Digital Design & Computer Architecture

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Chapter 1:

From Zero to One

Chapter 1 :: Topics

- **The Art of Managing Complexity**
- **Number Systems**
  - Binary Numbers
  - Hexadecimal Numbers
  - Bits, Bytes, Nibbles
  - Addition
  - Signed Numbers
  - Extension
- **Logic Gates**
- **Logic Levels**
- **CMOS Transistors**
- **Transistor-Level Gate Design**
- **Power Consumption**

Chapter 1: From Zero to One

The Art of Managing Complexity

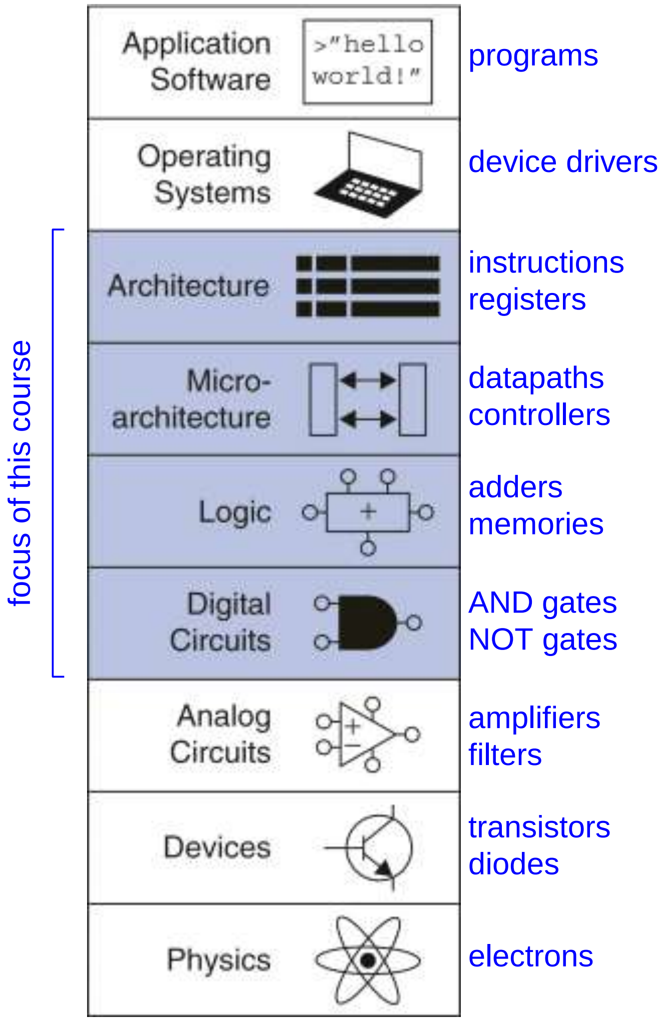
The Art of Managing Complexity

- How do we design things that are too big to fit in one person’s head at once?

- Abstraction
- Discipline
- The Three –y’s
  - Hierarchy
  - Modularity
  - Regularity

Abstraction

Hiding details when they aren't important



Discipline

- Intentionally restrict design choices
- Example: Digital discipline
  - Discrete voltages instead of continuous
  - Simpler to design than analog circuits – can build more sophisticated systems
  - Digital systems replacing analog predecessors: i.e., digital cameras, digital television, cell phones, CDs

The Three -y's

- **Hierarchy**
  –
- **Modularity**
  –
- **Regularity**
  –

Example: Model T Ford

- Famous early (1908) example of interchangeable parts.
  - Most previous cars were hand-crafted by skilled tradesmen.
  - Mass production on moving assembly lines greatly reduced cost.

en.wikipedia.org/wiki/Ford_Model_T#/media/File:1925_Ford_Model_T_touring.jpg

- Henry Ford:

  I will build a motor car for the great multitude. It will be large enough for the family, but small enough for the individual to run and care for. It will be constructed of the best materials, by the best men to be hired, after the simplest designs that modern engineering can devise. But it will be so low in price that no man making a good salary will be unable to own one – and enjoy with his family the blessing of hours of pleasure in God's great open spaces.

Example: Model T Ford

- **Hierarchy**
  - Car has chassis, wheels, seats, engine.
  - Engine has cylinders, spark plugs, exhaust, carburetor.
  - Carburator has air intake, inlet needle, feed pipe, coupling nut.

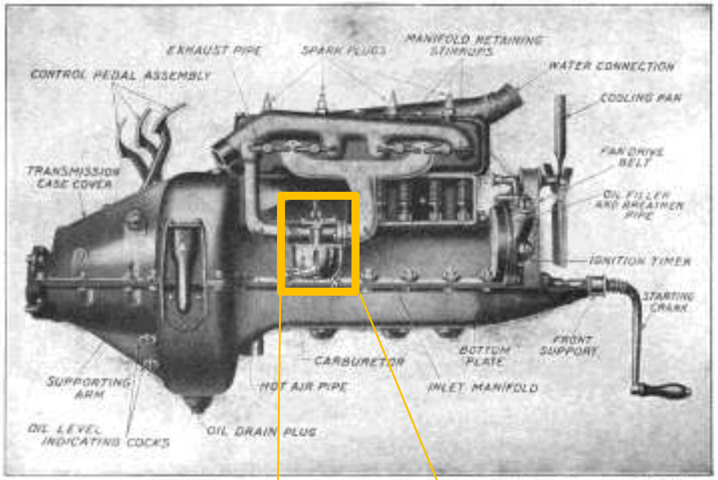


Fig. 8—Valve Side of the Ford Model T Unit Power Plant Showing Manifolds, Carburetor and Interior of One of the Valve Spring Chambers.

https://en.wikipedia.org/wiki/Ford_Model_T_engine#/media/File:Pagé_1917_Model_T_Ford_Car_Figure_08.png

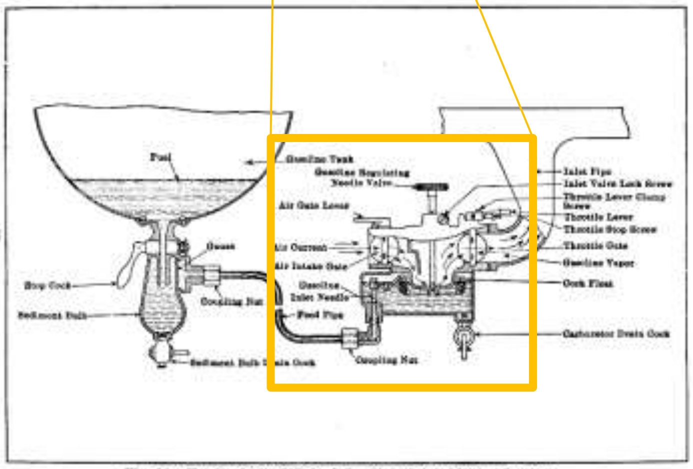


Fig. 14—The Ford Model T Fuel Supply and Gas Making System.

https://en.wikipedia.org/wiki/Ford_Model_T_engine#/media/File:Pagé_1917_Model_T_Ford_Car_Figure_14.png

Example: Model T Ford

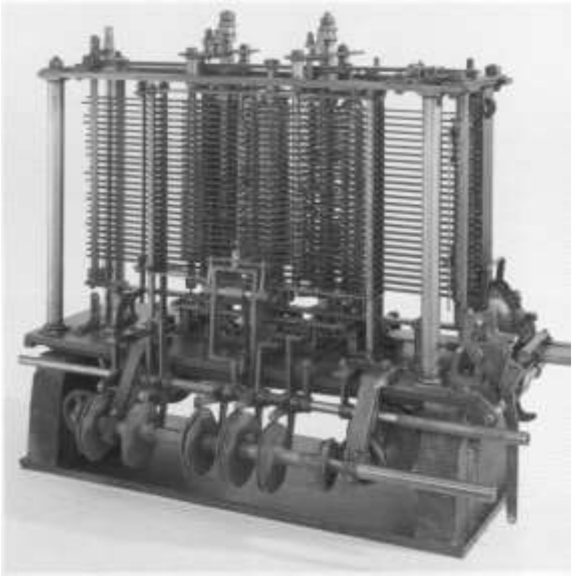
- **Modularity**
  - **Function of coupling nut:**
    - Hold fuel line to intake elbow
    - Prevent leaks
    - Easily removable
  - **Interface of coupling nut:**
    - Standardized diameter, thread pitch, torque
- **Regularity**
  - Interchangeable parts
    - Standard nut could be purchased from many suppliers
  - "Any customer can have a car painted any color that he wants so long as it is black." - Henry Ford

The Digital Abstraction

- Most physical variables are **continuous**
  - Voltage on a wire
  - Frequency of an oscillation
  - Position of a mass
- Digital abstraction considers **discrete subset** of values

The Analytical Engine

- Designed by Charles Babbage from 1834 – 1871
- Considered to be the first digital computer
- Built from mechanical gears, where each gear represented a discrete value (0-9)
- Babbage died before finishing it



Digital Discipline: Binary Values

- **Two discrete values:**
  - 1's and 0's
  - 1, TRUE, HIGH
  - 0, FALSE, LOW
- **1 and 0:** voltage levels, rotating gears, fluid levels, etc.
- Digital circuits use voltage levels
  - 0: low voltage (GND)
  - 1: high voltage (V_{DD})
- ***Bit***: *B*inary dig*it*

Number Systems: Binary Numbers

Number Systems

- Decimal numbers

Decimal numbers in digital systems mean any base 10 numbers, not just those with a decimal point.

1000's column
100's column
10's column
1's column

$$5374_{10} = 5 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 4 \times 10^0$$

five thousands | three hundreds | seven tens | four ones

- Binary numbers

8's column
4's column
2's column
1's column

$$1101_2 =$$

Counting in Binary

Binary	Decimal
0	0
1	1
10	2
11	3
100	4
101	5
…	

Powers of Two

- $2^0 =$
- $2^1 =$
- $2^2 =$
- $2^3 =$
- $2^4 =$
- $2^5 =$
- $2^6 =$
- $2^7 =$
- $2^8 =$
- $2^9 =$
- $2^{10} =$
- $2^{11} =$
- $2^{12} =$
- $2^{13} =$
- $2^{14} =$
- $2^{15} =$

Handy to memorize

Number Conversion

- Binary to decimal conversion:
  - Convert 10011_2 to decimal
  -
- Decimal to binary conversion:
  - Convert 47_{10} to binary
  -

Decimal to Binary Conversion

- Two methods:
  - **Method 1:** Find the largest power of 2 that fits, subtract and repeat
  - **Method 2:** Repeatedly divide by 2, remainder goes in next most significant bit

Decimal to Binary Conversion

53_{10}

Method 1: Find the largest power of 2 that fits, subtract and repeat

Method 2: Repeatedly divide by 2, remainder goes in next most significant bit

Binary Values and Range

- ***N*-digit decimal number**
  - How many values?
  - Range?
  - Example: 3-digit decimal number:
    -
    -

- ***N*-bit binary number**
  - How many values?
  - Range:
  - Example: 3-digit binary number:
    -
    -

Number Systems: Hexadecimal Numbers

Hexadecimal Numbers

Hex Digit	Decimal Equivalent	Binary Equivalent
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Hexadecimal Numbers

- Base 16
- Shorthand for binary

Hexadecimal to Binary Conversion

- Hexadecimal to binary conversion:
  - Convert $4AF_{16}$ (also written 0x4AF) to binary
  -

- Hexadecimal to decimal conversion:
  - Convert $4AF_{16}$ to decimal
  -

Hexadecimal and Binary Prefixes

- Hard to write subscripts in text files
- Some programming languages uses prefixes
  - Hex: 0x
    - 0x23AB = $23AB_{16}$
  - Binary: 0b
    - 0b1101 = 1101_2

Number Systems: Bytes, Nibbles, & All That Jazz

Bits, Bytes, Nibbles…

- Byte: 8 bits
  - Represents one of _____ values
  - [__, ___]
- Nibble: 4 bits
  - Represents one of _____ values
  - [__, ___]

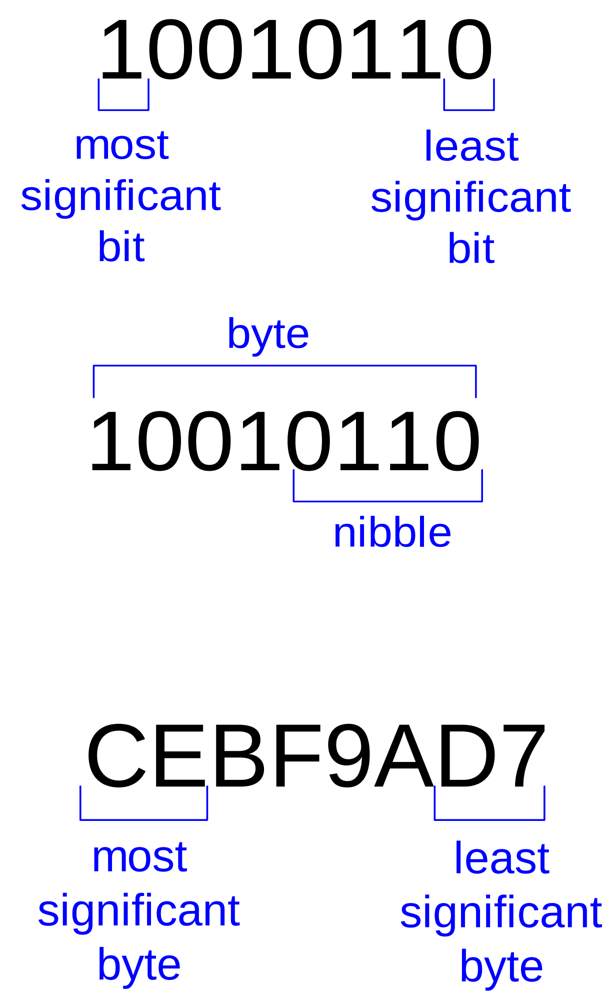
One binary digit is __ bit

One hex digit is ___ bits or ___ nibble

Two hex digits make ___ byte

Most significant on left

Least significant on right



Large Powers of Two

- $2^{10} = 1$ kilo $\approx 10^3$ (1024)
- $2^{20} = 1$ mega $\approx 10^6$ (1,048,576)
- $2^{30} = 1$ giga $\approx 10^9$ (1,073,741,824)
- $2^{40} = 1$ tera $\approx 10^{12}$
- $2^{50} = 1$ peta $\approx 10^{15}$
- $2^{60} = 1$ exa $\approx 10^{18}$

Estimating Powers of Two

- What is the value of 2^{24}?

- How large of a value can a 32-bit integer variable represent?

Number Systems: Addition

Addition

- Decimal

$$
\begin{array}{r l}
 & 11 \leftarrow \text{carries} \\
 & 3734 \\
+ & 5168 \\
\hline
 & 8902
\end{array}
$$

- Binary

$$
\begin{array}{r l}
 & 11 \leftarrow \text{carries} \\
 & 1011 \\
+ & 0011 \\
\hline
 & 1110
\end{array}
$$

Binary Addition Examples

- Add the following 4-bit binary numbers

$$\begin{array}{r} 1001 \\ +\ 0101 \\ \hline \end{array}$$

- Add the following 4-bit binary numbers

$$\begin{array}{r} 1011 \\ +\ 0110 \\ \hline \end{array}$$

Overflow

- Digital systems operate on a **fixed number of bits**
- Overflow: when result is too big to fit in the available number of bits
- See previous example of 11 + 6

Number Systems: Signed Numbers

Signed Binary Numbers

- Sign/Magnitude Numbers
- Two's Complement Numbers

Sign/Magnitude Numbers

- 1 sign bit, *N*-1 magnitude bits
- Sign bit is the most significant (left-most) bit
  - Positive number: sign bit = 0
  - Negative number: sign bit = 1

$$A: \{a_{N-1}, a_{N-2}, \dots a_2, a_1, a_0\}$$

$$A = (-1)^{a_{N-1}} \sum_{i=0}^{N-2} a_i \, 2^i$$

- Example, 4-bit sign/mag representations of ± 6:

  +6 =

  - 6 =

- Range of an *N*-bit sign/magnitude number:

Sign/Magnitude Numbers

Problems:

- Addition doesn't work, for example -6 + 6:

$$\begin{array}{r} 1110 \\ +\,0110 \\ \hline 10100 \end{array}$$ (wrong!)

- Two representations of 0 (± 0):

1000

0000

Two's Complement Numbers

- Don't have same problems as sign/magnitude numbers:
  - **Addition works**
  - **Single representation for 0**

Two's Complement Numbers

- msb has weight of -2^{N-1}

$$A = a_{N-1}(-2^{N-1}) + \sum_{i=0}^{N-2} a_i \, 2^i$$

- Most positive 4-bit number:
- Most negative 4-bit number:
- The most significant bit still indicates the sign (1 = negative, 0 = positive)
- Range of an *N*-bit two's complement number:

Reversing the Sign

- **Reverse the sign** of a two's complement number
- **Method:**
  1. Invert the bits
  2. Add 1
- **Example:** Reverse the sign of $3_{10} = 0011_2$
  1.
  2.

Historically, this reversing the sign method has been called: "Taking the Two's complement". But this terminology can be confusing, so we instead we call it "reversing the sign".

Two's Complement Examples

- Reverse the sign of $6_{10} = 0110_2$
  1.
  2.

- What is the decimal value of the two's complement number 1001_2?
  1.
  2.

Two's Complement Addition

- Add 6 + (-6) using two's complement numbers

$$\begin{array}{r} 0110 \\ +\ \ 1010 \\ \hline \end{array}$$

- Add -2 + 3 using two's complement numbers

$$\begin{array}{r} 1110 \\ +\ \ 0011 \\ \hline \end{array}$$

Subtraction

- Subtract a 2's complement number by reversing the sign and adding.
- Reverse sign by taking 2's complement

- Ex: 3 – 5 = 3 + (-5)

	0011	3
+	1011	-5
	1110	-2

Number System Comparison

Number System	Range
Unsigned	$[0, 2^N-1]$
Sign/Magnitude	$[-(2^{N-1}-1), 2^{N-1}-1]$
Two's Complement	$[-2^{N-1}, 2^{N-1}-1]$

For example, 4-bit representation:

	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Unsigned									0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Two's Complement	1000	1001	1010	1011	1100	1101	1110	1111	0000	0001	0010	0011	0100	0101	0110	0111								
Sign/Magnitude		1111	1110	1101	1100	1011	1010	1001	0000 1000	0001	0010	0011	0100	0101	0110	0111								

Number Systems: Extension

Increasing Bit Width

Extend number from *N* to *M* bits (*M* > *N*) :

- **Sign-extension** for 2's complement numbers
- **Zero-extension** for unsigned numbers

Sign-Extension

- Sign bit copied to msb's
- Number value is same
- **Example 1:**
  - 4-bit representation of 3 = **0**011
  - 8-bit sign-extended value: **0000**0011
- **Example 2:**
  - 4-bit representation of -5 = **1**011
  - 8-bit sign-extended value: **1111**1011

Zero-Extension

- Zeros copied to msb's
- Value changes for negative numbers
- **Example 1:**
  – 4-bit value = 0011 = 3_{10}
  – 8-bit zero-extended value: **0000**0011 = 3_{10}
- **Example 2:**
  – 4-bit value = 1011 = -5_{10}
  – 8-bit zero-extended value: **0000**1011 = 11_{10}

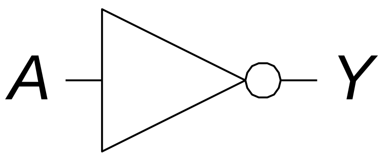
Logic Gates

Logic Gates

- **Perform logic functions:**
  - inversion (NOT), AND, OR, NAND, NOR, etc.
- **Single-input:**
  - NOT gate, buffer
- **Two-input:**
  - AND, OR, XOR, NAND, NOR, XNOR
- **Multiple-input**

Single-Input Logic Gates

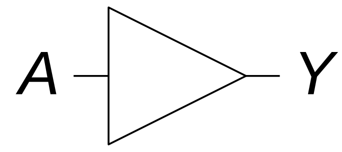
NOT



$$Y = \overline{A}$$

A	Y
0	
1	

BUF

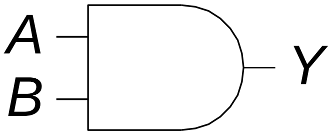


$$Y = A$$

A	Y
0	
1	

Two-Input Logic Gates

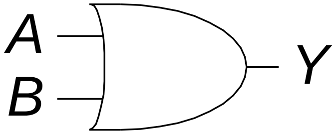
AND



$Y = AB$

A	B	Y
0	0	
0	1	
1	0	
1	1	

OR

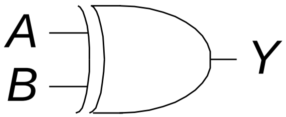


$Y = A + B$

A	B	Y
0	0	
0	1	
1	0	
1	1	

More Two-Input Logic Gates

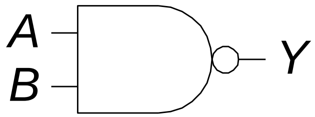
XOR



$$Y = A \oplus B$$

A	B	Y
0	0	
0	1	
1	0	
1	1	

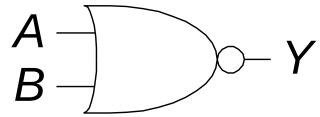
NAND



$$Y = \overline{AB}$$

A	B	Y
0	0	
0	1	
1	0	
1	1	

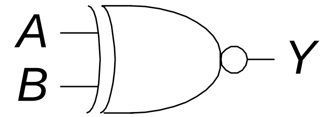
NOR



$$Y = \overline{A + B}$$

A	B	Y
0	0	
0	1	
1	0	
1	1	

XNOR

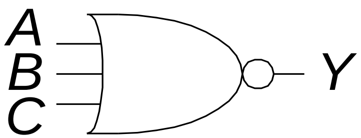


$$Y = \overline{A \oplus B}$$

A	B	Y
0	0	
0	1	
1	0	
1	1	

Multiple-Input Logic Gates

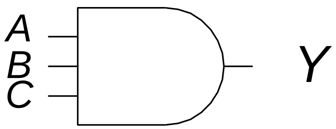
NOR3



$$Y = \overline{A+B+C}$$

A	B	C	Y
0	0	0	1
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

AND3



$$Y = ABC$$

A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Truth table rows are listed in binary order.

- Multi-input XOR: Odd parity

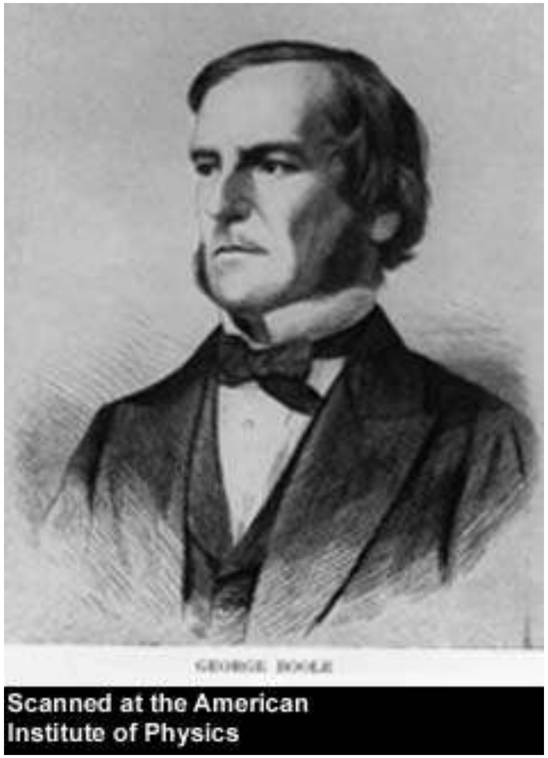
SystemVerilog Description

```
module gates(input  logic a, b, c,
             output logic y1, y2, y3, y4, y5);

  not  g1(y1, a);
  and  g2(y2, a, b);
  or   g3(y3, a, b, c);
  nand g4(y4, b, c);
  xor  g5(y5, a, c);
endmodule
```

George Boole, 1815-1864

- Born to working class parents
- Taught himself mathematics and joined the faculty of Queen's College in Ireland
- Wrote *An Investigation of the Laws of Thought* (1854)
- Introduced binary variables
- Introduced the three fundamental logic operations: AND, OR, and NOT



Logic Levels

Logic Levels

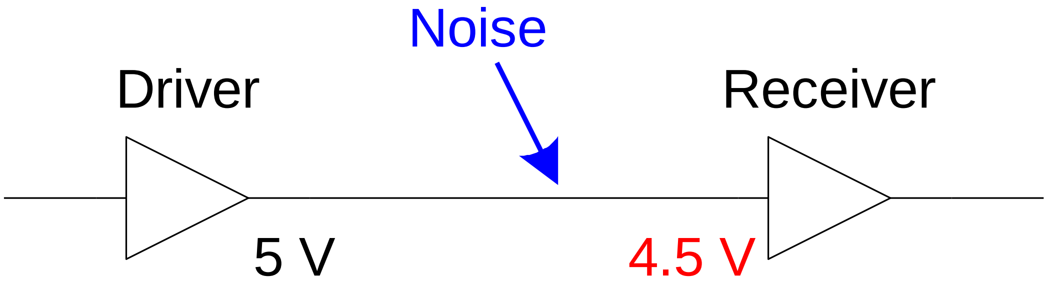
- Discrete voltages represent 1 and 0
- For example:
  - 0 = *ground* (GND) or 0 volts
  - 1 = V_{DD} or 5 volts
- What about 4.99 volts? Is that a 0 or a 1?
- What about 3.2 volts?

Logic Levels

- *Range* of voltages for 1 and 0
- Different ranges for inputs and outputs to allow for *noise*

What is Noise?

- **Anything that degrades the signal**
  - E.g., resistance, power supply noise, coupling to neighboring wires, etc.
- **Example:** a gate (driver) outputs 5 V but, because of resistance in a long wire, receiver gets 4.5 V

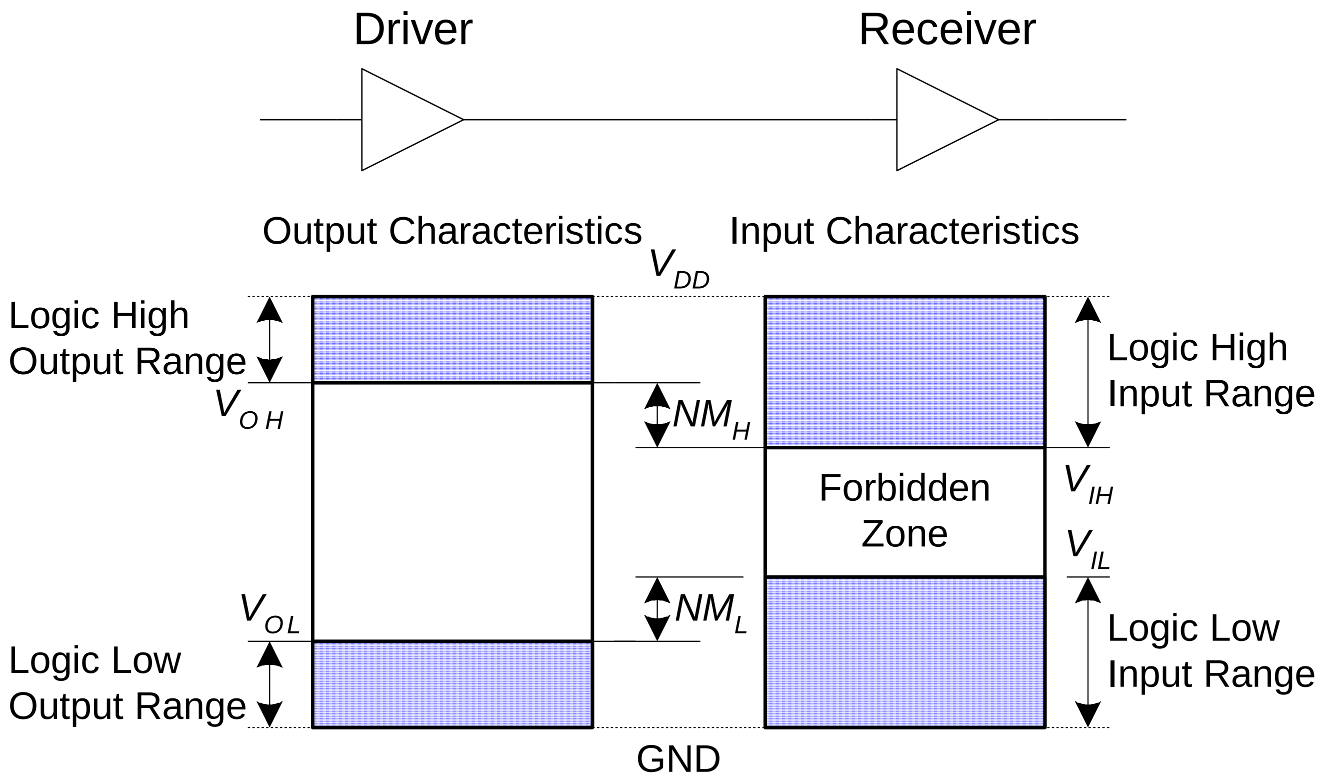


The Static Discipline

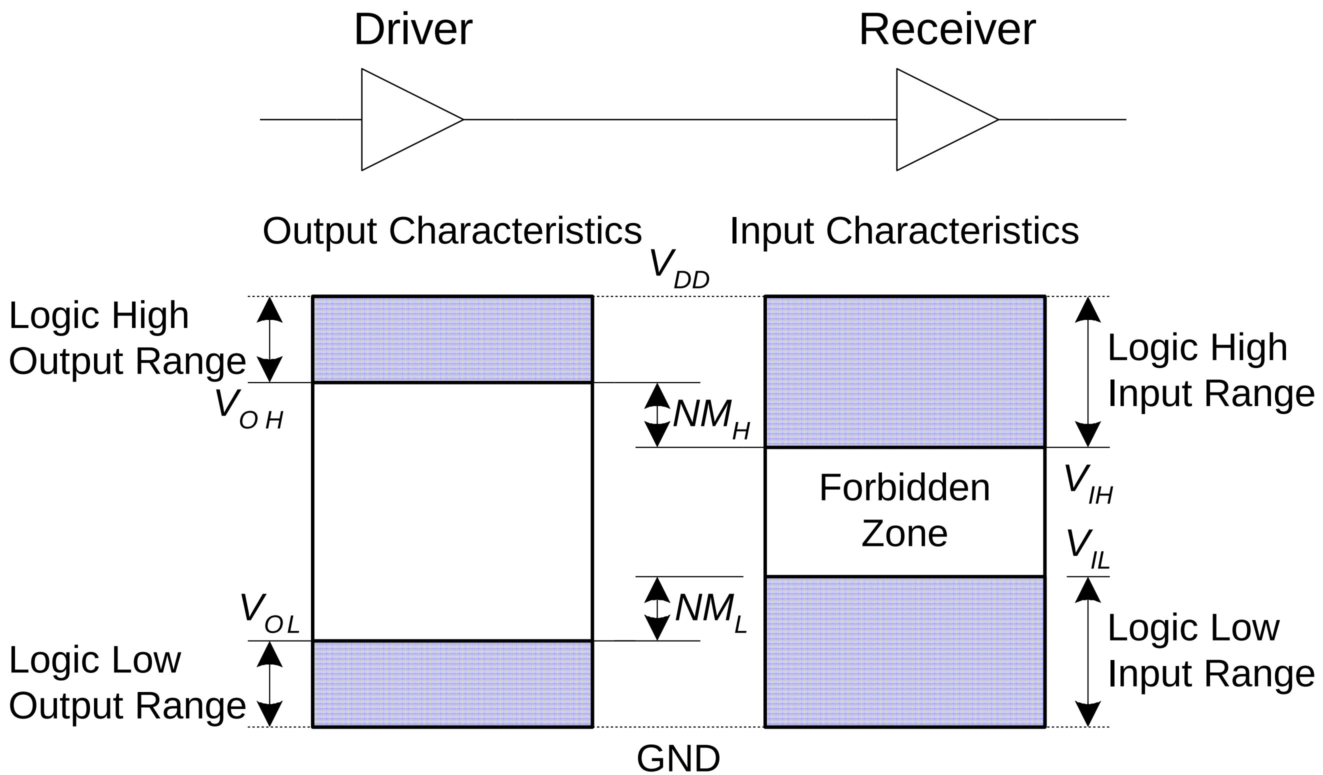
- With logically valid inputs, every circuit element must produce logically valid outputs

- Use limited ranges of voltages to represent discrete values

Noise Margins



Noise Margins

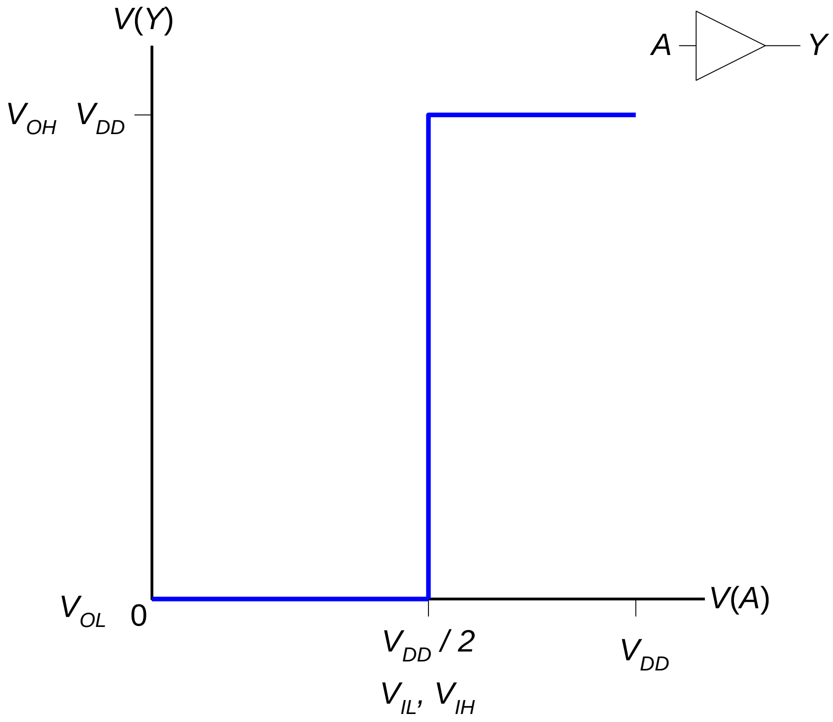


High Noise Margin: NM_H =

Low Noise Margin: NM_L =

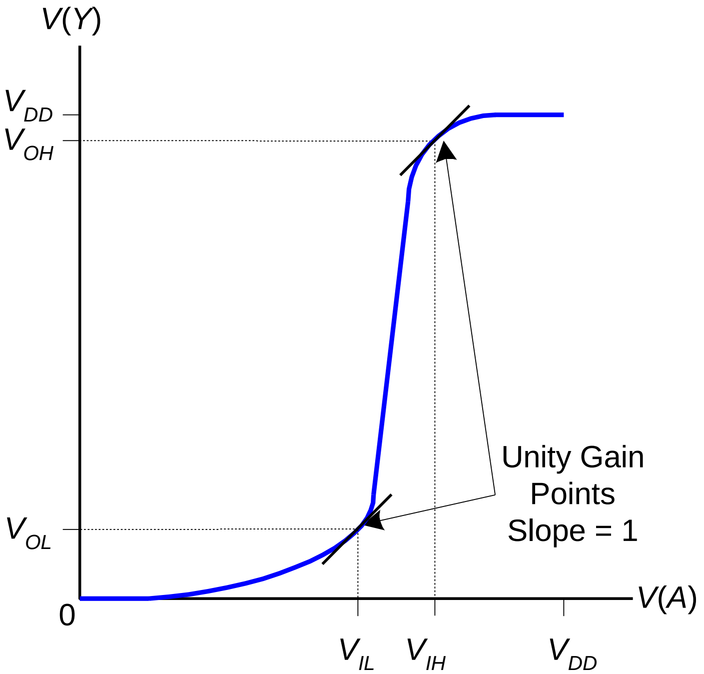
DC Transfer Characteristics

Ideal Buffer:



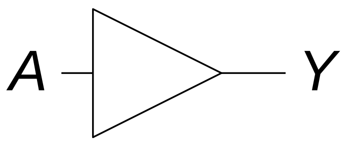
$$NM_H = NM_L = V_{DD}/2$$

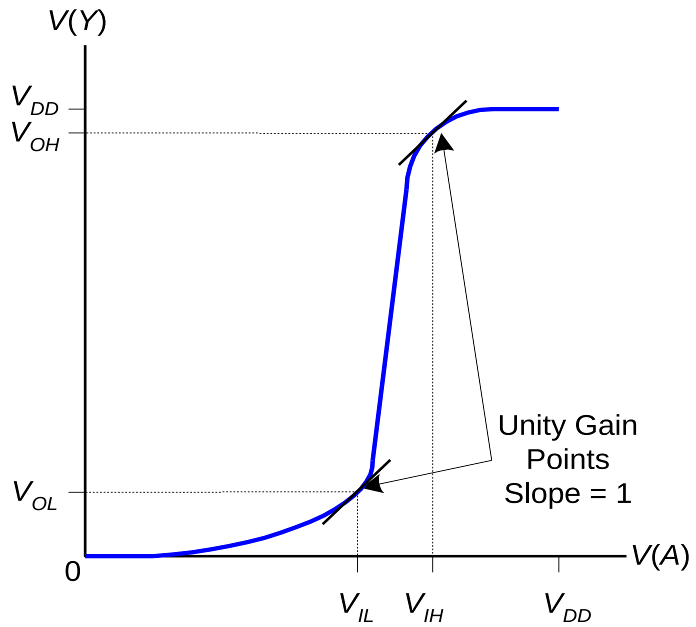
Real Buffer:

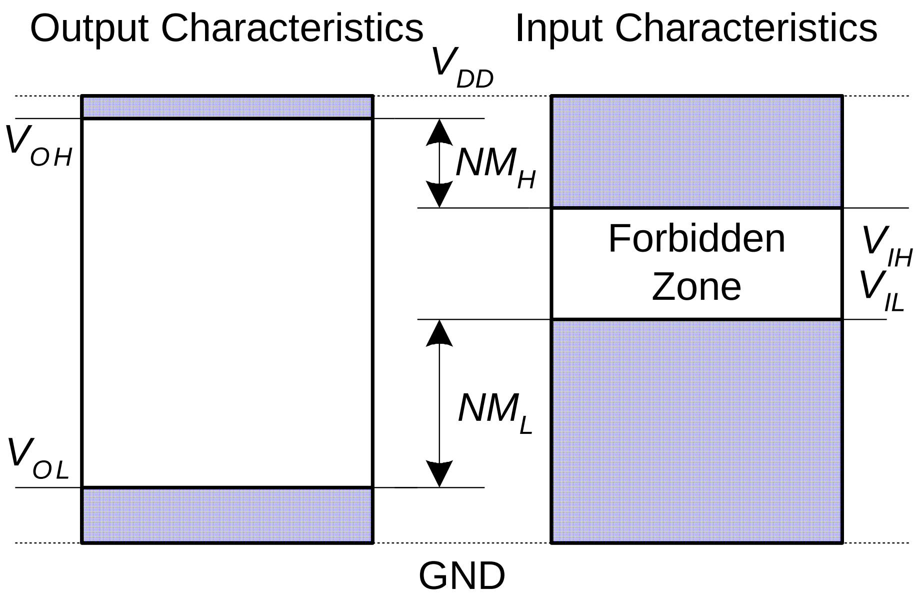


$$NM_H, NM_L < V_{DD}/2$$

DC Transfer Characteristics







V_{DD} Scaling

- In 1970's and 1980's, V_{DD} = 5 V
- V_{DD} has dropped
  - Avoid frying tiny transistors
  - Save power
- 3.3 V, 2.5 V, 1.8 V, 1.5 V, 1.2 V, 1.0 V, …
  - Be careful connecting chips with different supply voltages

V_{DD} Scaling

- In 1970's and 1980's, V_{DD} = 5 V
- V_{DD} has dropped
  - Avoid frying tiny transistors
  - Save power
- 3.3 V, 2.5 V, 1.8 V, 1.5 V, 1.2 V, 1.0 V, …
  - Be careful connecting chips with different supply voltages

Chips operate because they contain magic smoke

Proof: if the magic smoke is let out, the chip stops working

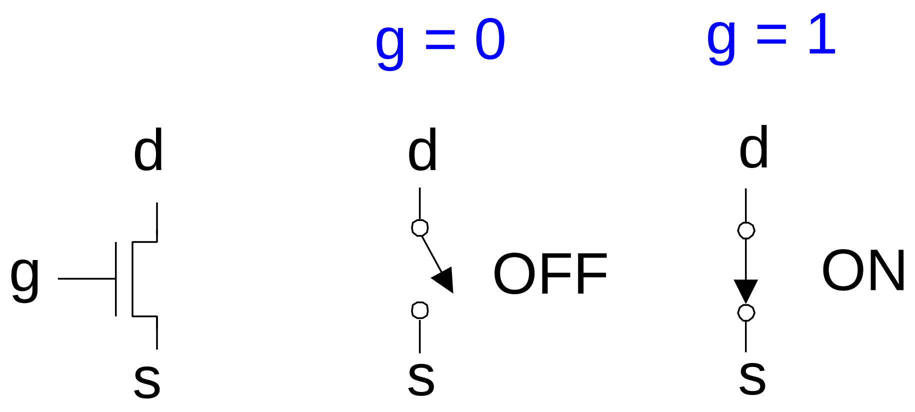
Logic Family Examples

Logic Family	V_{DD}	V_{IL}	V_{IH}	V_{OL}	V_{OH}
TTL	5 (4.75 - 5.25)	0.8	2.0	0.4	2.4
CMOS	5 (4.5 - 6)	1.35	3.15	0.33	3.84
LVTTL	3.3 (3 - 3.6)	0.8	2.0	0.4	2.4
LVCMOS	3.3 (3 - 3.6)	0.9	1.8	0.36	2.7

CMOS Transistors

Transistors

- Logic gates built from transistors
- 3-ported voltage-controlled switch
  - 2 ports connected depending on voltage of 3rd
  - d and s are connected (ON) when g is 1

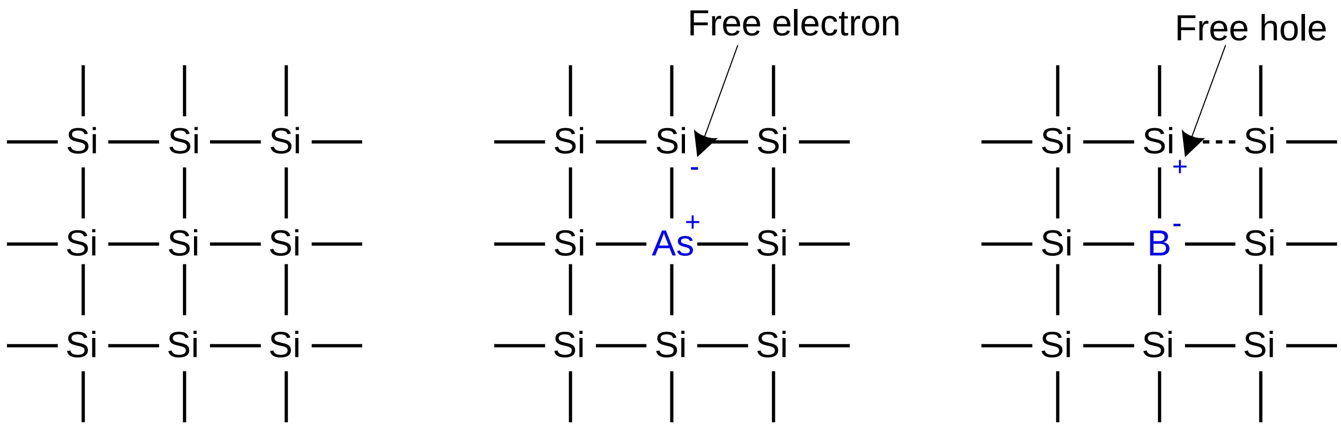


Robert Noyce, 1927-1990

- Nicknamed "Mayor of Silicon Valley"
- Cofounded Fairchild Semiconductor in 1957
- Cofounded Intel in 1968
- Co-invented the integrated circuit

Silicon

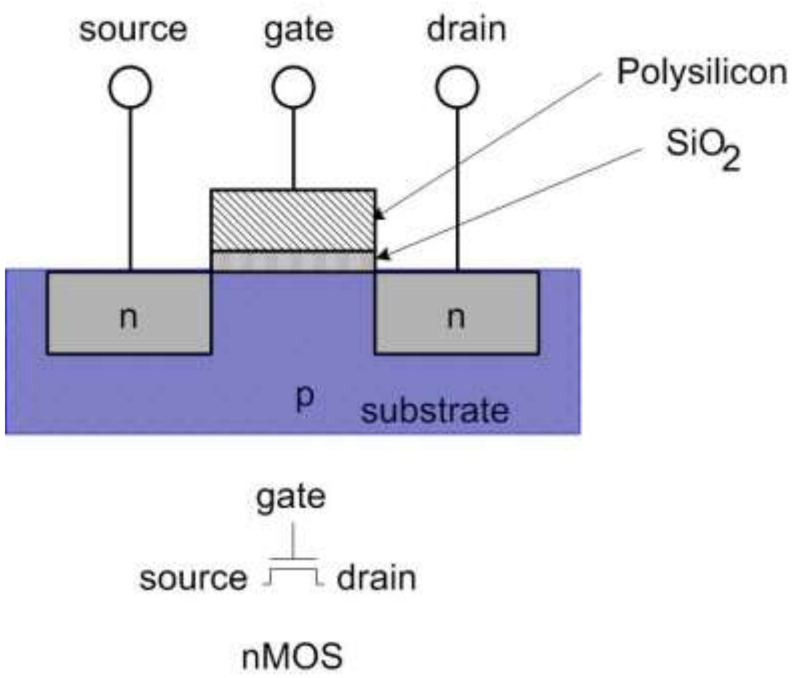
- Transistors built from silicon, a semiconductor
- Pure silicon is a poor conductor (no free charges)
- Doped silicon is a good conductor (free charges)
  - n-type (free ***n***egative charges, electrons)
  - p-type (free ***p***ositive charges, holes)



Silicon Lattice **n-Type** **p-Type**

MOS Transistors

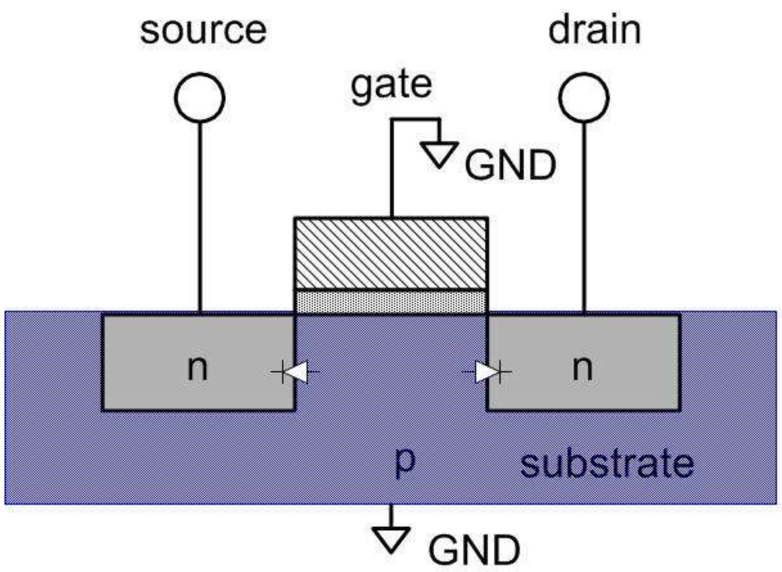
- **Metal oxide silicon (MOS) transistors:**
  - Polysilicon (used to be **metal**) gate
  - **Oxide** (silicon dioxide) insulator
  - Doped **silicon**



Transistors: nMOS

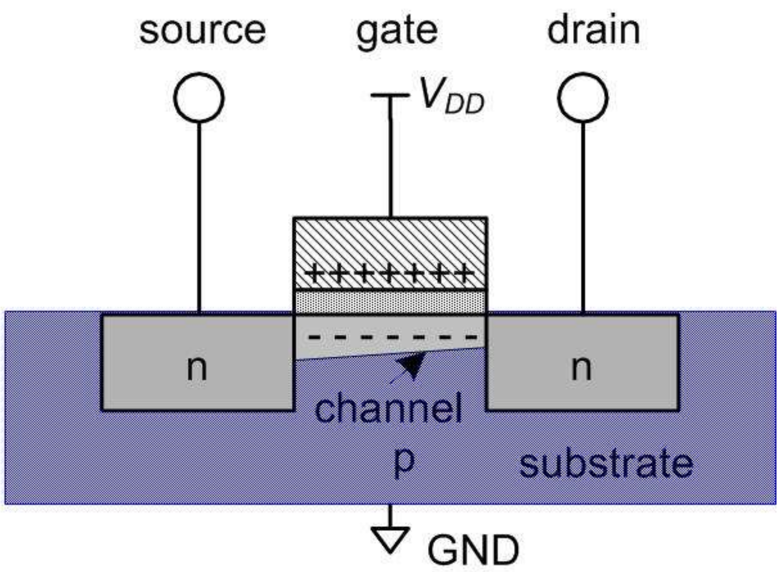
Gate = 0

OFF (no connection between source and drain)



Gate = 1

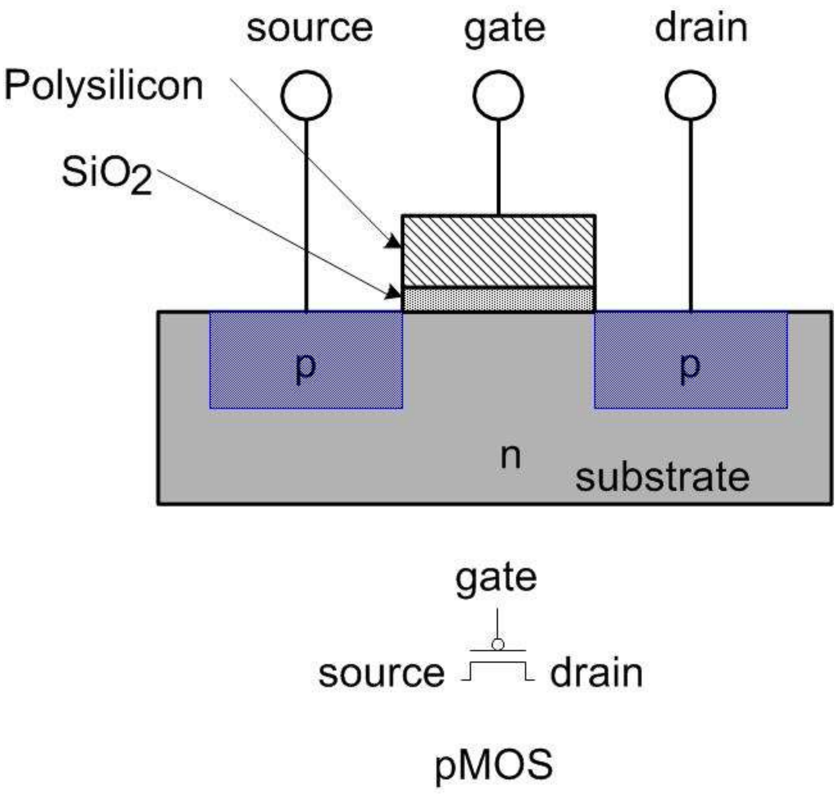
ON (channel between source and drain)



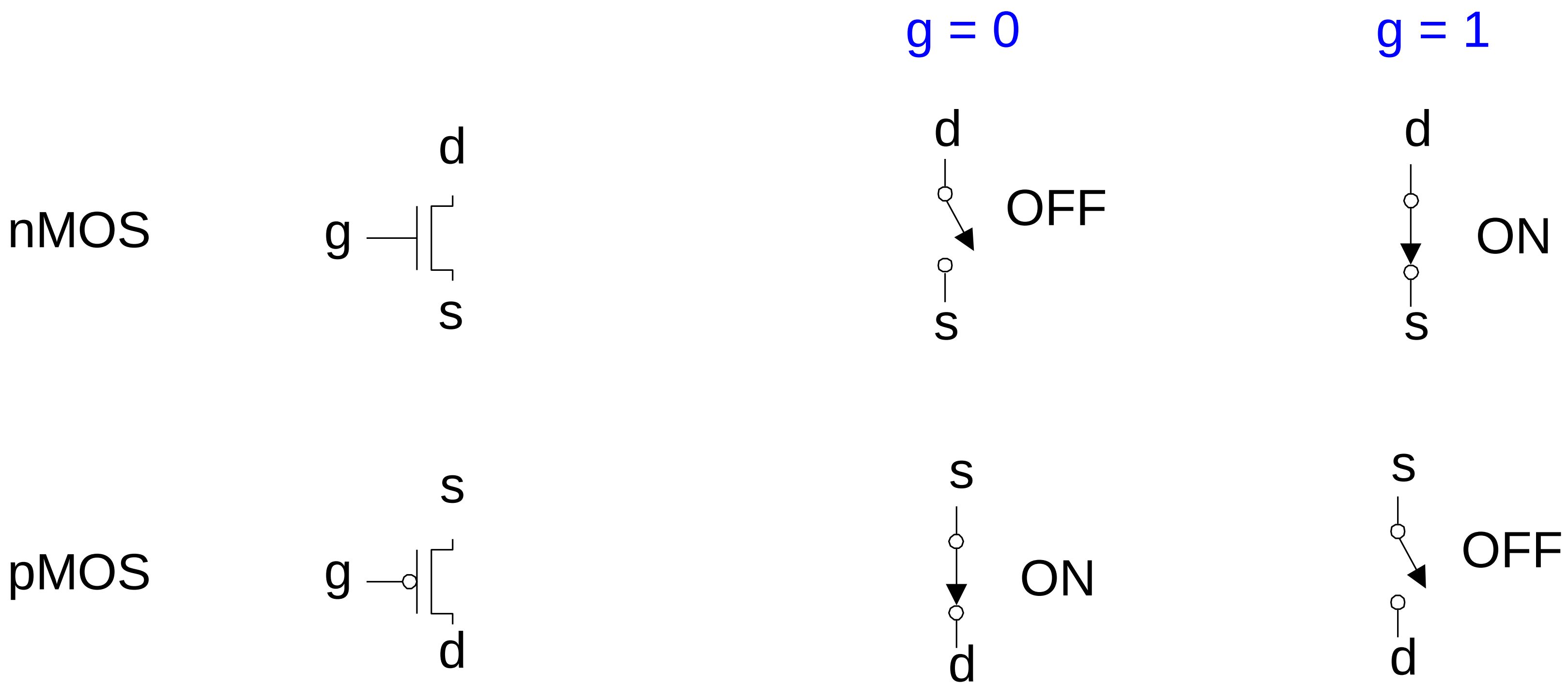
Transistors: pMOS

pMOS transistor is opposite

- **ON** when **Gate = 0**
- **OFF** when **Gate = 1**

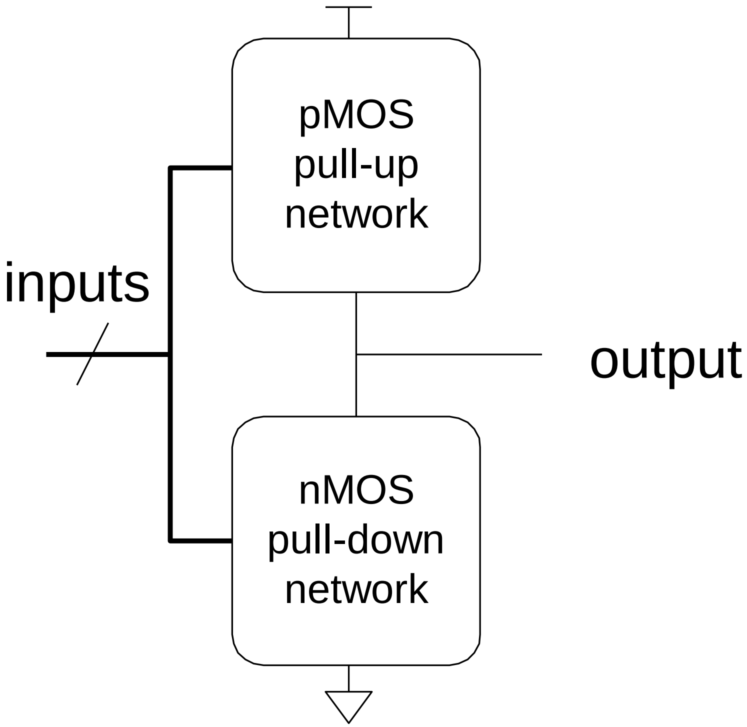


Transistor Function



Transistor Function

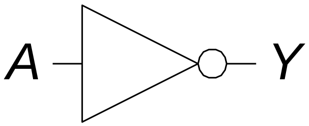
- **nMOS**: pass good **0**'s, so connect source to GND
- **pMOS**: pass good **1**'s, so connect source to V_{DD}



Gates from Transistors

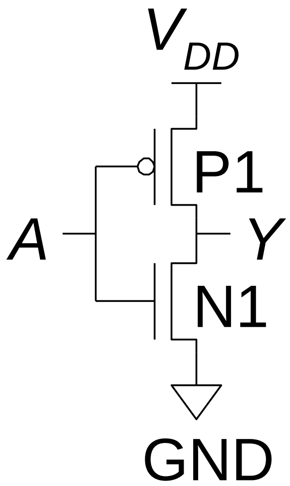
CMOS Gates: NOT Gate

NOT



$$Y = \overline{A}$$

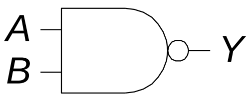
A	Y
0	1
1	0



A	P1	N1	*Y*
0			
1			

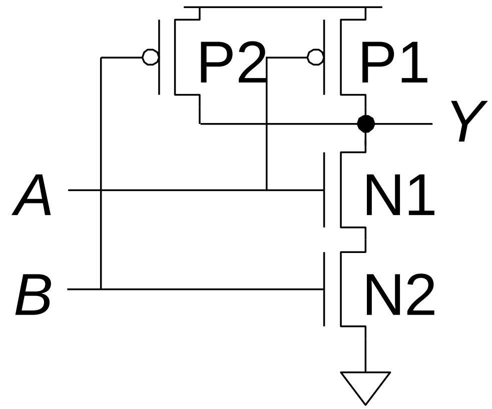
CMOS Gates: NAND Gate

NAND



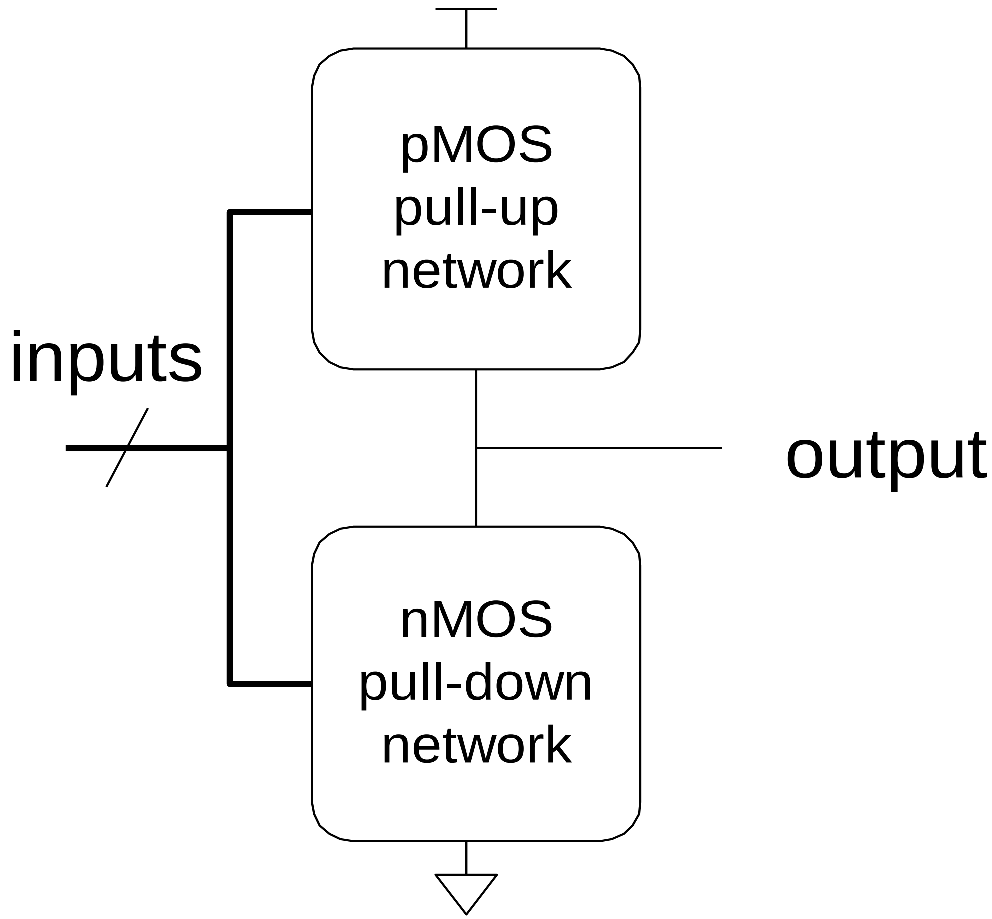
$Y = \overline{AB}$

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0



A	*B*	P1	P2	N1	N2	*Y*
0	0					
0	1					
1	0					
1	1					

CMOS Gate Structure



NOR3 Gate

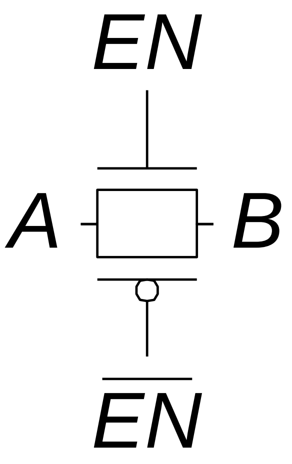
How do you build a three-input NOR gate?

AND2 Gate

How do you build a two-input AND gate?

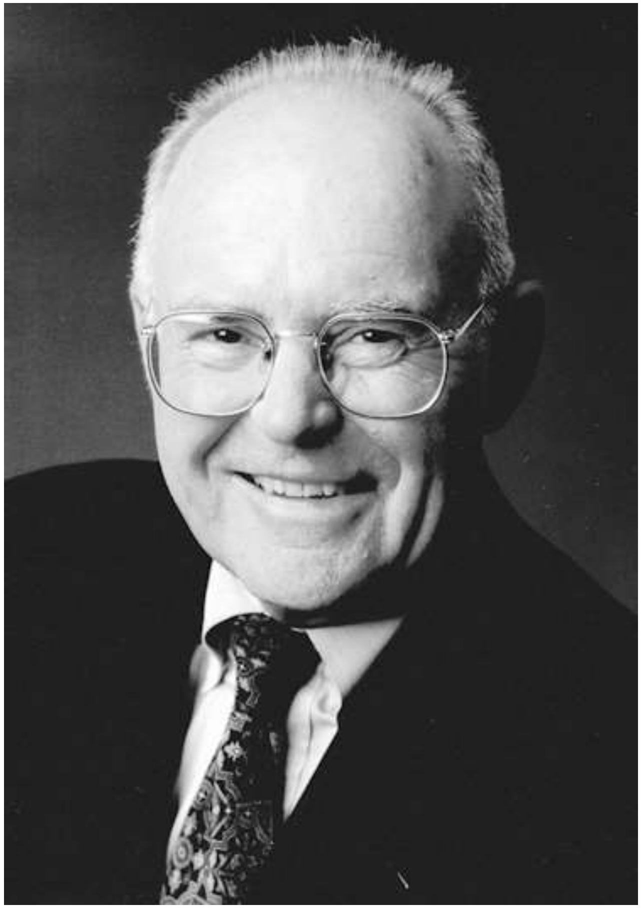
Transmission Gates

- nMOS pass 1's poorly
- pMOS pass 0's poorly
- Transmission gate is a better switch
  – passes both 0 and 1 well
- When *EN* = 1, the switch is ON:
  – $\overline{EN}$ = 0 and *A* is connected to *B*
- When *EN* = 0, the switch is OFF:
  – *A* is not connected to *B*

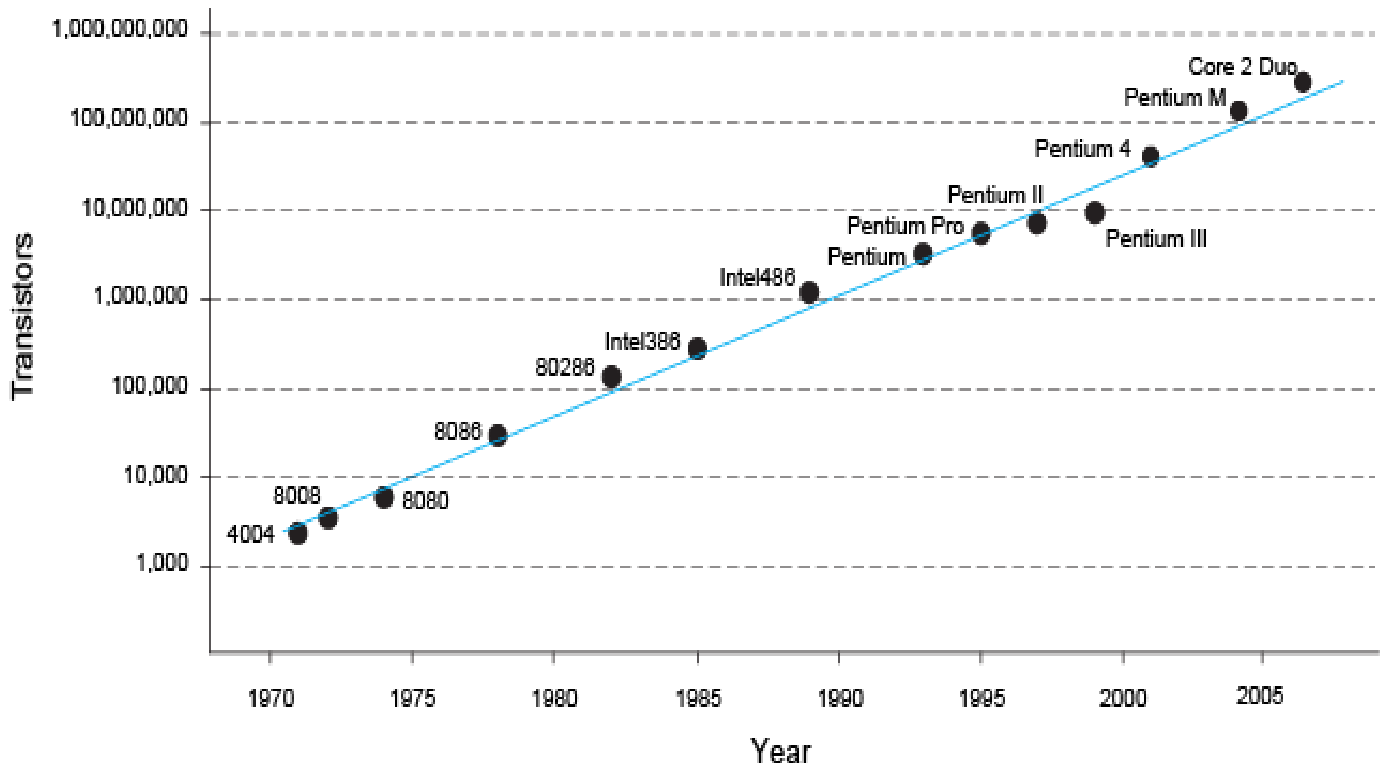


Gordon Moore, 1929-

- Cofounded Intel in 1968 with Robert Noyce.
- **Moore's Law:** number of transistors on a computer chip doubles every year (observed in 1965)
- Since 1975, transistor counts have doubled every two years.
- Corollaries: transistors get faster and lower power



Moore's Law



"If the automobile had followed the same development cycle as the computer, a Rolls-Royce would today cost $100, get one million miles to the gallon, and explode once a year . . ." (Robert Cringely, Infoworld)

– Robert Cringley

Power Consumption

Power Consumption

Power = Energy consumed per unit time

- Dynamic power consumption
- Static power consumption

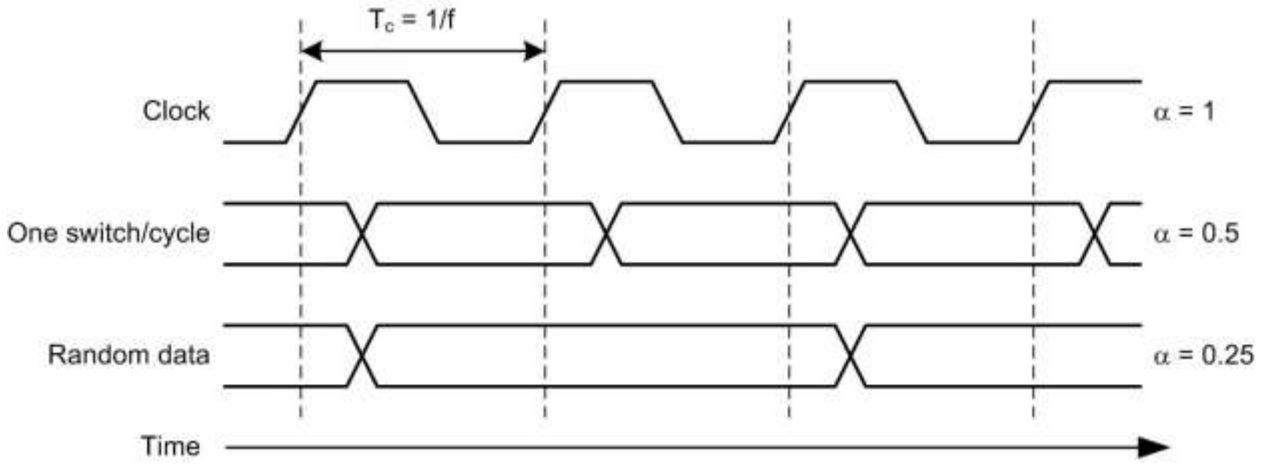
Dynamic Power Consumption

- **Power to charge transistor gate capacitances**
  - Energy required to charge a capacitance, C, to V_{DD} is CV_{DD}^2
  - Circuit running at frequency f (f cycles per second)
  - Capacitor is charged α times per cycle (discharging from 1 to 0 is free)
- **Dynamic power consumption:**

$$\boxed{P_{dynamic} = \alpha C V_{DD}^2 f}$$

Activty Factor α

- **α is the fraction of cycles that cap is charged**
  - Clock $\alpha = 1$
  - Data switching once per cycle $\alpha = 0.5$
  - Random data $\alpha = 0.25$
  - Typical data $\alpha = 0.1$



Static Power Consumption

- Power consumed when no gates are switching
- Caused by the *quiescent supply current*, I_{DD} (also called the *leakage current*)
- Static power consumption:

$$\boxed{P_{static} = I_{DD}V_{DD}}$$

Units

- Chips have tiny capacitances, small currents, and high frequencies.

Unit	Prefix	Value
Tera	T	10^{12}
Giga	G	10^{9}
Mega	M	10^{6}
Kilo	K	10^{3}
Mili	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Femto	f	10^{-15}

Power Consumption Example

- Estimate the power consumption of a mobile phone running Angry Birds
  - $V_{DD} = 0.8$ V
  - $C = 5$ nF
  - $f = 2$ GHz
  - $\alpha = 0.1$
  - $I_{DD} = 100$ mA

$$P = \alpha C V_{DD}^2 f + I_{DD} V_{DD}$$

$$= (0.1)(5 \text{ nF})(0.8 \text{ V})^2(2 \text{ GHz}) + (100 \text{ mA})(0.8 \text{ V})$$

$$= (0.64 + 0.08) \text{ W} \approx 0.72 \text{ W}$$

Power Consumption Example

- If the phone has a 8 W-hr battery, estimate its battery life sitting idle in your pocket.
  - V_{DD} = 0.8 V
  - I_{DD} = 100 mA

$$P_{static} = I_{DD}V_{DD} = 0.08 \text{ W}$$

Battery life = Capacity / Consumption

= (8 W-hr) / (0.08 W) = 100 hr (4 days)

About these Notes

Digital Design and Computer Architecture Lecture Notes