

# Summarizing and Displaying Continuous Data 1

EDUC 641: Unit 3 Part 1

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# Roadmap

<i>Research is a <u>partnership</u> of questions and data</i>		What types of data are collected?	
What kinds of questions can be asked of those data?	Descriptive questions	Categorical data	Continuous data
	Relational questions	<ul style="list-style-type: none"> <li>How many members of class have black hair?</li> <li>What proportion of the class attends full-time?</li> </ul>	<ul style="list-style-type: none"> <li>How tall are class members, on average</li> <li>How many hours per week do class members report studying, on average?</li> </ul>
		<ul style="list-style-type: none"> <li>Are male-identifying students more likely to study part-time?</li> <li>Are PrevSci PhD students more likely to be female-identifying?</li> </ul>	<ul style="list-style-type: none"> <li>Do people who say they study for more hours also think they'll finish their doctorate earlier?</li> <li>Are computer-literate students less anxious about statistics?</li> </ul>

# Class goals

- Describe and summarize quantitative data that are continuous
- Describe the purpose and compute the following measures of central tendency: mean, median and mode
- Describe the purpose and compute the following measures of variability: quartiles, inter-quartile range, range, variance and standard deviation
  - Describe conceptually the principles of skewness and kurtosis
- Create visualizations of quantitative data that are continuous using R
  - Includes constructing histograms, densities, stem-and-leaf, and box-and-whisker plots

# Life Expectancy Data

Suppose you are working for the World Health Organization and are investigating life expectancy across different regions.

Using this dataset, we can ask questions like:

- How does life expectancy compare in high- vs. middle- and low-income countries?
- Do individuals living in countries with more total years of attendance in school experience, on average, higher life expectancy?

**But**, before making comparisons between social/economic conditions, we want to start with describing our data, looking at all 183 nations in the dataset.

**Our first task:** Describe the distribution of regional life expectancy in 2015.

# Distributions

# Describing a Distribution

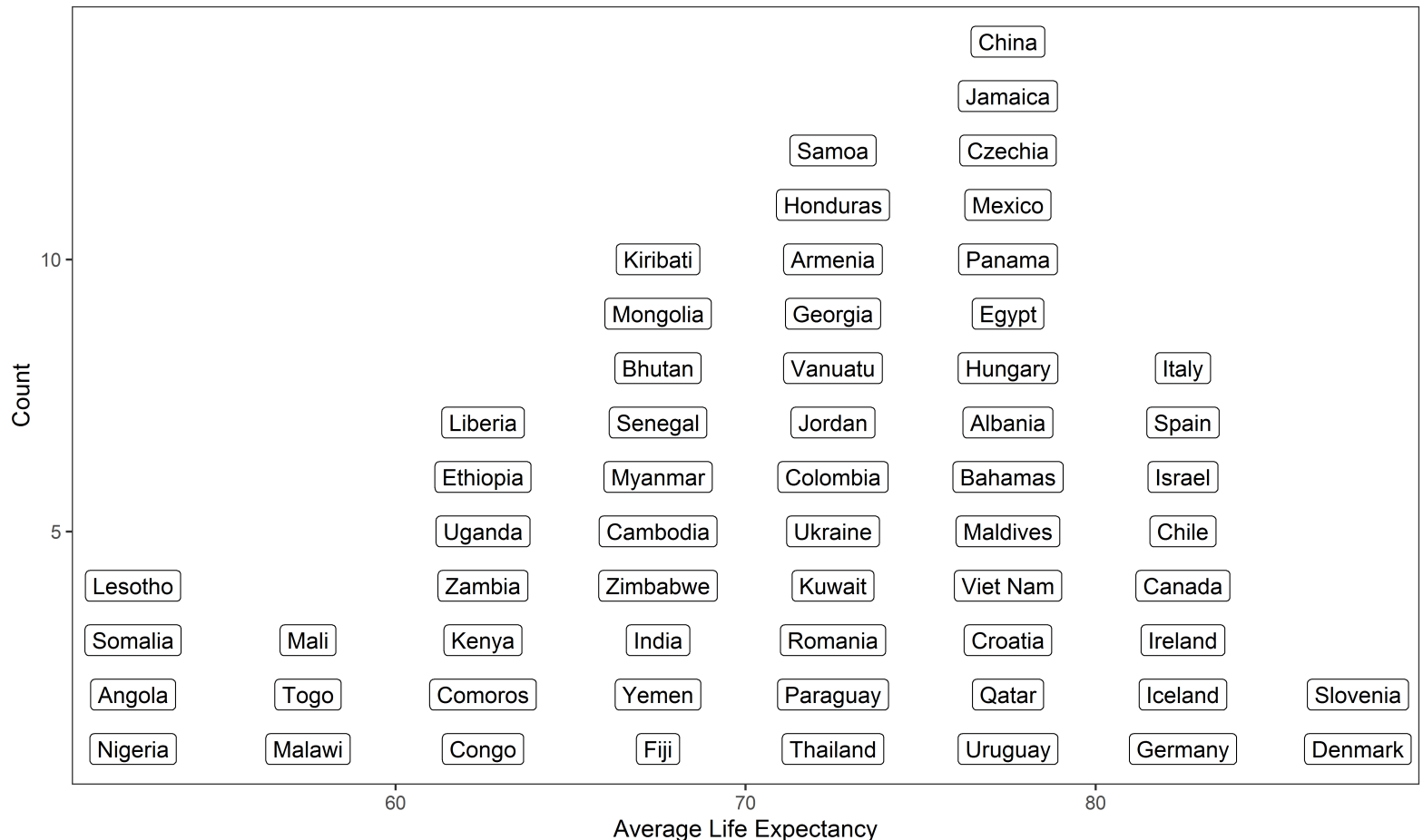
There are many ways to describe the characteristics of a distribution.

Today we are reviewing:

- What is a distribution?
- Measures of Central Tendency
  - Mean
  - Median
  - Mode
- Spread or Variability
  - Variance
  - Standard Deviation
  - Interquartile Range
  - Range
- Measures of Skewness
- Measures of Kurtosis

# What is a distribution?

A distribution describes the number of observations of variable that take a particular value.



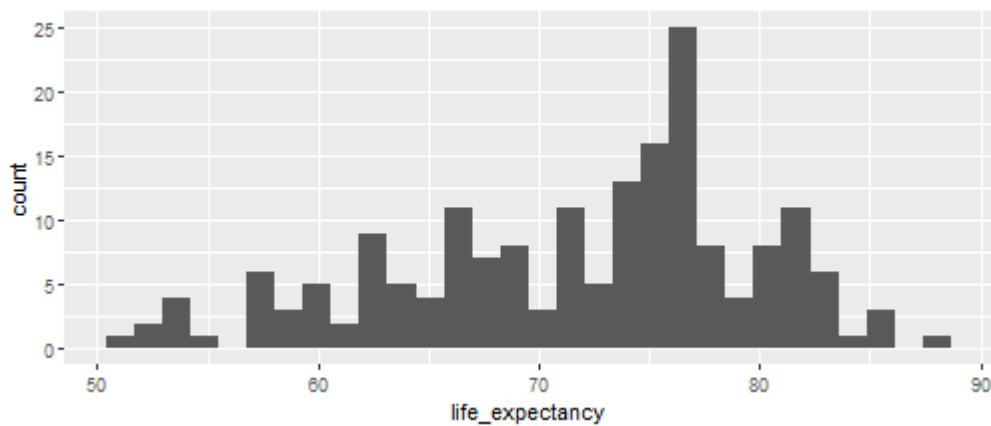
# Visualizing a Distribution

Some of the most common ways to visualize a distribution are:

## Histogram

- Groups the data into "bins" and shows the count of observations in each bin
- Can specify number of bins/bin width

```
ggplot(who, aes(life_expectancy)) +  
  geom_histogram()
```



*What can you already say about the shape of this distribution?*



# Stem-and-leaf plot

- The "leaf" represents the last digit of a number (typically the 1s place, unless data need to be rounded to the nearest ten)
- The "stem" contains all other digits of number and serves as a grouping category for observations

```
stem(who$life_expectancy)
```

```

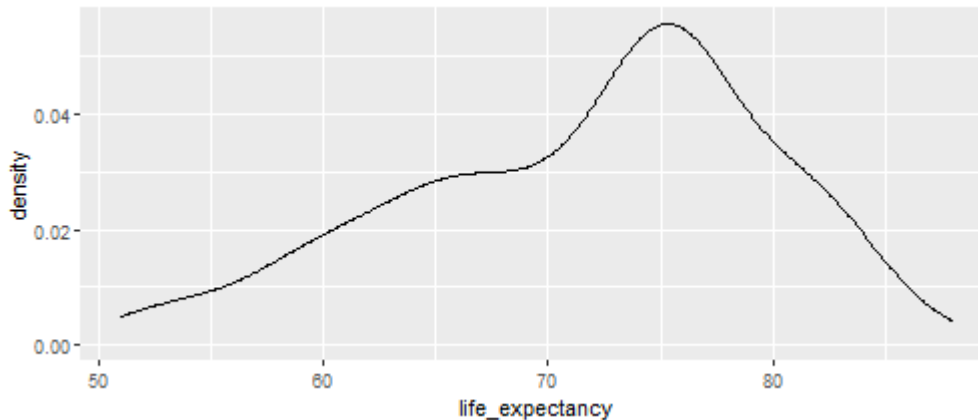
##
## The decimal point is at the |
##
## 50 | 0
## 52 | 0000
## 54 | 000
## 56 | 00
## 58 | 00000000
## 60 | 00000000
## 62 | 0000000000
## 64 | 0000000000
## 66 | 0000000000000000
## 68 | 00000000000000
## 70 | 00000000
## 72 | 00000000000000
## 74 | 00000000000000000000000000000000000000
## 76 | 00000000000000000000000000000000
## 78 | 00000000000000
## 80 | 00000000
## 82 | 00000000000000000000
## 84 | 000
## 86 | 0
## 88 | 0

```

# Density Plot

- Can think of this as a "smoothed out" histogram
- Values reflect the distribution's probability density function (not essential to understand their calculation)

```
ggplot(who, aes(life_expectancy)) +  
  geom_density()
```



- Any point on the curve records the probability that you would observe that value of the variable by randomly drawing one observation from your sample

# Measures of central tendency

# Mean

- Represents the **average**, or the sum of all observations divided by the number of observations.
- One of the most common forms of central tendency.

$$\text{mean}(71, 73, 76, 78, 79) = \frac{71 + 73 + 76 + 78 + 79}{5} = 75.4$$

Writing this out can be a little tedious. Since our datasets can often have hundreds or thousands of observations, we often use **summation notation**.

# Summation Notation

Summation (or sigma) notation is used to provide a concise expression for the sum of observations.

$$\sum_{i=1}^n x_i$$

- $i$  = index of summation (the unit that the observations are in; e.g., schools, students, numbers)
- $n$  = stopping point (number of observations)
- $x_i$  = summation element (what we are summing)


Our mean formula rewritten using summation notation:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

We use the bar on the  $X$  ( $\bar{X}$ ) to indicate that we are calculating the mean value of the variable.

# Summation Notation

Let's demonstrate using our WHO data of 183 countries to find the mean.


$$\bar{X} = \frac{1}{183} \sum_{i=1}^{183} X_i = \frac{13110}{183} = 71.64$$

The mean national life expectancy for these countries in 2015 is 71.64 years.

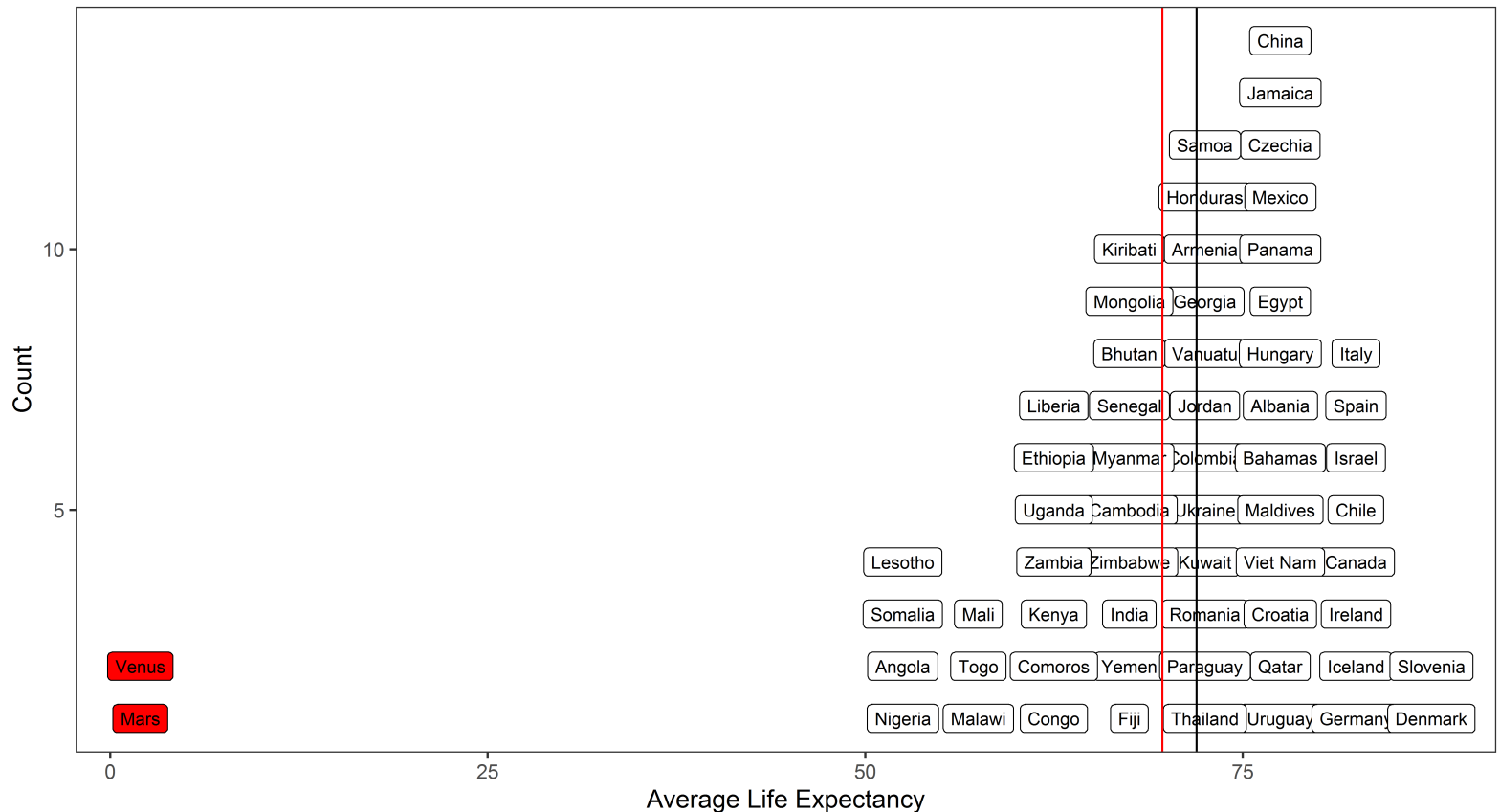
# Mean

- The mean represents the "balance point" of the distribution.
- Outliers, or observations with highly unusual values, can affect this balance point.
- Thus, the mean is vulnerable to outliers.




# Vulnerability of the Mean

- Humans on Mars and Venus have very low life expectancies (for now).
- Here, having Mars and Venus in this sample of countries greatly shifts our mean away from the "peak" of the distribution.



# Median

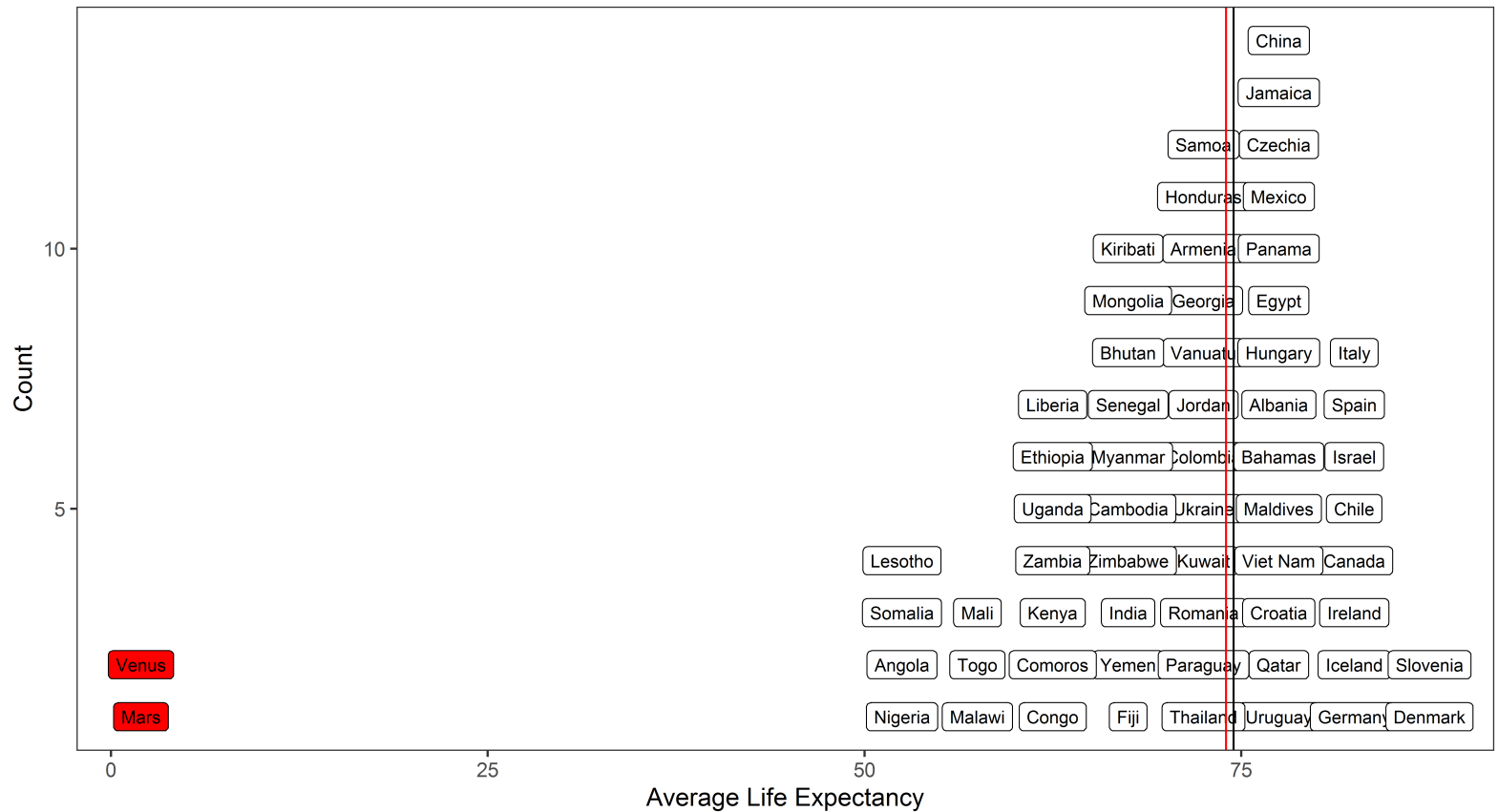
- Represents the **midpoint** of the distribution.
- Also called the *50th percentile* of a distribution or *2nd quartile*, meaning half of the observations are above the median and half are below.
- Particularly useful for describing the central tendency of skewed distributions (more on this later).
- To find the median, arrange the scores in ascending order and identify the middle value. If there are two middle values (even number of observations), use the average of the two.


$$\text{median}(71, 73, 76, 78, 79) = 76$$

$$\text{median}(71, 73, 76, 77, 78, 79) = 76.5$$

# Median

- Unlike the mean, the median is less affected by outliers.



# Mode

The mode simply refers to the **most frequent value** in the data.

What is the mode of the following data?

78, 79, 78, 77, 80, 79, 76, 74, 79, 78, 77, 78, 80, 79, 78, 76, 79, 77, 78, 78

It's easier if we sort the data first.

74, 76, 76, 77, 77, 77, **78, 78, 78, 78, 78, 78, 78**, 79, 79, 79, 79, 79, 80, 80

78 is the most frequent value in the data. Therefore, the mode is 78.

# Mode

Sometimes there may be more than one mode. Consider the following example:

| 4, 6, 6, 7, 7, 7, **8, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9, 9, 9, 9**, 10, 10

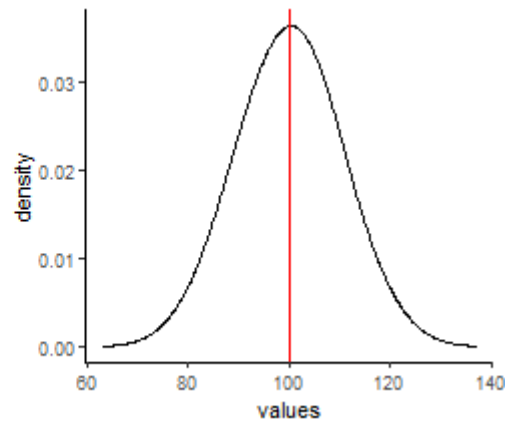
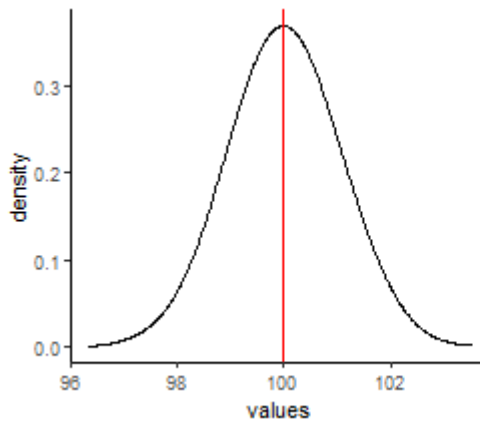
Both 8 and 9 are observed seven times. In this case, the data can be considered **bimodal** and both modes would be reported.

In general, the mode--as a statistic--is rarely used and is particularly vulnerable to idiosyncratic patterns in the data. However, the concept can be helpful in describing the overall shape of a distribution.

# Comparing Distributions

These distributions have the same central tendency mean = 100.

How are they different?



# Measures of variability

# Variability

- Central tendency is only one component of a distribution.
- What if we want to know how much variation there is in life expectancy across nations?
  - How much does the typical observation deviate from the mean?
- **Measures of Variability:**
  - Range
  - Interquartile Range
  - Variance
  - Standard Deviation



# Range

- Represents the difference between the highest value and lowest value in the dataset.
- Can provide a rough estimate of spread

|

$$\text{range}(71, 73, 76, 78, 79) = 79 - 71 = 8$$

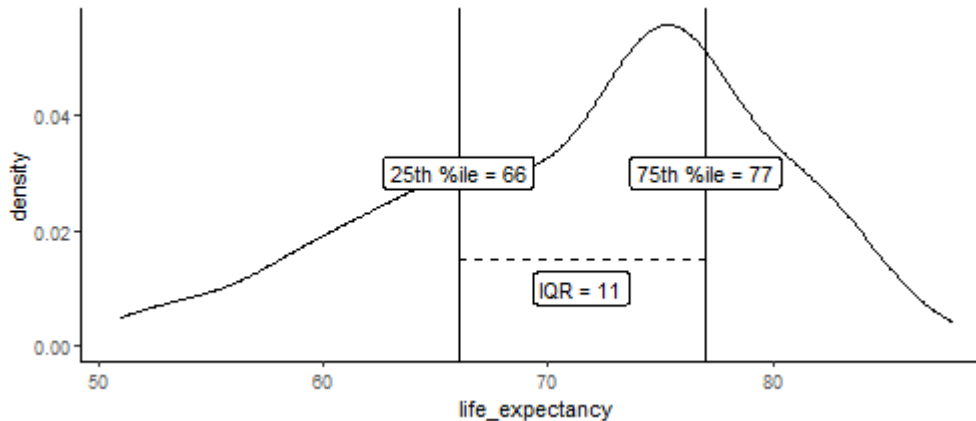
- Very vulnerable to outliers. One observation can make a big difference!

|

$$\text{range}(55, 73, 76, 78, 79) = 79 - 55 = 24$$

# Interquartile Range (IQR)

- Represents the difference between the 1st quartile (25th percentile) and the 3rd quartile (75th percentile).
- **Percentile** refers to the percentage of observations that fall at or below that score.
  - e.g., 25% of observed national life expectancies fall at or below 66 years of age.
- IQR summarizes the range of "average" scores in the dataset.



# Variance

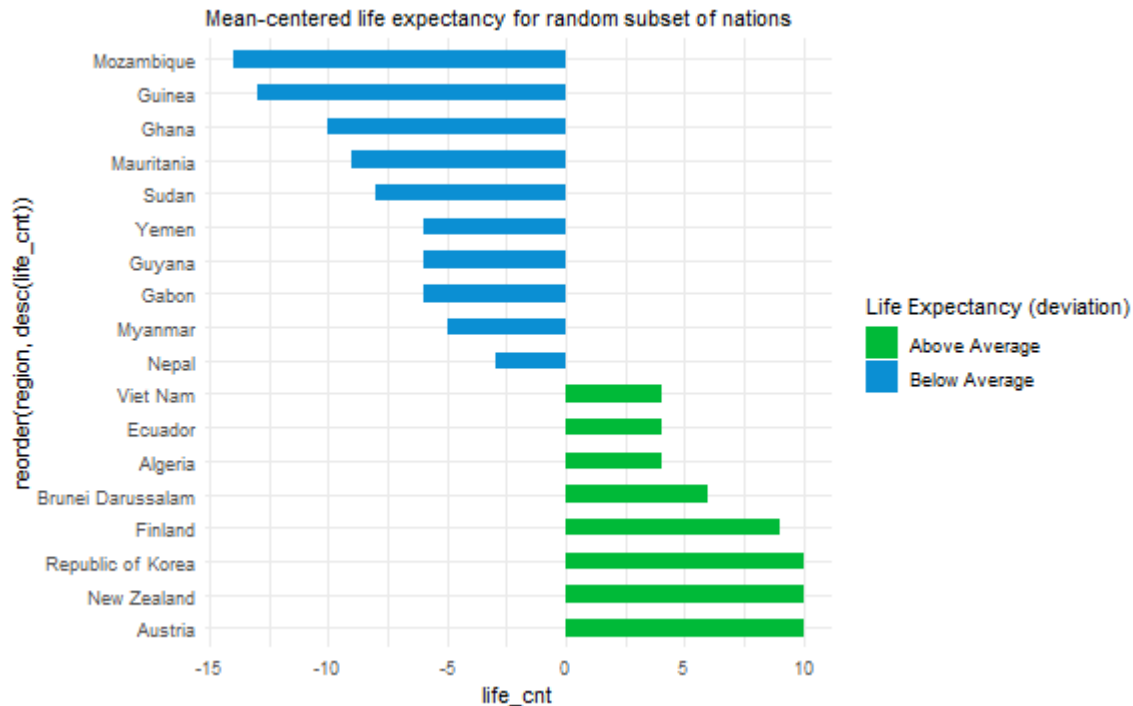
- Represents the **average squared deviation** (let's call this  $s^2$ ) of each observation from the mean.

$$s^2 = \frac{\sum_{i=1}^n x_i^2}{n}$$

- *Why do we square the deviation?*

# Variance

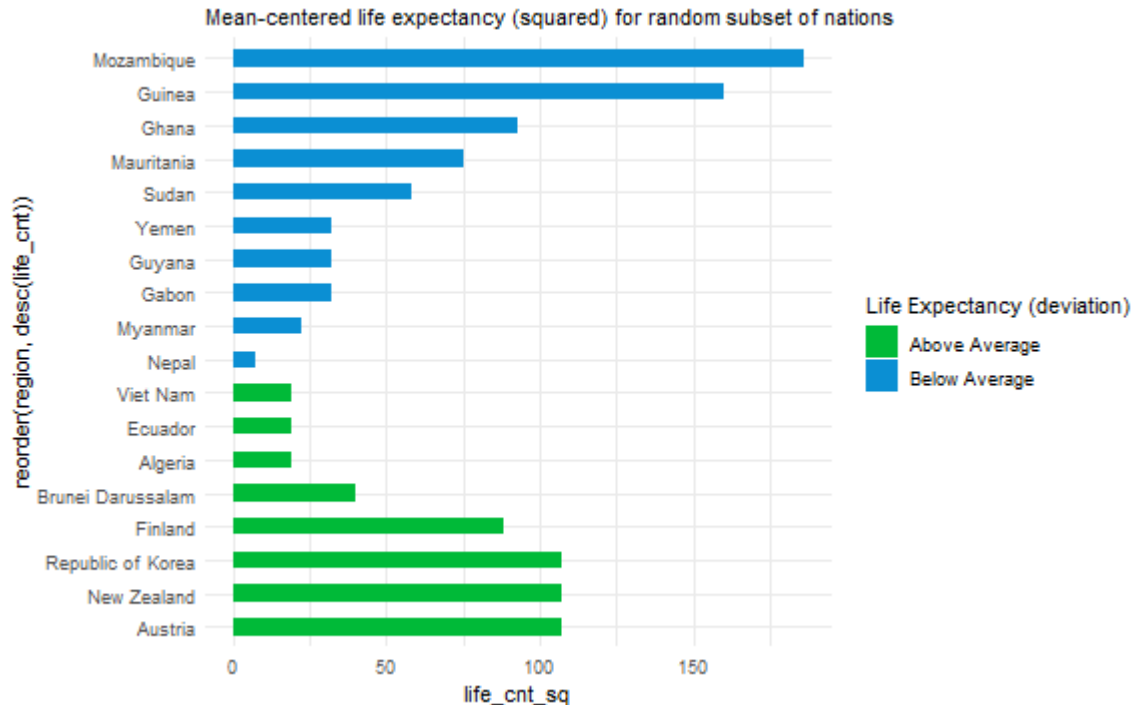
Here are some national life expectancies centered around the mean. Their value represents their difference relative to the sample mean.



Some observations fall below the mean so their deviation is a negative value. If we took the average of our positive and negative deviations, we'd just get zero!

# Variance

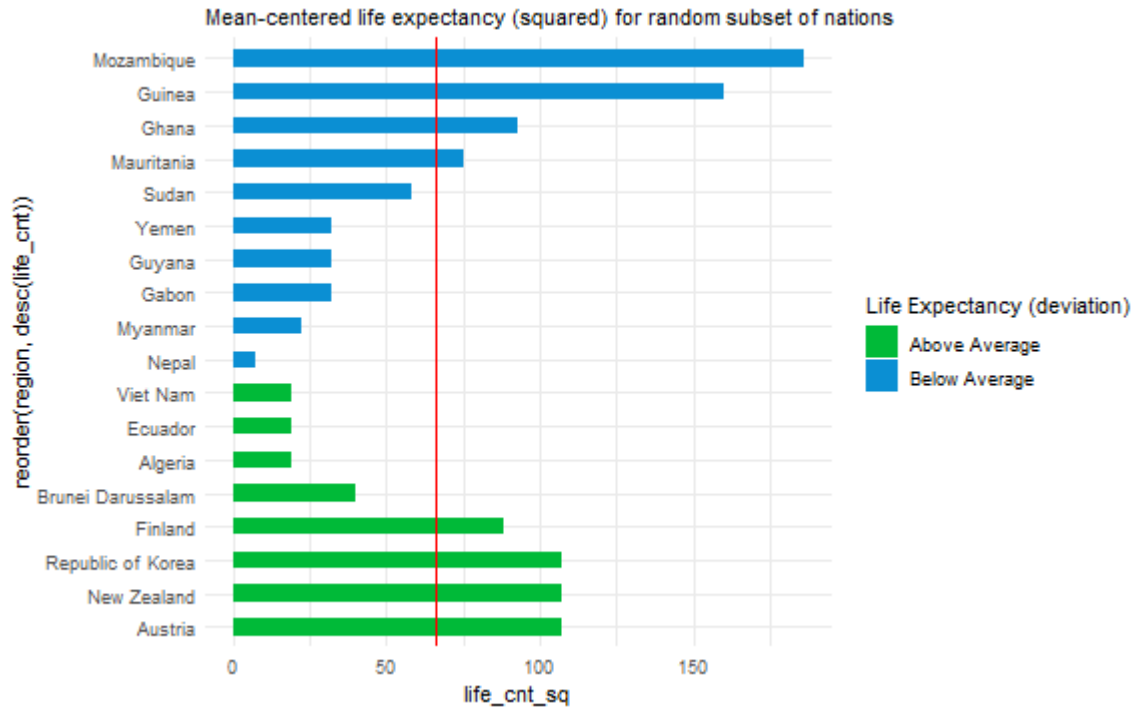
Since any number squared is positive, let's try squaring our deviations.



Now we can take the average of our squared deviations and get a non-zero value!

# Variance

Our variance (or average squared deviation) is approximately 66.43.



By itself, a variance of 66.43 is not very meaningful to us. It does not reflect the raw units of our original scale (it represents squared units).

# Standard Deviation

- If we take the square root of our sample variance, we can summarize average deviations back on our raw scale.
- The standard deviation represents the **positive square root of the variance**.

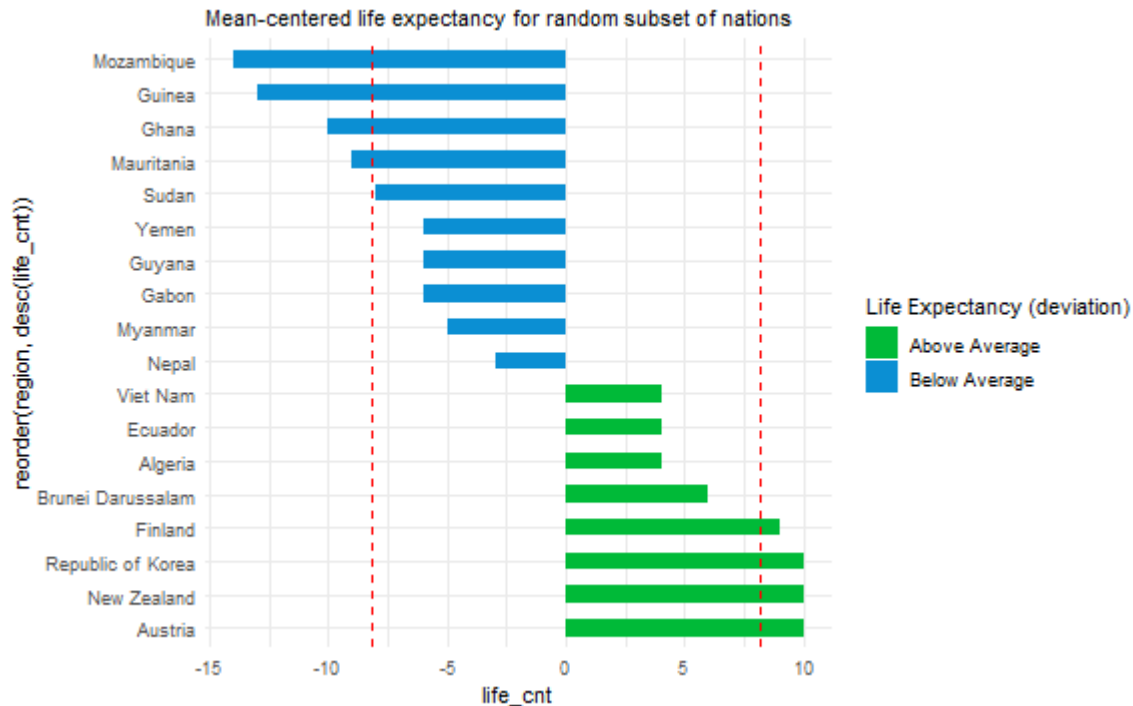
$$s = \sqrt{s^2}$$

$$\sqrt{66.43} = 8.15$$

The standard deviation of life expectancy in 2015 was approximately 8.15 *years*.

# Standard Deviation

Here is our original plot with our **standard deviation (SD)** plotted below and above the mean (i.e.,  $-1\text{ SD}$ ,  $+1\text{ SD}$ ).



The standard deviation of life expectancy in 2015 was approximately 8.15 years.



# Statistical Notation

Statistics usually have two notational forms:

- Population statistics (Greek alphabet) - Hypothetical values in the full universe of all possible values
- Sample statistics (Roman alphabet) - Actual values in our research sample

## Sample Statistics - Roman Alphabet

- Mean -  $\bar{x}$
- Standard Deviation -  $s$
- Variance -  $s^2$

## Population Statistics - Greek Alphabet

- Mean -  $\mu$
- Standard Deviation -  $\sigma$
- Variance -  $\sigma^2$

# Statistical Notation

Often times we use sample statistics to **estimate** population statistics.

An estimated value is denoted with a "hat."

## Sample Statistics → Population Estimates

- Mean -  $\bar{x} \rightarrow \hat{\mu}$
- Standard Deviation -  $s \rightarrow \hat{\sigma}$
- Variance -  $s^2 \rightarrow \hat{\sigma}^2$

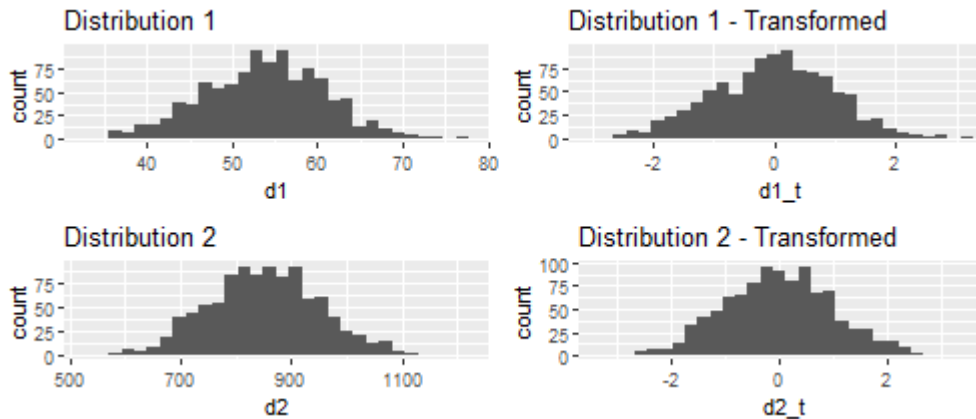
## Population Statistics - Greek Alphabet

- Mean -  $\mu$
- Standard Deviation -  $\sigma$
- Variance -  $\sigma^2$

*These Greek letters are conventions that statisticians have used over the years. There are many other of these notational conventions (for example, when to use capital- or lower-case Greek letters). There's no inherent reason why they are used (e.g., you could use  $\psi$  to represent the mean), but we tend to use the conventional symbols to reduce cognitive load. You'll want to get familiar with some of the most commonly used ones yourself.*

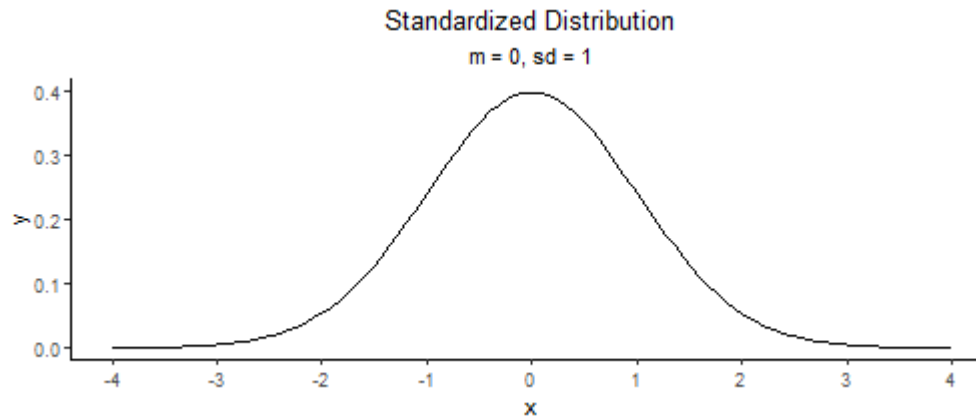
# Transformations

- A distribution can have any mean and any (positive) standard deviation.
- Sometimes it is helpful to standardize a distribution to a common mean and standard deviation so we can more easily compare them and identify extreme data points.



# Transformations

- The most common transformation is a z-transformation.
- A z-transformation rescales the distribution to a mean ( $\mu$ ) of 0 and a standard deviation ( $\sigma$ ) of 1.



# Transforming Distributions

- Any distribution can be standardized using a simple algorithm.

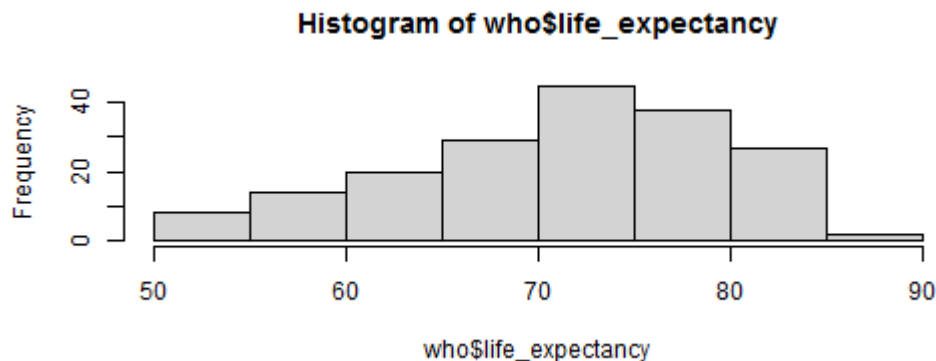
Each observation ( $i$ ) is transformed into a **z-score** using the following formula:

$$z_i = \frac{X_i - \mu}{\sigma}$$

- A z-score is calculated by **subtracting the mean** from each value and **dividing by the standard deviation**.
- An observation's z-score value is equal to its distance from the mean, in standard deviation units.

# Transforming Distributions

Here is a histogram of our life expectancy data.



We are going to create a new variable called `life_expectancy_zscore` using the linear function described on the previous slide.

```
who$life_expectancy_zscore ←  
  (who$life_expectancy - mean(who$life_expectancy)) /  
    sd(who$life_expectancy)
```

# Transforming Distributions

Let's look at the first few transformed values.

```
head(who$life_expectancy_zscore)
```

```
## [1] -0.8145996  0.7804065  0.5350210 -2.4096058  0.5350210  0.5350210
```

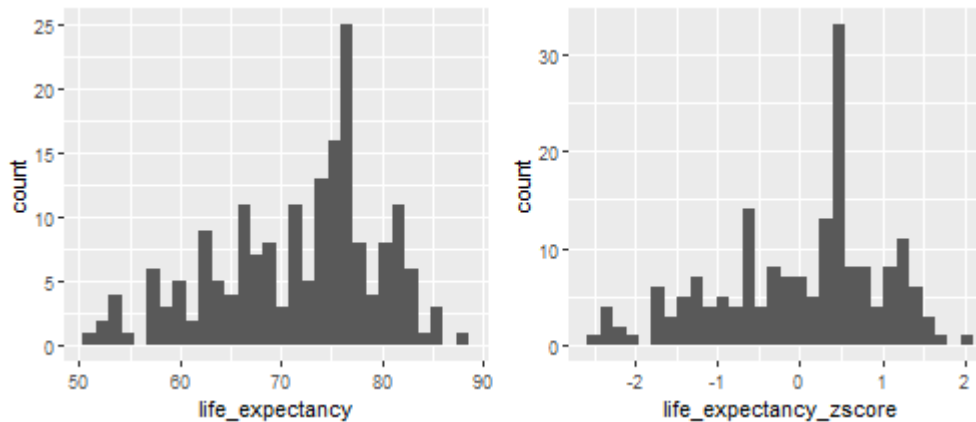
```
## Histogram of the new z-scores  
hist(who$life_expectancy_zscore)
```



We now have a mean of 0 and standard deviation of 1.

# Transforming Distributions

An important note about standardizing a distribution is that it changes the mean and standard deviation, but **does not change the overall shape**.

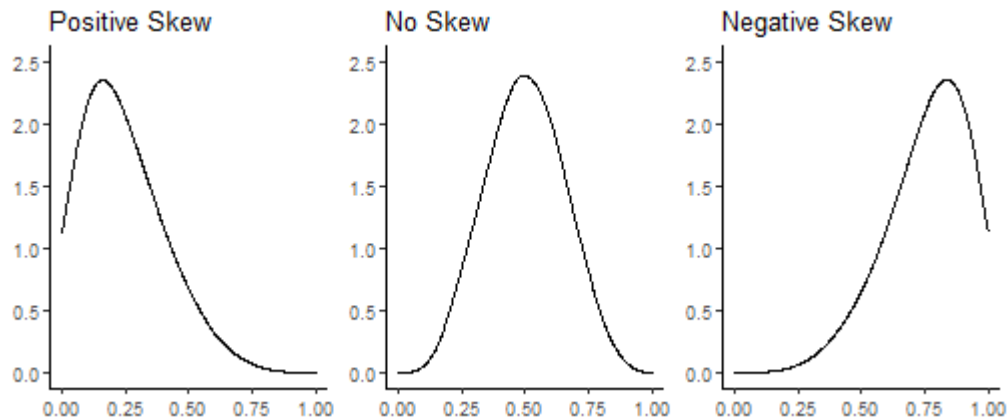




# Distributional Shape

## Skewness

- Not all distributions in the real world are symmetrical. Often they have asymmetry or **skewness**.
- Skewness describes how much a distribution is "bunched up" to the right or left.

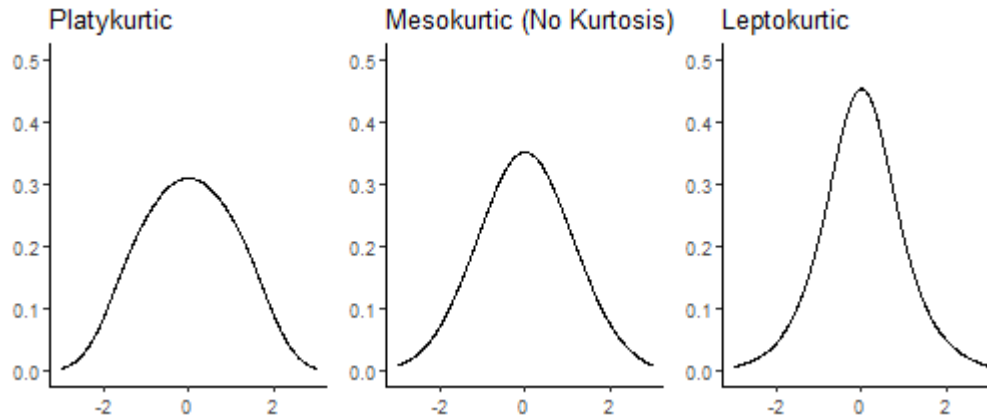


- **Positive Skew** - Distribution is skewed to the right with a "positive-pointing finger."
- **Negative Skew** - Distribution is skewed to the left with a "negative-pointing finger."

# Distributional Shape

## Kurtosis

- **Kurtosis** describes how much the values concentrate around the center (the "pointiness" of a distribution)
  - **Leptokurtic** distributions have more values around the center and have a "taller" peak.
  - **Platykurtic** distributions do not have a prominent peak and have a flatter top.



# Moments of a distribution

Each of these statistics we've covered describe a different "moment of a distribution."

## 1. Mean

$$\mu = \frac{\Sigma(x_i)}{N}$$

## 2. Variance

$$\sigma^2 = \frac{\Sigma(X_i - \mu)^2}{N}$$

## 3. Skewness

$$skewness(X) = \frac{1}{N\sigma^3} \Sigma(X_i - \mu)^3$$

## 4. Kurtosis

$$kurtosis(X) = \frac{1}{N\sigma^4} \Sigma(X_i - \mu)^4 - 3$$

We do not need to know how to calculate skew and kurtosis but we do need to have a sense of how to interpret them.

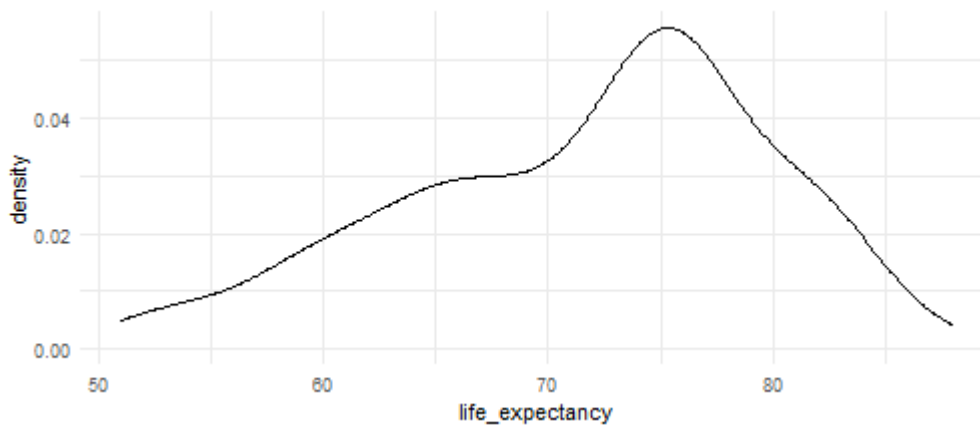
# Evaluating Skewness and Kurtosis

- Skewness and kurtosis can be evaluated visually and summarized quantitatively.
  - Positive skew (skewed to the right) → Skewness > 0
  - Negative skew (skewed to the left) → Skewness < 0
  - Leptokurtic → Kurtosis > 0
  - Platykurtic → Kurtosis < 0

Note: In this course we will focus visual analysis for severe skew or kurtosis. Future courses will discuss how to correct for skew or kurtosis, as necessary.

# Summarizing Data

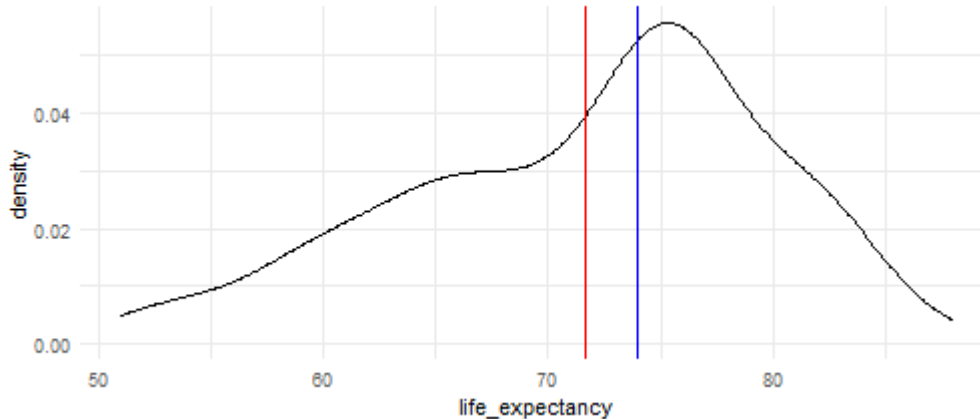
- Here is a density plot of the WHO life expectancy data for 2015.
- How would you characterize the distribution?



- Is there severe skew?
- Is there severe kurtosis?

# Summarizing Data

The distribution of life expectancy has a slightly negative skew and no excess kurtosis. The skewness and kurtosis are not severe.



The **mean** national life expectancy in 2015 was **71.62** years.

The **median** national life expectancy in 2015 was **74** years.

- Due to the slight skew, the median appears to better capture the "center" of our distribution.

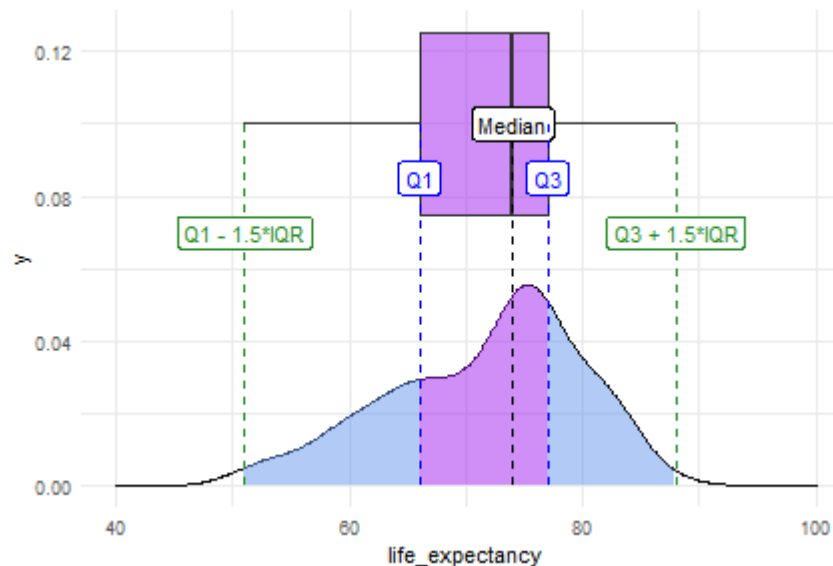
# Other Visualizations

Although histograms and density plots are useful for evaluating the shape of a distribution, they do not efficiently summarize the central tendency and variance.

## Box and Whisker Plot (Boxplot)

- Provides five helpful numbers of a distribution
  - Q1, Q2 (Median), Q3
  - Lower Fence ( $Q1 - 1.5 \times IQR$ )
  - Upper Fence ( $Q3 + 1.5 \times IQR$ )

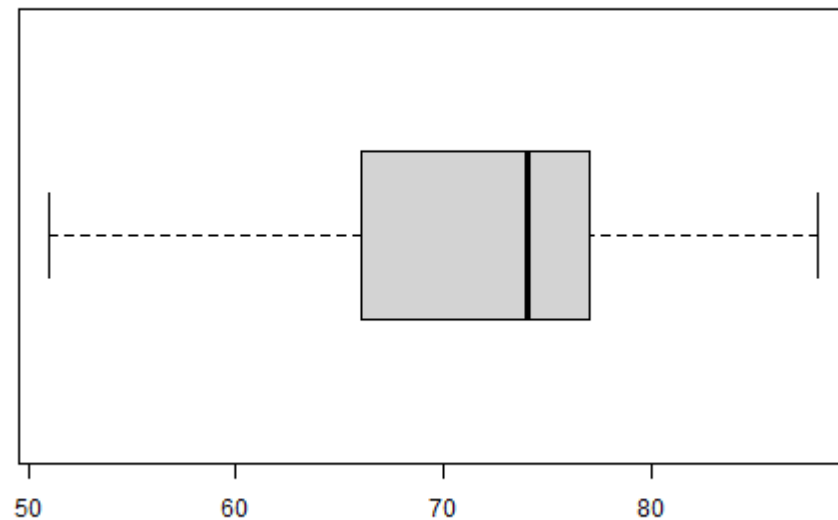
Here is a sample boxplot of our WHO data overlayed on a density plot.



# Boxplots

A basic boxplot can quickly capture the median, the interquartile range and any outlying values (we don't have any of these in our data).

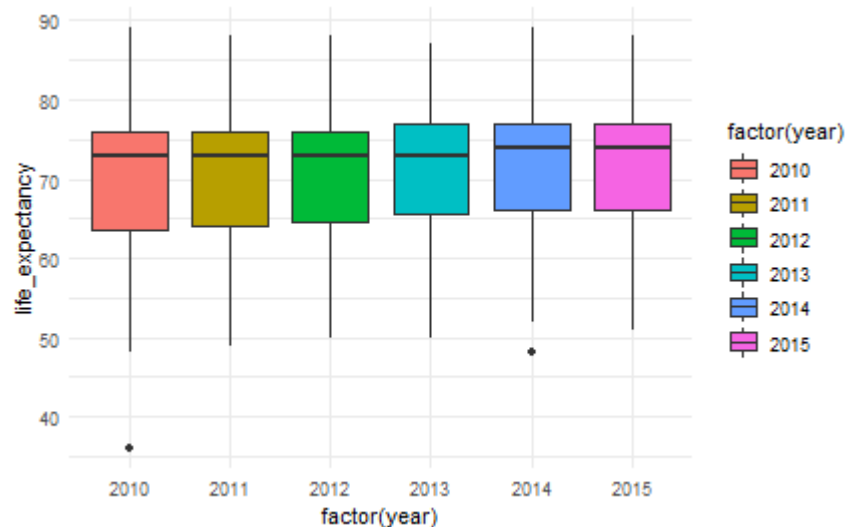
```
boxplot(who$life_expectancy, horizontal=T)
```





# Boxplots

However, we frequently visualize boxplots vertically with groups plotted along the x-axis for comparison.



Which part of the boxplot do you notice changing the most from year-to-year?

In general, it appears the life expectancy of Q1 increases more each year than the median or Q3. Thus, we might hypothesize that lower life expectancy countries made bigger gains from 2010 to 2015 than countries with higher than average life expectancies.

But more exploration would be needed to confirm this!

# Summing up

So we have characterized some features of the average national life expectancy among countries in the WHO database. However, we are still interested in asking more complex questions such as:

- How does life expectancy compare in high- vs. middle- and low-income countries?
- Do individuals living in countries with more total years of attendance in school experience, on average, higher life expectancy?

**That is where we will turn to in the next unit! But first, we need to learn more about how to make statistical inferences!**

# Synthesis and wrap-up

# Class goals

- Describe and summarize quantitative data that are continuous
- Describe the purpose and compute the following measures of central tendency: mean, median and mode
- Describe the purpose and compute the following measures of variability: quartiles, inter-quartile range, range, variance and standard deviation
  - Describe conceptually the principles of skewness and kurtosis
- Create visualizations of quantitative data that are continuous using R
  - Includes constructing histograms, densities, stem-and-leaf, and box-and-whisker plots

# To Dos

## Reading

- LSWR Chapter 5: descriptive statistics

## Optional follow-up

- Complete Module 8 (dataframes) in R Bootcamp
- Complete Module 10 (data management) in R Bootcamp

## Assignments

- Assignment #3 due November 7, 11:59pm