20170269 김은진 <home work 2>

Problem1)

$(\neg r \vee \neg f) \rightarrow (s \wedge l)$
$(s \rightarrow t)$
$\neg t$

$\therefore r$

steps	reasons
1) $(\neg r \vee \neg f) \rightarrow (s \wedge l)$	P1 (premise)
2) $(s \rightarrow t)$	P2 (premise)
3) $\neg t$	P3 (premise)
4) $\neg s$	Modus tollens of (2) & (3).
5) $[\neg(\neg r \vee \neg f) \vee (s \wedge l)]$ $\equiv (r \wedge f) \vee (s \wedge l)$.	Equivalences of (1)
6) $\dfrac{r \wedge f}{\therefore r}$ $\dfrac{s \wedge l}{\therefore s}$ $\equiv (r \vee s)$.	simplification of (5).
7) r	Disjunctive syllogism of (4) & (6).

Problem 2)

a) $C(\text{doug}) \wedge j(\text{doug})$
$\forall x (j(x) \rightarrow h(x))$

$\therefore \exists x (C(x) \wedge h(x))$.

Steps	Reasons.
1) $C(\text{doug}) \wedge j(\text{doug})$	P1
2) $\forall x (j(x) \rightarrow h(x))$	P2
3) $j(\text{doug}) \rightarrow h(\text{doug})$	UI of (2)
4) $j(\text{doug})$	and-elimination of (1).
5) $C(\text{doug})$	Simplification of (1)
6) $h(\text{doug})$	Modus ponens of (3) & (4)
7) $C(\text{doug}) \wedge h(\text{doug})$	conjunction of (5) & (6).
8) $\exists x (C(x) \wedge h(x))$	EG of (7).

b) $\exists x (C(x) \wedge W(x))$
$\forall x (W(x) \rightarrow P(x))$

$\therefore \exists x (C(x) \wedge P(x))$

Steps	Reasons
1) $\exists x (C(x) \wedge W(x))$	P1
2) $C(a) \wedge W(a)$	EI of (1)
3) $\forall x (W(x) \rightarrow P(x))$	P2
4) $W(a) \rightarrow P(a)$	UI of (3).
5) $W(a)$	and-elimination of (2)
6) $P(a)$	Modus ponens of (4) & (5)
7) $C(a)$	simplification of (2)
8) $C(a) \wedge P(a)$	conjunction of (6) & (7)
9) $\exists x (C(x) \wedge P(x))$	EG of (8).

c) $\forall x (C(x) \rightarrow P(x))$
$\forall x (P(x) \rightarrow W(x))$
$C(\text{zeke})$

$\therefore W(\text{zeke})$.

Steps	reasons
1) $\forall x (C(x) \rightarrow P(x))$	P1
2) $C(\text{zeke}) \rightarrow P(\text{zeke})$	UI of (1)
3) $\forall x (P(x) \rightarrow W(x))$	P2
4) $P(\text{zeke}) \rightarrow W(\text{zeke})$	UI of (3)
5) $C(\text{zeke}) \rightarrow W(\text{zeke})$	Hypothetical Syllogism of (2) & (4)
6) $C(\text{zeke})$	P3

7) $W(\text{Zeke})$ Modus ponens of (5)&(6).

d) $\forall x(J(x) \to f(x))$

$\exists x(J(x) \wedge \neg S(x))$

$\therefore \exists x(f(x) \wedge \neg S(x))$

Steps	Reasons
1) $\forall x(J(x) \to f(x))$	P1
2) $J(a) \to f(a)$	UI of (1)
3) $\exists x(J(x) \wedge \neg S(x))$	P2
4) $J(a) \wedge \neg S(a)$	EI of (3)
5) $\neg S(a)$	and-elimination of (4).
6) $J(a)$	simplification of (4).
7) $f(a)$	Modus ponens of (2)&(6).
8) $f(a) \wedge \neg S(a)$	conjunction of (5)&(7)
9) $\exists x(f(x) \wedge \neg S(x))$	EG of (8).

Problem 3)

$\forall x(P(x) \to (Q(x) \wedge S(x)))$

$\forall x(P(x) \wedge R(x))$

$\forall x(R(x) \wedge S(x))$.

Steps	Reasons
1) $\forall x(P(x) \to (Q(x) \wedge S(x)))$	P1
2) $P(a) \to (Q(a) \wedge S(a))$	UI of (1)
3) $\forall x(P(x) \wedge R(x))$	P2
4) $P(a) \wedge R(a)$	UI of (3)
5) $P(a)$	Simplification of (4)
6) $Q(a) \wedge S(a)$	Modus Ponens of (2)&(5).
7) $R(a)$	and-elimination of (4)
8) $S(a)$	and-elimination of (6)
9) $R(a) \wedge S(a)$	conjunction of (7)&(8)
10) $\forall x(R(x) \wedge S(x))$	UG of (9).

Problem 4)

a) $\neg(p \to q) \vee (r \to q)$

$\equiv \neg(\neg p \vee q) \vee (\neg r \vee q)$

$\equiv (p \wedge \neg q) \vee (\neg r \vee q)$

$\equiv [p \vee (\neg r \vee q)] \wedge [\neg q \vee (\neg r \vee q)]$

$\equiv (p \vee \neg r \vee q) \wedge (\neg q \vee \neg r \vee q)$.

b) $\neg(p \vee \neg q) \equiv \neg p \wedge q$.

c) $(p \wedge q) \vee (\neg q \wedge r)$

$\equiv [p \vee (\neg q \wedge r)] \wedge [q \vee (\neg q \wedge r)]$

$\equiv [(p \vee \neg q) \wedge (p \vee r)] \wedge [(q \vee \neg q) \wedge (q \vee r)]$

$\equiv (p \vee \neg q) \wedge (p \vee r) \wedge (q \vee \neg q) \wedge (q \vee r)$.

d) $(p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$

$\equiv [(p \vee (\neg p \wedge \neg q \wedge r)] \wedge [\neg q \vee (\neg p \wedge \neg q \wedge r)] \wedge [\neg r \vee (\neg p \wedge \neg q \wedge r)]$

$\equiv [(p \vee \neg p) \wedge (p \vee \neg q) \wedge (p \vee r)] \wedge [(\neg q \vee \neg p) \wedge (\neg q \vee \neg q) \wedge (\neg q \vee \neg r)] \wedge [(\neg r \vee \neg p) \wedge (\neg r \vee \neg q) \wedge (\neg r \vee r)]$

$\equiv (p \vee \neg p) \wedge (p \vee \neg q) \wedge (p \vee r) \wedge (\neg q \vee \neg p) \wedge (\neg q \vee \neg q) \wedge (\neg q \vee \neg r) \wedge (\neg r \vee \neg p) \wedge (\neg r \vee \neg q) \wedge (\neg r \vee r)$.

e) $(p\to q)\vee\neg(q\vee\neg r)$

$\equiv(\neg p\vee q)\vee(\neg q\wedge r)$

$\equiv(\neg q\wedge r)\vee(\neg p\vee q)$

$\equiv[\neg q\vee(\neg p\vee q)]\wedge[r\vee(\neg p\vee q)]$

$\equiv(\neg q\vee\neg p\vee q)\wedge(r\vee\neg p\vee q)$

f) $\neg(p\wedge q)\leftrightarrow(p\vee q)$

$\equiv(\neg p\vee\neg q)\leftrightarrow(p\vee q)$

$\equiv[(\neg p\vee\neg q)\to(p\vee q)]\wedge$
$[(p\vee q)\to(\neg p\vee\neg q)]$

$\equiv[\neg(\neg p\vee\neg q)\vee(p\vee q)]\wedge[\neg(p\vee q)\vee$
$(\neg p\vee\neg q)]$.

$\equiv[(p\wedge q)\vee(p\vee q)]\wedge[(\neg p\wedge\neg q)\vee$
$(\neg p\vee\neg q)]$.

$\equiv[(p(p\vee q))\wedge(q\vee(p\vee q))]\wedge$
$[(\neg p\vee(\neg p\vee\neg q))\wedge(\neg q\vee(\neg p\vee\neg q))]$

$\equiv(p\vee p\vee q)\wedge(q\vee p\vee q)\wedge(\neg p\vee\neg p\vee\neg q)$
$\wedge(\neg q\vee(\neg p\vee\neg q)$.

problem 6)

a) $a\wedge(b\leftrightarrow c)$

$\equiv a\wedge[(b\to c)\wedge(c\to b)]$.

$\equiv a\wedge[(\neg b\vee c)\wedge(\neg c\vee b)]$.

$\equiv a\wedge[(\neg b\wedge(\neg c\vee b))\vee(c\wedge(\neg c\vee b))]$

$\equiv a\wedge[((\neg b\wedge\neg c)\vee(\neg b\wedge b))\vee((c\wedge\neg c)\vee(c\wedge b))]$

$\equiv[(((\neg b\wedge\neg c)\vee(\neg b\wedge b))\wedge a)\vee(((c\wedge\neg r)\vee(c\wedge b)$
$))\wedge a)]$

$\equiv(\neg b\wedge\neg c\wedge a)\vee(\neg b\wedge b\wedge a)\vee(c\wedge\neg r\wedge a)$
$\vee(c\wedge b\wedge a)$.

b) $(a\to b)\wedge(\neg a\to\neg b)$.

$\equiv(\neg a\vee b)\wedge(a\vee b)$

$\equiv[\neg a\wedge(a\vee b)]\vee[b\wedge(a\vee b)]$

$\equiv(\neg a\wedge a)\vee(\neg a\wedge b)\vee(b\wedge a)\vee(b\vee b)$

c) $p\leftrightarrow q$

$\equiv(p\to q)\wedge(q\to p)$.

$\equiv(\neg p\vee q)\wedge(\neg q\vee p)$.

$\equiv(\neg p\wedge(\neg q\vee p)]\vee[q\wedge(\neg q\vee p)]$.

$\equiv[(\neg p\wedge\neg q)\vee(\neg p\wedge p)]\vee[(q\wedge\neg q)\vee$
$(q\wedge p)]$.

$\equiv(\neg p\wedge\neg q)\vee(\neg p\wedge p)\vee(q\wedge\neg q)\vee(q\wedge p)$

d) $(p\to q)\wedge\neg q$

$\equiv(\neg p\vee q)\wedge\neg q$.

$\equiv(\neg p\wedge\neg q)\vee(q\wedge\neg q)$