Lecture 7-2: Ensemble Learning Bagging

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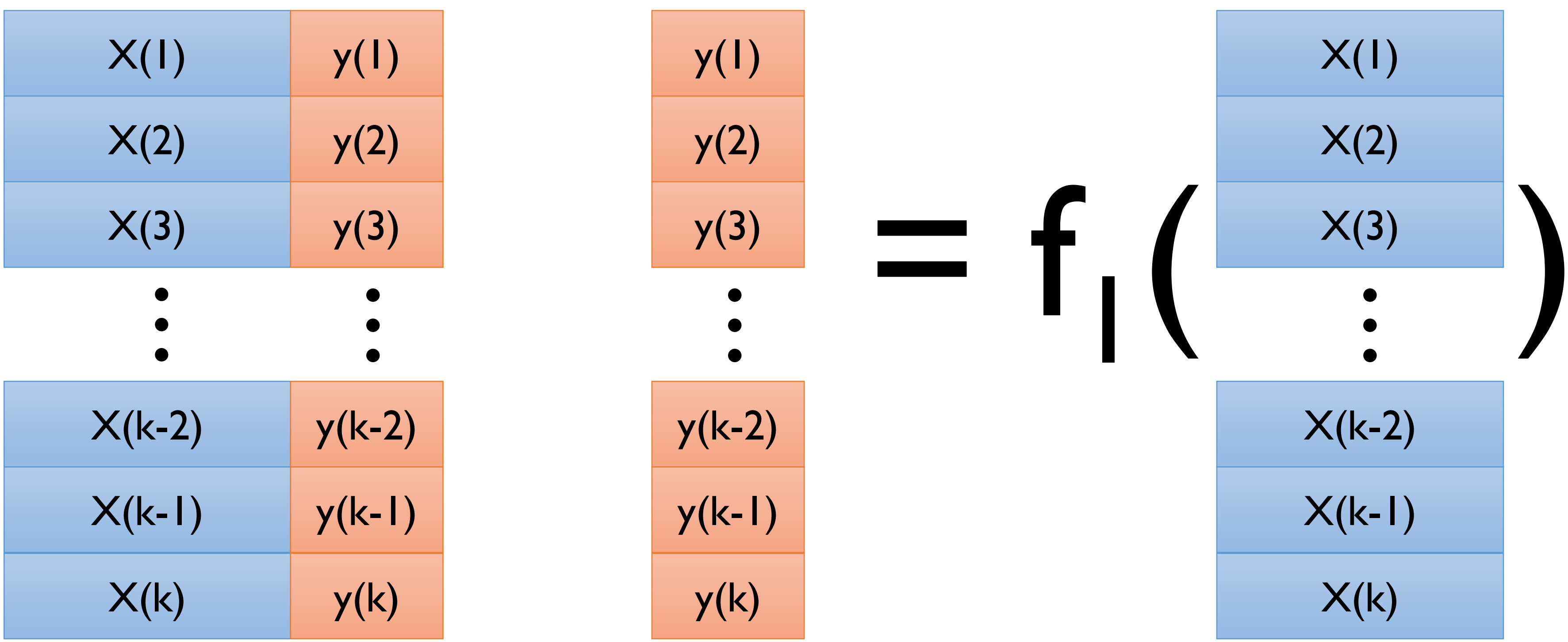
Sampling without Replacement

- K-fold data split
  - ✓ Entire data is split into k blocks; each classifier is trained only on different subset of (k-1) blocks



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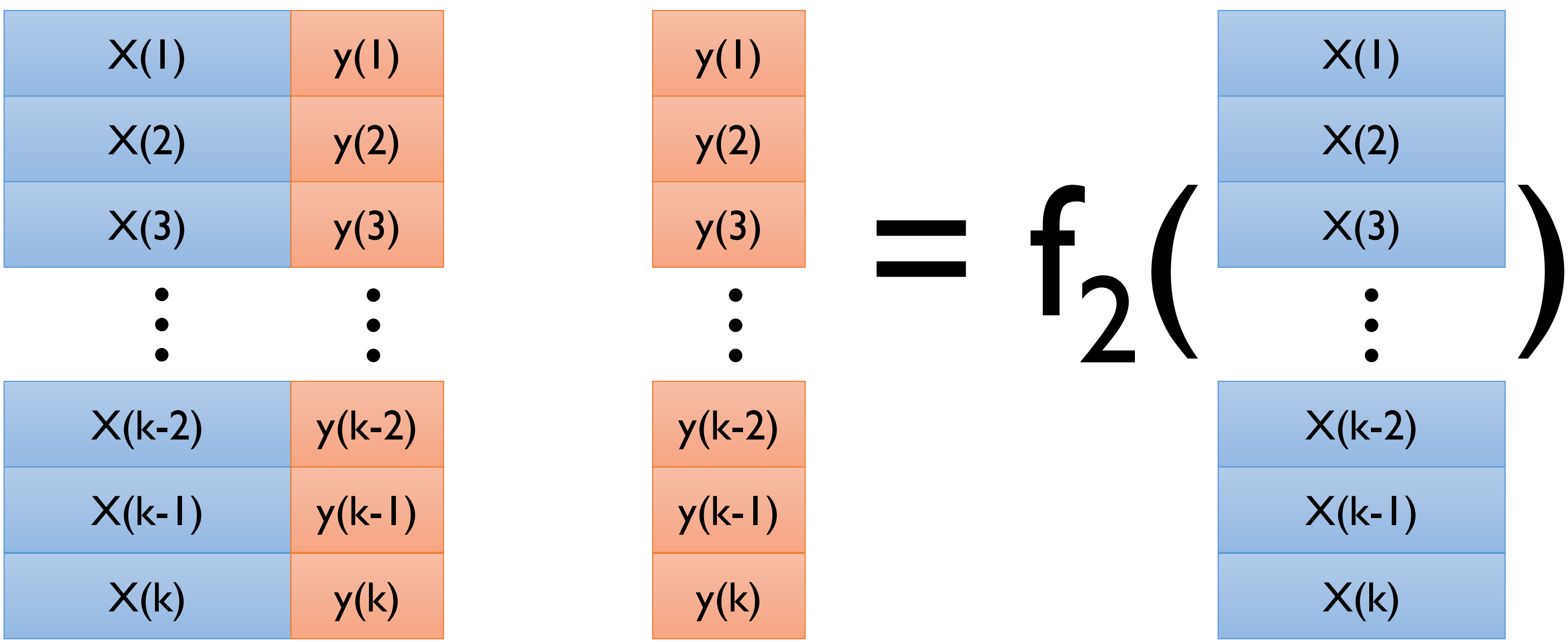

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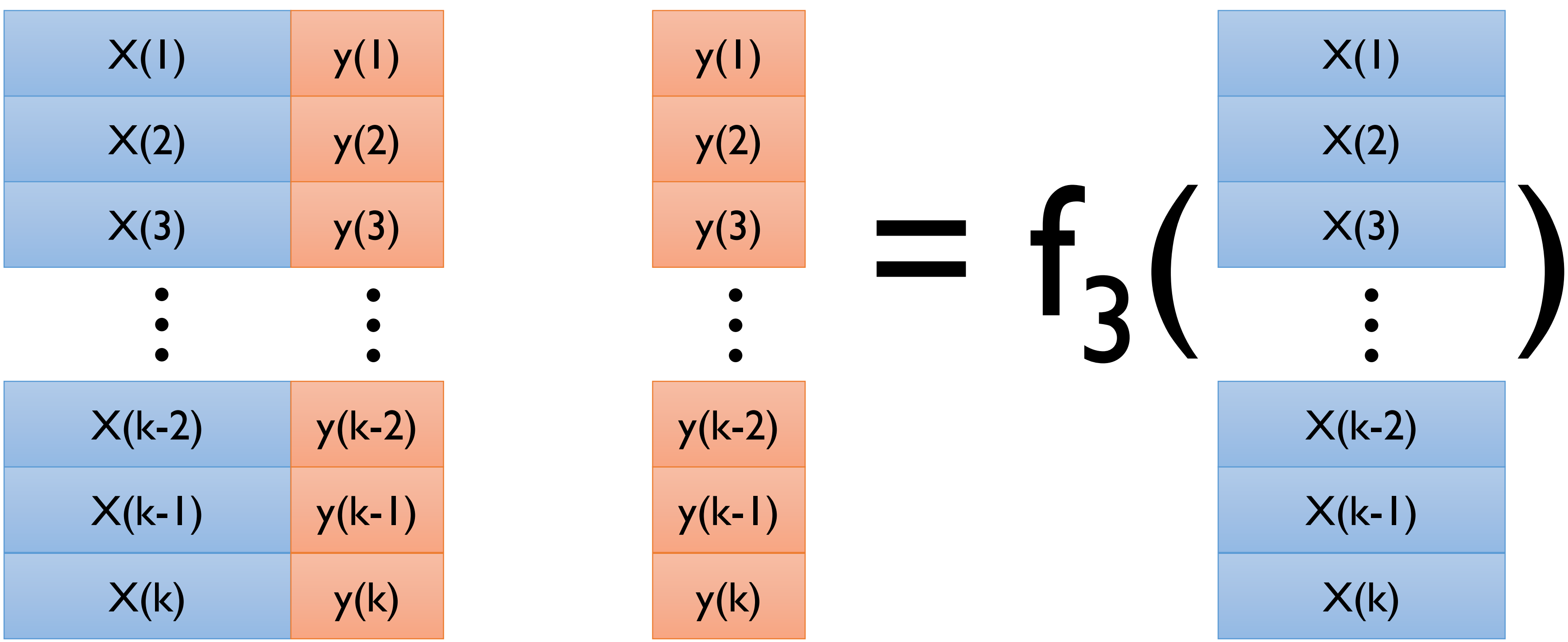

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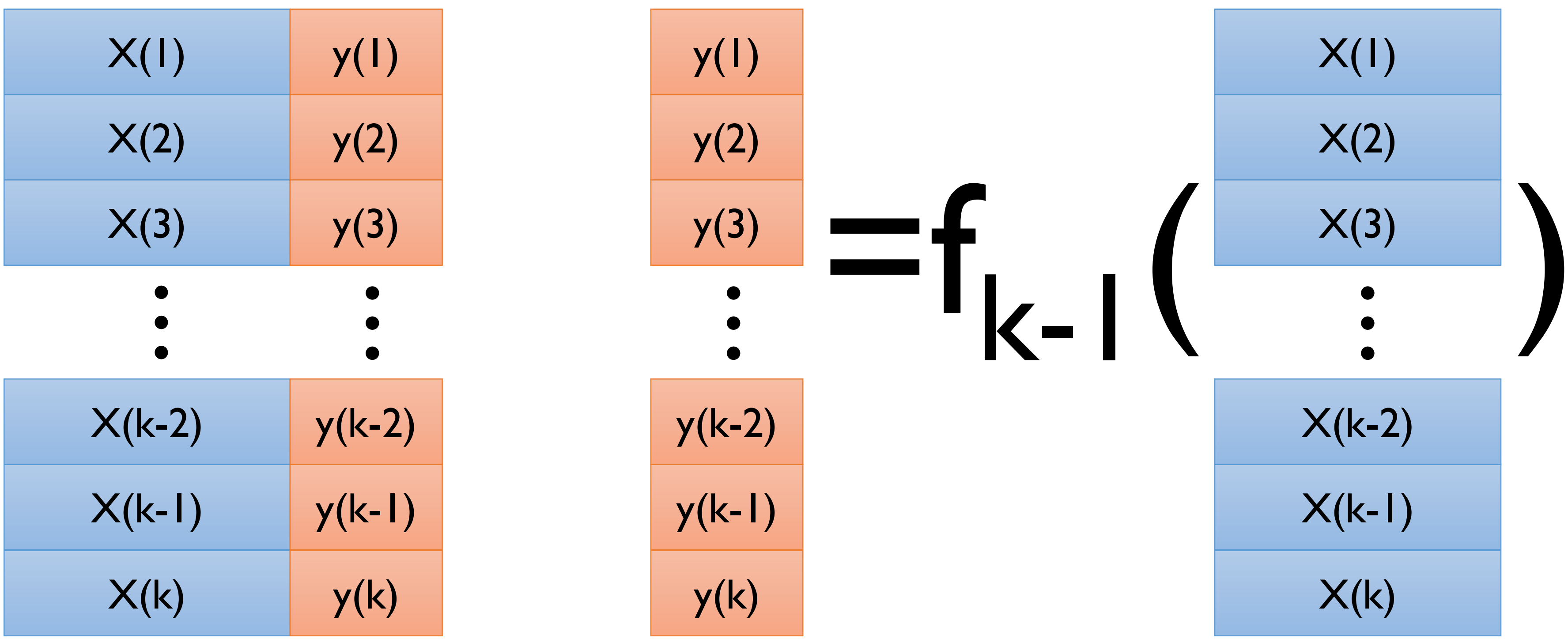

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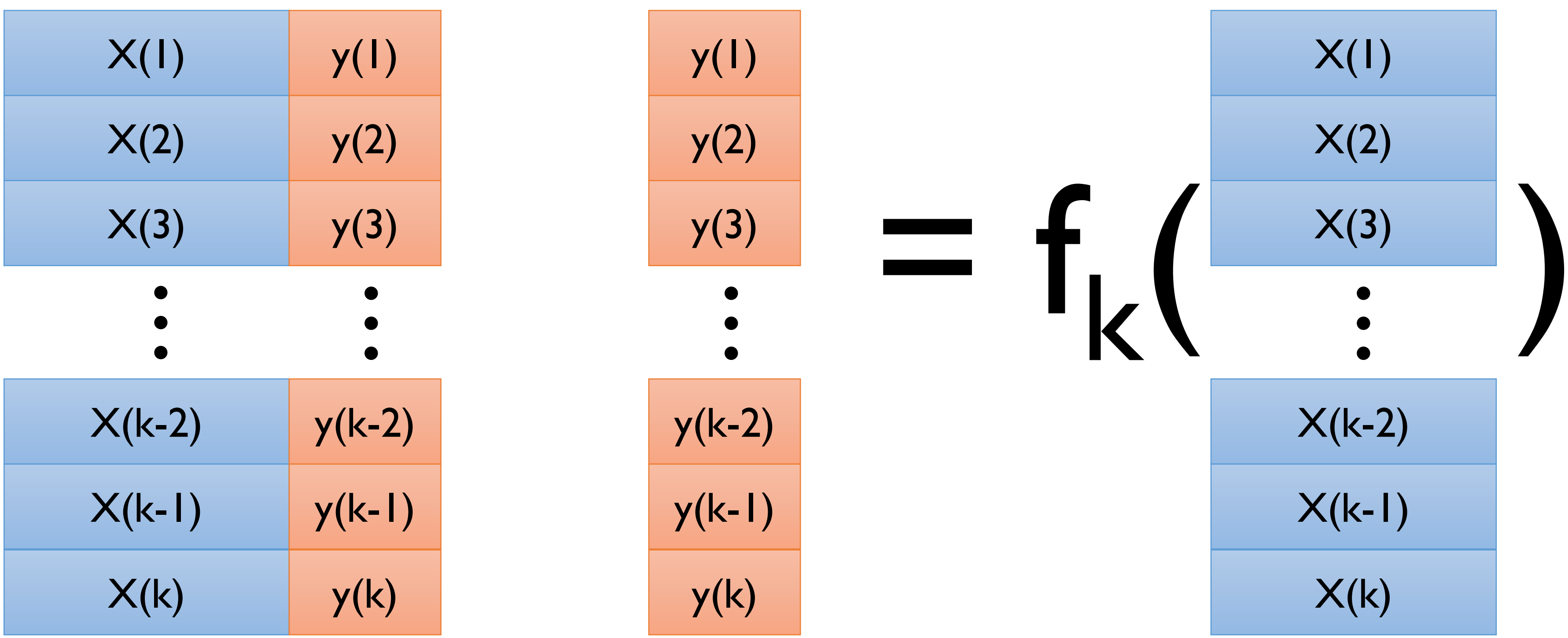


Sampling without Replacement

- K-fold data split
  - ✓ Entire data is split into k blocks; each classifier is trained only on different subset of (k-1) blocks
- Final output

$$\hat{y} = \delta\Big(f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_{k-1}(\mathbf{x}), f_k(\mathbf{x})\Big)$$

  - ✓ $\delta(\cdot)$: An aggregation function of individual outputs (ex: simple average)

Bootstrap Aggregating: Bagging

Breiman (1996)

- Main Idea
  - ✓ Each member of the ensemble is constructed from a different training dataset
  - ✓ Each dataset is generated by sampling from the total *N* data examples, choosing *N* items uniformly at random with replacement
  - ✓ Each dataset sample is known as a bootstrap

Original Dataset

x^1	y^1
x^2	y^2
x^3	y^3
x^4	y^4
x^5	y^5
x^6	y^6
x^7	y^7
x^8	y^8
x^9	y^9
x^{10}	y^{10}

Bootstrap 1

x^3	y^3
x^6	y^6
x^2	y^2
x^{10}	y^{10}
x^8	y^8
x^7	y^7
x^7	y^7
x^3	y^3
x^2	y^2
x^7	y^7

Bootstrap 2

x^7	y^7
x^1	y^1
x^{10}	y^{10}
x^1	y^1
x^8	y^8
x^6	y^6
x^2	y^2
x^6	y^6
x^4	y^4
x^9	y^9

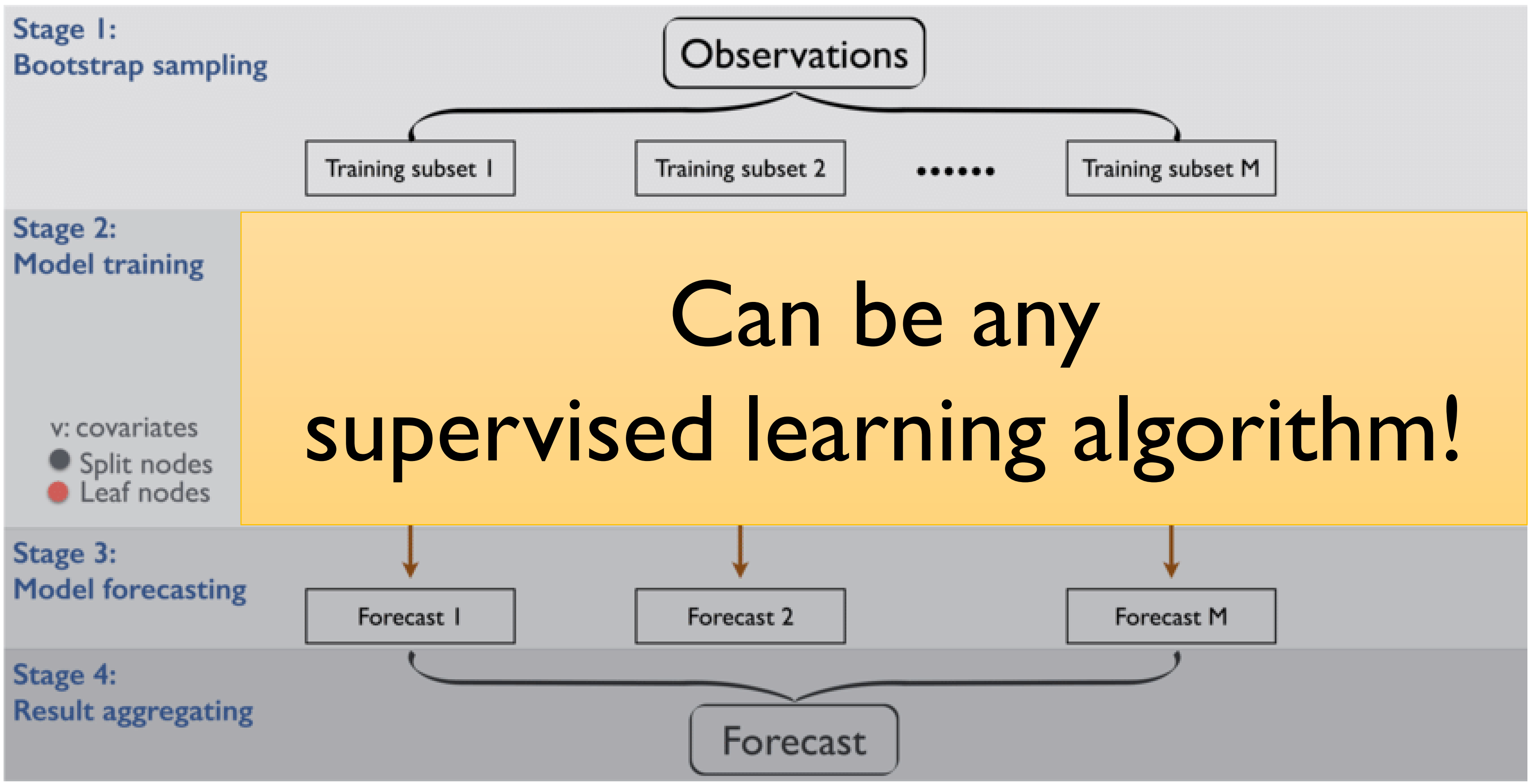
…

Bootstrap B

x^9	y^9
x^5	y^5
x^2	y^2
x^4	y^4
x^7	y^7
x^2	y^2
x^5	y^5
x^{10}	y^{10}
x^8	y^8
x^2	y^2

Bootstrap Aggregating: Bagging

- Bagging with Decision Tree



Bootstrap Aggregating: Bagging

- Result Aggregating
  - ✓ For classification problem
    - Majority voting

$$\hat{y}_{Ensemble} = arg\max_{i}\left(\sum_{j=1}^{n}\delta(\hat{y}_j = i), \quad i \in \{0,1\}\right)$$

Training Accuracy	Ensemble population	P(y=1) for a test instance	Predicted class label
0.80	Model 1	0.90	1
0.75	Model 2	0.92	1
0.88	Model 3	0.87	1
0.91	Model 4	0.34	0
0.77	Model 5	0.41	0
0.65	Model 6	0.84	1
0.95	Model 7	0.14	0
0.82	Model 8	0.32	0
0.78	Model 9	0.98	1
0.83	Model 10	0.57	1

$$\sum_{j=1}^{n}\delta(\hat{y}_j = 0) = 4$$

$$\sum_{j=1}^{n}\delta(\hat{y}_j = 1) = 6$$

$$\hat{y}_{Ensemble} = 1$$

Bootstrap Aggregating: Bagging

- Result Aggregating
  - ✓ For classification problem
    - Weighted voting (weight = training accuracy of individual models)

$$\hat{y}_{Ensemble} = arg\max_i \left(\frac{\sum_{j=1}^{n}(TrnAcc_j) \cdot \delta(\hat{y}_j = i)}{\sum_{j=1}^{n}(TrnAcc_j)}, \quad i \in \{0,1\} \right)$$

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$$\frac{\sum_{j=1}^{n}(TrnAcc_j) \cdot \delta(\hat{y}_j = 0)}{\sum_{j=1}^{n}(TrnAcc_j)} = 0.424$$

$$\frac{\sum_{j=1}^{n}(TrnAcc_j) \cdot \delta(\hat{y}_j = 1)}{\sum_{j=1}^{n}(TrnAcc_j)} = 0.576$$

$$\hat{y}_{Ensemble} = 1$$

Bootstrap Aggregating: Bagging

- Result Aggregating
  - ✓ For classification problem
    - Weighted voting (weight = predicted probability for each class)

$$\hat{y}_{Ensemble} = arg\max_i \left(\frac{1}{n} \sum_{j=1}^{n} P(y=i), \quad i \in \{0,1\} \right)$$

Training Accuracy	Ensemble population	P(y=1) for a test instance	Predicted class label
0.80	Model 1	0.90	1
0.75	Model 2	0.92	1
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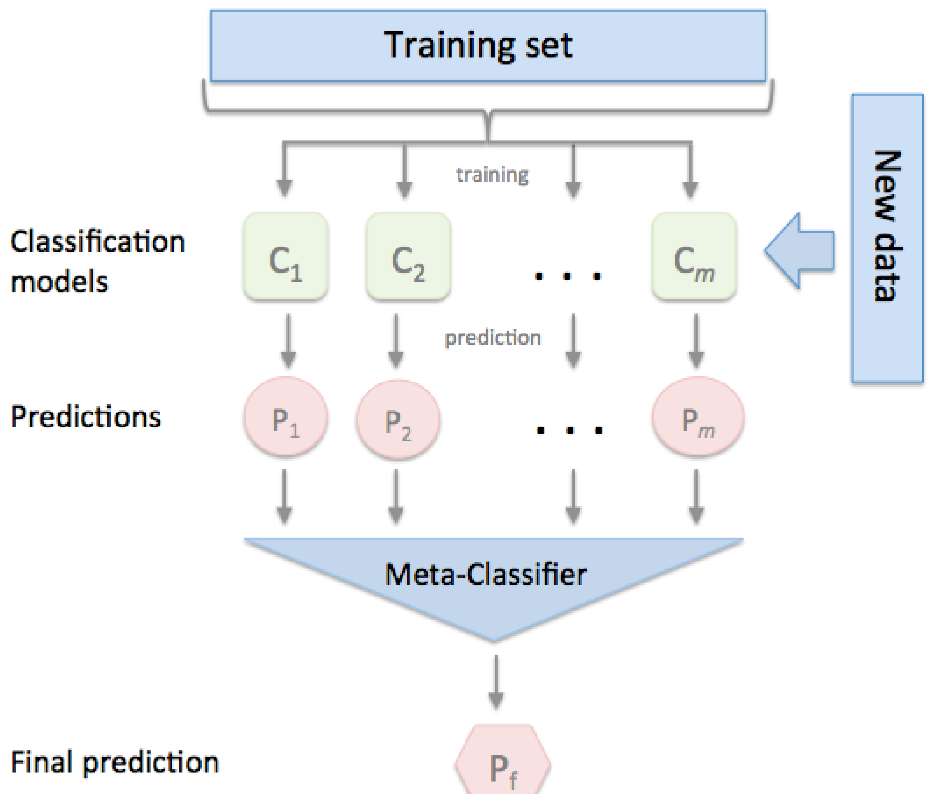
$$\frac{1}{n} \sum_{j=1}^{n} P(y=0) = 0.375$$

$$\frac{1}{n} \sum_{j=1}^{n} P(y=1) = 0.625$$

$$\hat{y}_{Ensemble} = 1$$

Bootstrap Aggregating: Bagging

- Result Aggregating: Stacking
  - ✓ Use another prediction model to aggregate the results
    - Input: Predictions made by ensemble members
    - Target: Actual true label



https://rasbt.github.io/mlxtend/user_guide/classifier/StackingClassifier/

Bootstrap Aggregating: Bagging

- Result Aggregating: Stacking
  - ✓ The winner of KDD-cup 2015
    - MOOC dropout prediction

∙Jeong-Yoon Lee, Winning Data Science Competitions

Bootstrap Aggregating: Bagging

- Bagging: Algorithm

Algorithm 1 Bagging

Input: Required ensemble size T
Input: Training set $S = \{(x_1, y_1), (x_2, y_2), ..., (x_N, y_N)\}$
for $t = 1$ to T **do**
  Build a dataset S_t, by sampling N items, randomly *with replacement* from S.
  Train a model h_t using S_t, and add it to the ensemble.
end for
For a new testing point (x', y'),
If model outputs are continuous, combine them by averaging.
If model outputs are class labels, combine them by voting.

Bootstrap Aggregating: Bagging

- Bagging: Illustration

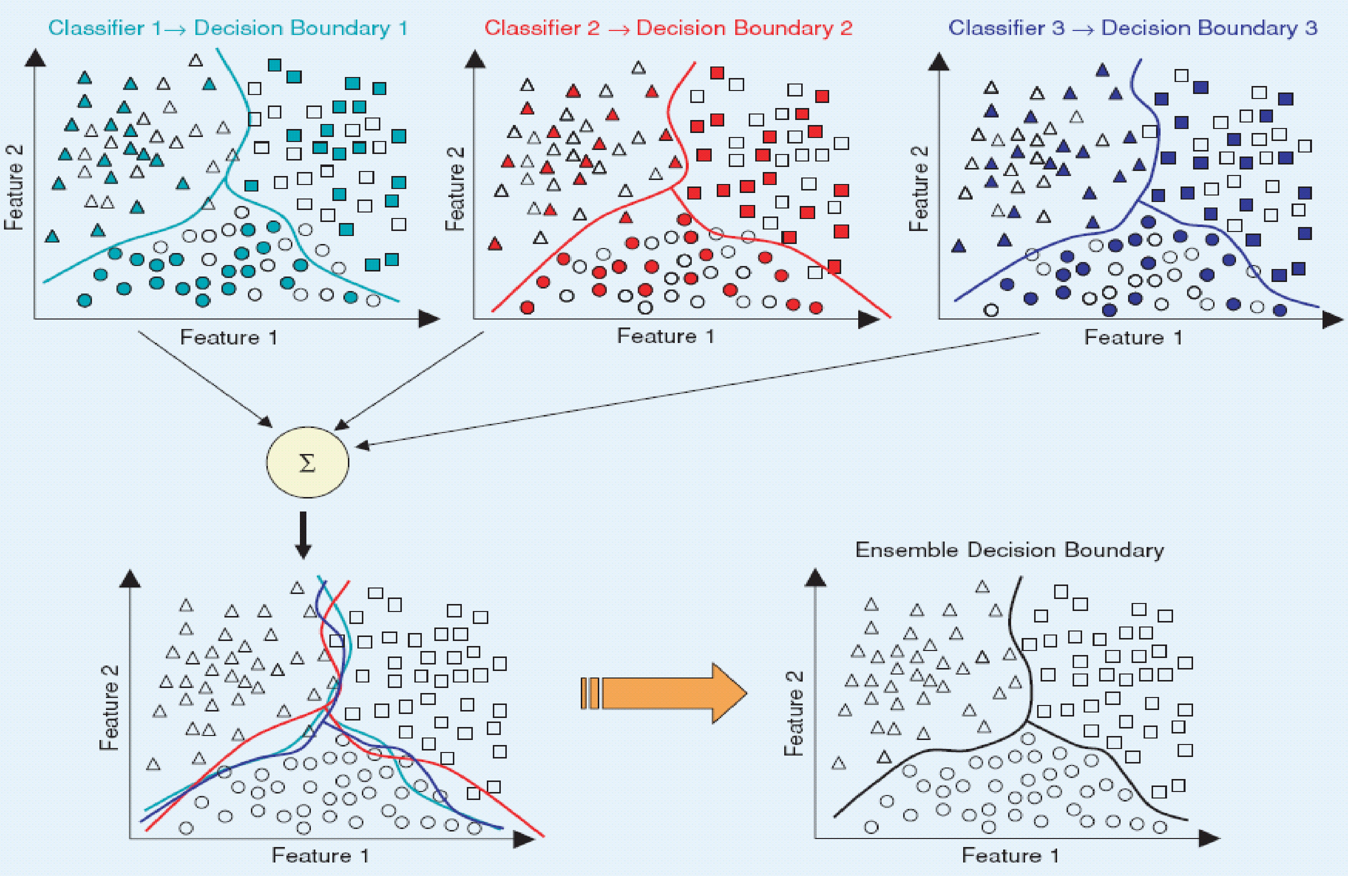
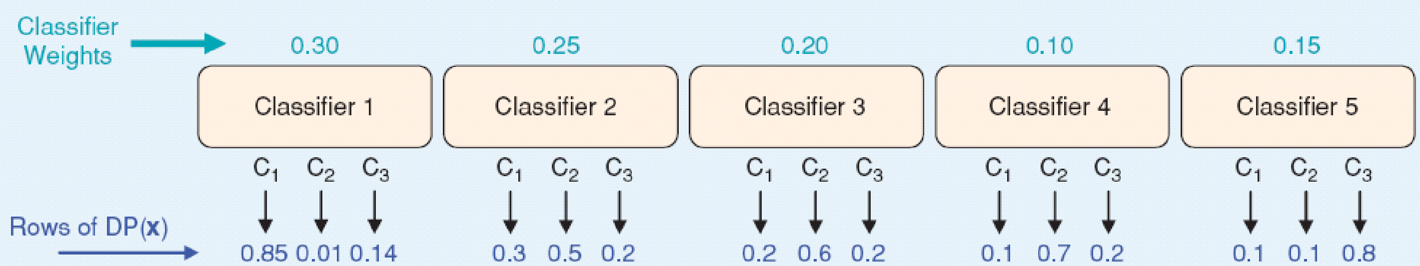


Figure 3. Combining classifiers that are trained on different subsets of the training data.

Bootstrap Aggregating: Bagging

- Aggregation examples

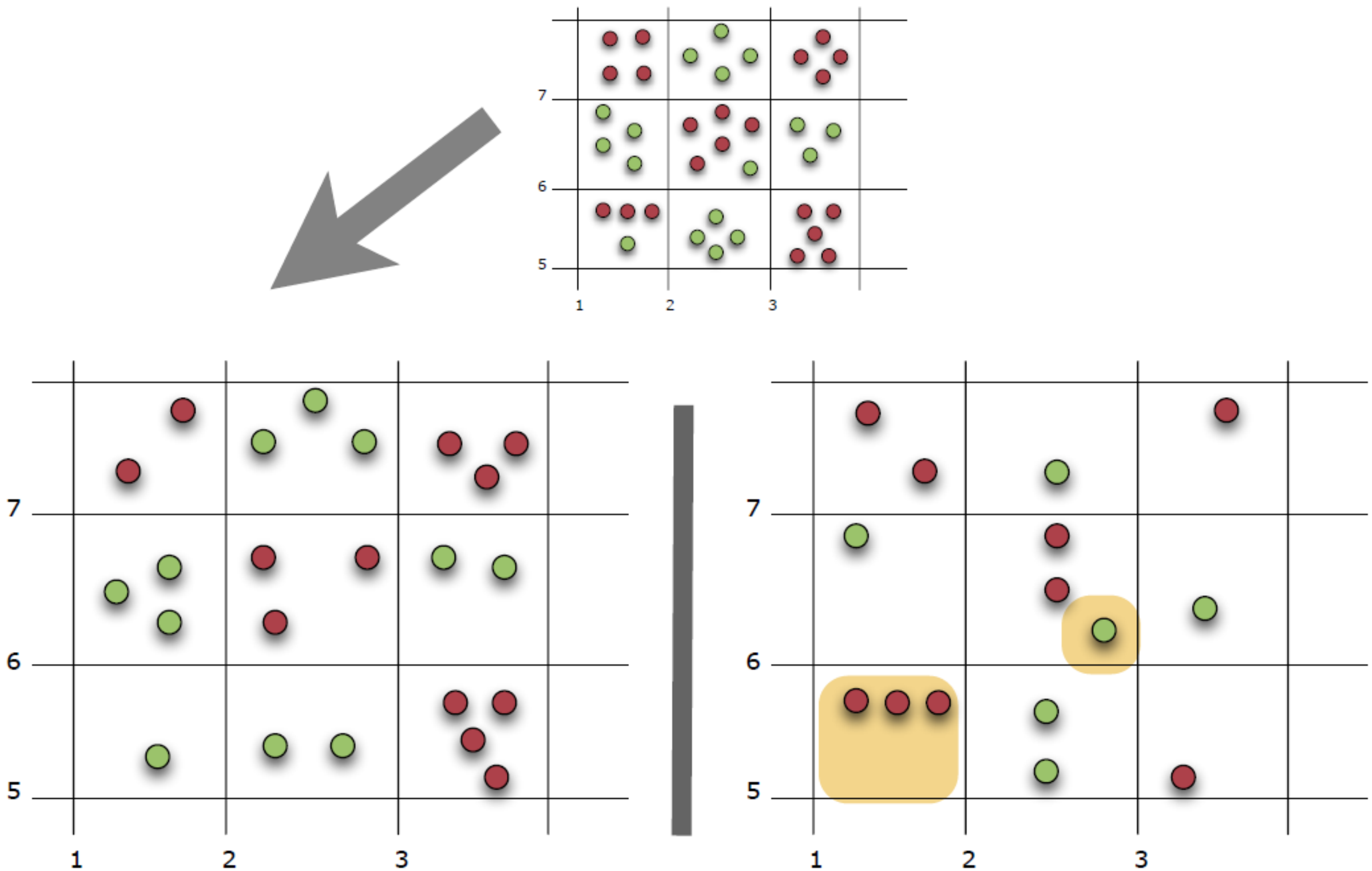


Product Rule:	**C_1: 0.0179,**	C_2: 0.00021,	C_3: 0.0009
Sum Rule:	C_1: 1.55,	**C_2: 1.91,**	C_3: 1.54
Mean Rule:	C_1: 0.310,	**C_2: 0.382,**	C_3: 0.308
Max Rule:	**C_1: 0.85,**	C_2: 0.7,	C_3: 0.8
Median Rule:	C_1: 0.2,	**C_2: 0.5,**	C_3: 0.2
Minimum Rule:	C_1: 0.1,	C_2: 0.01,	**C_3: 0.14**
Weighted Average	**C_1: 0.395,**	C_2: 0.333,	C_3: 0.272
Majority Voting:	C_1: 1,	**C_2: 3,**	C_3: 1 Vote
Weighted Majority Voting:	C_1: 0.3,	**C_2: 0.55,**	C_3: 0.15 Votes
Borda Count:	C_1: 5	**C_2: 6,**	C_3: 4 Votes

Figure 15. Example on various combination rules.

Bootstrap Aggregating: Bagging

- Out of bag error (OOB Error)
  - ✓ Use the training instances that are not sampled for validation

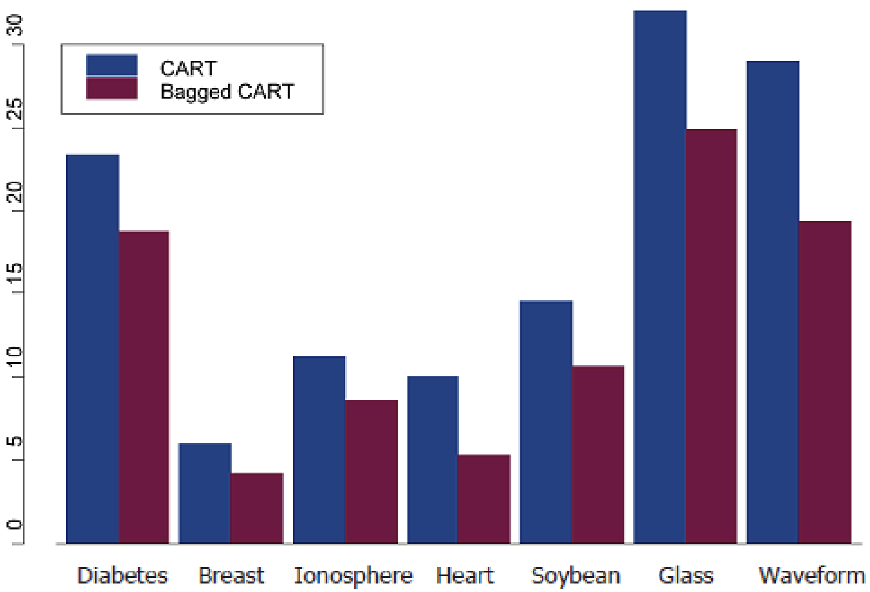


http://www.slideshare.net/oxdata/jan-vitek-distributedrandomforest522013

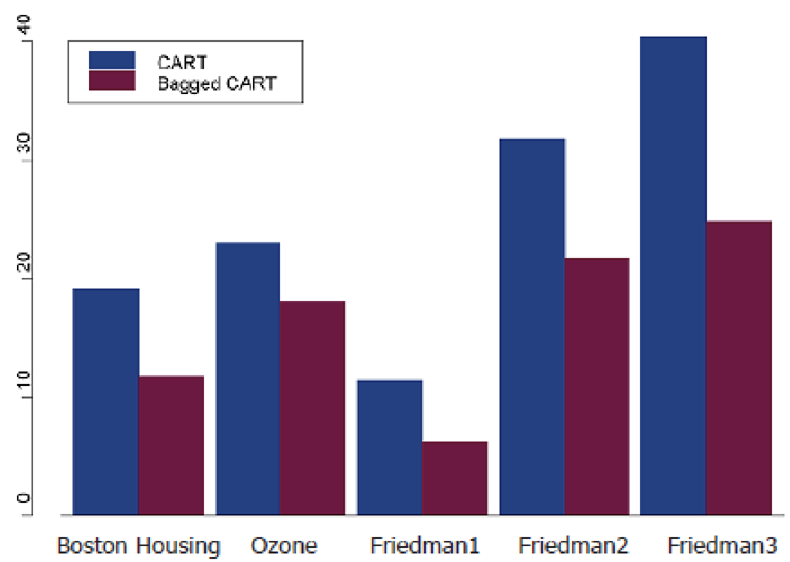
Bootstrap Aggregating: Bagging

- Bagged Trees vs. Single Tree

Classification



Regression




Q & A