<2017 0269 김은진 HW3>

3.6)

(a) $1-\int_0^{200} f(x)\,dx$

$= 1-\int_0^{200} \frac{20000}{(x+100)^3}\,dx$

$= 1-\left(20000\int_0^{200} (x+100)^{-3}dx\right)$

$= 1-20000\left[-\frac{1}{2}(x+100)^{-2}\right]_0^{200}$

$= 1+20000\times\frac{1}{2}\left[(x+100)^{-2}\right]_0^{200}$

$= 1+\left(\frac{1}{9}-1\right) = \frac{1}{9}.$

(b) $\int_{80}^{120} 20000(x+100)^{-3}dx$

$= 20000\left[-\frac{1}{2}(x+100)^{-2}\right]_{80}^{120}$

$= -10000\left(\frac{1}{48400}-\frac{1}{32400}\right) = 0.1020.$

3.27)

(a) $F(x) = \int_0^x \frac{1}{2000}e^{-t/2000}dt$

$= \frac{1}{2000}\left[-\frac{1}{2000}e^{-t/2000}\right]_0^x$

$= -(e^{-x/2000}-e^0) =$

$1-e^{-x/2000}.$

$\therefore F(x)=\begin{cases} 0, & x<0 \\ 1-e^{-x/2000}, & x\geq 0. \end{cases}$

(b) $P(X>1000) = 1-P(X\leq 1000)$

$= 1-F(1000) = 1-(1-e^{-1000/2000})$

$= 1-(1-e^{-\frac{1}{2}}) = e^{-\frac{1}{2}} = 0.6065$

(c) $P(X<2000) = F(2000) =$

$1-e^{-2000/2000} = 1-e^{-1} = 0.6321$

3.29)

(a) $F(x)=\int_{-\infty}^{\infty} f(x)dx = \int_1^{\infty} 3x^{-4}dx$

$= -\left[-x^{-3}\right]_1^{\infty} = 1.$

$f(x)\geq 0$ and $\int_{-\infty}^{\infty} f(x)dx = 1.$

$\therefore f(x)$ is a density function.

(b) $F(x)= \int_1^x f(t)dt = 3\int_1^x t^{-4}dt$

$= 3\left[-\frac{1}{3}t^{-3}\right]_1^x = -x^{-3}+1.$

$\therefore F(x)=\begin{cases} 0, & x\leq 1 \\ -x^{-3}+1, & x>1. \end{cases}$

(c) $P(X>4) = 1-P(X\leq 4)$

$= 1-F(4) = 1-(-4^{-3}+1)$

$= \frac{1}{64} = 0.0156.$

3.21)

(a) $k\int_0^1 f(x)\,dt = 1$

$= k[x^{\frac{1}{2}}]_0^1 = k\cdot\frac{2}{3}[x^{\frac{3}{2}}]_0^1 = 1$

$\Rightarrow \frac{2}{3}k\cdot 1 = 1$, $\underline{k = \frac{3}{2}}$

(b) $F(x) = \int_0^x \frac{3}{2}\sqrt{t}\,dt$

$= [t^{\frac{3}{2}}]_0^x = x^{\frac{3}{2}}$

$\therefore F(x) = \begin{cases} 0, & x \le 0 \\ x^{\frac{3}{2}}, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases}$

(c) $P(0.3 < X < 0.6)$

$= F(0.6) - F(0.3)$

$= (0.6)^{\frac{3}{2}} - (0.3)^{\frac{3}{2}} = 0.3004.$

3.33)

(a) $\int_0^1 ky^4(1^3 - 3y + 3y^2 - y^3)\,dy$

$= k\int_0^1 y^4(1^3 - 3y + 3y^2 - y^3)\,dy.$

$= k\int_0^1 y^4 - 3y^5 + 3y^6 - y^7\,dy.$

$= k[\frac{1}{5}y^5 - \frac{1}{2}y^6 + \frac{3}{7}y^7 - \frac{1}{8}y^8]_0^1$

$= k(\frac{1}{5} - \frac{1}{2} + \frac{3}{7} - \frac{1}{8}) = 1.$

$\Rightarrow k \times \frac{56 - 140 + 120 - 35}{280} = 1$, $k \times \frac{1}{280} = 1.$

$\therefore k = 280.$

(b)
$P(Y < 0.5) = \int_0^{0.5} f(y)\,dy$

$= 280\int_0^{0.5}(y^4 - 3y^5 + 3y^6 - y^7)\,dy.$

$= 280[\frac{1}{5}y^5 - \frac{1}{2}y^6 + \frac{3}{7}y^7 - \frac{1}{8}y^8]_0^{0.5}$

$= \frac{93}{256} = 0.3632$

(c) $P(Y > 0.8) = 1 - P(Y < 0.8).$

$= 1 - \int_0^{0.8} f(y)\,dy$

$= 1 - 280[\frac{1}{5}y^5 - \frac{1}{2}y^6 + \frac{3}{7}y^7 - \frac{1}{8}y^8]_0^{0.8}$

$= 1 - 0.9437\cdots = 0.0562.$

3.11)

(a) $f(x) \ge 0.$

$\int_{-1}^2 \frac{x^2}{3}dx = \frac{1}{3}\int_{-1}^2 x^2\,dx = \frac{1}{3}[\frac{x^3}{3}]_{-1}^2.$

$= \frac{8}{9} + \frac{1}{9} = 1.$

$\therefore f(x)$ is a density function.

(b) $P(0 < X \le 1)$

$F(x) = \frac{1}{3}\int_{-1}^x t^2\,dt = \frac{1}{3}[\frac{t^3}{3}]_{-1}^x$

$= \frac{1}{3}(\frac{x^3}{3} + 1).$

$\therefore F(x) = \begin{cases} 0, & x \le -1 \\ \frac{1}{3}(\frac{x^3}{3} + 1) & -1 < x < 2 \\ 1 & x \ge 2 \end{cases}$

$P(0 < X \le 1) = F(1) - F(0)$

$= \frac{1}{3}(\frac{1}{3} + 1) = \frac{4}{9}.$

3.13

(a) $\int_{\frac{5}{2}b}^{y} \frac{5}{8b}\, dt$

$= \left[\frac{5}{8b} t\right]_{\frac{5}{2}b}^{y}$

$= \frac{5}{8b} y - \frac{5}{8b} \times \frac{2b}{5}$

$= \frac{5}{8b} y - \frac{1}{4}$

$\therefore F(y) = \begin{cases} 0, & y \geq 2b \\ \frac{5}{8b} y - \frac{1}{4}, & \frac{5}{2}b \leq y \leq 2b \\ 1, & y > 2b \end{cases}$

(b) $P(y < b) = F(b).$

$= \frac{5}{8b} b - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{3}{8}.$