

20170269 김은진 Homework 4

Pg. 127 Q1)

a)  $\{-1, 1\}$     b)  $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

c)  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

d)  $\{\}$

Pg. 128 Q22)

a)  $\{(a, x, o), (a, x, l), (a, y, o), (a, y, l), (b, x, o), (b, x, l), (b, y, o), (b, y, l), (c, x, o), (c, x, l), (c, y, o), (c, y, l)\}$

b)  $\{(o, x, a), (o, x, b), (o, x, c), (o, y, a), (o, y, b), (o, y, c), (l, x, a), (l, x, b), (l, x, c), (l, y, a), (l, y, b), (l, y, c)\}$

c)  $\{(o, a, x), (o, a, y), (o, b, y), (o, c, x), (o, c, y), (l, a, x), (l, a, y), (l, b, x), (l, b, y), (l, c, x), (l, c, y), (o, b, a)\}$

d)  $\{(x, x, x), (x, x, y), (x, y, x), (x, y, y), (y, x, x), (y, x, y), (y, y, x), (y, y, y)\}$

Pg. 138 Q.2)

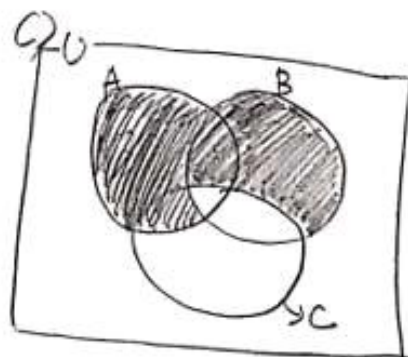
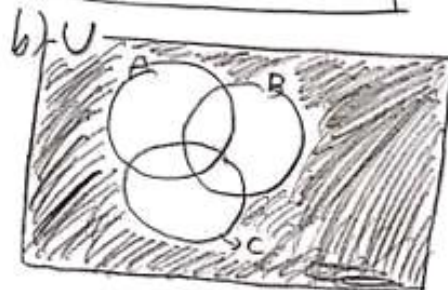
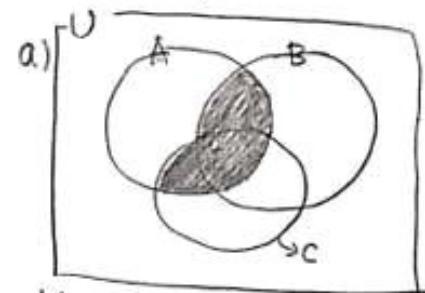
a)  $\{a, b, c, d, e, f, g, h\}$

b)  $\{a, b, c, d, e\}$

c)  $\{\}$

d)  $\{f, g, h\}$

Pg. 138 Q14)



Pg. 153 Q4)

a) Domain: The set of nonnegative integers

Range: The set of digits  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ .

b) Domain: the set of positive integers.

Range: the set of integers greater than 1.

c) Domain: the set of all bit strings.

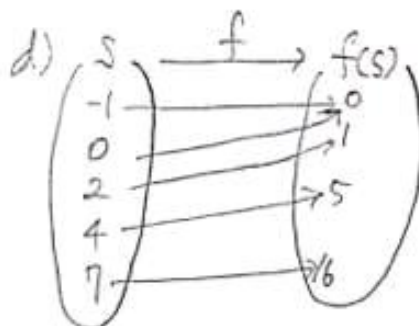
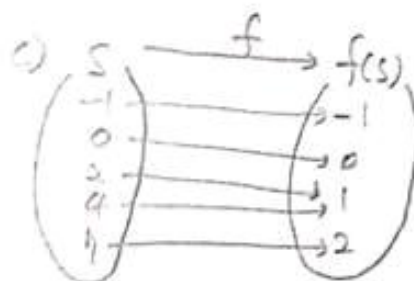
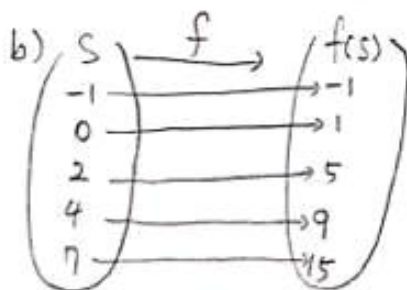
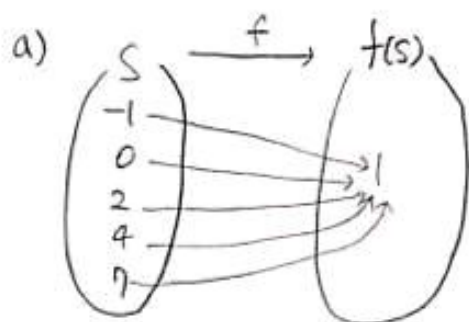
Range: the set of nonnegative integers

d) Domain: the set of all bit strings

Range: the set of nonnegative integers

(a bit string can have length 0).

Pg. 154 Q18)



Pg. 154 Q22)

-(f ∘ g)

$$= (f \circ g)(x) = f(g(x))$$

$$= (x^2 + 1) + 2 = x^2 + 3$$

-(g ∘ f)

$$= (g \circ f)(x) = g(f(x))$$

$$= (x + 2)^2 + 1 = x^2 + 4x + 4 + 1$$

$$= x^2 + 4x + 5$$

Example 1)

$$a_1 = \frac{1}{2}, a_2 = 2, a_3 = 4, a_4 = 8$$

$$a = \frac{1}{2}, r = 2$$

$$\therefore n\text{-th} = a_n = ar^{n-1}$$

$$= \frac{1}{2} \cdot (2)^{n-1} = \frac{2^n}{4}$$

$$a_{10} = \frac{2^{10}}{4} = 256$$

Example 2)

$$a_n = a + (n-1)d \quad \therefore d = \frac{3}{2}$$

$$a_6 = a + (5 \cdot \frac{3}{2}) = 5$$

$$a = -\frac{5}{2}$$

$$a_1 = -\frac{5}{2}$$

$$a_2 = -\frac{5}{2} + \frac{3}{2} = -1$$

$$a_3 = -\frac{5}{2} + 3 = \frac{1}{2}$$

$$a_n = -\frac{5}{2} + (n-1)\frac{3}{2}$$

Example 3)

$$a_4 = a + 3d = 93 \quad \text{--- (1) --- (2)}$$

$$a_8 = a + 7d = 65 \quad \text{--- (2) ---}$$

$$\begin{aligned} 3d - 7d &= 93 - 65 \\ -4d &= 28, \quad d = -7 \end{aligned}$$

$$a_4 = a + 3(-7) = 93$$

$$= a - 21 = 93$$

$$\therefore a = 114$$

$$a_1 = 114$$

$$a_2 = 114 - 7 = 107$$

$$a_3 = 114 - 14 = 100$$

$$a_n = 114 - (n-1)7$$

Example 4)  $a_n = ar^{n-1}$

$$a_5 = ar^4 = \frac{5}{4} \quad \text{--- (1) ---}$$

$$a_{12} = ar^{11} = 160 \quad \text{--- (2) ---}$$

$$\frac{(2)}{(1)} = r^7 = 128$$

$$2^7 = 128$$

$$\therefore r = 2$$

$$a_5 = a \cdot 2^4 = \frac{5}{4}$$

$$\Rightarrow a = \frac{5}{64}$$

$$a_{26} = \frac{5}{64} \cdot 2^{25} = 2621440$$

$$a_n = \frac{5}{64} \cdot 2^{n-1}$$

Example 5)

$$S_{35} = \sum_{i=1}^{35} a_i = \frac{35}{2} (a_1 + a_{35})$$

$$a_1 = \frac{1}{2} + 1 = \frac{3}{2}$$

$$a_2 = \frac{1}{2}(2) + 1 = 2$$

$$a_2 - a_1 = 2 - \frac{3}{2} = \left(\frac{1}{2}\right) \Rightarrow d$$

$$a_{35} = a + 34d = \frac{3}{2} + 17 = \frac{37}{2}$$

$$\therefore \sum_{i=1}^{35} a_i = \frac{35}{2} \left( \frac{3}{2} + \frac{37}{2} \right)$$

$$= 1700$$

Example 6)

$$a_n = 7 + (n-1)2$$

$$S_{14} = \sum_{i=1}^{14} a_i = \frac{14}{2} (a_1 + a_{14})$$

$$= 7 \cdot (7 + 33) = 280$$

$$S_{49} = \sum_{i=1}^{49} a_i = \frac{49}{2} (a_1 + a_{49})$$

$$= \frac{49}{2} \cdot (7 + 99)$$

$$= 2491$$

$$\sum_{i=15}^{49} = S_{49} - S_{14} = 2211$$

Example 7)

$$S_{20} = a \left( \frac{1-r^{20}}{1-r} \right)$$

$$a_1 = -6$$

$$\frac{a_2}{a_1} = \frac{12}{-6} = -2 = r$$

$$a_2 = 12$$

$$S_{20} = a \left( \frac{1-(-2)^{20}}{1-(-2)} \right)$$

$$= -6 \left( \frac{1-2^{20}}{3} \right) = 209150$$

Example 10)

$$\sum_{n=1}^{\infty} 10 \left( \frac{1}{2} \right)^{n-1}$$

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{10}{1-\frac{1}{2}} = \frac{10}{\frac{1}{2}} = 20$$

Example 8)

$$a_1 = 250$$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2}$$

$$a_2 = 100$$

$$= \frac{100}{250} = \frac{40}{100}$$

$$a_3 = 40$$

$$= \frac{2}{5}$$

$$S_{10} = 250 \left( \frac{1-\left(\frac{2}{5}\right)^{10}}{1-\left(\frac{2}{5}\right)} \right)$$

$$= 255.850$$

Example 9)

$$S_n = a \left( \frac{1-r^n}{1-r} \right)$$

$$S_4 = a \left( \frac{1-\left(\frac{1}{3}\right)^4}{1-\left(\frac{1}{3}\right)} \right) = \frac{26}{27}$$

$$= a \left( \frac{40}{27} \right) = \frac{26}{27}$$

$$\therefore a = \frac{23}{20}$$