

Debugging Sandwich Variance of DR ATE Estimator

--- 10월 12일 Version

<What To Do>

1) $\bar{Z}(\theta^*) = \frac{1}{n} \sum_{i=1}^n \psi_i(\theta^*)$ 일 때, # of obs n 에 따라 $E[\psi(\theta^*)\psi(\theta^*)^T]$ 와 “Sample Variance of $\bar{Z}(\theta^*) \times \sqrt{n}$ ” 비교!

2) 1st order Taylor expansion 부분 확인해보자!

$\Rightarrow J(\theta) = E\left[\frac{\partial}{\partial \theta} \psi(\theta)\right]$ 라 하면, $\frac{1}{\sqrt{N}} \sum_{i=1}^N \psi_i(\theta^*) \approx \frac{-\sqrt{N} \cdot J(\hat{\theta})(\hat{\theta} - \theta^*)}{\sqrt{N} \cdot J(\theta^*)(\hat{\theta} - \theta^*)}$ 이어야 한다!

(1) (2) (3)

\Rightarrow # of obs 마다 1000개의 Dataset 존재, 각 situation 마다 ①-②의 절대치들 average,

①-③의 절대치들 average 파악!

<Result>

①-① : # of obs = 100 일 때

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> var(z_true)*100
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	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]
[1,]	8.33887638	0.591302289	-0.937141031	-0.929222490	-0.414607920	-0.1619213086	-0.0164914329	-0.0262196840	-0.035351585
[2,]	0.59130229	1.650808643	-0.971208099	-0.004641558	-0.491840292	-0.0441023215	0.0093341579	0.0055168477	0.006057821
[3,]	-0.93714103	-0.971208099	0.951805928	0.110412217	0.483151047	0.0308951721	0.0015041551	0.0013805226	0.001937345
[4,]	-0.92922249	-0.004641558	0.110412217	0.110725919	0.049936637	0.0170629557	0.0028853038	0.0034789741	0.004616574
[5,]	-0.41460792	-0.491840292	0.483151047	0.049936637	0.486882139	0.0067039233	0.0077898462	0.0022969174	0.003742166
[6,]	-0.16192131	-0.044102321	0.030895172	0.017062956	0.006703923	0.9285252906	0.0006772722	0.0003498928	0.010946178
[7,]	-0.01649143	0.009334158	0.001504155	0.002885304	0.007789846	0.0006772722	0.1114492324	0.0548021816	0.007313380
[8,]	-0.02621968	0.005516848	0.001380523	0.003478974	0.002296917	0.0003498928	0.0548021816	0.0540410433	0.002706036
[9,]	-0.03535159	0.006057821	0.001937345	0.004616574	0.003742166	0.0109461778	0.0073133800	0.0027060362	0.108961426

$\underbrace{\quad}_{v_1} \quad \underbrace{\quad}_{v_0} \quad \underbrace{\quad}_{\alpha} \quad \underbrace{\quad}_{\beta}$

var(z_true)*1000

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]
[1,]	9.028783106	0.3564293440	-0.780087035	-1.0071698154	-0.341673446	0.017630242	0.0159535630	-0.0032031343	-0.021623555
[2,]	0.356429344	1.6139301959	-0.922485235	0.0221958786	-0.453002011	0.033440067	0.0081972178	-0.0008742965	0.003675494
[3,]	-0.780087035	-0.9224852352	0.898260004	0.0923499519	0.446899365	-0.047086926	0.0080945478	0.0065788188	0.008343814
[4,]	-1.007169815	0.0221958786	0.092349952	0.1195177386	0.041969267	-0.003033916	0.0001607829	0.0009956277	0.003687121
[5,]	-0.341673446	-0.4530020107	0.446899365	0.0419692670	0.488088341	0.015796592	0.0133245655	0.0091555385	0.001051984
[6,]	0.017630242	0.0334400671	-0.047086926	-0.0030339156	-0.015796592	1.003629790	-0.0092363143	-0.0055169147	-0.014601923
[7,]	0.015953563	0.0081972178	0.008094548	0.0001607829	0.013324565	-0.009236314	0.0970541589	0.0507500975	-0.001214042
[8,]	-0.003203134	-0.0008742965	0.006578819	0.0009956277	0.009155539	-0.005516915	0.0507500975	0.0542963504	-0.002965799
[9,]	-0.021623555	0.0036754940	0.008343814	0.0036871214	0.001051984	-0.014601923	-0.0012140416	-0.0029657992	0.108020193

ν_1 ν_0 α β

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> var(z_true)*10000
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	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]
[1,]	8.59396504	0.261329708	-0.805710033	-0.975105461	-0.403736229	-0.027688976	0.016538257	0.023586123	-0.035650746
[2,]	0.26132971	1.696686799	-1.011542332	0.030210882	-0.501206390	-0.045246902	-0.004744274	0.002060799	-0.006464125
[3,]	-0.80571003	-1.011542332	0.980890673	0.095727069	0.491700172	0.022845932	-0.006448335	-0.007432882	0.012123403
[4,]	-0.97510546	0.030210882	0.095727069	0.117472302	0.048770536	0.002002866	-0.002985418	-0.003143512	0.004504490
[5,]	-0.40373623	-0.501206390	0.491700172	0.048770536	0.475496376	-0.007940996	-0.009429928	-0.005190544	0.013684523
[6,]	-0.02768898	-0.045246902	0.022845932	0.002002866	-0.007940996	0.996983955	-0.007358215	0.003328952	-0.005447920
[7,]	0.01653826	-0.004744274	-0.006448335	-0.002985418	-0.009429928	-0.007358215	0.104879566	0.053539646	-0.001193861
[8,]	0.02358612	0.002060799	-0.007432882	-0.003143512	-0.005190544	0.003328952	0.053539646	0.052406559	-0.000372036
[9,]	-0.03565075	-0.006464125	0.012123403	0.004504490	0.013684523	-0.005447920	-0.001193861	-0.000372036	0.108453097

v_1 v_0 α β

```

> apply(nu_true_array,c(1,2),mean)
      [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]      [,8]      [,9]
[1,]  8.94880682  0.5071253788 -1.0129350504 -1.0109361808 -0.5017310531 -0.0095659301 -0.0101459287 -0.0028478307  0.0204396985
[2,]  0.507125379  1.6728490053 -1.0086641911  0.0020839476 -0.5023677878  0.0021134451 -0.0001777562 -0.0005371309  0.0001653495
[3,] -1.012935050 -1.0086641911  1.0092700440  0.1212922620  0.5004906028 -0.004256434  0.0008701605 -0.0001079156 -0.0026823053
[4,] -1.010936181  0.0020839476  0.1212922620  0.1212922620  0.0601241950  0.001791926  0.0011330054  0.0002185715 -0.0024447795
[5,] -0.501731053 -0.5023677878  0.5004906028  0.0601241950  0.5004906028 -0.004570849 -0.0001079156 -0.0001079156 -0.0020929759
[6,] -0.009565930  0.0021134450 -0.0042564344  0.0017919262 -0.0045708490  1.013062213 -0.0026823053 -0.0020929759  0.0018949461
[7,] -0.010145929 -0.0001777562  0.0008701605  0.0011330054 -0.0001079156 -0.0026823053  0.1053538494  0.0527845302  0.0005089425
[8,] -0.002847831 -0.0005371309 -0.0001079156  0.0002185715 -0.0001079156 -0.002092976  0.0527845302  0.0527845302  0.0008813136
[9,]  0.020439699  0.0001653495 -0.0026823053 -0.0024447795 -0.0020929759  0.001894946  0.0005089425  0.0008813136  0.1053816955

```

2)-① : # of obs 가 100 일 때

```
> apply(theta_hat_check,1,mean)
[1] 1.030045e+00; 4.898956e+00; 1.675442e-15 9.764651e-16 1.259291e-15 2.444525e-15; 1.168889e-01 5.702202e-02 4.201691e-02
> apply(theta_true_check,1,mean)
[1] 1.030045e+00; 5.009849e+00; 1.675442e-15 9.764651e-16 1.259291e-15 2.444525e-15; 1.767138e-01 8.356966e-02 6.169233e-02
```

Handwritten notes and annotations:

- Red arrow from "100" to the first two columns of the output.
- Red arrow from the first two columns of the output to the formula: $|\frac{1}{\sqrt{N}} \sum_{i=1}^N \psi_T(\theta^*) - (-\sqrt{N} \cdot J(\hat{\theta}) (\hat{\theta} - \theta^*))|$ 계산 결과 정리
- Blue arrow from the first two columns of the output to the formula: $|\frac{1}{\sqrt{N}} \sum_{i=1}^N \psi_T(\theta^*) - (-\sqrt{N} \cdot J(\theta^*) (\hat{\theta} - \theta^*))|$ 계산 결과 정리
- Blue bracket under the first two columns of the output, labeled ν_1 .
- Blue bracket under the next four columns of the output, labeled ν_0 .
- Blue bracket under the last three columns of the output, labeled β .
- Blue bracket under the last three columns of the output, labeled α .

2)-② : # of obs 가 1000개일 때

$$\left| \frac{1}{\sqrt{N}} \sum_{i=1}^N \psi_T(\theta^*) - (-\sqrt{N} \cdot J(\hat{\theta})(\hat{\theta} - \theta^*)) \right| \text{ 계산 결과 정리}$$

$$\left| \frac{1}{\sqrt{N}} \sum_{i=1}^N \psi_T(\theta^*) - (-\sqrt{N} \cdot J(\theta^*)(\hat{\theta} - \theta^*)) \right| \text{ 계산 결과 정리}$$

```

> apply(theta_hat_check, 1, mean)
[1] 3.065000e-01 1.550925e+01 6.953691e-14 2.072836e-14 3.793023e-14 2.193470e-14 3.594901e-02 1.837131e-02 1.456253e-02
> apply(theta_true_check, 1, mean)
[1] 3.065000e-01 1.551032e+01 6.953691e-14 2.072836e-14 3.793023e-14 2.193470e-14 3.647697e-02 1.864433e-02 1.482544e-02

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ν_1 ν_0 α β

⇒ 두 결과값은 비슷하나, “Outcome regression model” 관련 parameter 제외, 나머지 parameter set 들은 차이가 0에 가깝지 않은 것을 확인하였다.