

<b>A. DISCRETE -TIE STANDARD PID SYSTEM</b>		
$u(k)$	$= K_p e(k) + K_i T \sum_{n=1}^k e(n) + \frac{K_d}{T} [e(k) - e(k-1)]$	(1-11)
	$= \left( (K_p \cdot 10^m \cdot 10^{-m}) e(k) + (K_i \cdot 10^m \cdot 10^{-m} \cdot T) \sum_{n=1}^k e(n) + \frac{K_d \cdot 10^m}{T} [e(k) - e(k-1)] \right)$	(1-11)
	$K_p, K_i, K_d$ 소수점을 대비하여, $10^m$ 을 함으로써 정수를 만든다. 여기서 $m$ 은 $K_p, K_i, K_d$ 에서 소수점 자리수가 가장큰 파라미터중에 자릿수 이다	
	$= \left( K'_p e(k) + K'_i T \sum_{n=1}^k e(n) + \frac{K'_d}{T} [e(k) - e(k-1)] \right) \cdot 10^{-m}$	(1-12)
	$K'_p = K_p \cdot 10^m, K'_i = K_i \cdot 10^m, K'_d = K_d \cdot 10^m$ 이다. 그리고 $T < 1$ 이하에서 정수연산을 하게 되다면, Integral 항은 연속하여 0 이 될것이다. 따라서 이것을 해결하기 위하여 다음과 같이 나타낸다.	
	$= \left( \frac{K'_p}{T} e(k) + K'_i \sum_{n=1}^k e(n) + \frac{K'_d}{T^2} [e(k) - e(k-1)] \right) \cdot T \cdot 10^{-m}$	(1-13)
	각항에 $\frac{1}{T}$ 를 연산 하한다.. $\frac{1}{T}$ 은 형태이다. 다음 식에는 $e(k) = r_{plant} \cdot \frac{\hat{r} - \hat{y}(k)}{A_r^{bit}}$ 를 대입한다.	
	$= \left( \frac{K'_p}{T} \left( r_{plant} \cdot \frac{\hat{r} - \hat{y}(k)}{A_r^{bit}} \right) + K'_i \sum_{n=1}^k \left( r_{plant} \cdot \frac{\hat{r} - \hat{y}(n)}{A_r^{bit}} \right) + \frac{K'_d}{T^2} \left( r_{plant} \cdot \frac{[(\hat{r} - \hat{y}(k)) - (\hat{r} - \hat{y}(k-1))]}{A_r^{bit}} \right) \right) \cdot T \cdot 10^{-m}$	
	$= \left( \frac{K'_p}{T} (\hat{r} - \hat{y}(k)) + K'_i \sum_{n=1}^k (\hat{r} - \hat{y}(n)) + \frac{K'_d}{T^2} [(\hat{r} - \hat{y}(k)) - (\hat{r} - \hat{y}(k-1))] \right) \cdot \frac{r_{plant}}{A_r^{bit}} \cdot T \cdot 10^{-m}$	
	$\hat{e} = \hat{r} - \hat{y}$ 이므로 다음과 같이 나타낸다.	
	$= \left( \frac{K'_p}{T} \hat{e}(k) + K'_i \sum_{n=1}^k \hat{e}(n) + \frac{K'_d}{T^2} [\hat{e}(k) - \hat{e}(k-1)] \right) \cdot \frac{r_{plant}}{A_r^{bit}} \cdot T \cdot 10^{-m}$	
	$C = \frac{r_{plant}}{A_r^{bit}} \cdot T \cdot 10^{-m}$	
$u(k)$	$= C \cdot \left( \frac{K'_p}{T} \hat{e}(k) + K'_i \sum_{n=1}^k \hat{e}(n) + \frac{K'_d}{T^2} [\hat{e}(k) - \hat{e}(k-1)] \right)$	
	$\therefore \text{Sampling Time} = \frac{1}{\text{Sampling frequency}}$ 이므로, $\frac{1}{T} = f$ 이므로 다음과 같이 나타낼수 있다.	
$u(k)$	$= C \cdot \left( K'_p f \hat{e}(k) + K'_i \sum_{n=1}^k \hat{e}(n) + K'_d f^2 [\hat{e}(k) - \hat{e}(k-1)] \right)$	

<b>B. DISCRETE -TIME DIFFERERECE PID SYSTEM</b>		
$u(k)$	$= u(n-1) + b_0 \cdot e(n) + b_1 \cdot e(n-1) + b_2 \cdot e(n-2)$	(1-11)
	Equation (1-11) 은 [ ] 의 논문을 인용하여 시작한다.	
	$b_0 = K_p + \frac{T \cdot K_i}{2} + \frac{K_d}{T}$	(1-11)
	$b_0 = \frac{T}{2} \left( \frac{2K_p}{T} + K_i + \frac{2K_d}{T^2} \right)$	(1-12)
	$b_1 = \frac{K_i T}{2} - K_p - \frac{2K_d}{T}$	(1-13)
	$b_1 = \frac{T}{2} \left( K_i - \frac{2K_p}{T} - \frac{4K_d}{T^2} \right)$	
	$b_2 = \frac{K_d}{T}$	
	$= u(n-1) + \left( \frac{T}{2} \left( \frac{2K_p}{T} + K_i + \frac{2K_d}{T^2} \right) \right) \cdot e(n) + \left( \frac{T}{2} \left( K_i - \frac{2K_p}{T} - \frac{4K_d}{T^2} \right) \right) \cdot e(n-1) + \left( \frac{K_d}{T} \right) \cdot e(n-2)$	
	$= u(n-1) + \frac{T}{2} \left( \left( \frac{2K_p}{T} + K_i + \frac{2K_d}{T^2} \right) \cdot e(n) + \left( K_i - \frac{2K_p}{T} - \frac{4K_d}{T^2} \right) e(n-1) + \left( \frac{2K_d}{T^2} \right) \cdot e(n-2) \right)$	
	$= u(n-1) + \left( \frac{T}{2} \right) \left( \frac{r_{plant}}{A_r^{bit}} \right) \left( \left( \frac{2K_p}{T} + K_i + \frac{2K_d}{T^2} \right) \cdot \hat{e}(n) + \left( K_i - \frac{2K_p}{T} - \frac{4K_d}{T^2} \right) \hat{e}(n-1) + \left( \frac{2K_d}{T^2} \right) \cdot \hat{e}(n-2) \right)$	
$u(k)$	$= u(n-1) + \left( \frac{T}{2} \right) \left( \frac{r_{plant}}{A_r^{bit}} \right) \left( \left( \frac{2K_p}{T} + K_i + \frac{2K_d}{T^2} \right) \cdot \hat{e}(n) + \left( K_i - \frac{2K_p}{T} - \frac{4K_d}{T^2} \right) \hat{e}(n-1) + \left( \frac{2K_d}{T^2} \right) \cdot \hat{e}(n-2) \right)$	
	$= u(n-1) + \left( \frac{T}{2} \right) \left( \frac{r_{plant}}{A_r^{bit}} \right) (10^{-m}) \left( \left( \frac{2K'_p}{T} + K'_i + \frac{2K'_d}{T^2} \right) \cdot \hat{e}(n) + \left( K_i - \frac{2K_p}{T} - \frac{4K_d}{T^2} \right) \hat{e}(n-1) + \left( \frac{2K'_d}{T^2} \right) \cdot \hat{e}(n-2) \right)$	
$u(k)$	$= u(n-1) + \left( \frac{T}{2} \right) \left( \frac{r_{plant}}{A_r^{bit}} \right) (10^{-m}) \left( \left( \frac{2K'_p}{T} + K'_i + \frac{2K'_d}{T^2} \right) \cdot \hat{e}(n) + \left( K_i - \frac{2K_p}{T} - \frac{4K_d}{T^2} \right) \hat{e}(n-1) + \left( \frac{2K'_d}{T^2} \right) \cdot \hat{e}(n-2) \right)$	
	$= u(n-1) + \left( \left( \frac{1}{2f \cdot A_r^{bit}} \right) (r_{plant} \cdot 10^{-m}) \left( (2K'_p \cdot f + K'_i + 2K'_d \cdot f^2) \cdot \widehat{e(n)} \right. \right.$ $\left. \left. - (2K'_p \cdot f - K'_i + 4K'_d \cdot f^2) \widehat{e(n-1)} + (2K'_d f^2) \cdot \widehat{e(n-2)} \right) \right)$	