투빅스 18기 정규세션 3주차

Neural Network Basic Assignment

이름: 김은지

1. Sigmoid Function을 z에 대해 미분하세요.

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

Let $y = \sigma(z)$, then

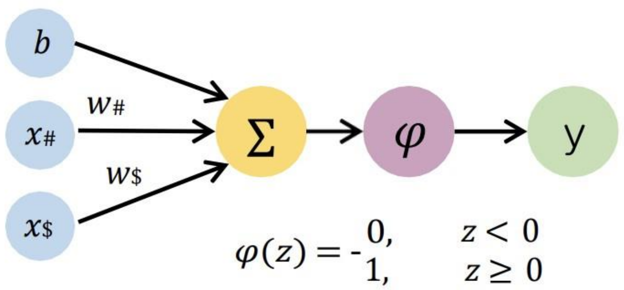
$$\frac{dy}{dz} = \frac{d}{dz}\sigma(z) = \frac{d}{dz}(1+e^{-z})^{-1} = -(1+e^{-z})^{-2}(-e^{-z})$$

$$= \frac{e^{-z}}{(1+e^{-z})^2} = \frac{1+e^{-z}-1}{(1+e^{-z})^2} = \frac{1}{1+e^{-z}} - \frac{1}{(1+e^{-z})^2}$$

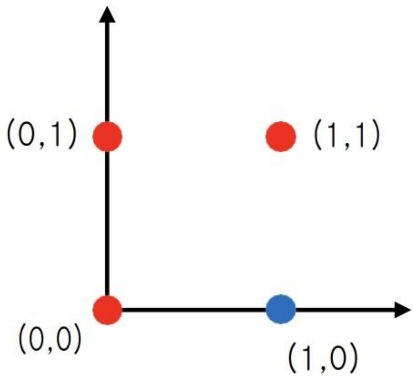
$$= \frac{1}{1+e^{-z}}\left(1 - \frac{1}{1+e^{-z}}\right)$$

$$= y(1-y)$$

2. 다음과 같은 구조의 Perceptron과 ●(=1), ●(=0)을 평면좌표상에 나타낸 그림이 있습니다.



$$\varphi(z) = \begin{cases} 0, & z < 0 \\ 1, & z \geq 0 \end{cases}$$



식: $z = x_\# W_\# + x_\$ W_\$ + b$

2-1. ●, ●을 분류하는 임의의 b, w를 선정하고 분류해보세요.

$W_\# = -0.4$, $W_\$ = 0.2$, $b = 0.5$ 라 하자.

$\varphi(z)$

i) (0,0)일때, $z = 0\cdot(-0.4) + 0\cdot(0.2) + 0.5 = 0.5 \geq 0 \Rightarrow 1$

ii) (0,1)일때, $z = 0\cdot(-0.4) + 1\cdot(0.2) + 0.5 = 0.7 \geq 0 \Rightarrow 1$

iii) (1,1)일때, $z = 1\cdot(-0.4) + 1\cdot(0.2) + 0.5 = 0.3 \geq 0 \Rightarrow 1$

iv) (1,0)일때, $z = 1\cdot(-0.4) + 0\cdot(0.2) + 0.5 = 0.1 \geq 0 \Rightarrow 1$

2-2. Perceptron 학습 규칙에 따라 임의의 학습률을 정하고 b, w를 1회 업데이트 해주세요.

Learning rate = 0.05 라 하자.

$b -$ Learning rate $\cdot (\Sigma(y - \varphi(z)))\cdot 1$

$b = 0.5 + 0.05\cdot(-1-1)\cdot 1 = 0.5 - 0.1 = 0.4$

$W -$ learning rate $(\Sigma(y - \varphi(z)))\cdot 1$

$W_\# = -0.4 + 0.05(-1-1)\cdot 1 = -0.4 - 0.1 = -0.5$

$W_\$ = 0.2 + 0.05(-1-1)\cdot 1 = 0.2 - 0.1 = 0.1$

$\varphi(z)$

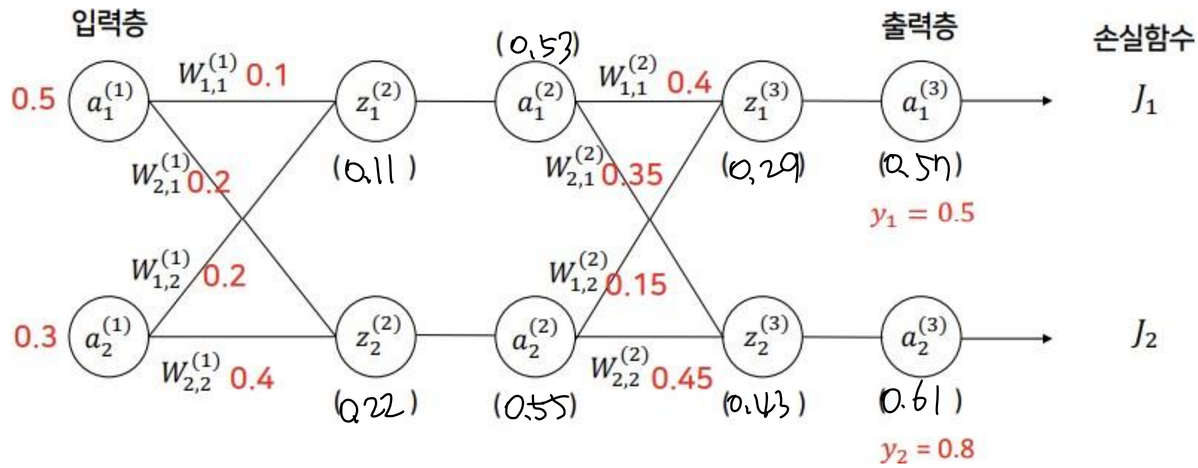
i) (0,0)일때, $z = 0\cdot(-0.5) + 0\cdot(0.1) + 0.4 = 0.4 \geq 0 \Rightarrow 1$

ii) (0,1)일때, $z = 0\cdot(-0.5) + 1\cdot(0.1) + 0.4 = 1.4 \geq 0 \Rightarrow 1$

iii) (1,1)일때, $z = 1\cdot(-0.5) + 1\cdot(0.1) + 0.4 = 0 \geq 0 \Rightarrow 1$

iv) (1,0)일때, $z = 1\cdot(-0.5) + 0\cdot(0.1) + 0.4 = -0.1 < 0 \Rightarrow 0$

3. 다음과 같이 입력과 가중치가 주어진 퍼셉트론이 있을 때, 아래의 물음에 답해주세요. 모든 문제는 풀이과정을 자세하게 적어주세요! (3-3까지 있습니다.)



3-1. FeedForward가 일어날 때, 각 노드가 갖는 값을 빈칸에 써주세요. 단, 활성화함수는 sigmoid 함수입니다. (모든 계산의 결과는 소수점 셋째자리에서 반올림하여 둘째자리까지만 써주세요.)

$z_1^{(2)} = a_1^{(1)} \times W_{1,1}^{(1)} + a_2^{(1)} \times W_{1,2}^{(1)}$
$= 0.5 \times 0.1 + 0.3 \times 0.2$
$= 0.11$

$z_2^{(2)} = a_1^{(1)} \times W_{2,1}^{(1)} + a_2^{(1)} \times W_{2,2}^{(1)}$
$= 0.5 \times 0.2 + 0.3 \times 0.4$
$= 0.22$

$z_1^{(3)} = a_1^{(2)} \times W_{1,1}^{(2)} + a_2^{(2)} \times W_{1,2}^{(2)}$
$= 0.53 \times 0.4 + 0.55 \times 0.15$
$\simeq 0.29$

$z_2^{(3)} = a_1^{(2)} \times W_{21}^{(2)} + a_2^{(2)} \times W_{2,2}^{(2)}$
$= 0.53 \times 0.35 + 0.55 \times 0.45$
$\simeq 0.43$

$a_1^{(2)} = \frac{1}{1+e^{-0.11}} \simeq 0.53$

$a_2^{(2)} = \frac{1}{1+e^{-0.22}} \simeq 0.55$

$a_1^{(3)} = \frac{1}{1+e^{-0.29}} \simeq 0.57$

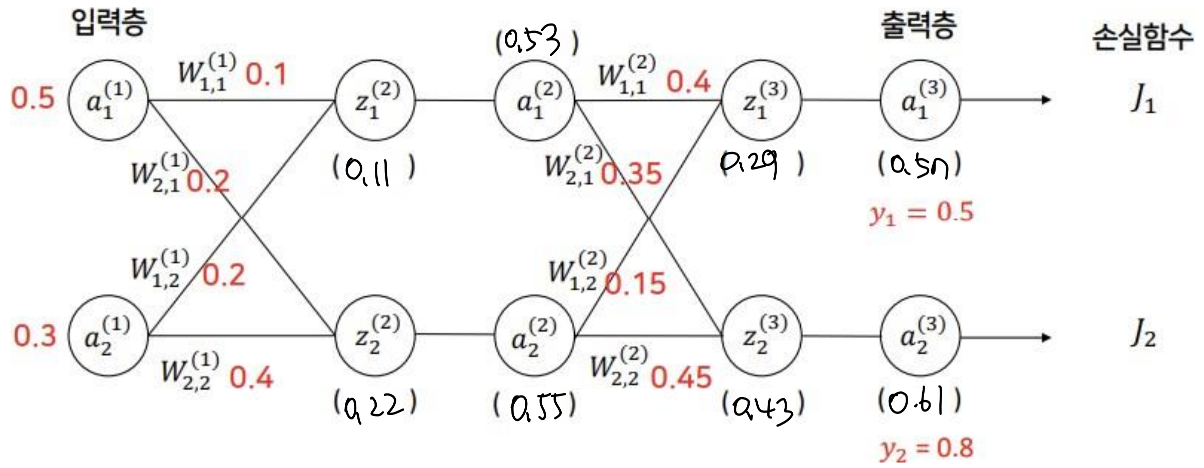
$a_2^{(3)} = \frac{1}{1+e^{-0.43}} \simeq 0.61$

3-2. 3-1에서 구한 값을 이용하여 손실함수 J_1과 J_2의 값을 구해주세요. (J_1과 J_2는 반올림하지 말고 써주세요.)

손실함수를 MSE로 지정.

$J_1 = \frac{1}{2}(a_1^{(3)} - y_1)^2 = \frac{1}{2}(0.57 - 0.5)^2 = 0.00245$

$J_2 = \frac{1}{2}(a_2^{(3)} - y_2)^2 = \frac{1}{2}(0.61 - 0.8)^2 = 0.01805$



3-3. 위에서 구한 값을 토대로, BackPropagation이 일어날 때 $W_{2,2}^{(2)}$과 $W_{2,1}^{(1)}$의 조정된 값을 구해주세요. 단, learning rate는 0.1입니다. (계산 과정에서 소수점 넷째자리에서 반올림하여 셋째자리까지만 써주시고, 마지막 결과인 $W_{2,1}^{(1)}$과 $W_{2,2}^{(2)}$의 값만 반올림하지 말고 써주세요.)

i) $W_{2,2}^{(2)}$

$$\frac{\partial J_1}{\partial W_{2,1}^{(2)}} + \frac{\partial J_2}{\partial W_{2,1}^{(2)}} \xrightarrow{\text{Chain rule}} \left(\frac{\partial J_1}{\partial a_1^{(3)}} \times \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} \times \frac{\partial z_1^{(3)}}{\partial a_2^{(2)}} \times \frac{\partial a_2^{(2)}}{\partial z_2^{(2)}} \times \frac{\partial z_2^{(2)}}{\partial W_{2,1}^{(1)}}\right)$$

$$+ \left(\frac{\partial J_2}{\partial a_2^{(3)}} \times \frac{\partial a_2^{(3)}}{\partial z_2^{(3)}} \times \frac{\partial z_2^{(3)}}{\partial a_2^{(2)}} \times \frac{\partial a_2^{(2)}}{\partial z_2^{(2)}} \times \frac{\partial z_2^{(2)}}{\partial W_{2,1}^{(1)}}\right)$$

$$= \left\{\left(\frac{1}{2} \cdot \frac{\partial}{\partial a_1^{(3)}}(a_1^{(3)}-y_1)^2\right) \times \left(\frac{\partial}{\partial z_1^{(3)}}\sigma(z_1^{(3)})\right) \times (W_{1,2}^{(2)}) \times \left(\frac{\partial}{\partial z_2^{(2)}}\sigma(z_2^{(2)})\right) \times (a_1^{(1)})\right\}$$

$$+ \left\{\left(\frac{1}{2} \cdot \frac{\partial}{\partial a_2^{(3)}}(a_2^{(3)}-y_2)^2\right) \times \left(\frac{\partial}{\partial z_2^{(3)}}\sigma(z_2^{(3)})\right) \times (W_{2,2}^{(2)}) \times \left(\frac{\partial}{\partial z_2^{(2)}}\sigma(z_2^{(2)})\right) \times (a_1^{(1)})\right\}$$

$$= \{(0.07)\times(0.206)\times(0.15)\times(0.172)\times(0.5)\}$$

$$+ \{(-0.19)\times(0.245)\times(0.45)\times(0.172)\times(0.5)\}$$

$$= -0.002$$

$$\therefore W_{2,1}^{\prime(1)} = W_{2,1}^{(1)} - 0.1 \cdot \left(\frac{\partial J_1}{\partial W_{2,1}^{(2)}} + \frac{\partial J_2}{\partial W_{2,1}^{(2)}}\right)$$

$$= 0.2 - 0.1(-0.002) = 0.2002$$

ii) $W_{2,2}^{(2)}$

$$\frac{\partial J_1}{\partial W_{2,2}^{(2)}} \xrightarrow{\text{Chain rule}} \frac{\partial J_1}{\partial a_2^{(3)}} \times \frac{\partial a_2^{(3)}}{\partial z_2^{(3)}} \times \frac{\partial z_2^{(3)}}{W_{2,2}^{(2)}}$$

$$= (a_2^{(3)} - y_2) \times \sigma(z_2^{(3)})(1-\sigma(z_1^{(3)})) \times a_2^{(2)}$$

$$= (0.61-0.8) \times (0.43 \times 0.57) \times 0.55$$

$$= -0.026$$

$$\therefore W_{2,2}^{(2)} = W_{2,2}^{(2)} - 0.1\left(\frac{\partial J_1}{\partial W_{2,2}^{(2)}}\right)$$

$$= 0.45 - 0.1(-0.026)$$

$$= 0.4526$$