Advanced Robotics

-Computer Problem Set 5-

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미래융합기술학과

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Content

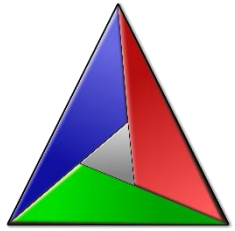
1. Introduction

① 프로젝트 목적

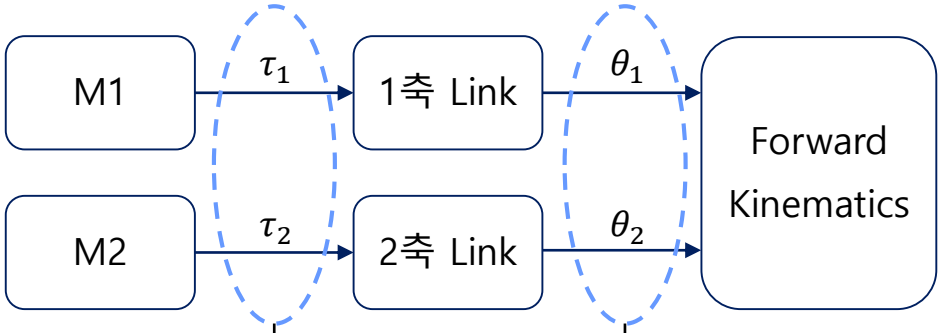
- Runge-Kutta법을 이해하고 2차 미분방정식을 1차 미분방정식의 쌍으로 간략화 하여 계산
- 토크를 사용하여 각도와 각속도를 계산 (Runge-Kutta법 이용)

② 사용 Tool 및 라이브러리

- Ubuntu
- GCC Compiler
- Python
- Matplotlib



2. Algorithm



$$\tau_i = \sum_{j=1}^{n} H_{ij}\alpha_j + \sum_{j=1}^{n} h_{ijk}\omega_j\omega_k + G_i$$

$$\tau^T = [-\sin(t)/6, -\cos(t)/30]$$

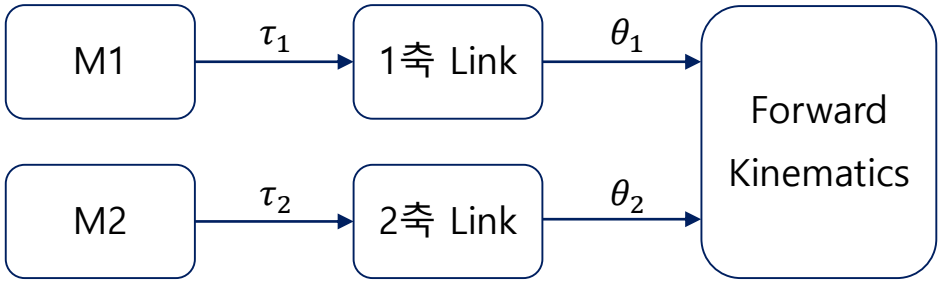
or

$$\tau^{\mathrm{T}} = [-10\sin(\mathrm{t})/3, -2\cos(\mathrm{t})/3]$$

$$\begin{bmatrix}\theta_1\\ \theta_2\\ \theta_3\\ \theta_4\end{bmatrix} = \begin{bmatrix}X_1\\ X_2\\ X_3\\ X_4\end{bmatrix}$$

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\\ \dot{X}_4\end{bmatrix} = \begin{bmatrix} X_2 \\ \frac{1}{H_{11}\cdot H_{22} - H_{21}\cdot H_{12}}[H_{22}(\tau_1 - h_{112}X_2X_4 - h_{122}X_4^2) - H_{12}(\tau_2 - h_{211}X_2^2)] \\ X_4 \\ \frac{1}{H_{11}\cdot H_{22} - H_{21}\cdot H_{12}}[-H_{21}(\tau_1 - h_{112}X_2X_4 - h_{122}X_4^2) + H_{11}(\tau_2 - h_{211}X_2^2)] \end{bmatrix} = \begin{bmatrix} X_2 \\ f_1(t, X_1, X_2, X_3, X_4) \\ X_3 \\ f_2(t, X_1, X_2, X_3, X_4) \end{bmatrix}$$

2. Algorithm



$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\\ \dot{X}_4\end{bmatrix} = \begin{bmatrix} X_2 \\ \frac{1}{H_{11}\cdot H_{22} - H_{21}\cdot H_{12}}[H_{22}(\tau_1 - h_{112}X_2X_4 - h_{122}X_4^2) - H_{12}(\tau_2 - h_{211}X_2^2)] \\ X_4 \\ \frac{1}{H_{11}\cdot H_{22} - H_{21}\cdot H_{12}}[-H_{21}(\tau_1 - h_{112}X_2X_4 - h_{122}X_4^2) + H_{11}(\tau_2 - h_{211}X_2^2)] \end{bmatrix} = \begin{bmatrix} X_2 \\ f_1(t, X_1, X_2, X_3, X_4) \\ X_4 \\ f_2(t, X_1, X_2, X_3, X_4) \end{bmatrix}$$

$$K_1 = hX_{2n}$$
$$L_1 = hf_1(t_n, X_{1n}, X_{2n}, X_{3n}, X_{4n})$$
$$B_1 = hX_{4n}$$
$$C_1 = hf_2(t_n, X_{1n}, X_{2n}, X_{3n}, X_{4n})$$
$$K_2 = h(X_{2n} + L_1)$$
$$L_2 = hf_1(t_{n+1}, X_{1n} + K_1, X_{2n} + L_1, X_{3n} + B_1, X_{4n} + C_1)$$
$$B_2 = h(X_{4n} + C_1)$$
$$C_2 = hf_2(t_{n+1}, X_{1n} + K_1, X_{2n} + L_1, X_{3n} + B_1, X_{4n} + C_1)$$

$$\omega(t) = \dot{\theta}(t)$$

$$\theta_{1_{n+1}} = \theta_{1_n} + \frac{1}{2}(K_1 + K_2)$$
$$\omega_{1_{n+1}} = \omega_{1_n} + \frac{1}{2}(L_1 + L_2)$$
$$\theta_{2_{n+1}} = \theta_{2_n} + \frac{1}{2}(B_1 + B_2)$$
$$\omega_{2_{n+1}} = \omega_{2_n} + \frac{1}{2}(C_1 + C_2)$$

3. Source Analysis

```
H11 = cos(theta2) + (5 / 3)
H12 = H21 = (cos(theta2) / 2) + (1 / 3)
H22 = 1 / 3

h112 = -sin(theta2)
h122 = -sin(theta2) / 2
h211 = sin(theta2) / 2

tau1 = -sin(time) / 6
tau2 = -cos(time) / 30
```

h = 0.05

h = 0.01

```
tau1 = (-10 * sin(time)) / 3
tau2 = (-2 * cos(time)) / 3
```

```
X1 = theta1
X2 = omega1
X3 = theta2
X4 = omega2
```

$$\begin{bmatrix}\theta_1\\ \theta_2\\ \theta_3\\ \theta_4\end{bmatrix} = \begin{bmatrix}X_1\\ X_2\\ X_3\\ X_4\end{bmatrix}$$

```
X1_dot = X2
z = (H11 * H22) - (H21 * H12)
a = tau1 - (h112 * X2 * X4) - (h122 * X4**2)
b = tau2 - (h211 * X2**2)
X2_dot = ((H22 * a) - (H12 * b)) / z
X3_dot = X4
X4_dot = ((H11 * b) - (H21 * a)) / z
```

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\\ \dot{X}_4\end{bmatrix} = \begin{bmatrix}X_2\\ \frac{1}{H_{11}\cdot H_{22} - H_{21}\cdot H_{12}}\left[H_{22}(\tau_1 - h_{112}X_2X_4 - h_{122}X_4^2) - H_{12}(\tau_2 - h_{211}X_2^2)\right]\\ X_4\\ \frac{1}{H_{11}\cdot H_{22} - H_{21}\cdot H_{12}}\left[-H_{21}(\tau_1 - h_{112}X_2X_4 - h_{122}X_4^2) + H_{11}(\tau_2 - h_{211}X_2^2)\right]\end{bmatrix}$$

3. Source Analysis

```
K1 = h * X1_dot
L1 = h * X2_dot
B1 = h * X3_dot
C1 = h * X4_dot

a = tau1 - (h112 * (X2 + L1) * (X4 + C1)) - (h122 * pow((X4 + C1),2))
b = tau2 - (h211 * pow((X2 + L1),2))
X2_dot_2 = ((H22 * a) - (H12 * b)) / z
X4_dot_2 = ((H11 * b) - (H21 * a)) / z

K2 = h * (X1_dot + L1)
L2 = h * X2_dot_2
B2 = h * (X3_dot + C1)
C2 = h * X4_dot_2
```

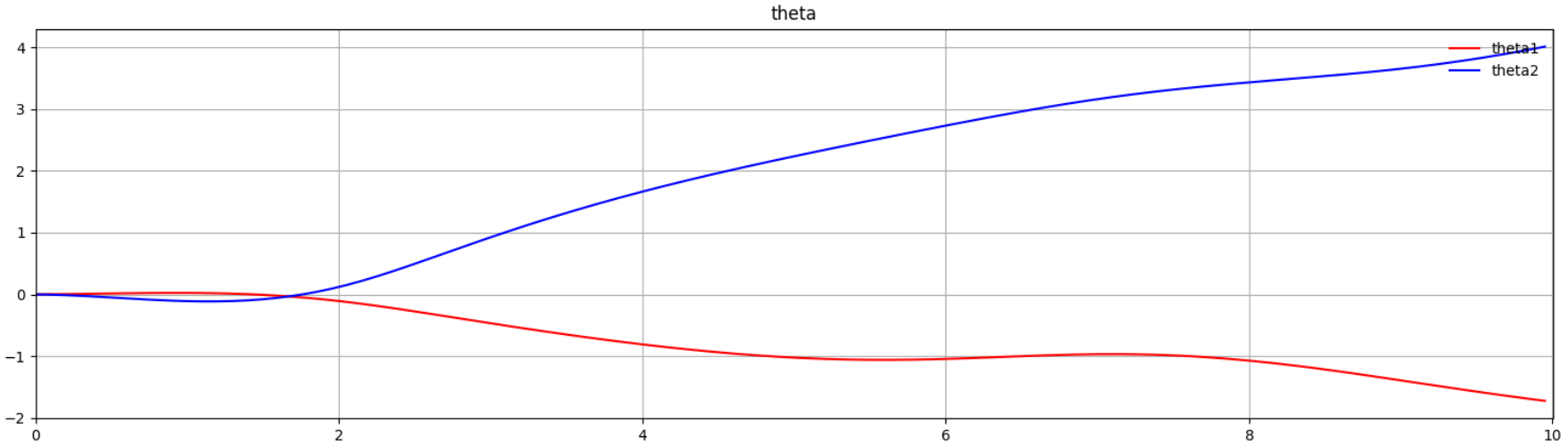
$$K_1 = hX_{2n}$$
$$L_1 = hf_1(t_n, X_{1n}, X_{2n}, X_{3n}, X_{4n})$$
$$B_1 = hX_{4n}$$
$$C_1 = hf_2(t_n, X_{1n}, X_{2n}, X_{3n}, X_{4n})$$
$$K_2 = h(X_{2n} + L_1)$$
$$L_2 = hf_1(t_{n+1}, X_{1n} + K_1, X_{2n} + L_1, X_{3n} + B_1, X_{4n} + C_1)$$
$$B_2 = h(X_{4n} + C_1)$$
$$C_2 = hf_2(t_{n+1}, X_{1n} + K_1, X_{2n} + L_1, X_{3n} + B_1, X_{4n} + C_1)$$

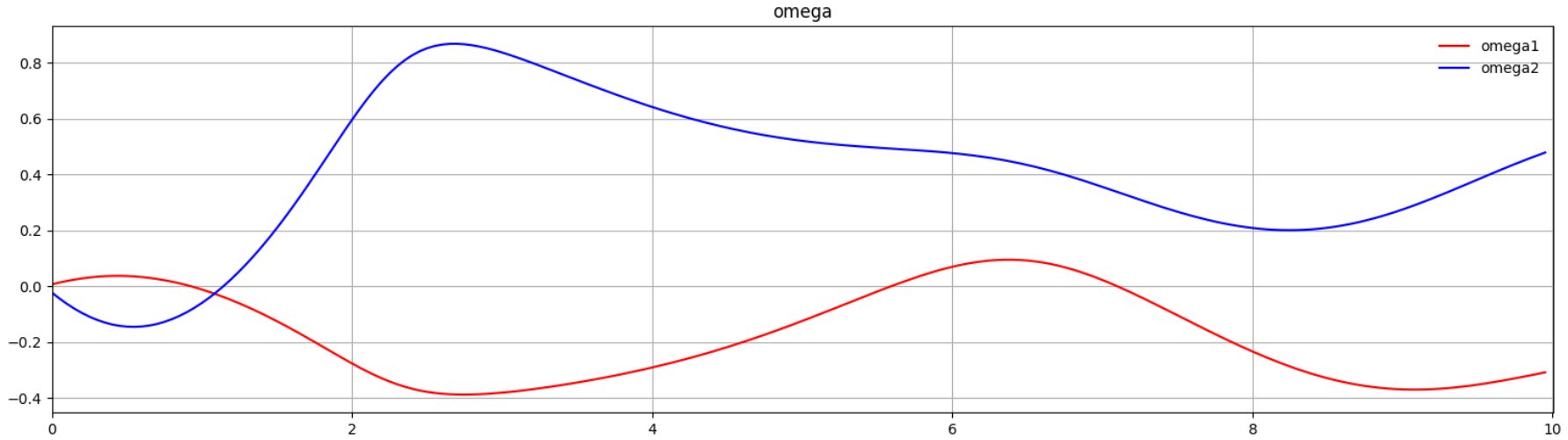
```
theta1 += (K1 + K2) / 2
omega1 += (L1 + L2) / 2

theta2 += (B1 + B2) / 2
omega2 += (C1 + C2) / 2
```

$$\theta_{1_{n+1}} = \theta_{1_n} + \frac{1}{2}(K_1 + K_2)$$
$$\omega_{1_{n+1}} = \omega_{1_n} + \frac{1}{2}(L_1 + L_2)$$
$$\theta_{2_{n+1}} = \theta_{2_n} + \frac{1}{2}(B_1 + B_2)$$
$$\omega_{2_{n+1}} = \omega_{2_n} + \frac{1}{2}(C_1 + C_2)$$

4. Result

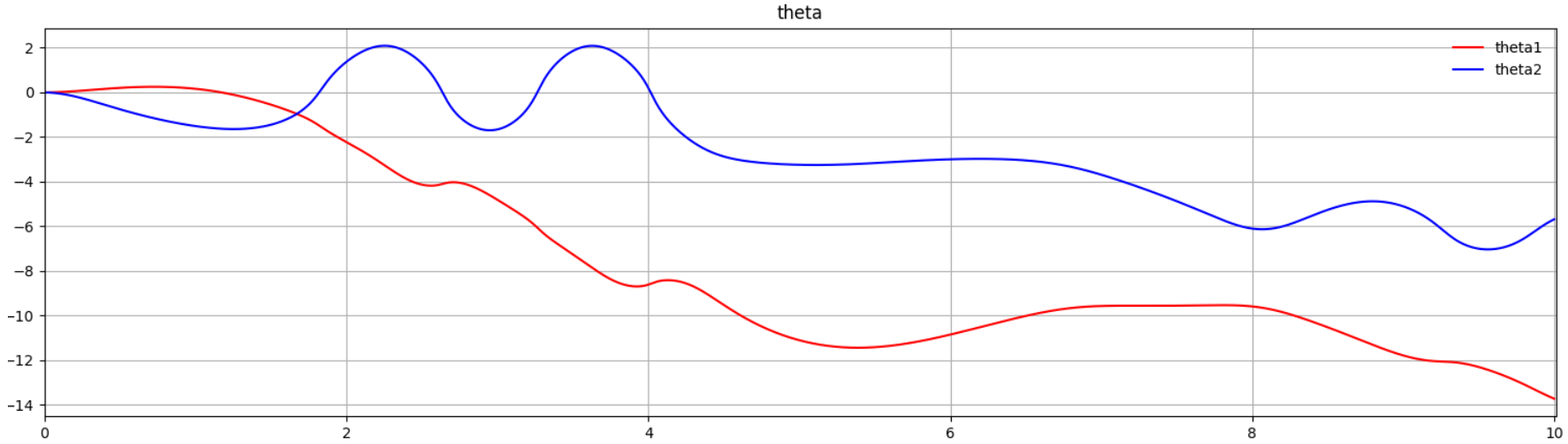
① Problem 1 (h=0.05, $\tau^T = [-sin(t)/6, -cos(t)/30]$)

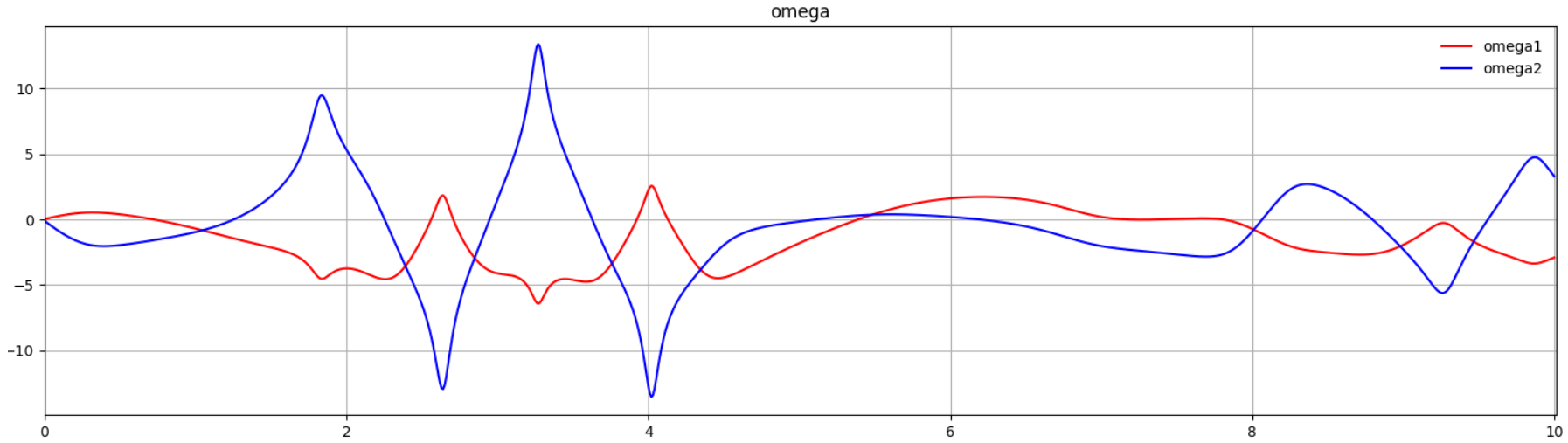




4. Result

② Problem 2 (h=0.01, $\tau^T = [-10sin(t)/3, -2cos(t)/3]$)





Q & A

감사합니다.