SAT SOLVER ON NONOGRAM

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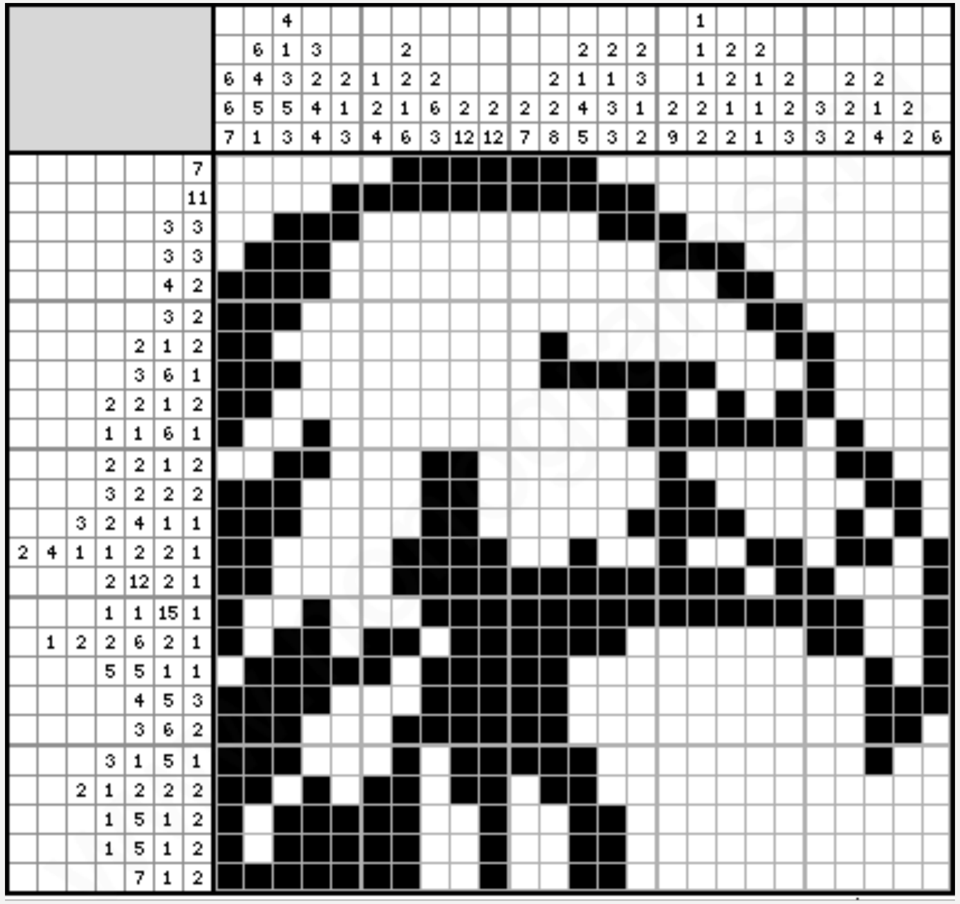
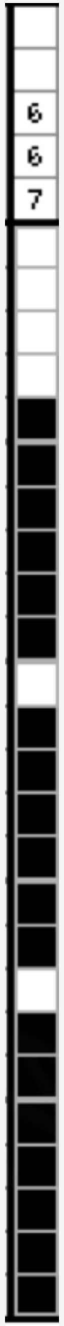
OUTLINE

- Introduction
- 2 Encoding Method
- Experiment

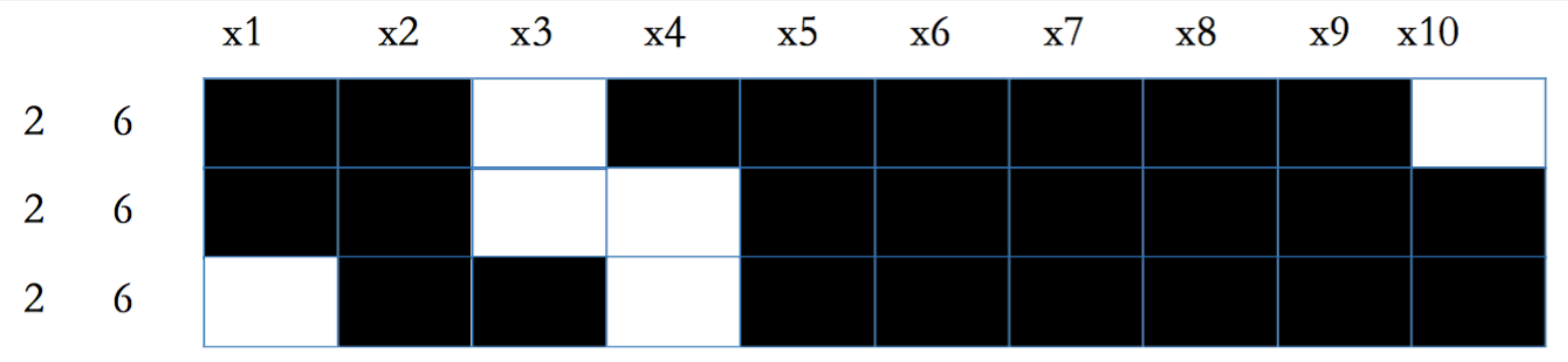
WHAT IS NONOGRAM?

NP-complete problem





ENCODING METHOD 1



$(x1 \land \mathbf{x2} \land \sim x3 \land x4 \land x5 \land x6 \land x7 \land x8 \land x9 \land \sim x10\ ) \lor$

$(x1 \land \mathbf{x2} \land \sim x3 \land \sim x4 \land x5 \land x6 \land x7 \land x8 \land x9 \land x10\ ) \lor$

$(\sim x1 \land \mathbf{x2} \land x3 \land \sim x4 \land x5 \land x6 \land x7 \land x8 \land x9 \land x10\ ) \lor$

ENCODING METHOD 1

- DNF to CNF

DNF

$(x1 \wedge x2 \wedge \sim x3 \wedge x4 \wedge x5 \wedge x6 \wedge x7 \wedge x8 \wedge x9 \wedge \sim x10) \vee$

$(x1 \wedge x2 \wedge \sim x3 \wedge \sim x4 \wedge x5 \wedge x6 \wedge x7 \wedge x8 \wedge x9 \wedge x10) \vee$

$(\sim x1 \wedge x2 \wedge x3 \wedge \sim x4 \wedge x5 \wedge x6 \wedge x7 \wedge x8 \wedge x9 \wedge x10) \vee$

CNF

$(y1 \vee y2 \vee y3) \wedge$
$(\sim y1 \vee x1) \wedge (\sim y1 \vee x2) \wedge (\sim y1 \vee \sim x3) \wedge (\sim y1 \vee x4) \wedge (\sim y1 \vee x5) \wedge$
$(\sim y1 \vee x6) \wedge (\sim y1 \vee x7) \wedge (\sim y1 \vee x8) \wedge (\sim y1 \vee x9) \wedge (\sim y1 \vee \sim x10) \wedge$
$(\sim y2 \vee x1) \wedge (\sim y2 \vee x2) \wedge (\sim y2 \vee \sim x3) \wedge (\sim y2 \vee \sim x4) \wedge (\sim y2 \vee x5) \wedge$
$(\sim y2 \vee x6) \wedge (\sim y2 \vee x7) \wedge (\sim y2 \vee x8) \wedge (\sim y2 \vee x9) \wedge (\sim y2 \vee x10) \wedge$
$(\sim y3 \vee \sim x1) \wedge (\sim y3 \vee x2) \wedge (\sim y3 \vee x3) \wedge (\sim y3 \vee \sim x4) \wedge (\sim y3 \vee x5) \wedge$
$(\sim y3 \vee x6) \wedge (\sim y3 \vee x7) \wedge (\sim y3 \vee x8) \wedge (\sim y3 \vee x9) \wedge (\sim y3 \vee x10)$

ENCODING METHOD 2

- Starting position as variable (e.g. U1=1, U2=5, U3=7)

$U1 = < a_1, a_2, a_3, \cdots, a_{10} >$
$U2 = < b_1, b_2, b_3, \cdots, b_{10} >$
$U3 = < c_1, c_2, c_3, \cdots, c_{10} >$

If a_i is true, it means $U1>=I$ is True，U2, U3 are the same，e.g. $U1=<1,1,1,0,0,0,0,0,0,0>$ represent $U1>=1$, $U1>=2$, $U1>=3$, $U1<4$, $U1<5\cdots$，then we know U1=3

ENCODING METHOD 2

- **Nonogram's features :**

(1) Every starting position is between 1~LENGTH

(2) In every row and column the second starting position is greater than the first one, and the third one is greater than the second one and so on

(3) Every block fall between the starting position and segment length should be black

(4) Every block fall between the first segment length and the next starting position should left empty

ENCODING METHOD 2

- $unary(< x_1, \dots, x_n >, [a, b]) \; = \wedge_1^n (x_{i-1} \leftarrow x_i) \wedge x_a \wedge \neg x_{b+1}$
  - E.g. U1 = <x_1, x_2, x_3, x_4, x_5> fall between 2~5
  - $(\neg x_1 \vee x_0) \wedge (\neg x_2 \vee x_1) \wedge (\neg x_3 \vee x_2) \wedge (\neg x_4 \vee x_3) \wedge (\neg x_5 \vee x_1) \wedge x_2 \wedge \neg x_6$
  - (U1<1 or U1>=0) and (U1<2 or U1>=1) and (U1<3 or U1>=2) and … and U1>=2 and U1<6

- $block(U_1, U_2, < x_1, \dots, x_n >) = \wedge_{i=1}^n \big(\neg U_1(i) \wedge U_2(i)\big) \rightarrow x_i$
  - Mean if U1< x_i <= U2 then x_i=1
  - E.g. let U_1= <1,1,1,0,0,0,0>

    U_2= <1,1,1,1,1,0,0>

    X = <0,0,0,1,1,0,0>
- $leq(< x_1, \dots, x_n >, < y_1, \dots, y_n >) = \wedge_{i=1}^n x_i \rightarrow y_i$
  - Mean X<=Y
  - E.g. Let X= <1,1,1,0,0,0,0>

    Y= <1,1,1,?,?,?,?>

ENCODING METHOD 2 - EXAMPLE

- **Constraint number of a row is :** <3, 1, 2>
- **Let** X = <x1, x2, x3,…, x10>、$\overline{X}$ = <~x1,~x2,~x3,…, ~x10> 、
- **Let** U1, U2, U3 as starting position
- **4 features encoding :**

(1) **U1,U2,U3 is between 1~10 :** unary(U1, [1,10]), unary(U2, [1, 10]), unary(U3, [1, 10])

(2) **U1+Length <= U2 :** leq($U1^{+3}$, U2), leq($U2^{+1}$,U3), leq($U3^{+1}$,10)

(3) **X=1 (black) range :** block($U1^{-1}$, $U1^{+2}$, X), block($U2^{-1}$, $U2^{+0}$, X), block($U3^{-1}$, $U3^{+1}$, X)

(4) **X=0 (white) range :** block(0,U1,$\overline{X}$), block($U1^{+3}$,U2,$\overline{X}$), block($U2^{+1}$,U3,$\overline{X}$), block($U3^{+2}$,10,$\overline{X}$)

$U1^{+5}$ represent U1+5 = <1,1,1,1,1,1,1,1,0,0> = 3+5 = 8

$U1^{-2}$ represent U1-2 = <1,0,0,0,0,0,0,0,0,0> = 3-2 =1

EXPERIMENT

Method / Test case	Method1 run time (second)	Method2 run time (second)
5*5 puzzle1	0.002	0.014
10*10 puzzle1	0.104	0.002
25*25 puzzle1	∞	3.802
25*25 puzzle2	∞	4.790
25*25 puzzle3	∞	3.633
25*25 puzzle4	∞	3.833
25*25 puzzle5	∞	26.982

EXPERIMENT

Method / Test case	Method1 Variable/Clause	Method2 Variable/Clause
5*5 puzzle1	60/180	116/311
10*10 puzzle1	377/2780	472/1349
25*25 puzzle1	180230/4490150	9508/34234
25*25 puzzle2	245342/6117950	9562/34434
25*25 puzzle3	206815/5154775	9562/34433
25*25 puzzle4	261970/6533650	9373/33686
25*25 puzzle5	209123/5212475	9535/34342

REFERENCE

- [1] Ueda, Nobuhisa; Nagao, Tadaaki (1996), NP-completeness results for NONOGRAM via Parsimonious Reductions, TR96-0008, Technical Report, Department of Computer Science, Tokyo Institute of Technology, CiteSeerX 10.1.1.57.5277

- [2] Amit Metodi, Michael Codish, Vitaly Lagoon, Peter J. Stuckey (2011), Boolean Equi-propagation for Optimized SAT Encoding

- [3] Naoyuki Tamura, Tomoya Tanjo, and Mutsunori Banbara, Solving Constraint Satisfaction Problems with SAT Technology

THANK YOU FOR LISTERNING ☺