20170269 김은진 Homework

9.2) 96% CI for μ.

$\Rightarrow \sigma = 40,\ n = 30,\ \bar{X} = 780.$

$\Rightarrow$ 96% CI for $\mu = (\bar{X} \pm z_{0.02} \frac{40}{\sqrt{30}})$

$\Rightarrow (780 \pm 2.055 \frac{40}{\sqrt{30}}) \Rightarrow (780 \pm \boxed{15.0075})$

$\therefore 764.0075 < \mu < 795.0075$

9.3) 95% CI for μ

$\Rightarrow n = 75,\ \bar{X} = 0.31,\ \sigma = 0.0015.$

95% CI for $\mu = (\bar{X} \pm z_{0.025} \frac{0.0015}{\sqrt{75}})$

$= (0.31 \pm 1.96 \frac{0.0015}{\sqrt{75}}) = (0.31 \pm 0.0003)$

$= (0.3097, 0.3103)$

$\therefore 0.3097 < \mu < 0.3103.$

9.6) $n = 30,\ \sigma = 40,\ \bar{X} = 780.$

$\lceil n \rceil = \left(\frac{z_{0.02}\sigma}{\varepsilon} \right)^2$ $\varepsilon \rightarrow$ Error.

$\Rightarrow \left(\frac{2.055(40)}{10} \right)^2 = \lceil 67.56847 \approx$

68

9.7) $n = 75,\ \bar{X} = 0.31,\ \sigma = 0.0015.$

$\lceil n \rceil = \left(\frac{z_{0.025}\sigma}{\varepsilon} \right)^2 = \left(\frac{1.96(0.0015)}{0.0005} \right)^2$

$\lceil 34.57447 \approx 35$

9.12) 99% CI for μ.

$n = 10,\ \bar{X} = 230,\ S = 15.$

99% CI for μ.

$\Rightarrow \sigma$ is unknown, $n < 30$

$\Rightarrow (\bar{X} \pm t_{0.005,9} \frac{15}{\sqrt{10}})$

$\Rightarrow (230 \pm 3.250 \cdot \frac{15}{\sqrt{10}})$

$\Rightarrow (230 \pm 15.4161)$

$\Rightarrow (214.5839,\ 245.4161)$

$\therefore (214.5839 < \mu < 245.4161).$

9.13) $n = 12.\ \bar{X} = 48.50.\ S = 1.5.$

90% CI for μ.

σ is unknown, $n < 30.$

$\Rightarrow (\bar{X} \pm t_{0.05,11} \frac{1.5}{\sqrt{12}}).$

$\Rightarrow (48.50 \pm 1.796 \cdot \frac{1.5}{\sqrt{12}})$

$= (48.5 \pm 0.7777)$

$\Rightarrow (47.723,\ 49.277)$

$\therefore (47.723 < \mu < 49.277).$

9.14) $n = 15.\ \bar{X} = 3.79.\ S = 0.9426$

95% CI for $\mu = (\bar{X} \pm t_{0.025,14} \frac{0.9426}{\sqrt{15}}).$

$\Rightarrow (3.79 \pm 0.5186)$

$= (3.2714,\ 4.3086)$

$\therefore (3.2714 < \mu < 4.3086).$

9.35)

1	2
$n_1 = 25$	$n_2 = 36$
$\sigma_1 = 5$	$\sigma_2 = 3$
$\bar{x}_1 = 80$	$\bar{x}_2 = 75$

94% CI for $\mu_1 - \mu_2$

$\Rightarrow (\bar{X}_1 - \bar{X}_2) \pm z_{0.03} \sqrt{\frac{25}{25} + \frac{9}{36}}$

$\Rightarrow (80-75) \pm 2.102$

$\Rightarrow (2.898, 7.102)$

$\Rightarrow 2.898 < \mu_1 - \mu_2 < 7.102$

9.36)

A	B
$n_A = 50$	$n_B = 50$
$\sigma_A = 5.6$	$\sigma_B = 6.3$
$\bar{x}_A = 78.3$	$\bar{x}_B = 87.2$

95% CI for $\mu_1 - \mu_2$

$\Rightarrow (\bar{X}_A - \bar{X}_B) \pm z_{0.025} \sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}$

$\Rightarrow (78.3 - 87.2) \pm 1.96 \sqrt{\frac{(5.6)^2}{50} + \frac{(6.3)^2}{50}}$

$\Rightarrow -8.9 \pm 2.3364$

$\Rightarrow (-11.2364 < \mu_1 - \mu_2 < -6.5636)$.

9.41) σ = unknown. $n_1, n_2 < 30$

99% CI for $\mu_2 - \mu_1$, $v = n_1 + n_2 - 2 = 28$

$\Rightarrow (\bar{X}_2 - \bar{X}_1) \pm t_{0.005,28}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$\left(S_p^2 = \frac{(13 \cdot 1.5) + (15 \cdot 1.8)}{28} \Rightarrow S_p = 1.2887 \right)$.

$\Rightarrow (19 - 17) \pm 2.763 \cdot 1.2887 \cdot \sqrt{\frac{1}{14} + \frac{1}{16}}$

$\Rightarrow 2 \pm 1.303$

$\therefore (0.697, 3.303)$.

$\Rightarrow (0.697 < \mu_2 - \mu_1 < 3.303)$.

9.50) σ is unknown, $n_1, n_2 < 30$.

95% CI for $\mu_1 - \mu_2$. ($v = 19$). $\hookrightarrow n_1 + n_2 - 2$.

$\Rightarrow (\bar{X}_1 - \bar{X}_2) \pm t_{0.025,19} \cdot s_p \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$\Rightarrow (1.98 - 1.30) \pm 2.093 \cdot 0.4161 \cdot \sqrt{\frac{1}{8} + \frac{1}{13}}$

$\left(S_p^2 = \frac{(7 \cdot 0.51^2) + (12 \cdot 0.35^2)}{19} \Rightarrow S_p = 0.4161 \right)$.

$\Rightarrow (1.98 - 1.30) \pm 0.3913$

$\Rightarrow (0.68) \pm 0.3913$

$\Rightarrow (0.2887, 1.0713)$.

$\therefore 0.2887 < \mu_1 - \mu_2 < 1.0713$

9.43) $\sigma_1^2 \neq \sigma_2^2$, σ_1; σ_2 is unknown.

$\Rightarrow$ 95% CI for $\mu_A - \mu_B$

$\Rightarrow (36300-38100) \pm t_{0.025,v}\sqrt{\frac{S_A^2}{n_A}+\frac{S_B^2}{n_B}}$

$\llcorner V = \dfrac{(w_A+w_B)^2}{\frac{w_A}{n_A-1}+\frac{w_B}{n_B-1}} = 21.1839$

$\approx 21.$

$\Rightarrow (36300-38100) \pm 0.280 \cdot 2276.8765$

$\Rightarrow -1800 \pm 637.5254$

$\Rightarrow (-2437.5254, -1162.4746)$

$\therefore (-2437.5254 < \mu_A - \mu_B < -1162.4746)$

9.45)

$d = -7, -2, 4, 3, -6, -4, 1, -9, -5.$

$d^2 = 49; 4; 16, 9, 36, 16, 1, 81, 25.$

$\bar{d} = -2.778$

95% CI for $\mu_D = (\bar{d} \pm t_{0.025,8} \frac{S_d}{\sqrt{n}})$

$S_d = \frac{1}{8}\left[235 - \frac{625}{9}\right]$

$= 20.7$

$\Rightarrow (-2.778 \pm 2.306 \cdot \frac{\sqrt{20.7}}{\sqrt{9}})$

$\Rightarrow (-2.778 \pm 3.497) \Rightarrow (-6.275, 0.719)$

$\therefore -6.275 < \mu_D < 0.719.$

9.44). $n=8.$

$d = -2300, -1300, -1000, 900, 600, -3600,$
$-800, -1400.$

$d^2 = 529000, 1690000, 1000000, 810000,$
$360000, 12960000, 640000, 1960000.$

$\bar{d} = -1112.5$

99% CI for μ_D. =

$(\bar{d} \pm t_{0.005,7} \frac{S_d}{\sqrt{8}})$

$(S_d^2 = \frac{1}{8}\left[23810000 - \frac{79210000}{8}\right]$

$= 1738593.75)$

$\Rightarrow (-1112.5 \pm 3.499 \cdot \frac{1318.5574}{\sqrt{8}})$

$\Rightarrow (-1112.5 \pm 1631.1653)$

$= (-2743.6653, 518.6653)$

$\therefore (-2743.6653 < \mu_D < 518.6653)$

9.53)

(a) $n=200$.

$\hat{p} = \frac{114}{200} = 0.57$, $\hat{q} = 1-0.57 = 0.43$.

96% CI for P

$\Rightarrow \left(\hat{p} \pm z_{0.02}\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$

$= \left(0.57 \pm 2.055\sqrt{\frac{0.57\times0.43}{200}}\right)$

$= (0.57 \pm 0.0719)$

$= (0.4981, 0.6419)$

$= 0.4981 < P < 0.6419$.

(b) error, $\varepsilon \le z_{0.002}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 0.0719$

9.51) $n = 1000$. $\hat{P} = \frac{228}{1000} = 0.228$,

$\hat{q} = 0.772$.

99% CI for P.

$= \left(\hat{p} \pm z_{0.005}\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$

$= \left(0.0228 \pm 2.575\sqrt{\frac{0.228\times0.772}{1000}}\right)$.

$= (0.228 \pm 0.0342)$

$= (0.1938 < P < 0.2622)$

9.113) $n=400$ $\hat{p} = \frac{17}{400} = 0.0425$,

$\hat{q} = 0.9575$

$\rightarrow$ two sided

(a) 95% CI for p

$= \left(\hat{p} \pm z_{0.025}\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$

$= \left(0.0425 \pm 1.96\sqrt{\frac{0.0425\times0.0975}{400}}\right) =$

$0.0227 < P < 0.0623$.

(b) one sided.

95% CI for P.

$= \left(\hat{p} \pm z_{0.05}\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$.

$= \left(\hat{p} \pm 1.645\sqrt{\frac{0.0425\times0.0975}{400}}\right)$.

$= (\hat{p} \pm 0.0053)$

$= (0.0425 \pm 0.0053)$

= lower bound : 0.0372

upper bound : 0.0478.

9.66)

1	2
$n_1 = 250$	$n_2 = 175$
$\hat{P}_1 = \frac{80}{250} = 0.32$	$\hat{P}_2 = \frac{40}{175} = 0.2286$
$\hat{q}_1 = 0.68$.	$\hat{q}_2 = 0.7714$.

90% CI for $(P_1 - P_2)$

$= (\hat{P}_1 - \hat{P}_2) \pm z_{0.05}\sqrt{\frac{\hat{P}_1\hat{q}_1}{n_1} + \frac{\hat{P}_2\hat{q}_2}{n_2}}$

$= (0.0914) \pm 1.645 \cdot \sqrt{\frac{0.2176}{250} + \frac{0.1763}{175}}$ $\longrightarrow$

$= (0.0914) \pm 0.0713$

$\Rightarrow 0.0201 < P_1 - P_2 < 0.1627.$

9.69).

1	2
$n_1 = 1000$	$n_2 = 760$
$\hat{p}_1 = \frac{274}{1000} = 0.274$	$\hat{p}_2 = \frac{274}{760} = 0.316$
$\hat{q}_1 = 0.726$	$\hat{q}_2 = 0.684.$

95% CI for $P_1 - P_2$

$\Rightarrow (\hat{P}_1 - \hat{P}_2) \pm z_{0.025} \left(\sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} \right)$

$= (0.042) \pm 1.96 \cdot \sqrt{\frac{0.2}{1000} + \frac{0.2161}{760}}$

$= -0.089 < P_1 - P_2 < 0.0013.$

$\therefore$ not significant.

9.72) $n=20, \bar{x}=72, s^2=16.$

98% CI for σ^2

$= \frac{19 \cdot 16}{\chi^2_{0.01,19}} < \sigma^2 < \frac{19 \cdot 16}{\chi^2_{0.99,19}}$

$\Rightarrow 8.4 < \sigma^2 < 39.827.$

9.109) $n = 20, \ s^2 = 0.045.$

95% CI for σ^2

$\Rightarrow \frac{19 \cdot 0.045}{\chi^2_{0.025,19}} < \sigma^2 < \frac{19 \cdot 0.045}{\chi^2_{0.975,19.}}$

$= \frac{19 \cdot 0.045}{32.852} < \sigma^2 < \frac{19 \cdot 0.045}{8.907}$

$= 0.0260 < \sigma^2 < 0.095.$

9.78).

1	2
$n_1 = 12$	$n_2 = 12$
$s_1^2 = (5000)^2$	$s_2^2 = (6100)^2.$
$\bar{x}_1 = 36300$	$\bar{x}_2 = 38100.$

$v_1 = 11$

$v_2 = 11.$

90% CI for $\frac{\sigma_1^2}{\sigma_2^2}$

$= \frac{S_1^2}{S_2^2} \frac{1}{f_{0.05}(v_1, v_2)} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{S_1^2}{S_2^2} \frac{1}{f_{0.95}(v_1, v_2)}.$

$\Rightarrow \frac{25000000}{37210000} \cdot \frac{1}{2.82} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{25000000}{37210000} \cdot 2.82$

$= 0.2382 < \frac{\sigma_1^2}{\sigma_2^2} < 1.8947$