\dfrac{\partial\psi_{\alpha_0}}{\partial\alpha_2} = -\dfrac{h(L_T;\alpha)\times C_T}{(1+h(L_T;\alpha))^2}$

2) $\psi_{\alpha_1} = \{A_T - (h(L_T;\alpha)/(1+h(L_T;\alpha))\} B_T$

① $\dfrac{\partial\psi_{\alpha_1}}{\partial\alpha_0} = -\dfrac{h(L_T;\alpha)\times B_T}{(1+h(L_T;\alpha))^2}$ ② $\dfrac{\partial\psi_{\alpha_1}}{\partial\alpha_1} = -\dfrac{h(L_T;\alpha)\times B_T^2}{(1+h(L_T;\alpha))^2}$ ③ $\dfrac{\partial\psi_{\alpha_1}}{\partial\alpha_2} = -\dfrac{h(L_T;\alpha)\times B_T\times C_T}{(1+h(L_T;\alpha))^2}$

3) $\psi_{\alpha_2} = \{A_T - (h(L_T;\alpha)/(1+h(L_T;\alpha))\} C_T$

① $\dfrac{\partial\psi_{\alpha_2}}{\partial\alpha_0} = -\dfrac{h(L_T;\alpha)\times C_T}{(1+h(L_T;\alpha))^2}$ ② $\dfrac{\partial\psi_{\alpha_2}}{\partial\alpha_1} = -\dfrac{h(L_T;\alpha)\times B_T\times C_T}{(1+h(L_T;\alpha))^2}$ ③ $\dfrac{\partial\psi_{\alpha_2}}{\partial\alpha_2} = -\dfrac{h(L_T;\alpha)\times C_T^2}{(1+h(L_T;\alpha))^2}$

⇒ 이때, $h(L_T;\alpha)/(1+h(L_T;\alpha))^2 = \dfrac{h(L_T;\alpha)}{1+h(L_T;\alpha)} \times \dfrac{1}{1+h(L_T;\alpha)} = P(A_T=1 \mid \underset{\sim}{L_T}) \times (1-P(A_T=1 \mid \underset{\sim}{L_T}))$

그러면, $a_{11} = \left[\begin{array}{l} -\dfrac{h(L_T;\alpha)}{(1+h(L_T;\alpha))^2}(1, B_T, C_T)^T \\ -\dfrac{h(L_T;\alpha)}{(1+h(L_T;\alpha))^2}(B_T, B_T^2, B_TC_T)^T \\ -\dfrac{h(L_T;\alpha)}{(1+h(L_T;\alpha))^2}(C_T, B_TC_T, C_T^2)^T \end{array}\right]$

Coding → $W = \text{diag}\left(\dfrac{h(L_i;\alpha)}{(1+h(L_T;\alpha))^2}\right) = \text{diag}(ps * (1-ps))$

X = Design matrix $\begin{pmatrix} 1 & \underset{\sim}{L_1} \\ 1 & \underset{\sim}{L_2} \\ & \vdots \\ 1 & \underset{\sim}{L_n} \end{pmatrix}$

⇒ - crossprod (sqrt(W), X) / nrow(data)

4) $\psi_0 = (Y_T-\mu_0)(1-A_T)\,h(L_T;\alpha)$

① $\dfrac{\partial\psi_0}{\partial\alpha_0} = (Y_T-\mu_0)(1-A_T)\,h(L_T;\alpha)$ ② $\dfrac{\partial\psi_0}{\partial\alpha_1} = (Y_T-\mu_0)(1-A_T)\,h(L_T;\alpha)\times B_T$

③ $\dfrac{\partial\psi_0}{\partial\alpha_2} = (Y_T-\mu_0)(1-A_T)\,h(L_T;\alpha)\times C_T$

$\hat{\mu}_0$ = result_lm_obj $ fitted_values [E==0]

그러면, $a_{21} = (Y_T-\mu_0)(1-A_T)\,\underline{h(L_T;\alpha)}\,(1, B_T, C_T)^T$

ps / (1-ps) 로 계산

$$B(\xi) = E\left[\begin{array}{ccc|cc} \psi_{\alpha 0}^2 & \psi_{\alpha 0}\psi_{\alpha 1} & \psi_{\alpha 0}\psi_{\alpha 2} & \psi_{\alpha 0}\psi_1 & \psi_{\alpha 0}\psi_0 \\ \psi_{\alpha 0}\psi_{\alpha 1} & \psi_{\alpha 1}^2 & \psi_{\alpha 1}\psi_{\alpha 2} & \psi_{\alpha 1}\psi_1 & \psi_{\alpha 1}\psi_0 \\ \psi_{\alpha 0}\psi_{\alpha 2} & \psi_{\alpha 1}\psi_{\alpha 2} & \psi_{\alpha 2}^2 & \psi_{\alpha 2}\psi_1 & \psi_{\alpha 2}\psi_0 \\ \hline \psi_1\psi_{\alpha 0} & \psi_1\psi_{\alpha 1} & \psi_1\psi_{\alpha 2} & \psi_1^2 & \psi_1\psi_0 \\ \psi_0\psi_{\alpha 0} & \psi_0\psi_{\alpha 1} & \psi_0\psi_{\alpha 2} & \psi_0\psi_1 & \psi_0^2 \end{array}\right]$$

$\psi_1 = \text{weight}(A_T)(Y_T - \mu_0)$

$\psi_0 = \text{weight}(1 - A_T)(Y_T - \mu_0)$

$\Rightarrow \begin{pmatrix} \psi_1 \\ \psi_0 \end{pmatrix} (\psi_1 \;\; \psi_0)$ 방법으로 계산!

b21_1 $\leftarrow \psi_1(\psi_{\alpha 0}, \psi_{\alpha 1}, \psi_{\alpha 2}) = A_T(Y_T - \mu_1) \,|\, \{A_T - e_T\}(1, B, C)$

b22_2 $\leftarrow \psi_0(\psi_{\alpha 0}, \psi_{\alpha 1}, \psi_{\alpha 2}) = (1 - A_T)(Y_T - \mu_0)\, h(L_T; \alpha) \,|\, \{A_i - e_T\}(1, B, C)$

⇒ 마지막에 nrow(data)로 나누어줌.