BLM2041 Signals and Systems

Syllabus

The Instructors:

Doç. Dr. Ali Can Karaca

ackaraca@yildiz.edu.tr

Dr. Öğr. Üyesi Erkan Uslu

euslu@yildiz.edu.tr

DEFINITION(S) OF SYSTEM

A system can be broadly defined as an integrated set of elements that accomplish a defined objective.

People from different engineering disciplines have different perspectives of what a "system" is.

For example,

- software ennngineers often refer to an integrated set of computer programs as a "system"
- electrical engineers might refer to complex integrated circuits or an integrated set of electrical units as a "system"

As can be seen, "system" depends on one's perspective, and the "integrated set of elements that accomplish a defined objective" is an appropriate definition.

Information Systems:

Fundamentals

Definition(s) of system

- A system is an assembly of parts where:
  - The parts or components are connected together in an organized way.
  - The parts or components are affected by being in the system (and are changed by leaving it).
  - The assembly does something.
  - The assembly has been identified by a person as being of special interest.
- Any arrangement which involves the handling, processing or manipulation of resources of whatever type can be represented as a system.
- Some definitions on online dictionaries
  - http://en.wikipedia.org/wiki/System
  - http://dictionary.reference.com/browse/systems
  - http://www.businessdictionary.com/definition/system.html

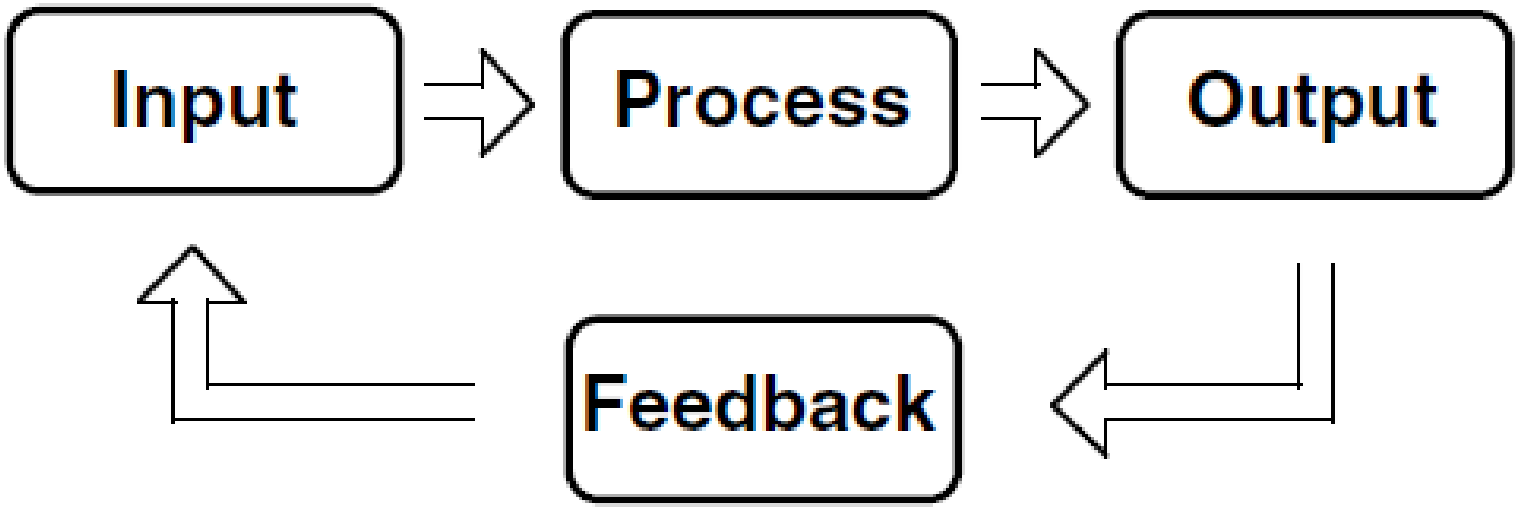
Definition(s) of system

- A system is defined as multiple parts working together for a common purpose or goal.
- Systems can be large and complex
  - such as the air traffic control system or our global telecommunication network.
- Small devices can also be considered as systems
  - such as a pocket calculator, alarm clock, or 10-speed bicycle.

Definition(s) of system

- Systems have inputs, processes, and outputs.
- When feedback (direct or indirect) is involved, that component is also important to the operation of the system.
- To explain all this, systems are usually explained using a model.
- A model helps to illustrate the major elements and their relationship, as illustrated in the next slide

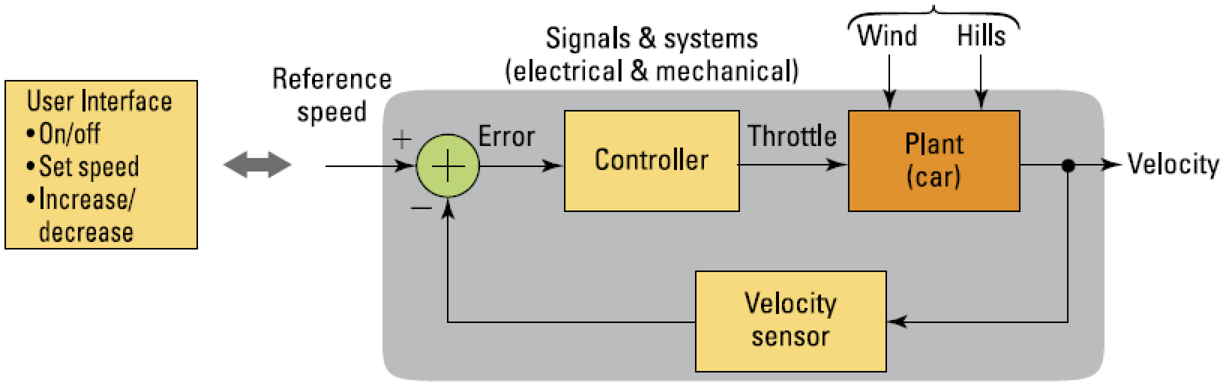
A systems model



Arabalardaki Hız Sabitleme Sistemi

Giriş işareti: Referans hız, rüzgar ve yokuş olma durumu

Çıkış işareti: Arabanın hızı

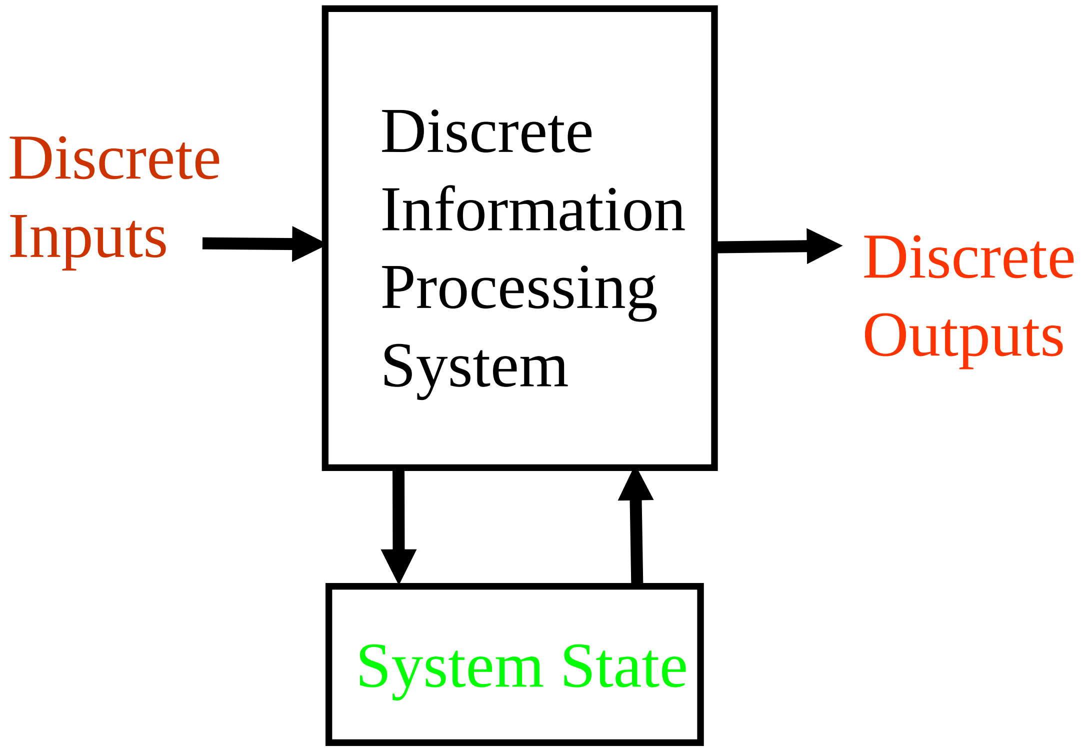


Sistem

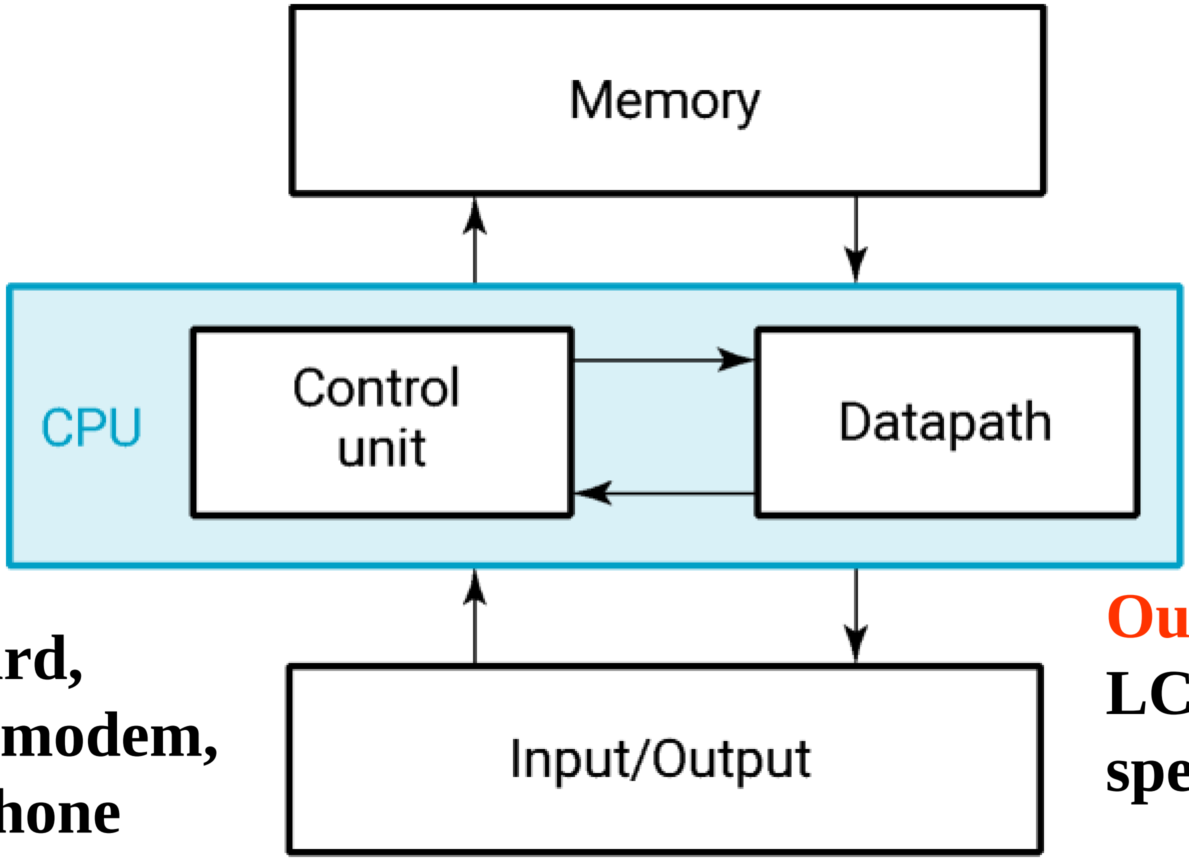
Ref: *Signals and Systems for Dummies*

Digital System

- **Takes a set of discrete information (inputs) and discrete internal information (system state) and generates a set of discrete information (outputs).**



A Digital Computer Example



Inputs: Keyboard, mouse, modem, microphone

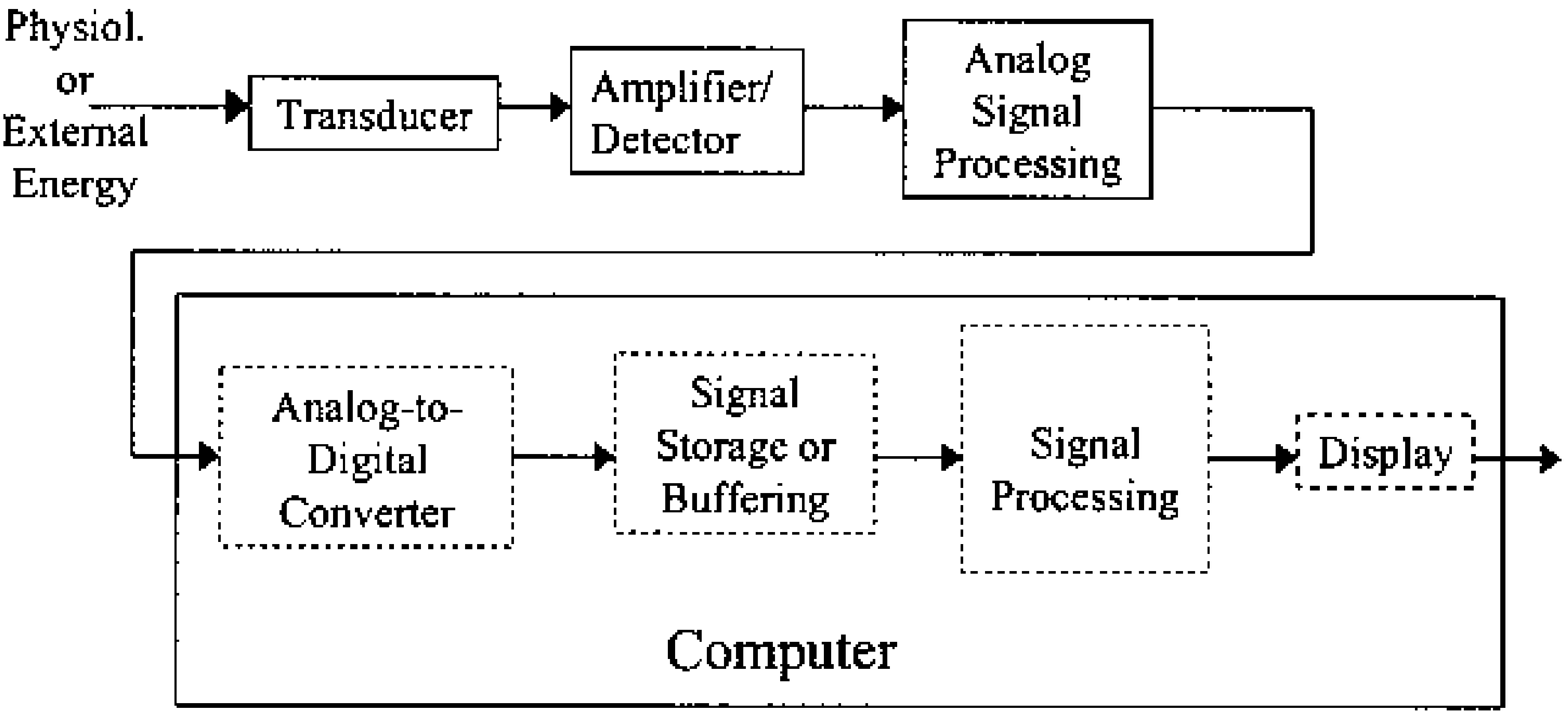
Outputs: CRT, LCD, modem, speakers

Synchronous or Asynchronous?

Signal

- **An information variable represented by physical quantity.**
- **For digital systems, the variable takes on discrete values.**
- **Two level, or binary values are the most prevalent values in digital systems.**
- **Binary values are represented abstractly by:**
  - **digits 0 and 1**
  - **words (symbols) False (F) and True (T)**
  - **words (symbols) Low (L) and High (H)**
  - **and words On and Off.**
- **Binary values are represented by values or ranges of values of physical quantities**

A typical measurement system



Transducers

- A "transducer" is a device that converts energy from one form to another.
- In signal processing applications, the purpose of energy conversion is to transfer information, not to transform energy.
- In physiological measurement systems, transducers may be
  - input transducers (or sensors)
    - they convert a non-electrical energy into an electrical signal.
    - for example, a microphone.
  - output transducers (or actuators)
    - they convert an electrical signal into a non-electrical energy.
    - For example, a speaker.

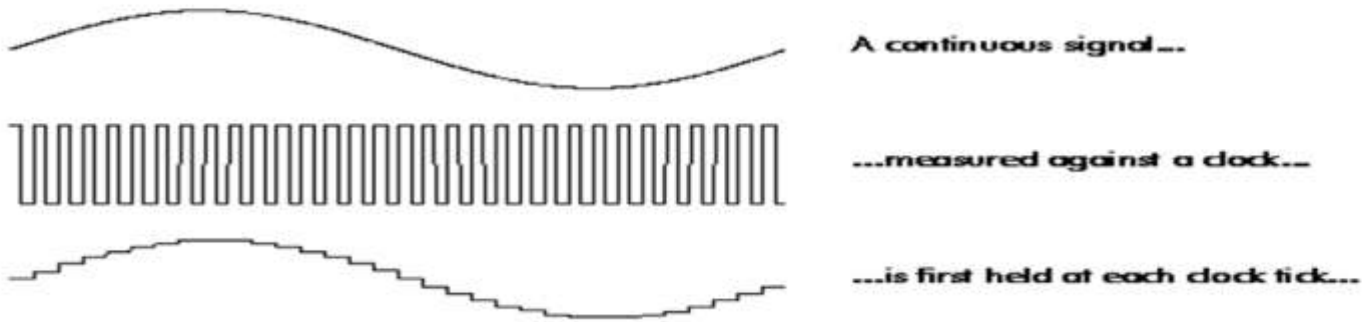
Analogue signal

- The analogue signal
  - a continuous variable defined with infinite precision

  is converted to a discrete sequence of measured values which are represented digitally
- Information is lost in converting from analogue to digital, due to:
  - inaccuracies in the measurement
  - uncertainty in timing
  - limits on the duration of the measurement
- These effects are called quantisation errors

Digital signal

- The continuous analogue signal has to be held before it can be sampled

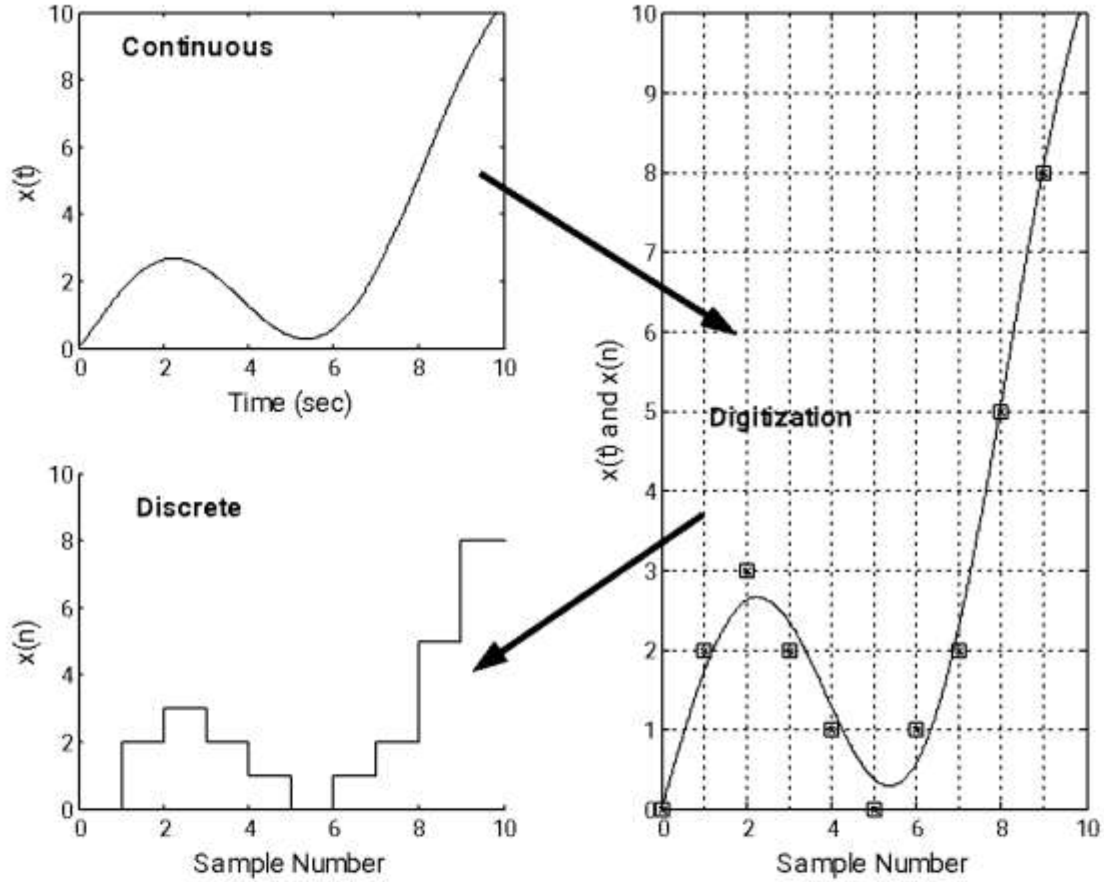


- Otherwise, the signal would be changing during the measurement
- Only after it has been held can the signal be measured, and the measurement converted to a digital valu

Signal Encoding: Analog-to Digital Conversion

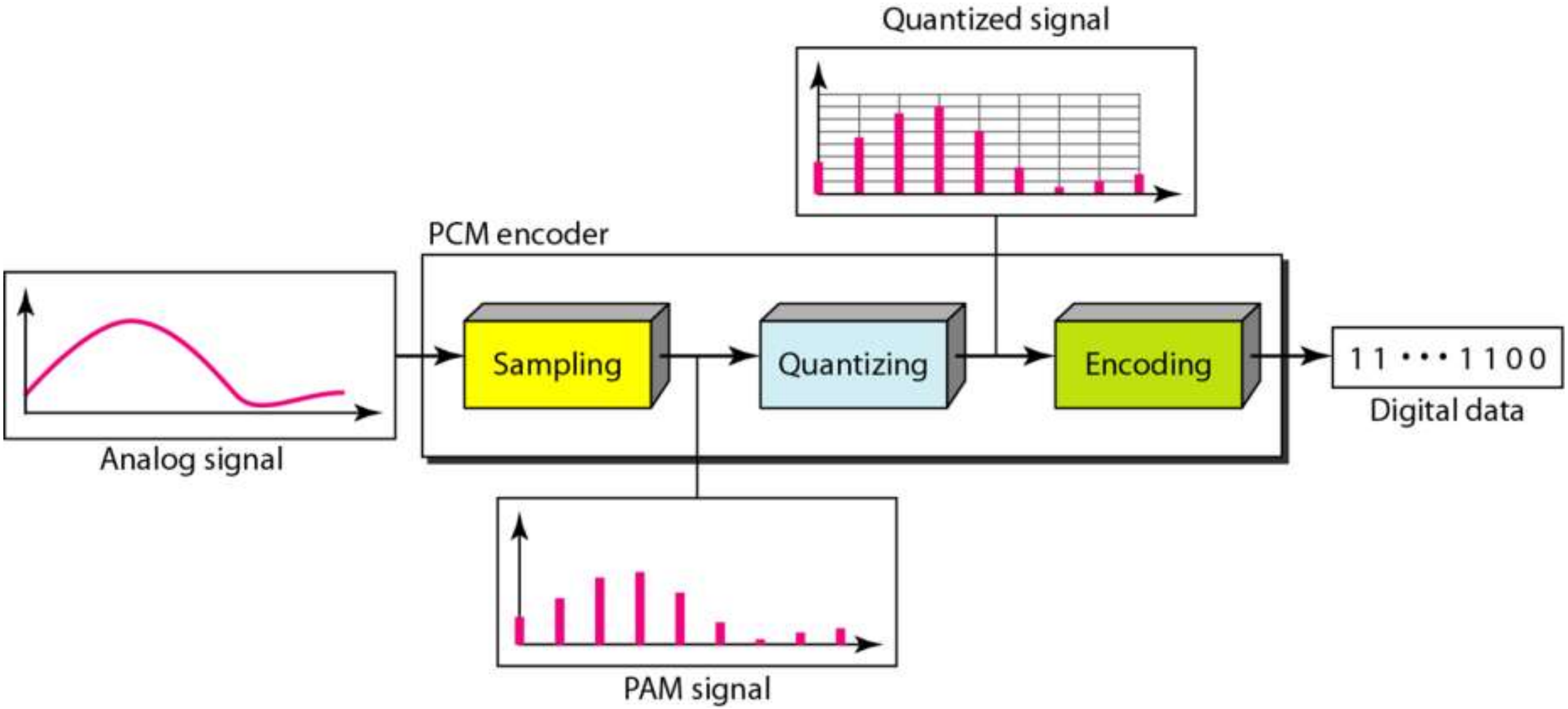
Continuous (analog) signal $\leftrightarrow$ Discrete signal

x(t) = f(t) $\leftrightarrow$ Analog to digital conversion $\leftrightarrow$ x[n] = x [1], x [2], x [3], ... x[n]



Analog-to Digital Conversion

- ADC consists of four steps to digitize an analog signal:
  1. Filtering
  2. Sampling
  3. Quantization
  4. Binary encoding
- Before we sample, we have to filter the signal to limit the maximum frequency of the signal as it affects the sampling rate.
- Filtering should ensure that we do not distort the signal, ie remove high frequency components that affect the signal shape.

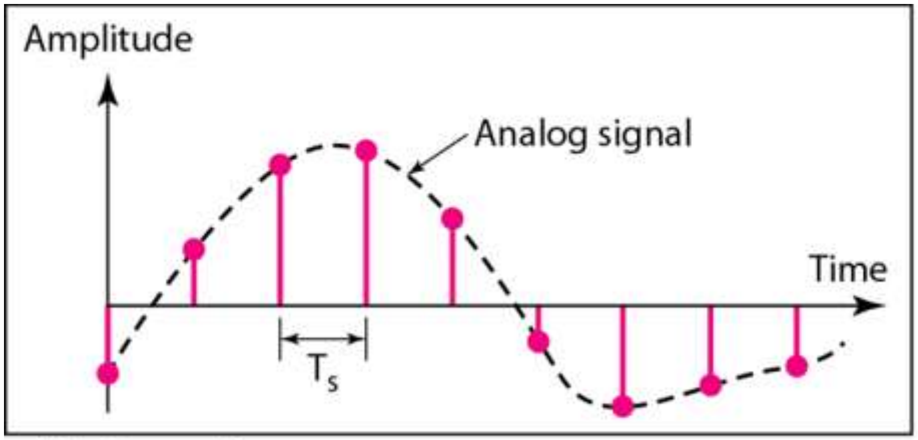

Quantized signal
PCM encoder
Sampling
Quantizing
Encoding
1 1 • • • 1 1 0 0
Digital data
Analog signal
PAM signal

Sampling

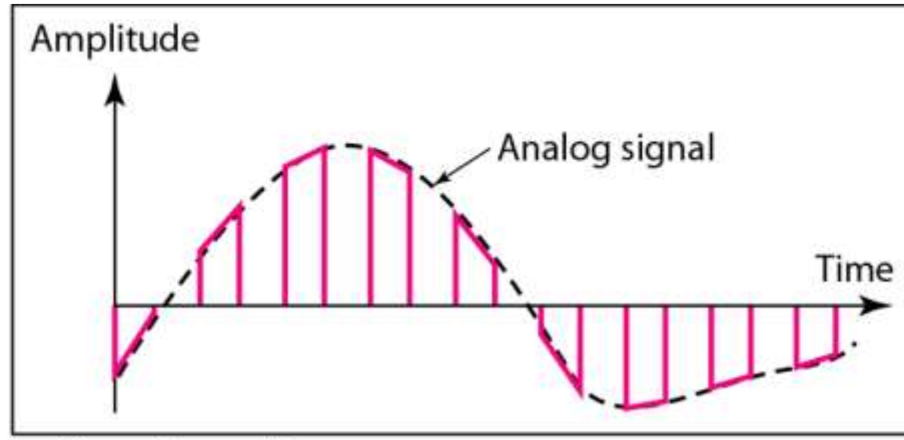
- The sampling results in a discrete set of digital numbers that represent measurements of the signal
  - usually taken at equal intervals of time
- Sampling takes place after the hold
  - The hold circuit must be fast enough that the signal is not changing during the time the circuit is acquiring the signal value
- We don't know what we don't measure
- In the process of measuring the signal, some information is lost

Sampling

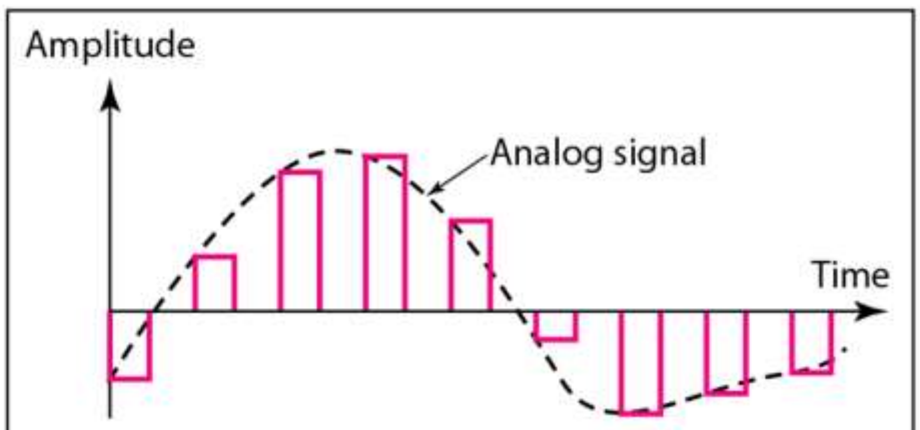
- Analog signal is sampled every T_S secs.
- T_s is referred to as the sampling interval.
- $f_s = 1/T_s$ is called the sampling rate or sampling frequency.
- There are 3 sampling methods:
  - Ideal - an impulse at each sampling instant
  - Natural - a pulse of short width with varying amplitude
  - Flattop - sample and hold, like natural but with single amplitude value
- The process is referred to as pulse amplitude modulation PAM and the outcome is a signal with analog (non integer) values



a. Ideal sampling

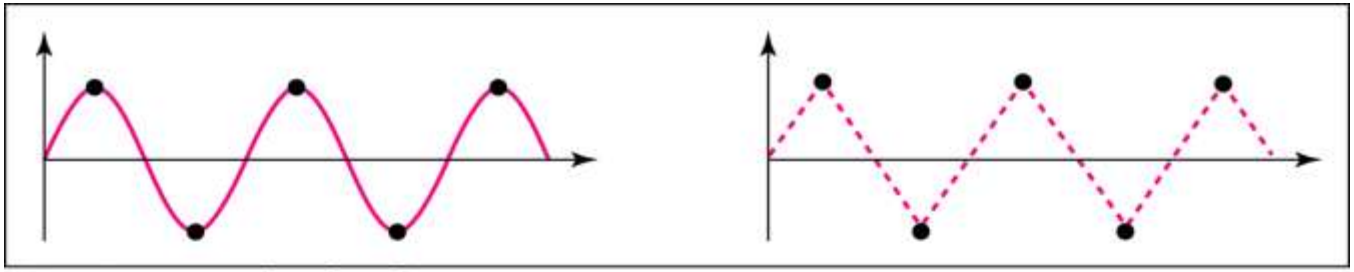


b. Natural sampling

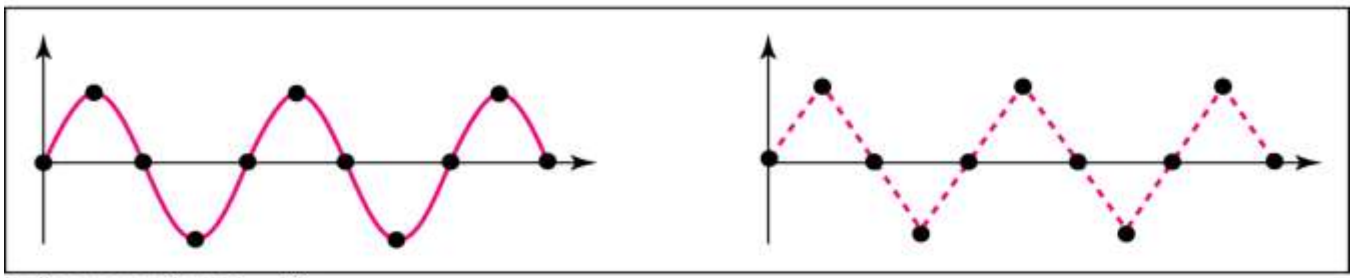


c. Flat-top sampling

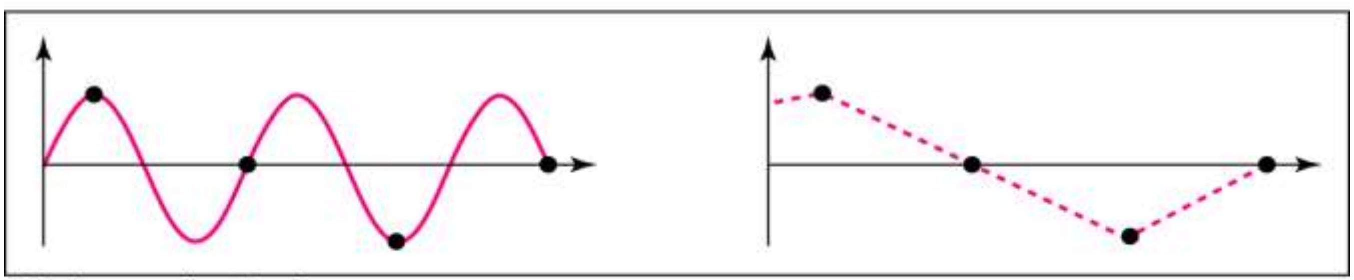
Recovery of a sampled sine wave for different sampling rates



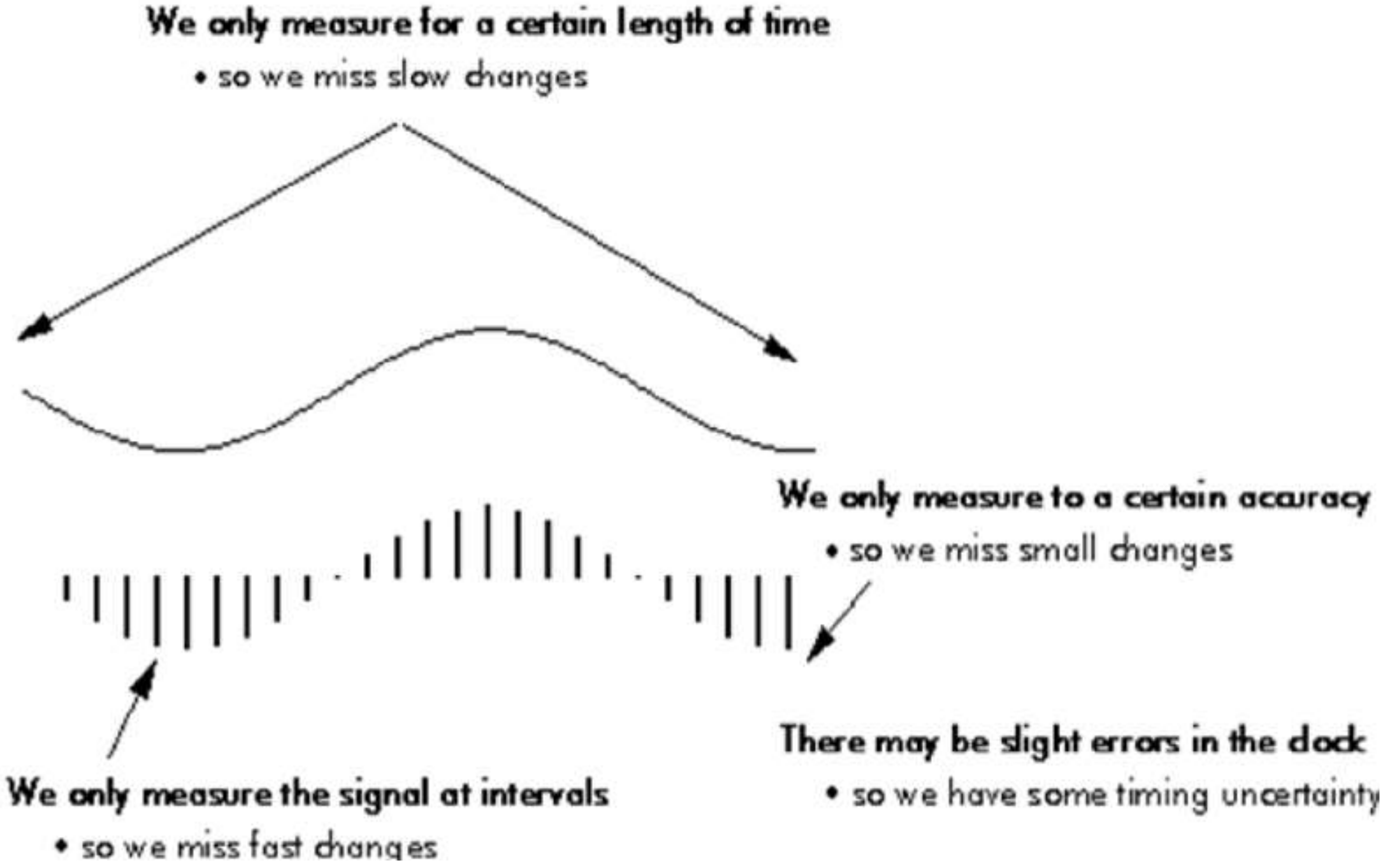
a. Nyquist rate sampling: $f_s = 2\,f$

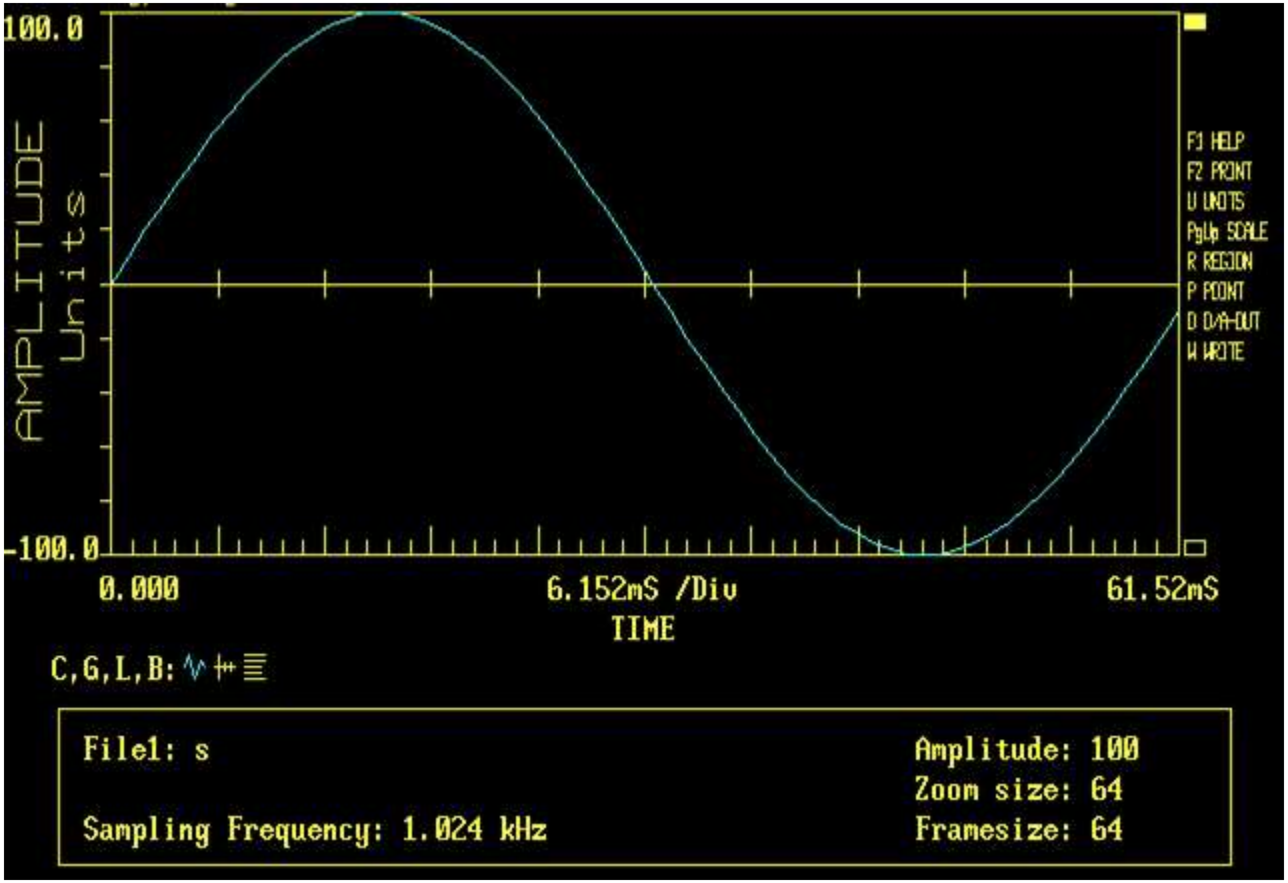


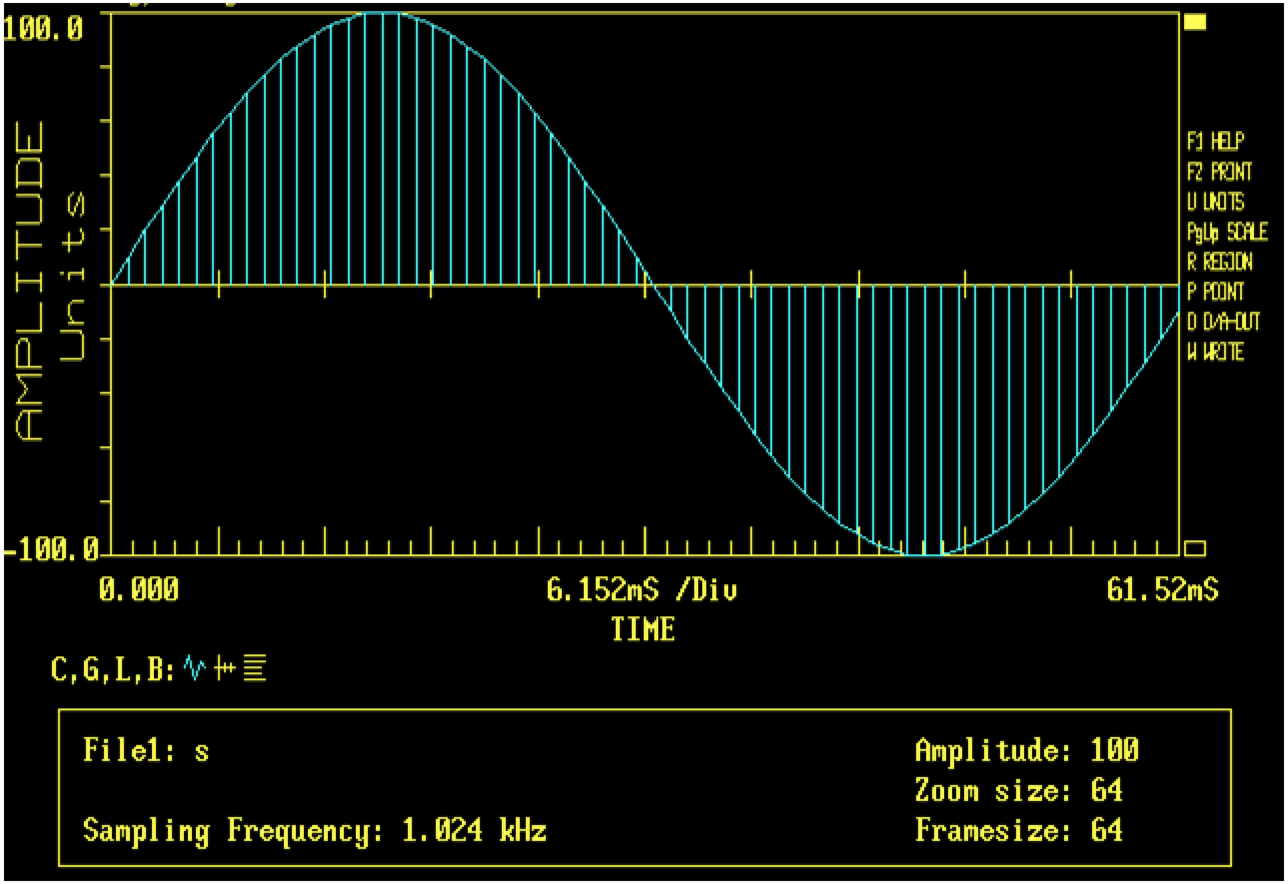
b. Oversampling: $f_s = 4\,f$



c. Undersampling: $f_s = f$


We only measure for a certain length of time
• so we miss slow changes
We only measure to a certain accuracy
• so we miss small changes
There may be slight errors in the clock
• so we have some timing uncertainty
We only measure the signal at intervals
• so we miss fast changes


100.0
-100.0
AMPLITUDE
Units
F1 HELP
F2 PRINT
U UNITS
PgUp SCALE
R REGION
P POINT
D D/A-OUT
W WRITE
0.000
6.152mS /Div
61.52mS
TIME
C,G,L,B:
File1: s
Sampling Frequency: 1.024 kHz
Amplitude: 100
Zoom size: 64
Framesize: 64



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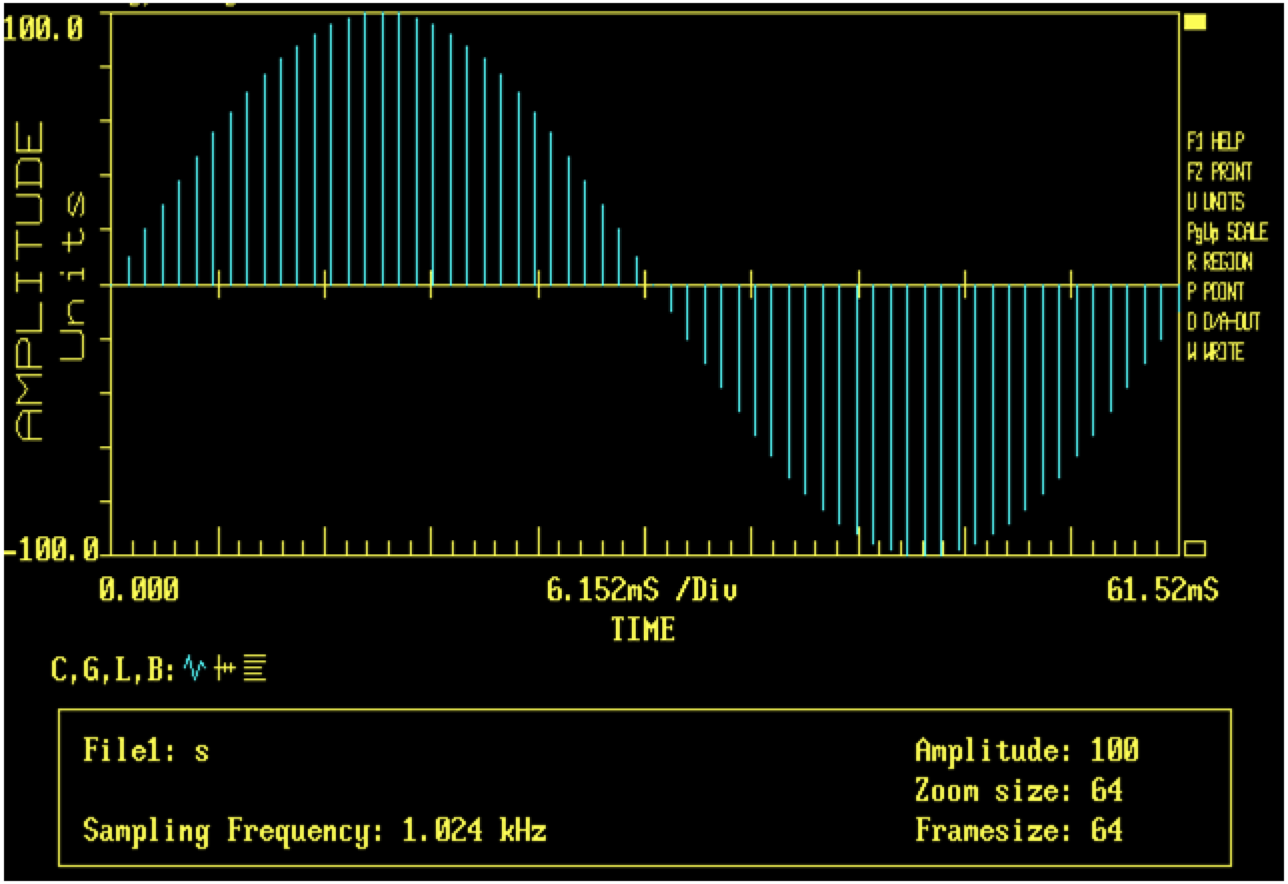
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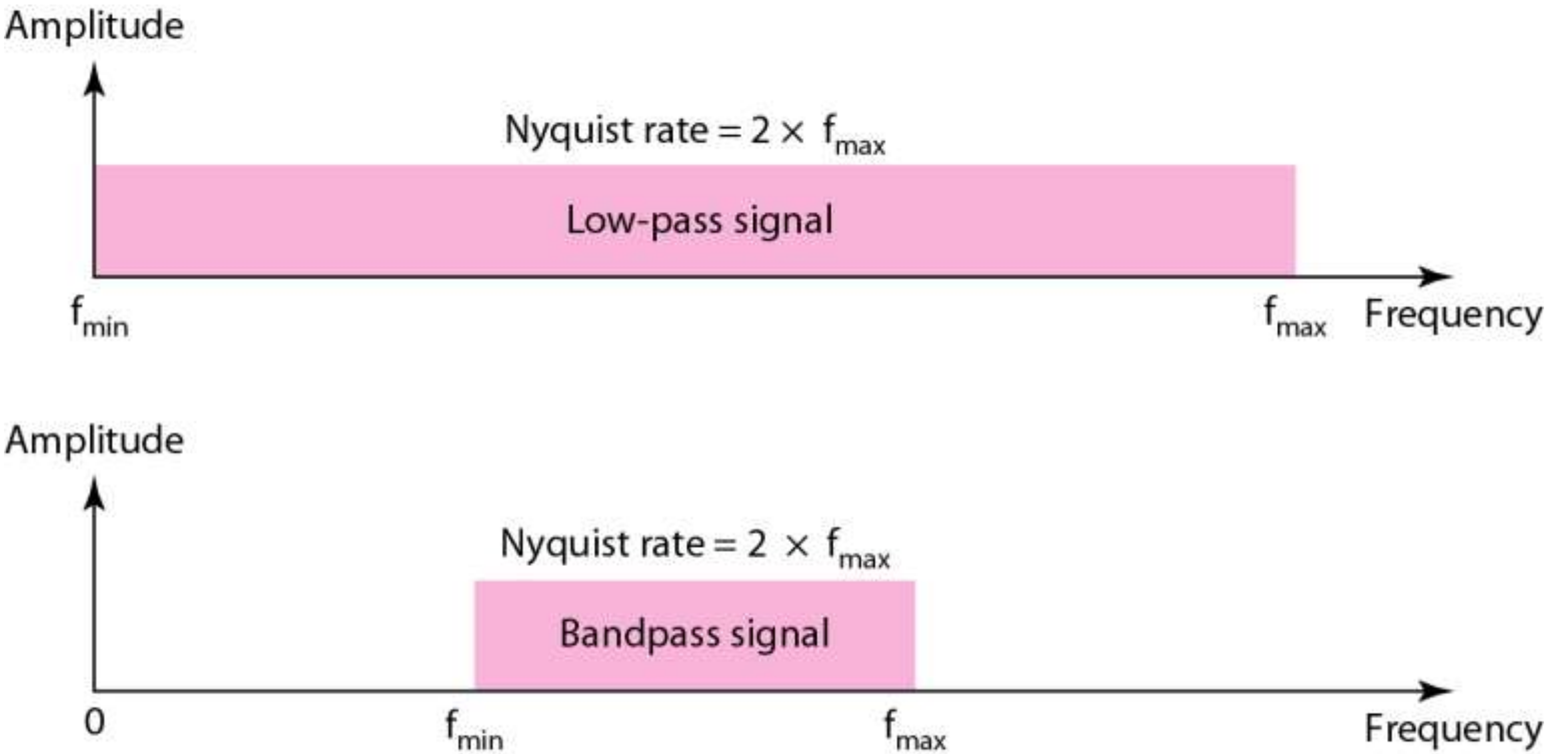
Sampling Theorem

$$F_s \geq 2f_m$$

According to the Nyquist theorem, the sampling rate must be at least 2 times the highest frequency contained in the signal.

Nyquist sampling rate for low-pass and bandpass signals



Quantization

- Sampling results in a series of pulses of varying amplitude values ranging between two limits: a min and a max.
- The amplitude values are infinite between the two limits.
- We need to map the *infinite* amplitude values onto a finite set of known values.
- This is achieved by dividing the distance between min and max into L zones, each of height Δ.

$$\Delta = (max - min)/L$$

Quantization Levels

- The midpoint of each zone is assigned a value from 0 to L-1 (resulting in L values)
- Each sample falling in a zone is then approximated to the value of the midpoint.

Quantization Zones

- Assume we have a voltage signal with amplitutes V_{min}=-20V and V_{max}=+20V.
- We want to use L=8 quantization levels.
- Zone width $\Delta = (20 - -20)/8 = 5$
- The 8 zones are: -20 to -15, -15 to -10, -10 to -5, -5 to 0, 0 to +5, +5 to +10, +10 to +15, +15 to +20
- The midpoints are: -17.5, -12.5, -7.5, -2.5, 2.5, 7.5, 12.5, 17.5

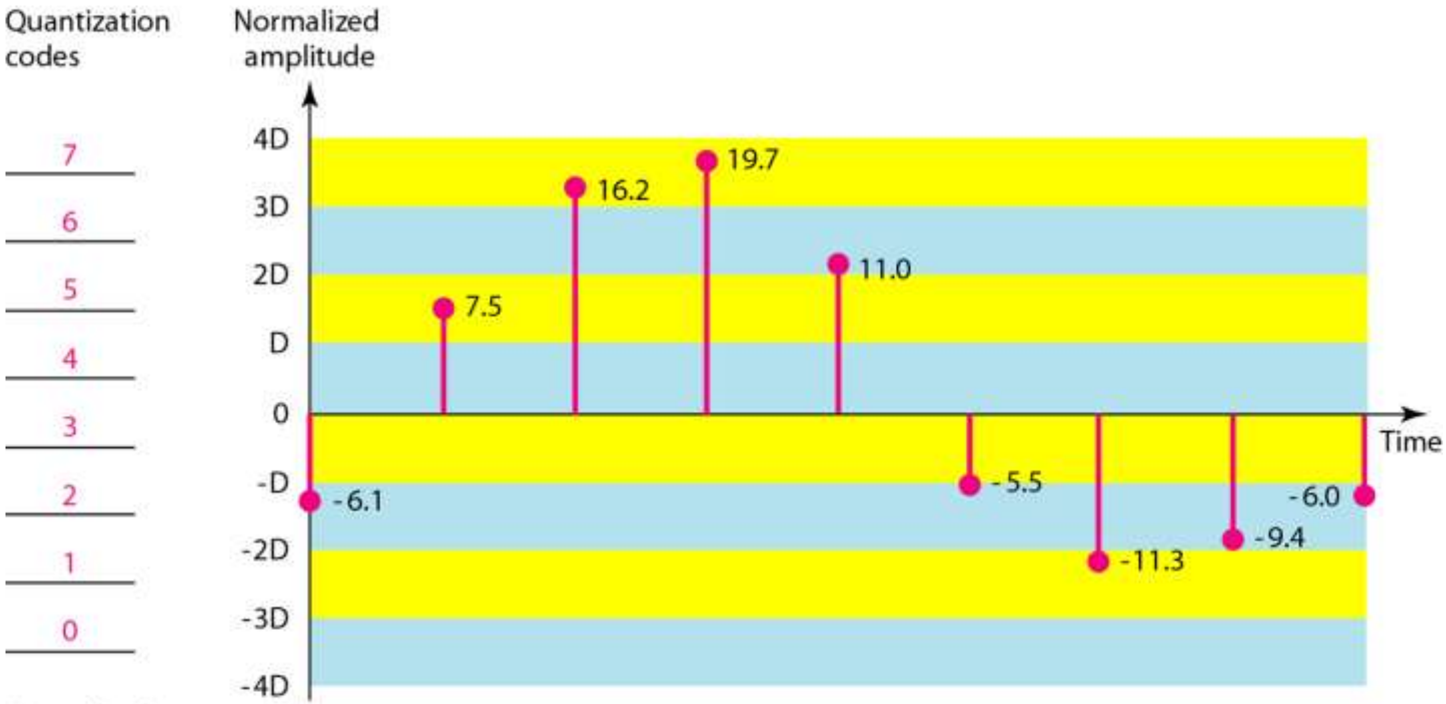
Assigning Codes to Zones

- Each zone is then assigned a binary code.
- The number of bits required to encode the zones, or the number of bits per sample as it is commonly referred to, is obtained as follows:

$$n_b = \log_2 L$$

- Given our example, $n_b = 3$
- The 8 zone (or level) codes are therefore: 000, 001, 010, 011, 100, 101, 110, and 111
- Assigning codes to zones:
  - 000 will refer to zone -20 to -15
  - 001 to zone -15 to -10, etc.

Quantization and encoding of a sampled signal



Normalized PAM values	-1.22	1.50	3.24	3.94	2.20	-1.10	-2.26	-1.88	-1.20
Normalized quantized values	-1.50	1.50	3.50	3.50	2.50	-1.50	-2.50	-1.50	-1.50
Normalized error	-0.38	0	+0.26	-0.44	+0.30	-0.40	-0.24	+0.38	-0.30
Quantization code	2	5	7	7	6	2	1	2	2
Encoded words	010	101	111	111	110	010	001	010	010

Quantization Error

- When a signal is quantized, we introduce an error
  - the coded signal is an approximation of the actual amplitude value.
- The difference between actual and coded value (midpoint) is referred to as the quantization error.
- The more zones, the smaller Δ
  - which results in smaller errors.
- BUT, the more zones the more bits required to encode the samples
  - higher bit rate

Analog-to-digital Conversion

Example An 12-bit analog-to-digital converter (ADC) advertises an accuracy of ± the least significant bit (LSB). If the input range of the ADC is 0 to 10 volts, what is the accuracy of the ADC in analog volts?

<u>Solution:</u>

If the input range is 10 volts then the analog voltage represented by the LSB would be:

$$V_{LSB} = \frac{V_{max}}{2^{\text{Nu bits}}} = \frac{10}{2^{12}} = \frac{10}{4096} = .0024 \text{ volts}$$

Hence the accuracy would be ± 0.0024 volts.

Steps for digitization/reconstruction of a signal

- Band limiting (LPF)
- Sampling / Holding
- Quantization
- Coding

These are basic steps for A/D conversion

- D/A converter
- Sampling / Holding
- Image rejection

These are basic steps for reconstructing a sampled digital signal

Digital data: end product of A/D conversion and related concepts

- Bit: least digital information, binary 1 or 0
- Nibble: 4 bits
- Byte: 8 bits, 2 nibbles
- Word: 16 bits, 2 bytes, 4 nibbles
- Some jargon:
  - integer, signed integer, long integer, 2s complement, hexadecimal, octal, floating point, etc.


MP3 Audio
Recording
ADC
00000
00000
DSP
00000
MEMORY
DAC
00000
00000
Playback

Example

- Hertz = clock cycles per second (frequency)
  - 1MHz = 1,000,000Hz
  - Processor speeds are measured in MHz or GHz.
- Byte = a unit of storage
  - 1KB = 2^{10} = 1024 Bytes
  - 1MB = 2^{20} = 1,048,576 Bytes
  - Main memory (RAM) is measured in MB
  - Disk storage is measured in GB for small systems, TB for large systems.

Number of Bits Required

- Given M elements to be represented by a binary code, the minimum number of bits, *n*, needed, satisfies the following relationships:

  $2^n \geq M > 2^{(n-1)}$

  $n = \lceil \log_2 M \rceil$ where $\lceil x \rceil$, called the *ceiling function,* is the integer greater than or equal to x.

- Example: How many bits are required to represent decimal digits with a binary code?
  - 4 bits are required ($n = \lceil \log_2 9 \rceil = 4$)

Number of Elements Represented

- Given n digits in radix r, there are r^n distinct elements that can be represented.
- But, you can represent m elements, $m < r^n$
- Examples:
  - You can represent 4 elements in radix $r = 2$ with $n = 2$ digits: (00, 01, 10, 11).
  - You can represent 4 elements in radix $r = 2$ with $n = 4$ digits: (0001, 0010, 0100, 1000).