20170269 김은진 <Homework 3>

Example 1)

(a) $P_1 \rightarrow P_2$ is true.

Direct proof: Suppose n is even, then $n=2k$ for some integer k.

consequently, $n-1 = 2k-1$
$= 2(k-1)+1$
$= 2m+1$, where $m = k-1$.

$\therefore$ $n-1$ is odd

(b) $P_2 \rightarrow P_3$ is true.

Direct proof: suppose $n-1$ is odd, then $n-1 = 2k+1$. for some integer k.

consequently, $n = 2k+2$, $n^2 = (2k+2)^2$
$= 4k^2+8k+4$.
$= 2(2k^2+4k+2)$
$= 2m$, Where $m = 2k^2+4k+2$.

$\therefore$ n^2 is even.

Example 2)

Proof by contraposition

$\Rightarrow$ If $\frac{1}{x}$ is rational, then x is rational

Suppose $\frac{1}{x}$ is rational, then $\frac{1}{x} = \frac{p}{q}$ for some integer p, q with $q \neq 0$.

consequently, since $\frac{1}{x} \neq 0$, we know that $p \neq 0$.

$x = \frac{1}{(\frac{1}{x})} = \frac{1}{(\frac{p}{q})} = \frac{q}{p}$

$\therefore$ x is rational

Example 3)

proof by contraposition

$x+y \geq 2$: p
$x \geq 1$ or $y \geq 1$: q

$\rightarrow$ $\neg q \rightarrow \neg p$.

"If $x<1$ and $y<1$ then $x+y<2$"

$\longrightarrow$

suppose $(x<1)$ and $(y<1)$.

Consequently, $x+y < 1+1 = 2$.

$\therefore$ $x+y<2$.

Example 4)

Direct proof

Suppose r and s are rational numbers.

then $r = \frac{a}{b}$, $s = \frac{c}{d}$ for some integer a, b, c, d with $b \neq 0$, $d \neq 0$.

consequently, $r+s = \frac{a}{b} + \frac{c}{d}$

$= \frac{ad+bc}{bd} = \frac{p}{q}$.

where $p = ad+bc$,
$q = bd$ and $q \neq 0$.

$\therefore$ $r+s$ is rational.

Example 5)

Direct proof

suppose $b = ar$ and $c = bs$ for some integers r & s.

consequently, $c = bs$
$= ars$
$= ak$, where $k = rs$.

$\therefore$ a devides c.

Example 6)

Proof by contraposition

$\rightarrow$ "If n is odd then (n^3+5) is even."

Suppose n is odd then $n = 2k+1$ for some integer k.

Consequently, $n^3+5 = (2k+1)^3+5$.
$= 8k^3+12k^2+6k+6$
$= 2(4k^3+6k^2+3k+3)$
$= 2m$, where $m = (4k^3+6k^2+3k+3)$

$\therefore$ (n^3+5) is even.

Example 7)

$\boxed{\text{proof by contraposition}}$

"If a is odd, then a^3+a^2+a is odd"

Suppose a = odd, then $a=(2k+1)$ for some integer k.

consequently,

$$a^3+a^2+a = (2k+1)^3+(2k+1)^2+(2k+1)$$
$$=(8k^3+12k^2+6k+1)+(4k^2+4k+1)+(2k+1)$$
$$=8k^3+12k^2+12k+3$$
$$=2(4k^3+8k^2+6k+1)+1$$
$$=2m+1, \text{ where } m=4k^3+8k^2+6k+1$$

$\therefore a^3+a^2+a$ is odd.

Example 8)

$\boxed{\text{Proof by contradiction}}$

Suppose not the sum of two number is always odd.

Suppose x and y are even number, $x=2a$, $y=2b$ for some integer a and b.

Consequently,

$$x+y=2a+2b$$
$$=2(a+b)$$
$$=2m, \text{ where is } m=a+b$$

$\therefore x+y$ is always even.
(contradiction).

Example 9)

$\boxed{\text{proof by contradiction}}$

Suppose not The sum of two positive numbers is always negative.

Suppose a and b are positive number such that $a+b<0$

consequently, $a<-b$.

Since a is smaller than $-b$, then a must be a negative number
(contradiction)

Example 10)

$\boxed{\text{proof by contradiction}}$

Suppose not The diffrence of any rational number and any irrational number is rational.

Suppose

$$r=\frac{a}{b} \quad r-s=\frac{c}{d}$$

for some integer a, b, c, d and $b\neq0$, $d\neq0$.

consequently,

$$s=\frac{ad-bc}{bd}.$$

Since $s=\frac{ad-bc}{bd}$ then s is rational.
(contradiction).

Example 11)

$\boxed{\text{proof by contradiction}}$

suppose not $\sqrt{2}$ is rational.

suppose $\sqrt{2}=\frac{m}{n}$ for some integer m, n and $n\neq0$.

then $2=\frac{m^2}{n^2}$, $m^2=2n^2$ $\therefore$ m is even

consequently, $m=2k$ for some integer k.

then $(2k)^2=2n^2$
$$4k^2=2n^2$$
$$n^2=2k^2 \quad \therefore n \text{ is even} \longrightarrow$$

Consequently,

m and n are even and they have a common factor.

(contradiction)

Example (2).

$\boxed{\text{proof by contradiction}}$

Suppose not $1+3\sqrt{2}$ is rational.

Suppose 1 is rational number.

$3\sqrt{2}$ is irrational number.

$1+3\sqrt{2} = \frac{m}{n}$ for some integer n, m. $n \neq 0$.

Consequently,

$$1+3\sqrt{2} = \frac{m}{n}$$

$$3\sqrt{2} = \frac{m}{n} - 1$$

$$3\sqrt{2} = \frac{m-n}{n}$$

$$\sqrt{2} = \boxed{\frac{m-n}{3n}}$$

↳ rational number

$\therefore \sqrt{2}$ must be rational

(contradiction)

Example (3)

$\boxed{\text{Proof by Equivalence}}$

p: n is odd. q: $(n-1)$ is even

$(p \leftrightarrow q)$ → Case 1 : $p \to q$

→ case 2 : $q \to p$.

proof for case 1: $p \to q$.

Suppose $n = 2k-1$ for some integer k.

$n-1 = 2k-1$

$= 2k-2$

$= 2(k-1)$

$= 2m$, where m is $k-1$.

$\therefore n-1$ is even

Proof for case 2: $q \to p$

Suppose $n-1 = 2k$ for some integer k

$n-1 = 2k$

$n = 2k+1$

$\therefore n =$ odd.

Consequently, for every integer.

n is odd if and only if $n-1$ is even.

Example (4)

$\boxed{\text{Proof by Equivalence}}$

p: $(x+y)^2 = x^2+y^2$ q: $x=0$ or $y=0$.

$(p \leftrightarrow q)$ → case 1 : $p \to q$.

→ case 2 : $q \to p$.

Proof for case 1 : $p \to q$.

$$(x+y)^2 = x^2+2xy+y^2$$

$$= x^2+y^2+0$$

$\therefore x$ or $y = 0$.

proof for case 2 : $q \to p$

$$(x+y)^2 = x^2+\underbrace{2xy}_{0\,(x \text{ or } y=0)}+y^2$$

$$= x^2+y^2$$

$$\therefore (x+y)^2 = x^2+y^2.$$

consequently, $x, y \in \mathbb{R}$, $(x+y)^2 = x^2+y^2$ if and only if $x=0$ or $y=0$.

Example 15)

$\boxed{\text{Proof by Equivalance}}$

Proof for case 1: $\boxed{\text{Direct proof}}$

Suppose a is even, $a=2k$ for some integer k.

consequently,

$$a^3+a^2+a=(2k)^3+(2k)^2+2k$$
$$=8k^3+4k^2+2k$$
$$=2(4k^3+2k^2+k)$$
$$=2m, \text{ where } m \text{ is } 4k^3+2k^2+k$$

Proof for case 2: $\boxed{\text{Proof by contraposition}}$

"If a is odd, a^3+a^2+a is odd"

Suppose a is odd, $a=2k+1$ for some integer k.

consequently, $a^3+a^2+a=(2k+1)^3+(2k+1)^2+(2k+1)$

$$=8k^3+12k^2+6k+1+4k^2+4k+1+2k+1$$
$$=8k^3+16k^2+12k+3$$
$$=2(4k^3+8k^2+6k+1)+1$$
$$=2n+1, \text{ where } n=4k^3+8k^2+6k+1.$$

$\therefore a^3+a^2+a$ is odd.

Example 16)

$\boxed{\text{Proof by Existense}}$

suppose x is Mid point between a and b.

consequently, $x=\frac{a+b}{2}$,

$\therefore a<x<b$.

Example (17)

$\boxed{\text{proof by Existense}}$

Suppose $m=3$, $n=2$

consequently,

$$2m+3n=2\cdot3+3\cdot2$$
$$=6+6$$
$$=12$$

$\therefore m=3, n=2$ exist for $2m+3n=12$.

Example (18)

$\boxed{\text{Proof by Existense}}$

$f(x)=x^3-3x^2+2x-4$.

$f(2)=(2)^3-3(2^2)+(2\cdot2)-4=-4$.

$f(3)=(3)^3-3(3^2)+(2\cdot3)-4=2$.