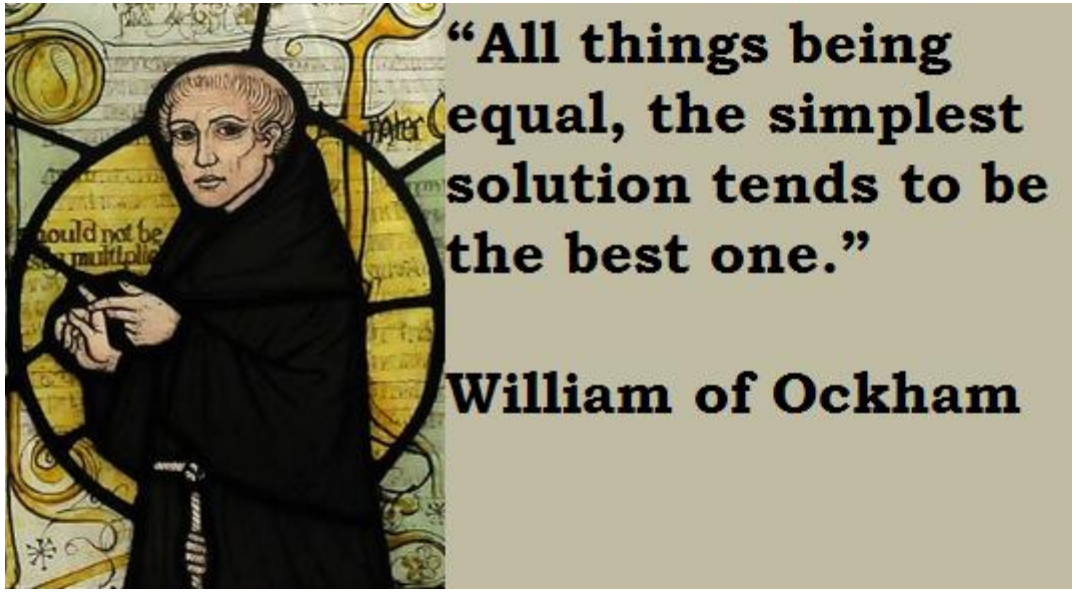

Lecture 6: Dimensionality Reduction

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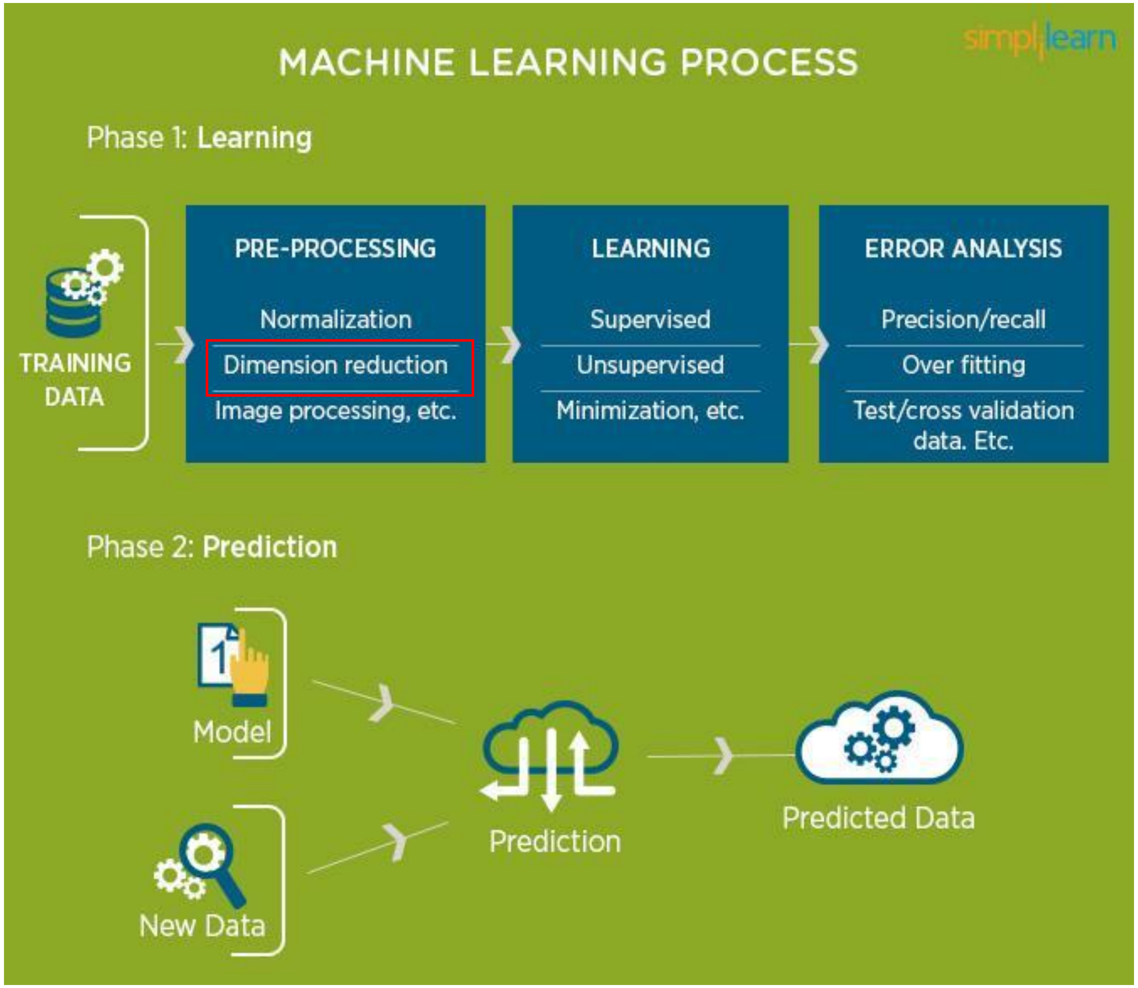
AGENDA

Data Analytics Process

- Process of Business Analytics with Machine Learning



https://www.simplilearn.com/what-is-machine-learning-and-why-it-matters-article

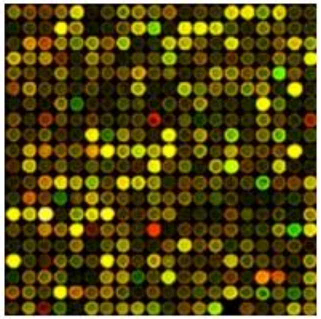
High-dimensional Data

- Examples of high dimensional data

Document classification:
Billions of documents x Thousands/
Millions of words/bigrams matrix

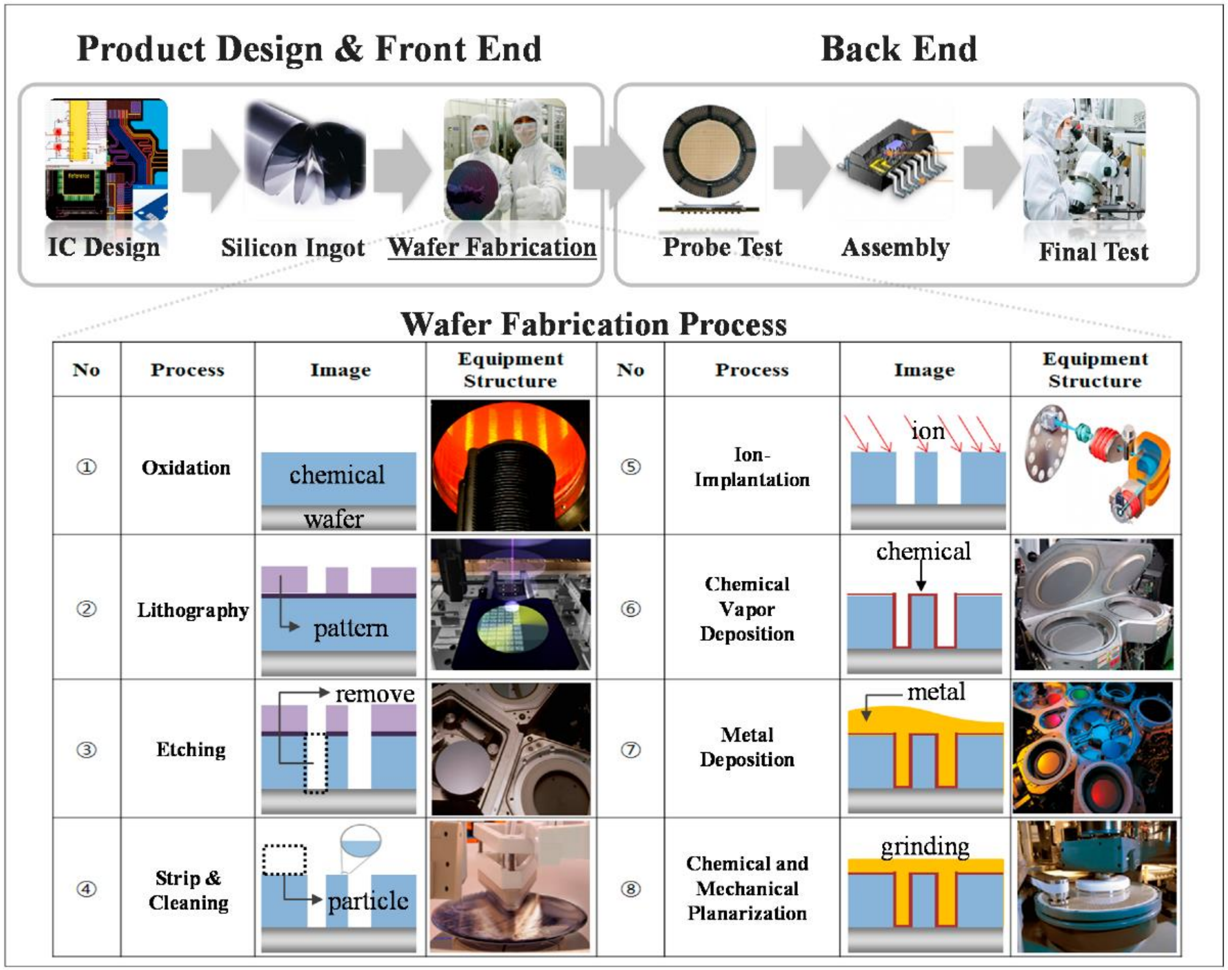
Recommendation systems:
480,189 users x 17,770 movies matrix

Clustering gene expression profiles:
10,000 genes x 1,000 conditions



High-dimensional Data

- Examples of high dimensional data

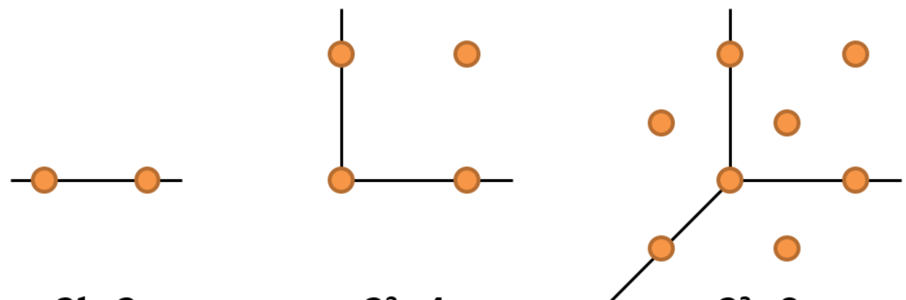


Park, S. H., Kim, S., & Baek, J. G. (2018). Kernel-Density-Based Particle Defect Management for Semiconductor Manufacturing Facilities. Applied Sciences, 8(2), 224.

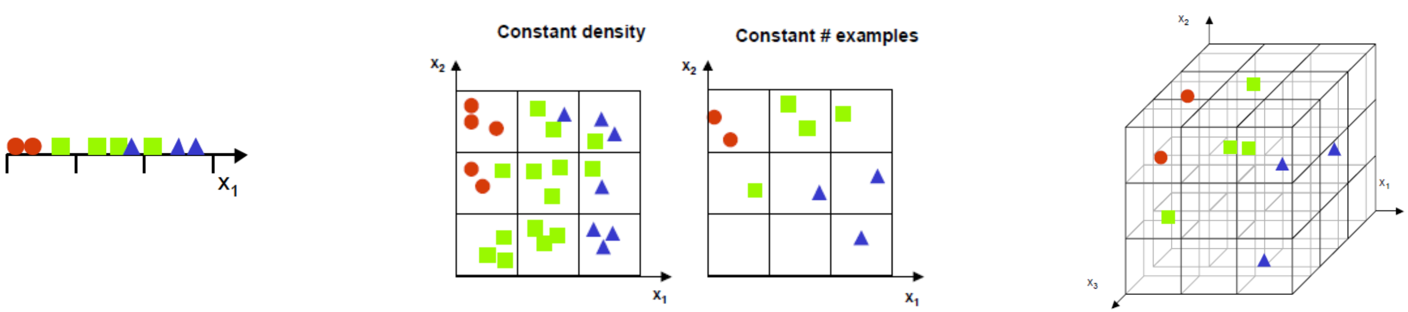
Dimensionality Reduction: Overview

- Curse of dimensionality
  - ✓ The number of instances increases exponentially to achieve the same explanation ability when the number of variables increases

“If there are various logical ways to explain a certain phenomenon, the simplest is the best” - Occam’s Razor

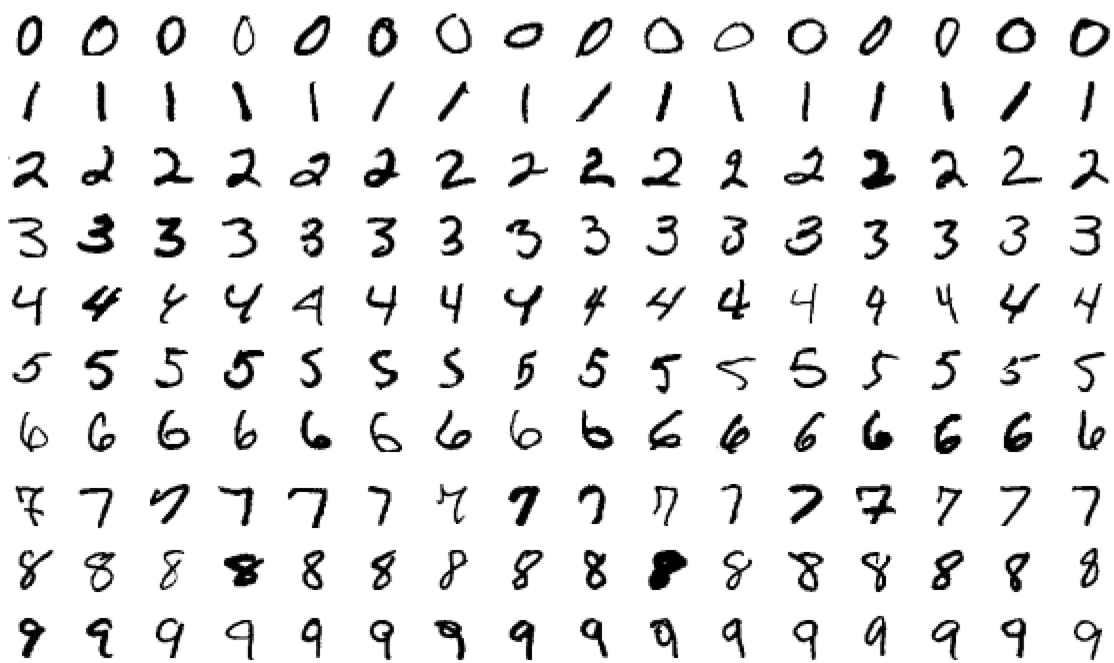


$2^1=2$ $2^2=4$ $2^3=8$



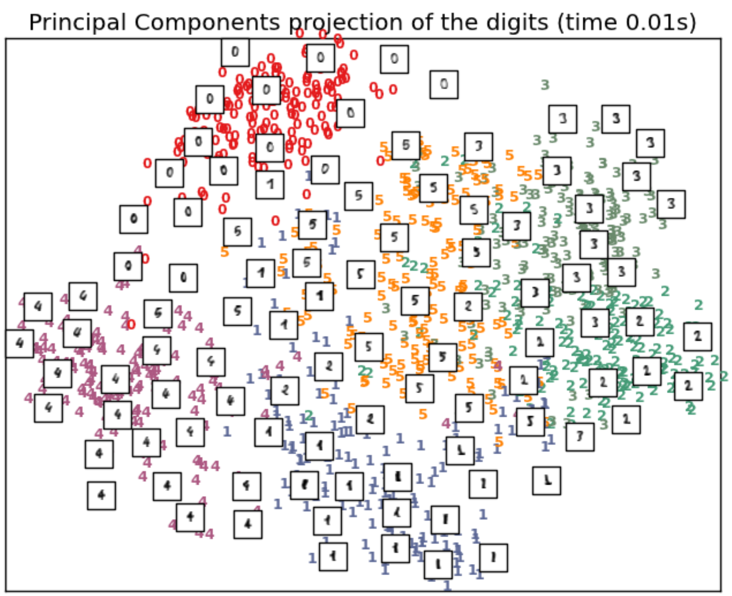
Dimensionality Reduction: Overview

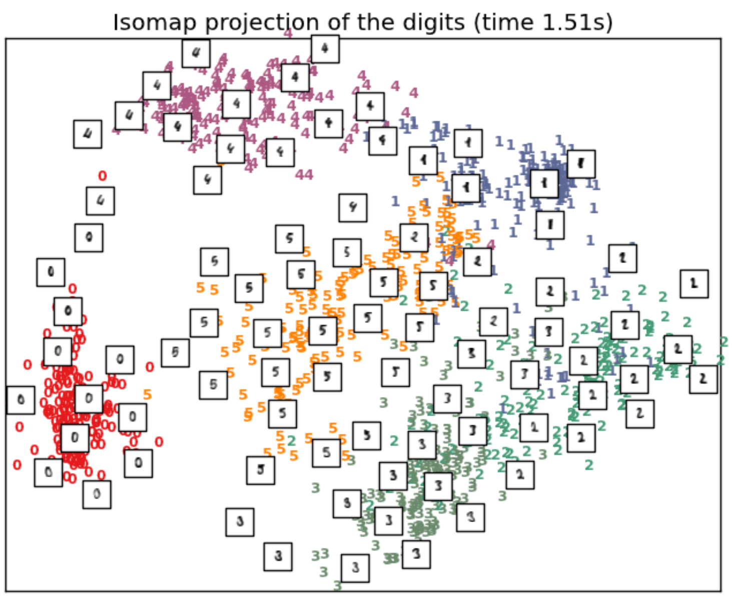
- Curse of dimensionality
  - ✓ Sometimes, an intrinsic dimension is relatively low compared to the original dimension.
    - Ex: handwritten digits in a 16 by 16 pixel (256 dimensions)



Dimensionality Reduction: Overview

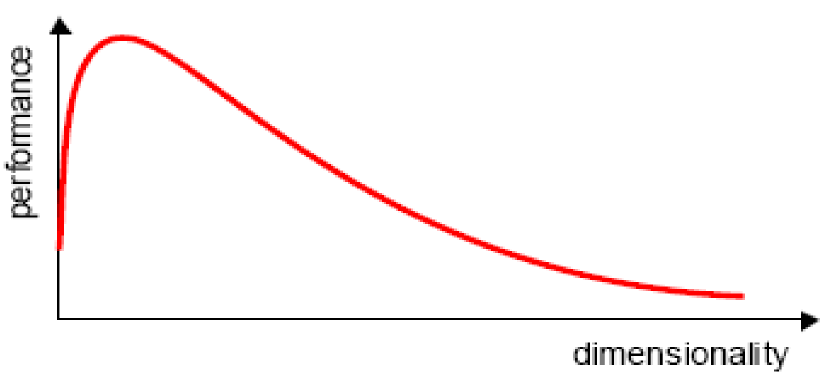
- Curse of dimensionality
  - ✓ Sometimes, an intrinsic dimension is relatively low compared to the original dimension.
    - Ex: handwritten digits in a 16 by 16 pixel (256 dimensions)
    - Reduced to two dimensions by PCA and ISOMAP





Dimensionality Reduction: Overview

- Curse of dimensionality
  - ✓ Problems caused by high-dimensionality
    - Increase the probability of having noise in data → degenerate the prediction performance
    - Increase computational burden for training/applying prediction models
    - Require more number of examples to secure generalization ability of prediction model
  - ✓ To resolve the curse of dimensionality
    - Utilize domain knowledge
    - Use a regularization term in objective function
    - Employ a quantitative reduction technique

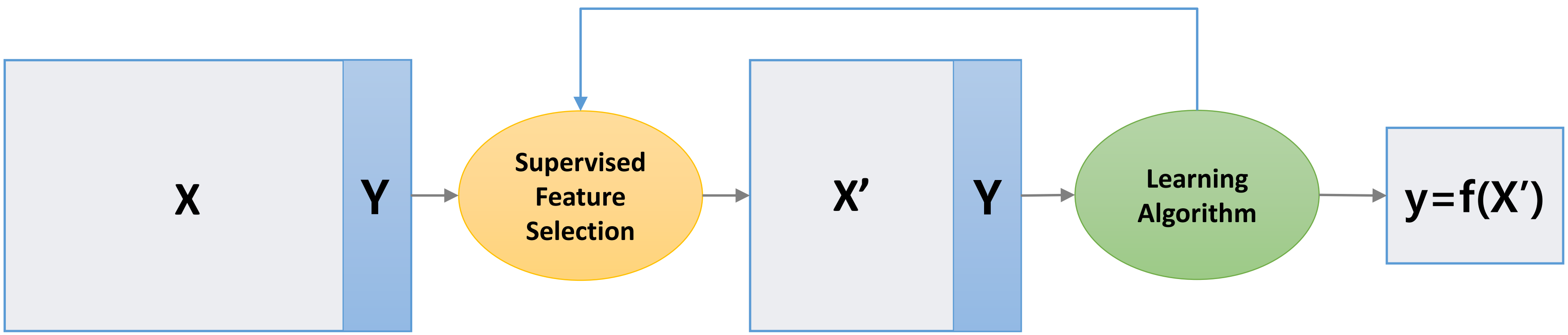


Dimensionality Reduction: Overview

- Backgrounds
  - ✓ Theoretically, model performance improves when the number of variables increases (Under variable independence condition)
  - ✓ In reality, model performance degenerates due to variable dependence, existence of noise, etc.
- Purpose
  - ✓ Identify a subset of variables that best fit the model
- Effect
  - ✓ Remove correlations between variables
  - ✓ Simplified post-processing
  - ✓ Remove redundant or unnecessary variables while keeping relevant information
  - ✓ Visualization can be possible

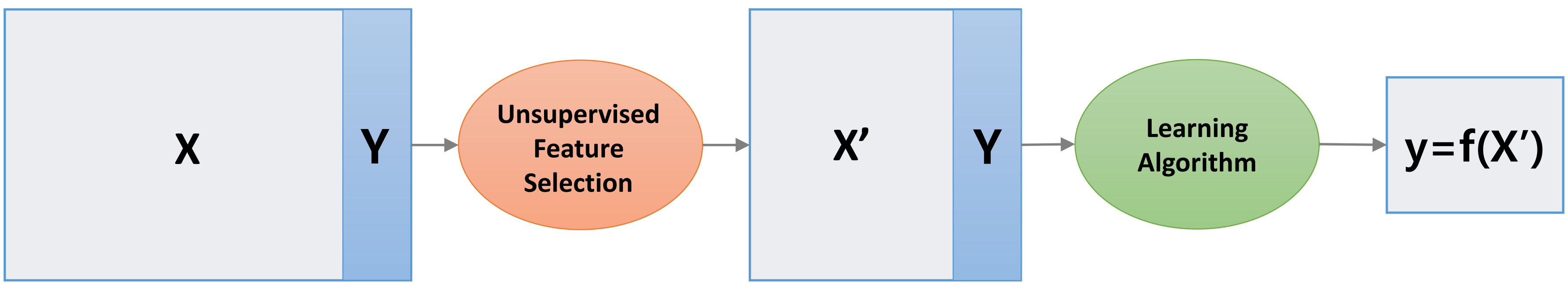
Dimensionality Reduction: Overview

- Supervised vs. Unsupervised Dimensionality Reduction
  - ✓ Supervised dimensionality reduction
    - Use data mining models to verify the reduced dimensions
    - Dimensionality reduction results can be different according to the data mining algorithms employed



Dimensionality Reduction: Overview

- Supervised vs. Unsupervised Dimensionality Reduction
  - ✓ Unsupervised dimensionality reduction
    - Find a set of coordinate systems in a lower dimension that preserve the information (e.g., variance, distance, etc.) in the original input space as much as possible
    - Do not use data mining models during the process
    - Dimensionality reduction results are identical if the data and method is same



Dimensionality Reduction: Overview

- Dimensionality reduction techniques
  - ✓ Variable/feature selection
    - Select a subset of variables from the original variable set
    - Filter – Variable selection and model training are independent
    - Wrapper – Variable selection is done to optimizes the result of the considered data mining model
  - ✓ Variable/feature extraction
    - Extract a new smaller set of variables that preserve the characteristics of the original data
    - Performance metric that is independent from data mining models is used

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \\ x_N \end{bmatrix} \xrightarrow{\text{feature selection}} \begin{bmatrix} x_{i_1} \\ x_{i_2} \\ \\ x_{i_M} \end{bmatrix} \qquad \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \\ x_N \end{bmatrix} \xrightarrow{\text{feature extraction}} \begin{bmatrix} y_1 \\ y_2 \\ \\ y_M \end{bmatrix} = f\left(\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \\ x_N \end{bmatrix}\right)$$

Dimensionality Reduction: Overview

- Selection vs. Extraction
  - ✓ Conceptual difference between variable selection and variable extraction

X_1	X_2	X_3	…	X_n
…	…	…	…	…
…	…	…	…	…
…	…	…	…	…
…	…	…	…	…
…	…	…	…	…

Variable selection

X_1	X_5	X_8
…	…	…
…	…	…
…	…	…
…	…	…
…	…	…

Variable extraction

Z_1	Z_2	Z_3
…	…	…
…	…	…
…	…	…
…	…	…
…	…	…

$Z_1 = X_1 + 0.2{*}X_2$

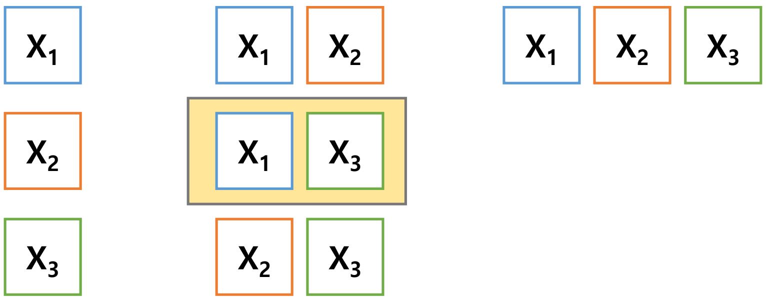
$Z_2 = X_3 - 2{*}X_5$

$Z_3 = X_4 + X_6 - X_9$

AGENDA

Exhaustive Search

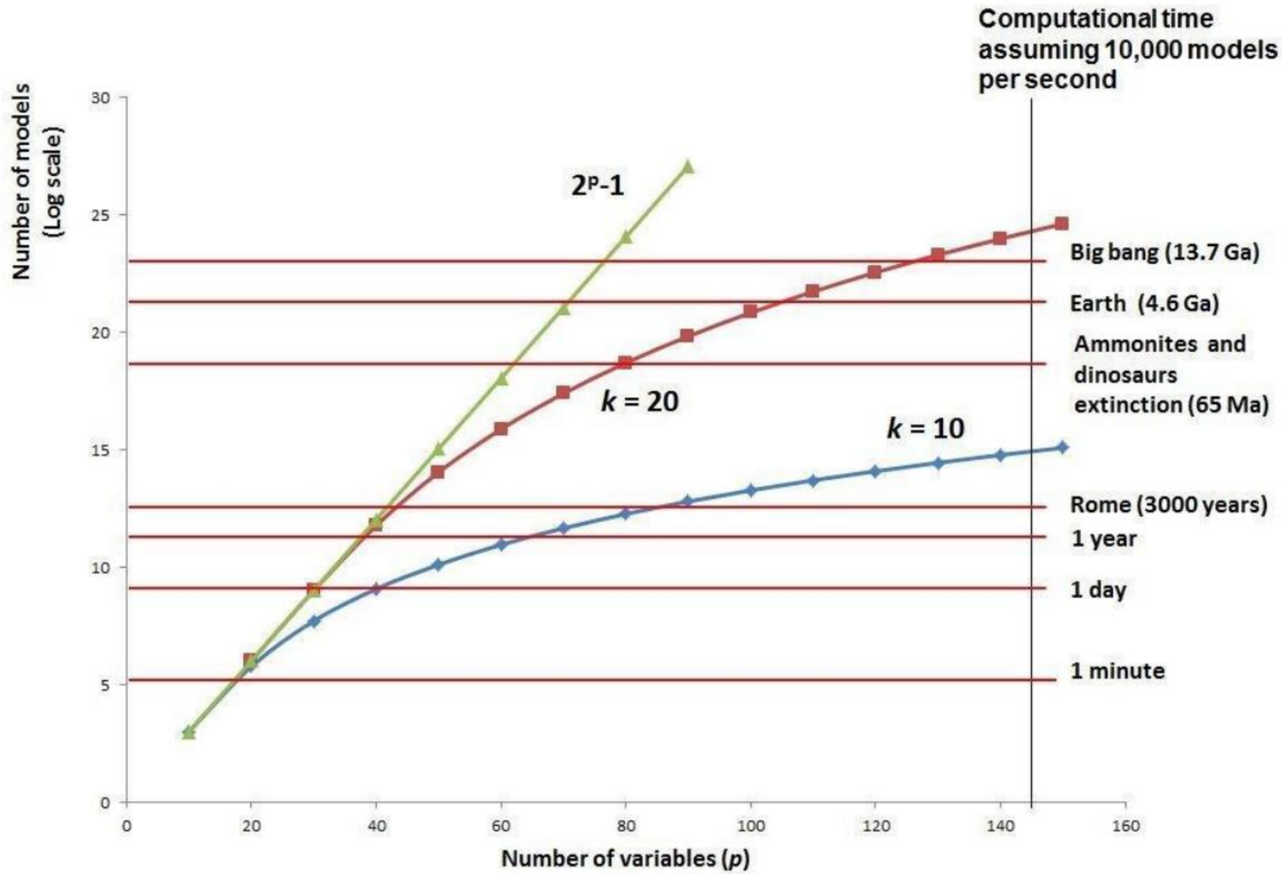
- Exhaustive search
  - ✓ Search all possible combinations
    - Ex) 3 variables X_1 X_2 X_3
    - A total of 7 possible subsets are tested



  - ✓ Performance criteria for variable selection
    - Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), Adjusted R^2, Mallow's C_p, etc.

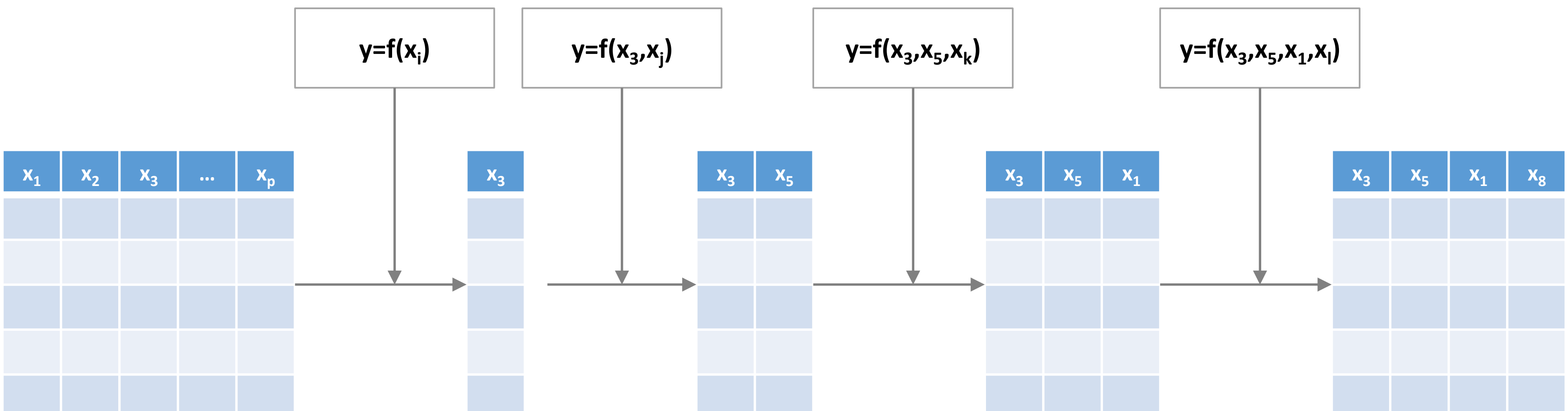
Exhaustive Search

- Exhaustive search
  - ✓ Assume that we have a computer that can evaluate 10,000 models/second



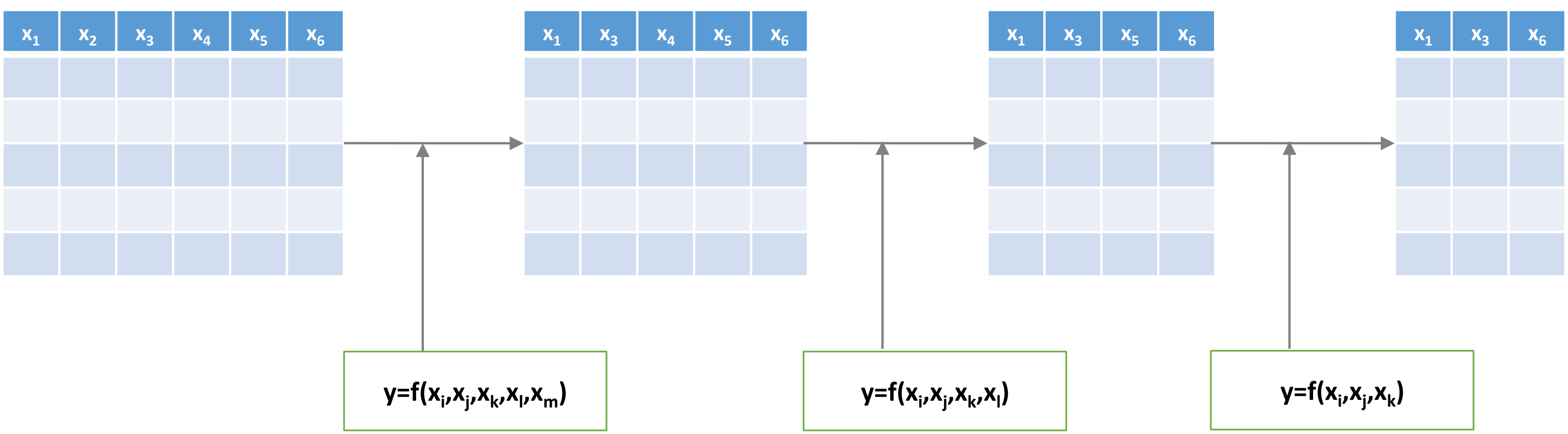
Forward Selection

- Forward selection
  - ✓ From the model with no variable, significant variables are sequentially added
  - ✓ Once the variable is selected, it will never be removed (The number of variables gradually increases)



Backward Elimination

- Backward Elimination
  - ✓ From the model with all variables, irrelevant variables are sequentially removed
  - ✓ Once a variable is removed, it will never be selected (The number of variables gradually decreases)

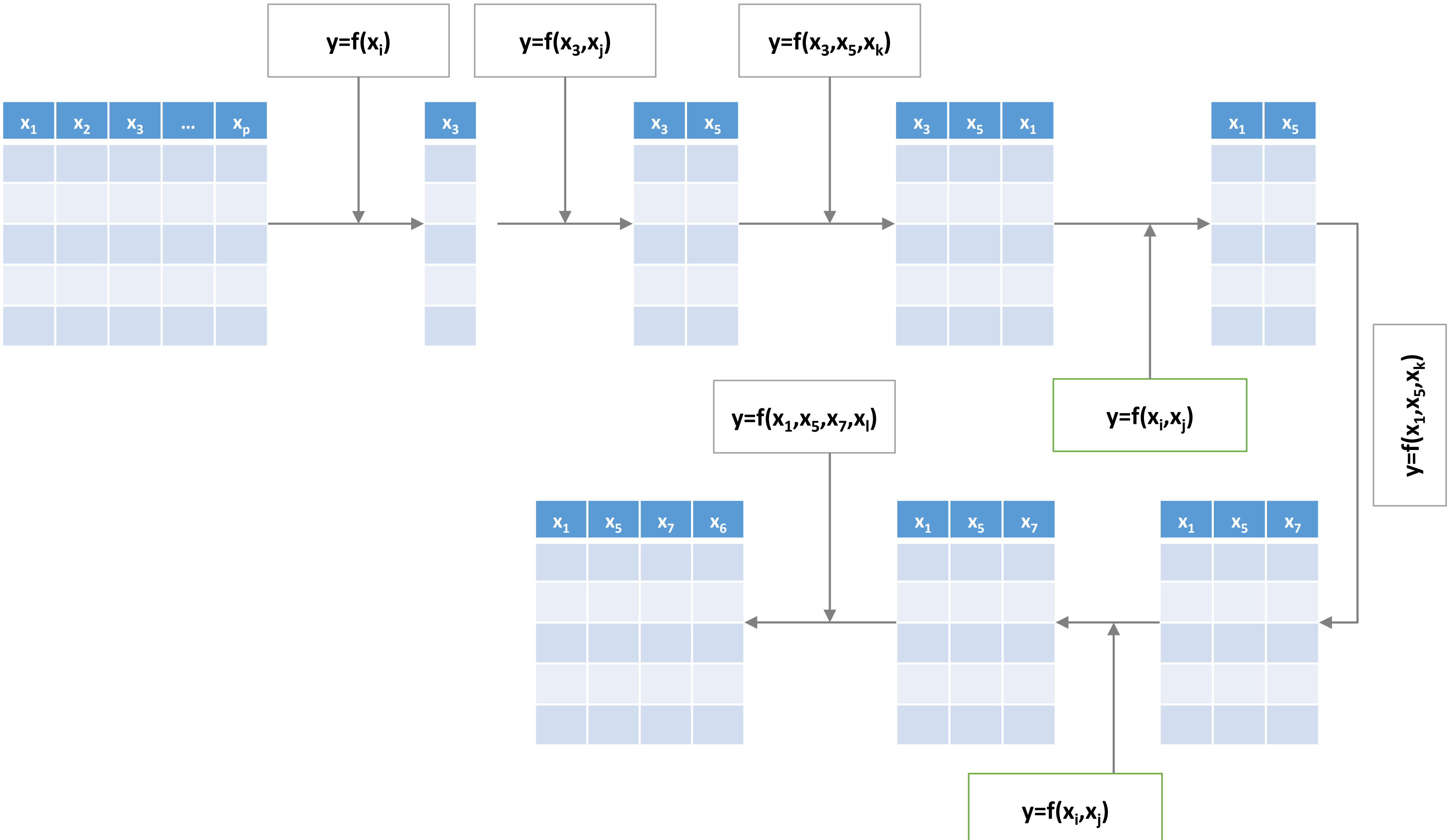


Stepwise Selection

- Stepwise Selection
  - ✓ From the model with no variable, conduct the forward selection and backward elimination alternately
  - ✓ Takes longer time than forward selection/backward elimination, but has more chances to find the optimal set of variables
  - ✓ Variables that is either selected/removed can be reconsidered for selection/removal
  - ✓ The number of variables increases in the early period, but it can either increase or decrease

Stepwise Selection

- Stepwise selection example



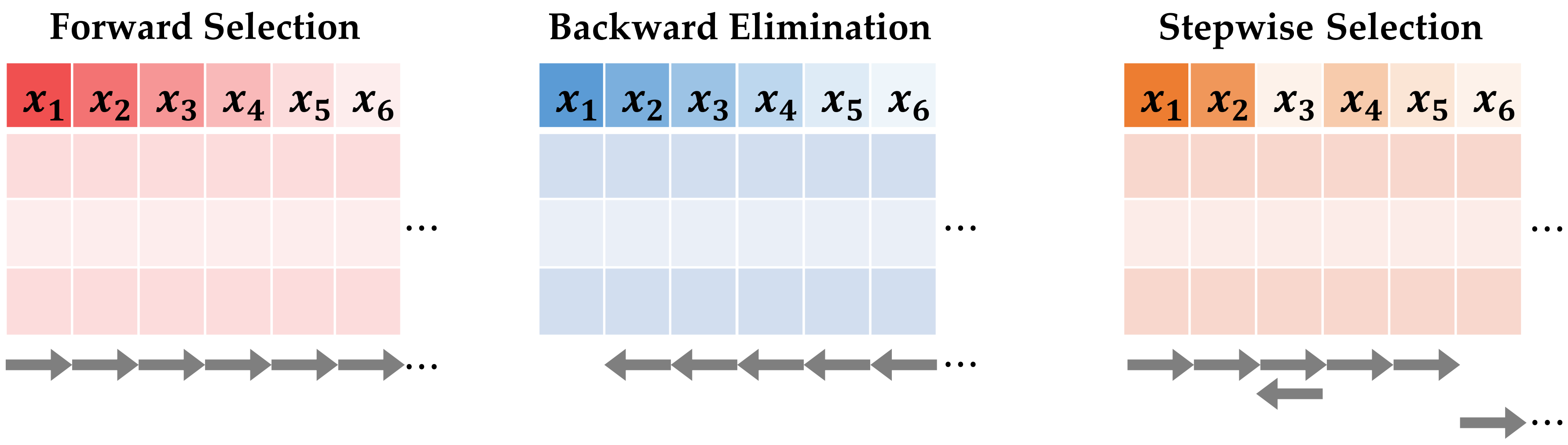
Stepwise Selection

- Stepwise Selection
  - ✓ Stepwise selection process

> - Start with model with no predictors.
> - Add variable with largest F-statistic (provided P less than some cut-off).
> - Refit with this variable added. Recompute all F statistics for adding one of the remaining variables and add variable with largest F statistic.
> - At each step after adding a variable try to eliminate any variable not significant at some level (that is, do BACKWARD elimination till that stops).
> - After doing the backwards steps take another FORWARD step.
> - Continue until every remaining variable is significant at cut-off level and every excluded variable is insignificant OR until variable to be added is same as last deleted variable.

Comparison among FS/BE/SS

- Illustrative Example



Performance Metrics

- Akaike Information Criteria (AIC)
  - ✓ Sum of squared error (SSE) with the number of variables as a penalty term

$$AIC = n \cdot ln\left(\frac{SSE}{n}\right) + 2k$$

- Bayesian Information Criteria (BIC)
  - ✓ SSE, number of variables, standard deviation obtained by the model with all variables

$$BIC = n \cdot ln\left(\frac{SSE}{n}\right) + \frac{2(k+2)n\sigma^2}{SSE} - \frac{2n^2\sigma^4}{SSE^2}$$

Performance Metrics

- Adjusted R^2
  - ✓ Simple R^2 increases when the number of variable increases

$$\begin{aligned} \text{Model 1}: \quad & y = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k + \epsilon \\ \text{Model 2}: \quad & y = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k + \ldots + \beta_{k+m} x_{k+m} \epsilon \end{aligned}$$

$$R^2(M2) \geq R^2(M1)$$

  - ✓ Use the adjusted R^2 that account for the number of variables (k)

$$\text{Adjusted } R^2 = 1 - \left(\frac{n-1}{n-k-1}\right)(1 - R^2) = 1 - \frac{n-1}{n-k-1}\frac{SSE}{SST}$$

Genetic Algorithm

Siedlecki, W. Sklansky (1989)

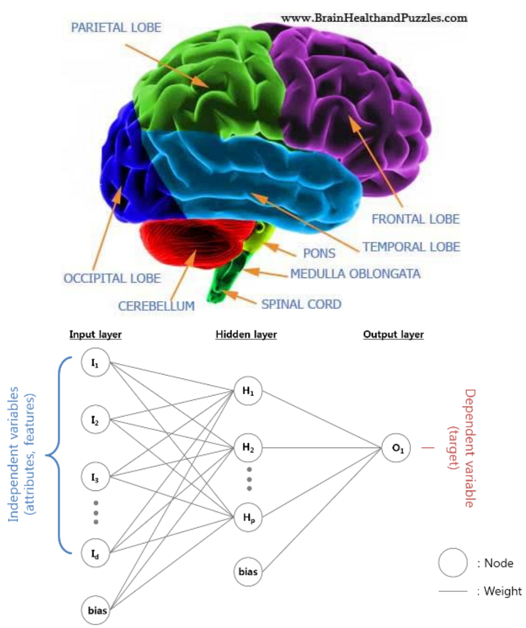
- Limitations of the previous variable selection methods
  - ✓ Exhaustive search: guarantee the optimal subset, but takes too long time (practically impossible for many tasks)
  - ✓ Local search (forward/backward/stepwise): efficient search but the search space is very limited, which leads to a low probability of finding the optimal solution

- Idea
  - ✓ Improve the performance of local searches with a little additional computational time!

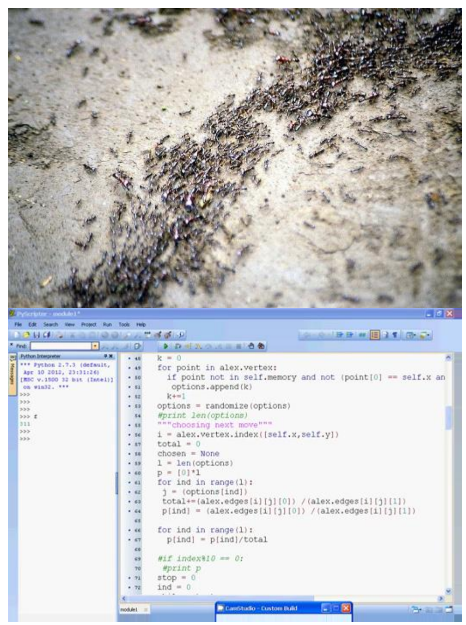
Genetic Algorithm

- Meta-Heuristic Approach
  - ✓ Solve a complex problem by doing trials and errors efficiently
  - ✓ Among the optimization algorithms, many of them mimic the way of a natural system works

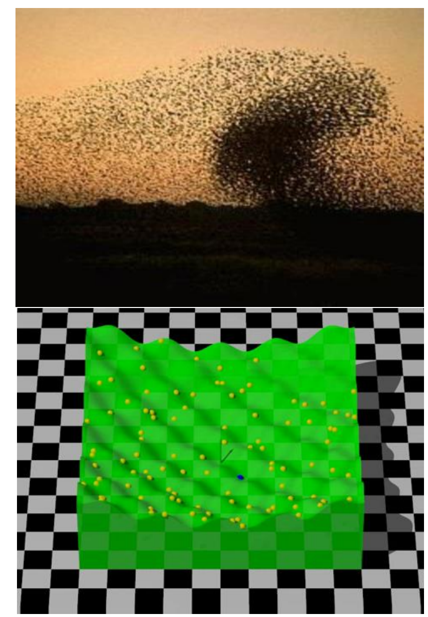
Artificial Neural Networks



Ant Colony Algorithm

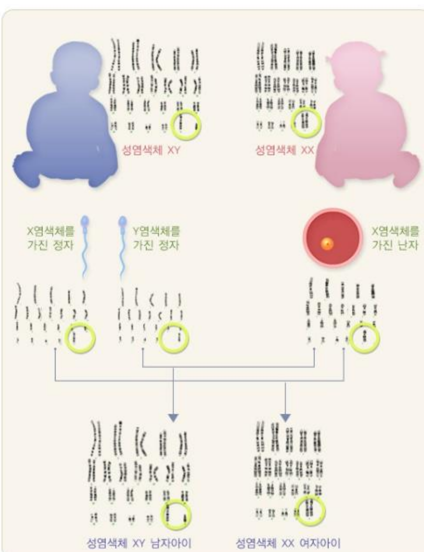


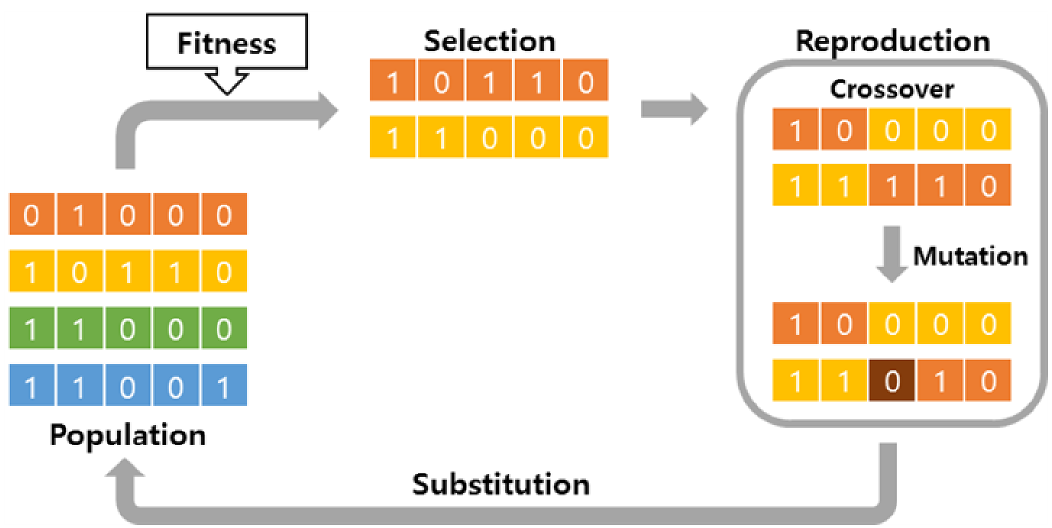
Particle Swarm Optimization



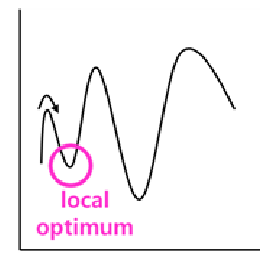
Genetic Algorithm

- An Evolutionary Algorithm that mimics the Reproduction of Creatures
  - ✓ Find a superior solutions and preserve by repeating the reproduction process
    - Selection: Select a superior solution to improve the quality
    - Crossover: Search various alternatives based on the current solutions
    - Mutation: Give a chance to escape the local optima

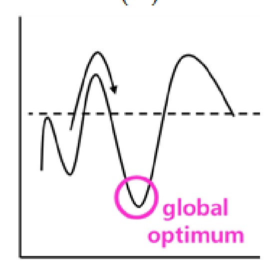




(a)



(b)



(c)

Genetic Algorithm

- Genetic Algorithm for Feature Selection

Step 1: Initialize Chromosomes

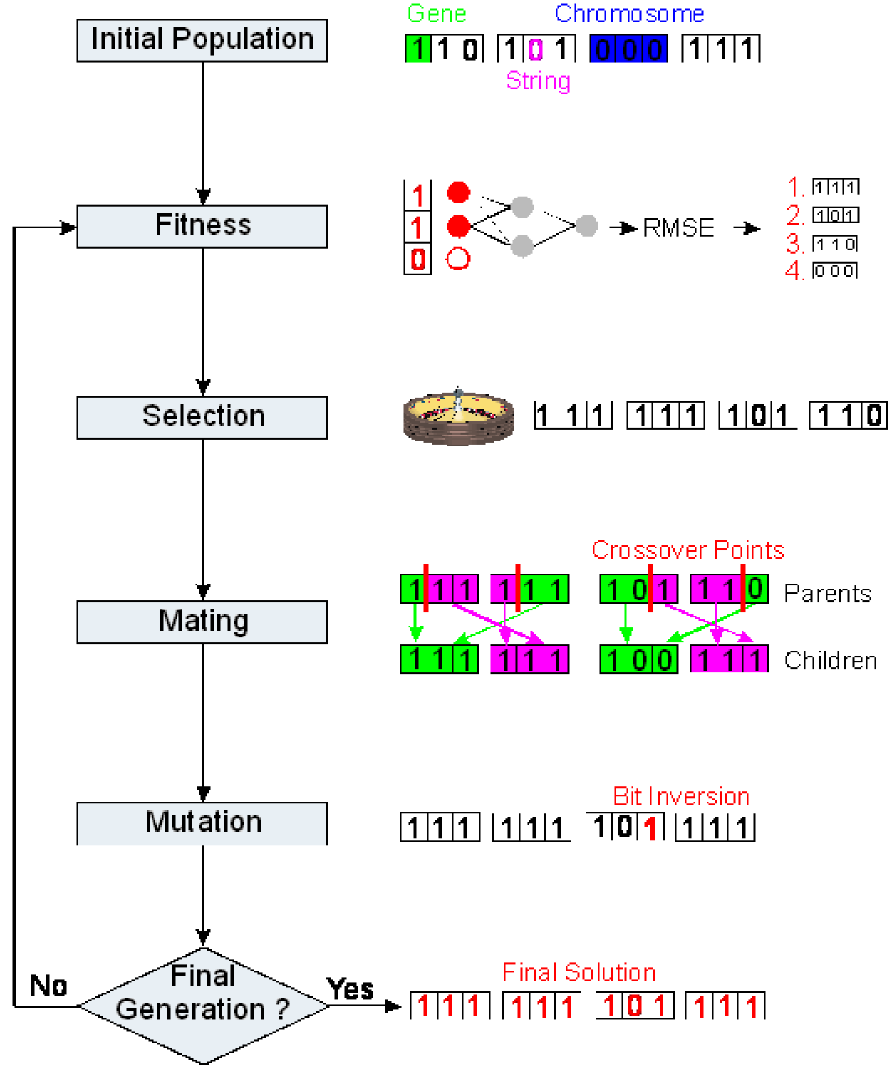
Step 2:
Model training based on chromosomes

Step 3:
Fitness evaluation

Step 4:
Selection good chromosomes

Step 5:
Create next generation: Crossover & Mutation

Step 6:
Select the final variable set



GA Step 1: Initialization

- Encoding Chromosomes
  - ✓ Genetic algorithm can be used not only for variable selection, but for a wide range of optimization problems
  - ✓ Encoding scheme can be different for different tasks
  - ✓ Binary encoding is commonly used for variable selection

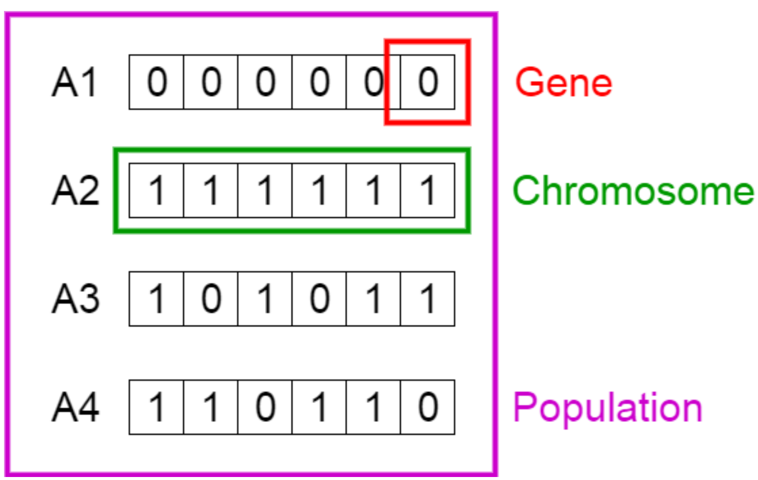
Chromosome — Gene

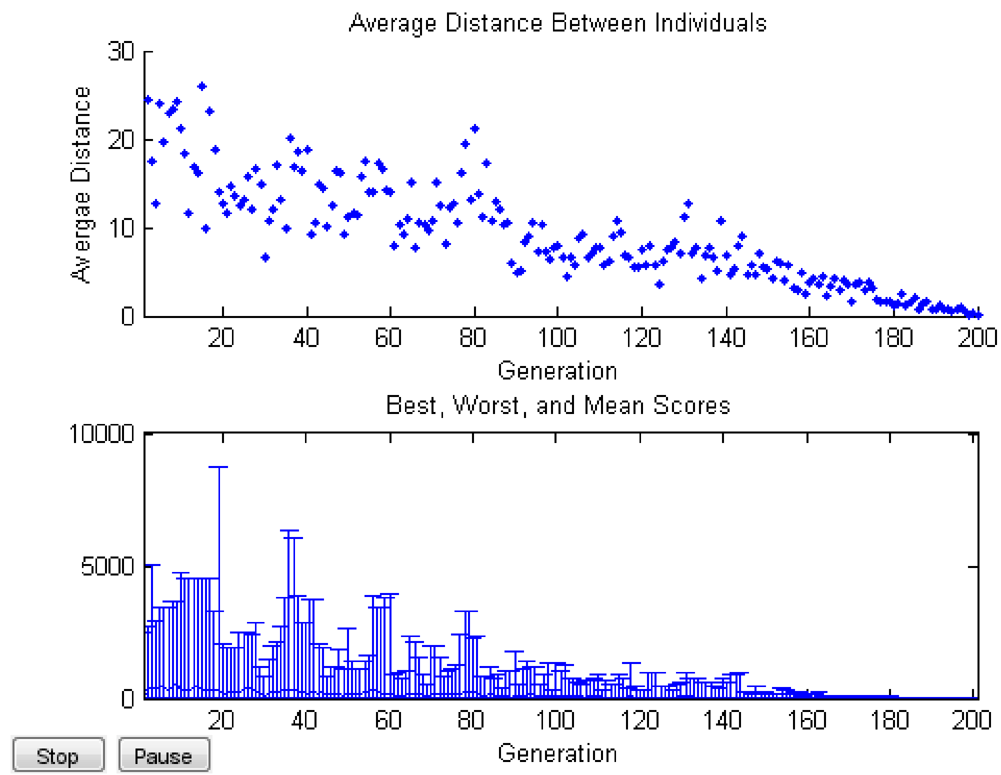
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	…	x_d
1	0	0	1	0	1	1	0	…	1

1: Use the corresponding variable in the modeling
0: Do not use the variable

GA Step 1: Initialization

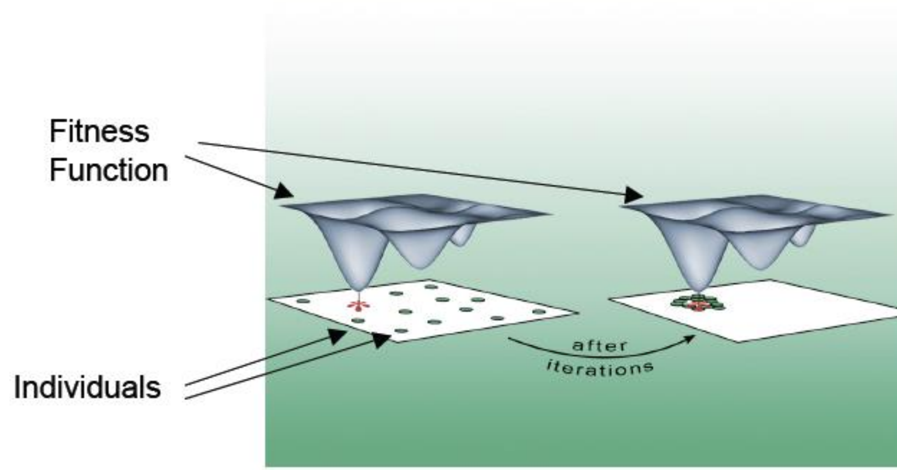
- Parameter Initialization
  - ✓ The number of chromosome (population)
  - ✓ Fitness function
  - ✓ Crossover mechanism
  - ✓ The rate of mutation
  - ✓ Stopping criteria
    - minimum fitness improvement
    - maximum iterations, etc.





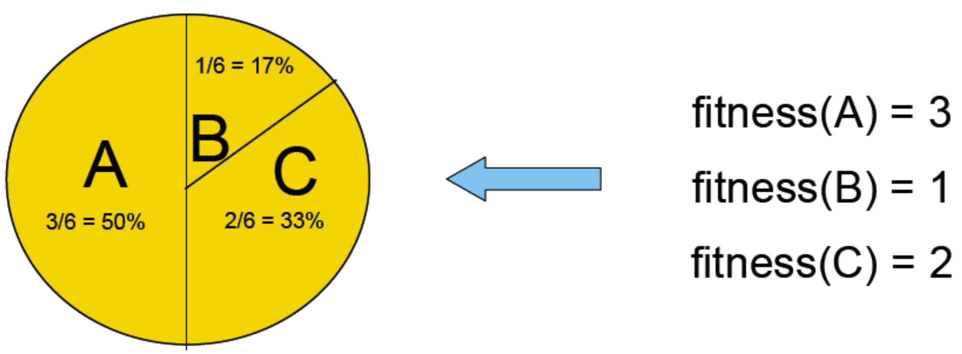
GA Step 3: Fitness Evaluation

- Fitness Function
  - ✓ A criterion that determines which chromosomes are better than others
  - ✓ In general, the higher the fitness value, the better the chromosomes
  - ✓ Common criteria that are embedded in the fitness function
    - If two chromosomes have the same fitness value, the one with fewer variables is preferred
    - If two chromosomes use the same number of variables, the one with higher predictive performance is preferred
  - ✓ In case of multiple linear regression
    - Adjusted R2
    - Akaike information criterion (AIC)
    - Bayesian information criterion (BIC)



GA Step 4: Selection

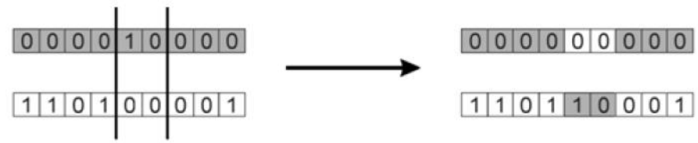
- Selection
  - ✓ Select superior chromosomes in the current population to reproduce the population of the next generation
  - ✓ Deterministic selection
    - ▪ Select only top N% of chromosomes
    - ▪ Bottom (100-N)% chromosomes are never selected
  - ✓ Probabilistic selection
    - ▪ Use the fitness value of each chromosome as the selection weight
    - ▪ All chromosomes can be selected with different probabilities



GA Step 5: Crossover & Mutation

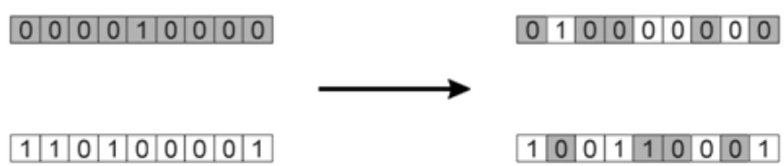
- Crossover (Reproduction)
  - ✓ Two child chromosomes are produced from two parent chromosomes
  - ✓ The number of crossover points can vary from 1 to n (total number of genes)

Crossover point = 2



Crossover points = 3

Crossover points = N



Assume array: [0.35, 0.62, 0.18, 0.42, 0.83, 0.76, 0.39, 0.51, 0.36]

GA Step 5: Crossover & Mutation

- Mutation
  - ✓ Genetic operator used to maintain diversity from one generation of a population of chromosomes to the next
  - ✓ Alters one or more gene values in a chromosome from its initial state, which result in entirely new gene values being added to the gene pool
  - ✓ By mutation, the current solution can have a chance to escape from the local optima
  - ✓ A too mutation rate can increase the time to converge (0.01 can be a good choice)

Consider the two original off-springs selected for mutation.

Original offspring 1	1	1	0	1	1	1	1	0	0	0	0	1	1	1	1	0
Original offspring 2	1	1	0	1	1	0	0	1	0	0	1	1	0	1	1	0

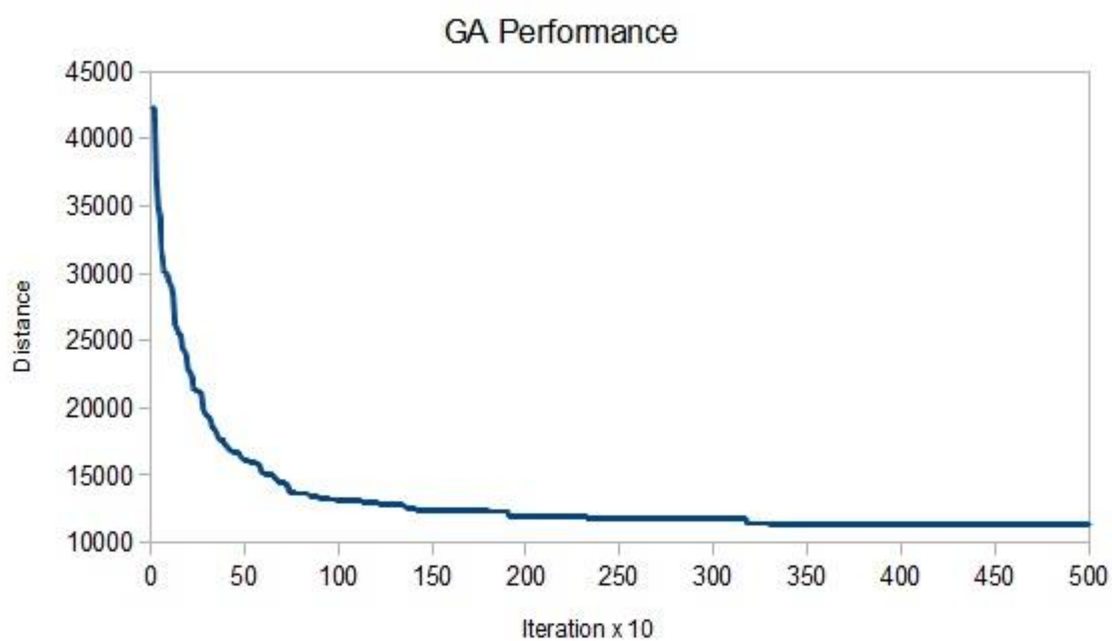
Invert the value of the chosen gene as **0** to **1** and **1** to **0**

The Mutated Off-spring produced are :

Mutated offspring 1	1	1	0	0	1	1	1	0	0	0	0	1	1	1	1	0
Mutated offspring 2	1	1	0	1	1	0	1	1	0	0	1	1	0	1	0	0

GA Step 5: Find the Best Solution

- Find the best variable subset
  - ✓ Select the chromosome with the highest fitness value after the stopping criteria are satisfied.
  - ✓ Generally, significant fitness improvement occurs in the early stages, which becomes marginal after some generations



AGENDA

Revisit MLR

- Multiple Linear Regression
  - ✓ Formulation

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 \cdots + \hat{\beta}_d x_d$$

  - ✓ Objective function (should be minimized)

$$\frac{1}{2}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \frac{1}{2}\sum_{i=1}^{n}\left(y_i - \sum_{j=0}^{d}\hat{\beta}_j x_{ij}\right)^2$$

Revisit Logistic Regression

- Logistic Regression
  - ✓ Formulation

$$log(Odds) = log\Big(\frac{p}{1-p}\Big) = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 \cdots + \hat{\beta}_d x_d$$

  - ✓ Objective function (should be minimized)

$$-\sum_{i=1}^{n}\left(y_i \log\left(\frac{1}{1+\exp(-\sum_{j=0}^{d}\hat{\beta}_j x_j)}\right) + (1-y_i)\log\left(\frac{\exp(-\sum_{j=0}^{d}\hat{\beta}_j x_j)}{1+\exp(-\sum_{j=0}^{d}\hat{\beta}_j x_j)}\right)\right)$$

Ridge Regression

- Ridge Linear Regression

$$\frac{1}{2}\sum_{i=1}^{n}\left(y_i - \sum_{j=0}^{d}\hat{\beta}_j x_{ij}\right)^2 + \lambda\sum_{j=1}^{d}\hat{\beta}_j^2$$

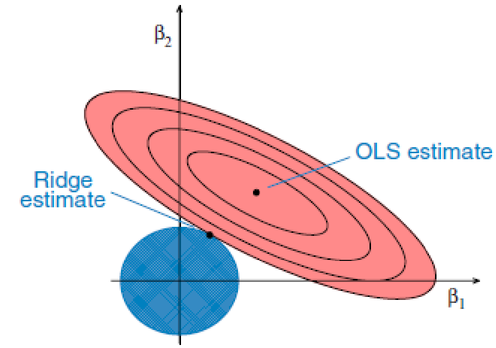
- Ridge Logistic Regression

$$-\sum_{i=1}^{n}\left(y_i \log\left(\frac{1}{1+\exp(-\sum_{j=0}^{d}\hat{\beta}_j x_j)}\right) + (1-y_i)\log\left(\frac{\exp(-\sum_{j=0}^{d}\hat{\beta}_j x_j)}{1+\exp(-\sum_{j=0}^{d}\hat{\beta}_j x_j)}\right)\right) + \lambda\sum_{j=1}^{d}\hat{\beta}_j^2$$

Ridge Regression

- Ridge (Logistic) Regression
  - ✓ Add L_2 nom penalty for the objective function

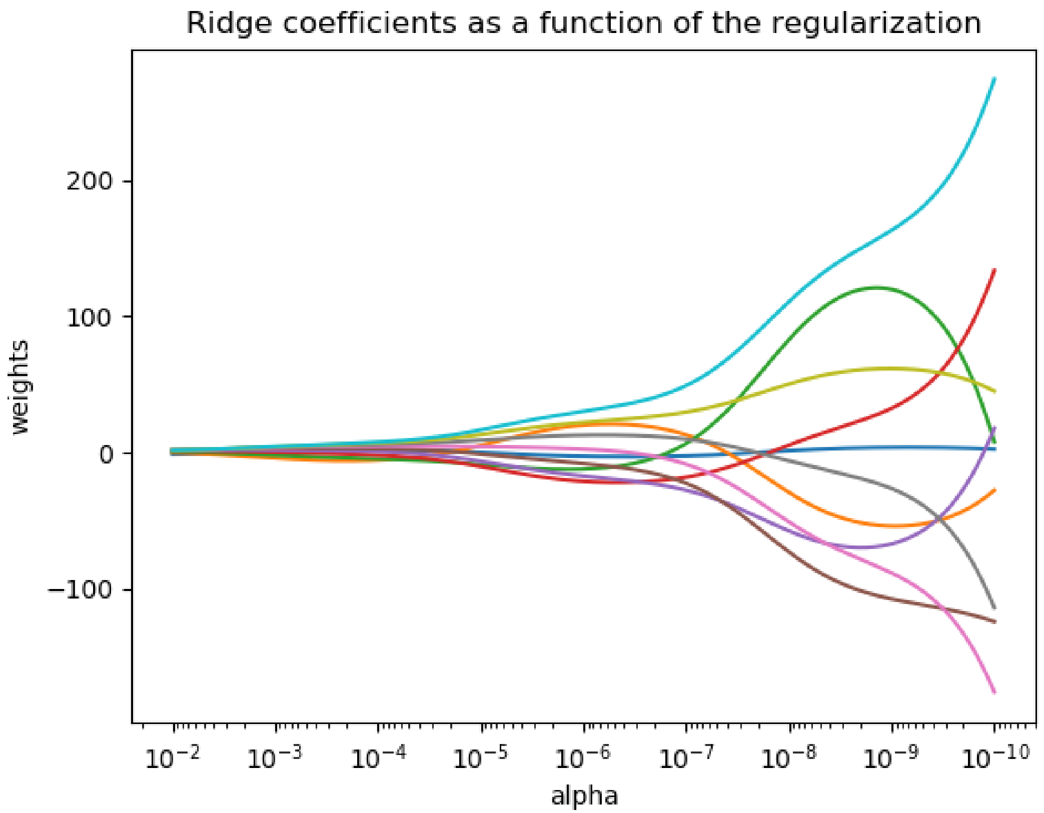
$$\lambda \sum_{j=1}^{d} \hat{\beta}_j^2$$



  - ✓ Properties
    - If two models have the same performance, smaller regression coefficients are preferred
    - Regression coefficients can be very small, but hard to make them exactly 0 ➔ not for variable selection
    - Work well when input variables have high correlations

Ridge Regression

- Ridge (Logistic) Regression
  - ✓ Example of estimated regression coefficients according to different λ



http://scikit-learn.org/stable/auto_examples/linear_model/plot_ridge_path.html#sphx-glr-auto-examples-linear-model-plot-ridge-path-py

LASSO

- LASSO: Least Absolute Shrinkage and Selection Operator

  ✓ Multiple Linear Regression

$$\frac{1}{2}\sum_{i=1}^{n}\left(y_i - \sum_{j=0}^{d}\hat{\beta}_j x_{ij}\right)^2 + \lambda\sum_{j=1}^{d}|\hat{\beta}_j|$$

  ✓ Logistic Regression

$$-\sum_{i=1}^{n}\left(y_i \log\left(\frac{1}{1+\exp(-\sum_{j=0}^{d}\hat{\beta}_j x_j)}\right) + (1-y_i)\log\left(\frac{\exp(-\sum_{j=0}^{d}\hat{\beta}_j x_j)}{1+\exp(-\sum_{j=0}^{d}\hat{\beta}_j x_j)}\right)\right) + \lambda\sum_{j=1}^{d}|\hat{\beta}_j|$$

LASSO

- LASSO: Least Absolute Shrinkage and Selection Operator
  - ✓ Ridge gives L_2 norm penalty while LASSO gives L_1 norm penalty
  - ✓ Can make the coefficients of irrelevant variables 0 → can do variable selection
  - ✓ The number of selected variables (variables with non-zero coefficients) vary according to λ

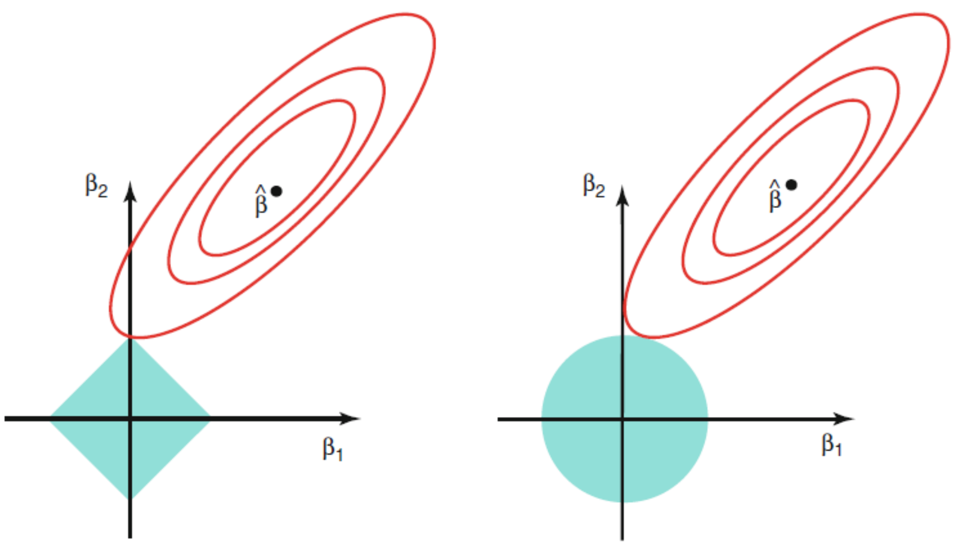
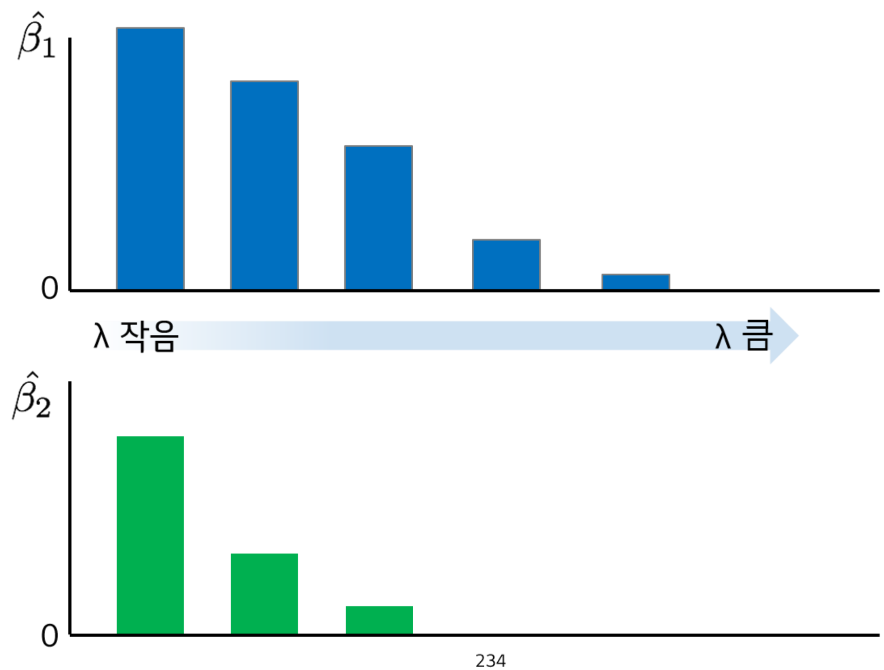
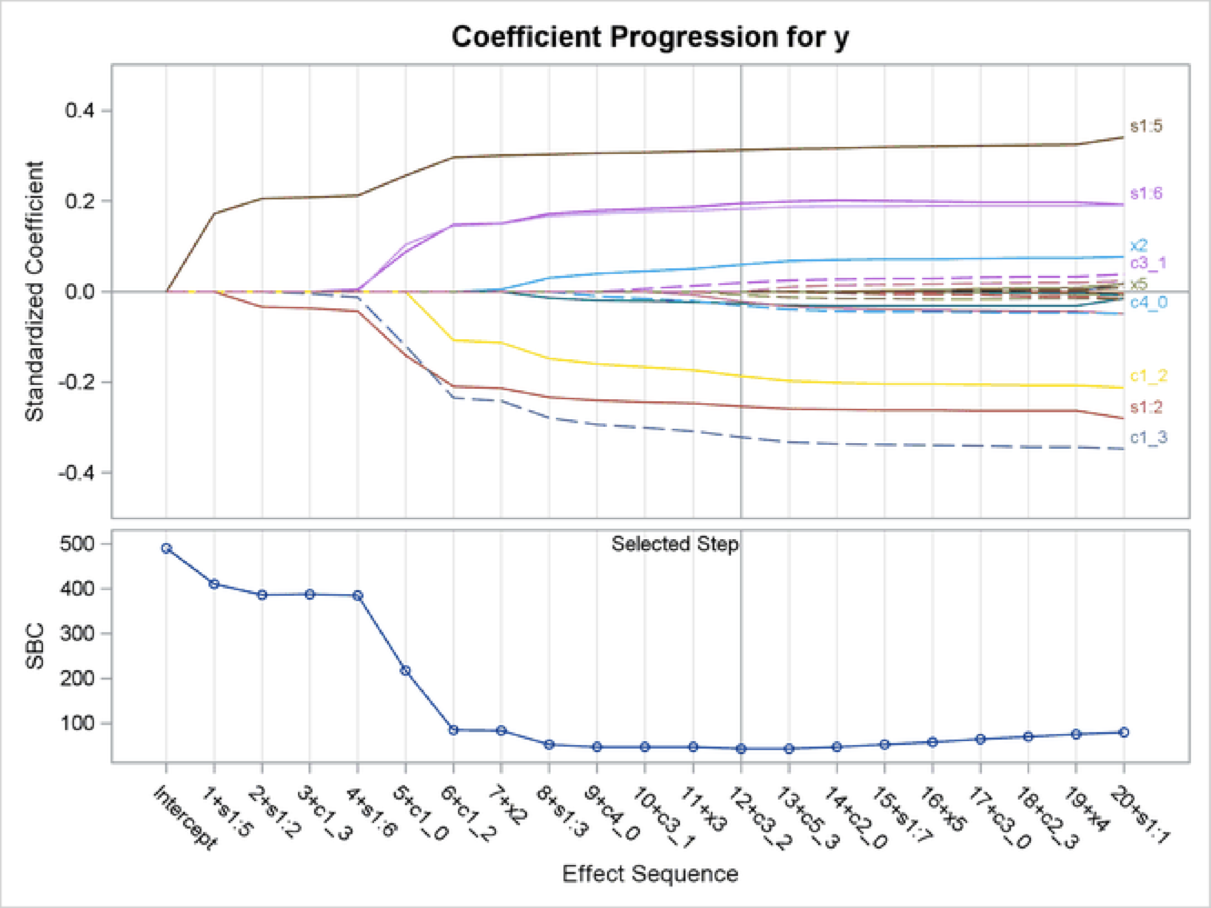


FIGURE 6.7. *Contours of the error and constraint functions for the lasso* (left) *and ridge regression* (right). *The solid blue areas are the constraint regions, $|\beta_1| + |\beta_2| \leq s$ and $\beta_1^2 + \beta_2^2 \leq s$, while the red ellipses are the contours of the RSS.*



LASSO

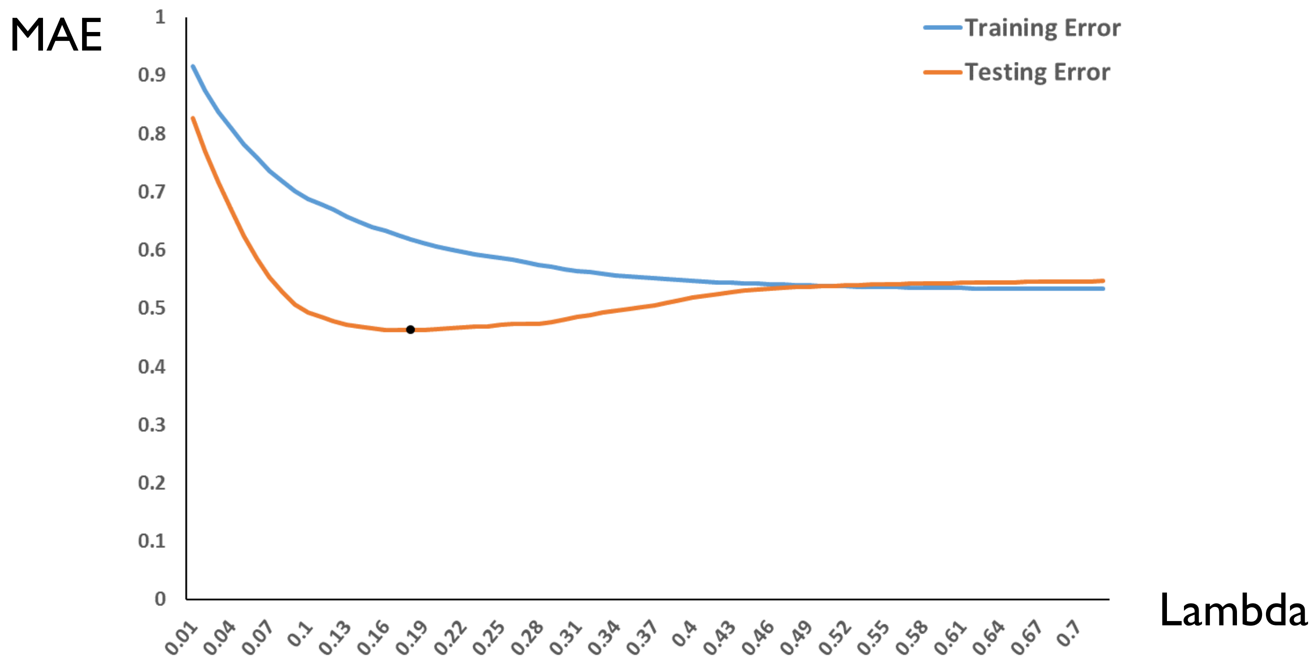
- LASSO: Least Absolute Shrinkage and Selection Operator
  - ✓ Example of estimated regression coefficients according to different λ



http://support.sas.com/documentation/cdl/en/statug/68162/HTML/default/viewer.htm#statug_glmselect_examples08.htm

LASSO

- LASSO: Least Absolute Shrinkage and Selection Operator
  - ✓ determine the best λ with the highest regression performance



  - ✓ Limitation: Both variable selection and regression performance degenerate if variables are highly correlated

Elastic Net

- Elastic Net
  - ✓ Can have advantages of both Ridge (considering correlation between variables) and LASSO (variable selection ability)
  - ✓ Multiple Linear Regression

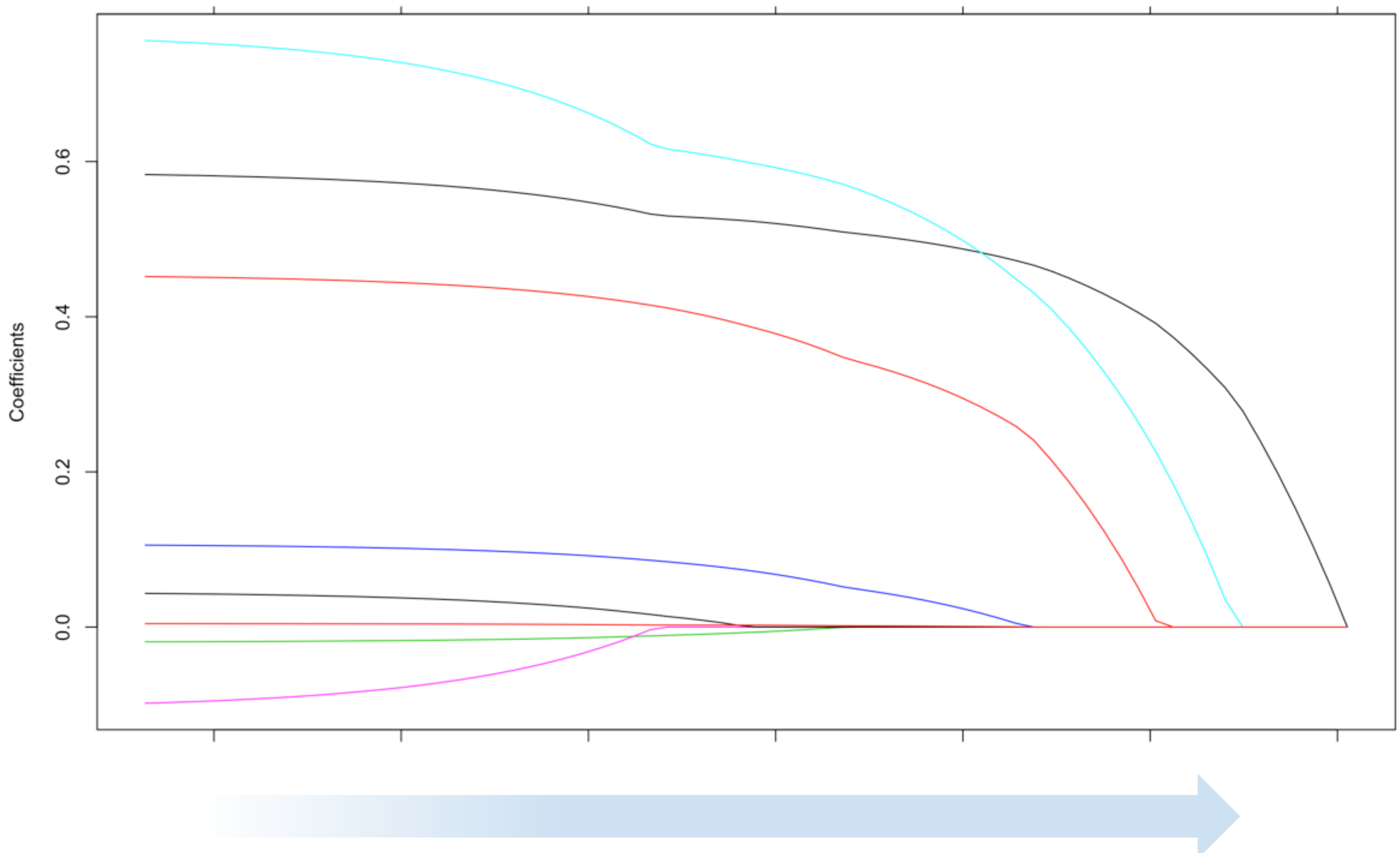
$$\frac{1}{2}\sum_{i=1}^{n}\left(y_i - \sum_{j=0}^{d}\hat{\beta}_j x_{ij}\right)^2 + \lambda_1 \sum_{j=1}^{d}|\hat{\beta}_j| + \lambda_2 \sum_{j=1}^{d}\hat{\beta}_j^2$$

  - ✓ Logistic Regression

$$-\sum_{i=1}^{n}\left(y_i \log\left(\frac{1}{1+\exp(-\sum_{j=0}^{d}\hat{\beta}_j x_j)}\right) + (1-y_i)\log\left(\frac{\exp(-\sum_{j=0}^{d}\hat{\beta}_j x_j)}{1+\exp(-\sum_{j=0}^{d}\hat{\beta}_j x_j)}\right)\right)$$

$$+\lambda_1 \sum_{j=1}^{d}|\hat{\beta}_j| + \lambda_2 \sum_{j=1}^{d}\hat{\beta}_j^2$$

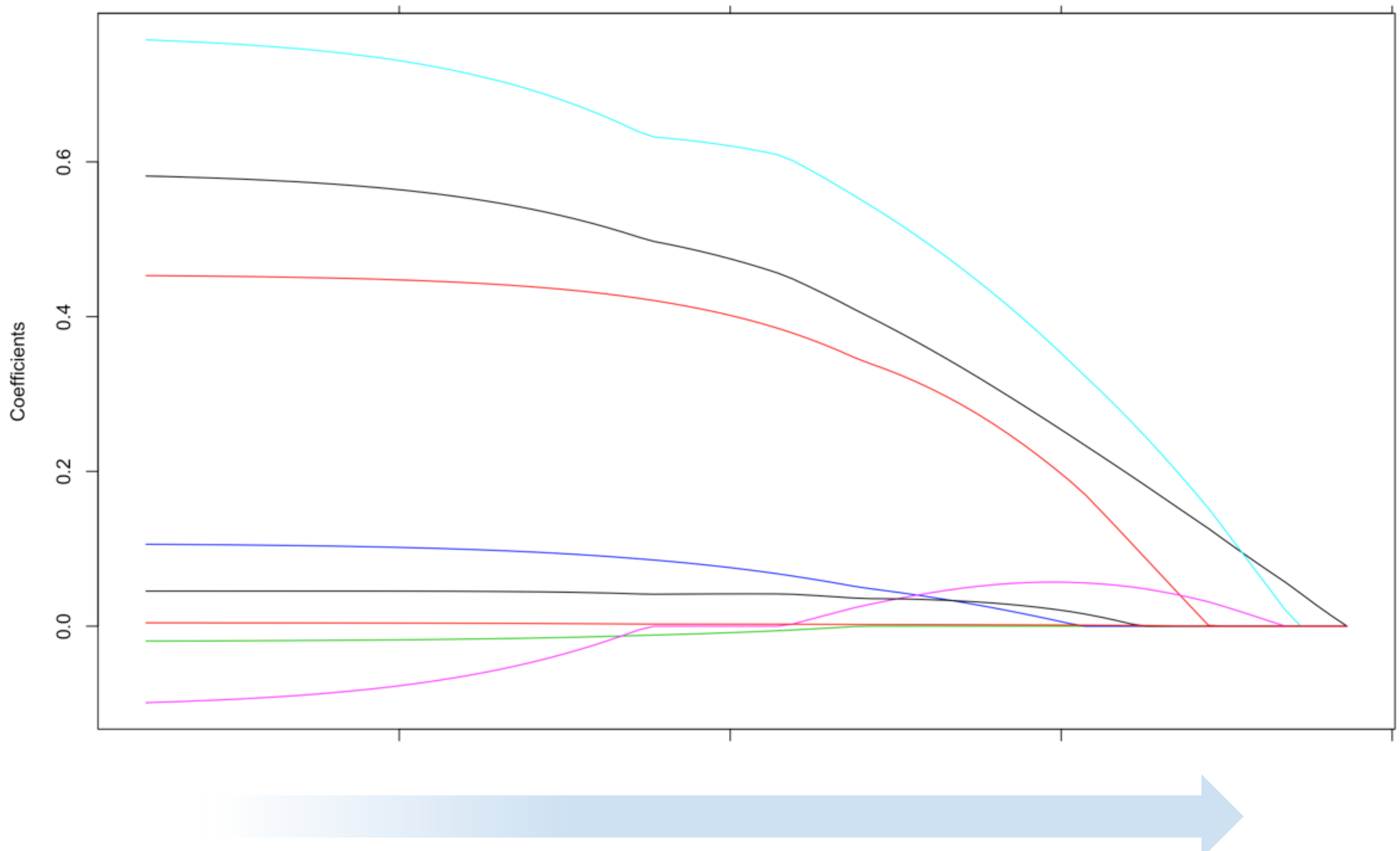
Elastic Net



λ_1 Increases

Number of variable decreases

Elastic Net

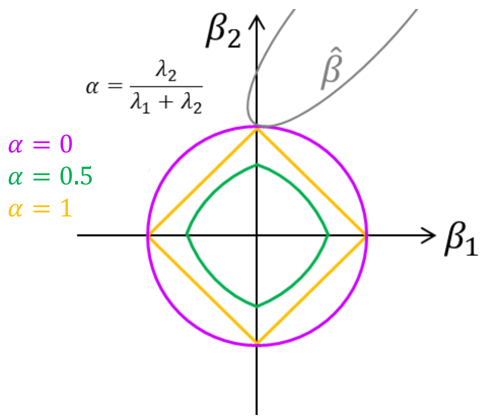


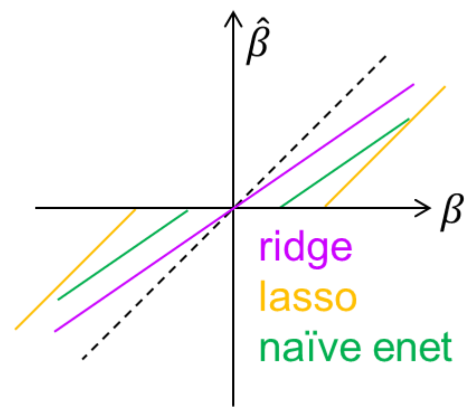
λ_2 Increases

Little impact on variable selection

Empirical Study

- Compare four variable selection methods and three shrinkage methods
  - ✓ Variable selection: Forward selection, Backward elimination, Stepwise selection, GA
  - ✓ Shrinkage: Ridge, Lasso, Elasitic Net





Ridge	$\hat{\beta} = \min_{\beta} \lvert Y - X\beta \rvert^2 + \lambda_1 \lvert \beta \rvert^2$	shrinkage
Lasso	$\hat{\beta} = \min_{\beta} \lvert Y - X\beta \rvert^2 + \lambda_2 \lvert \beta \rvert^1$	shrinkage, variable selection
Elastic net	$\hat{\beta} = \min_{\beta} \lvert Y - X\beta \rvert^2 + \lambda_2 \lvert \beta \rvert^1 + \lambda_1 \lvert \beta \rvert^2$	shrinkage, variable selection, grouping effect

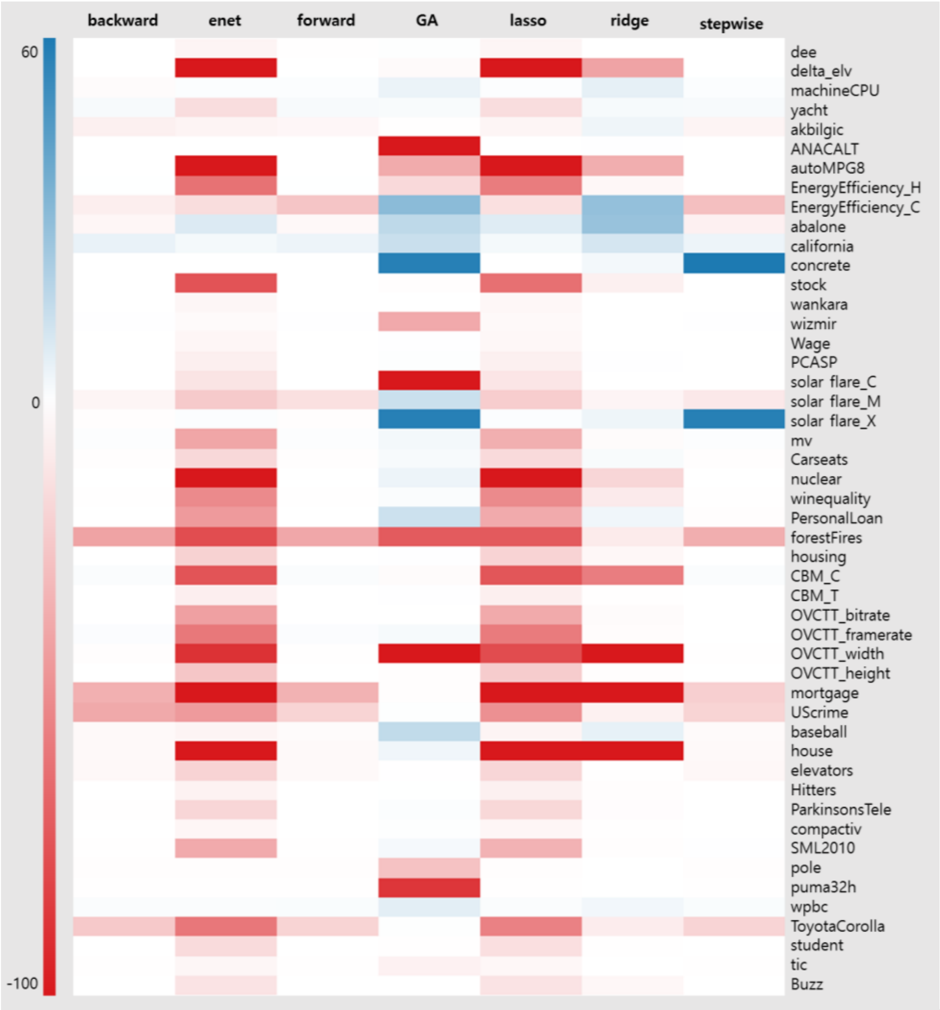
Empirical Study

- Data sets: 49 regression data sets

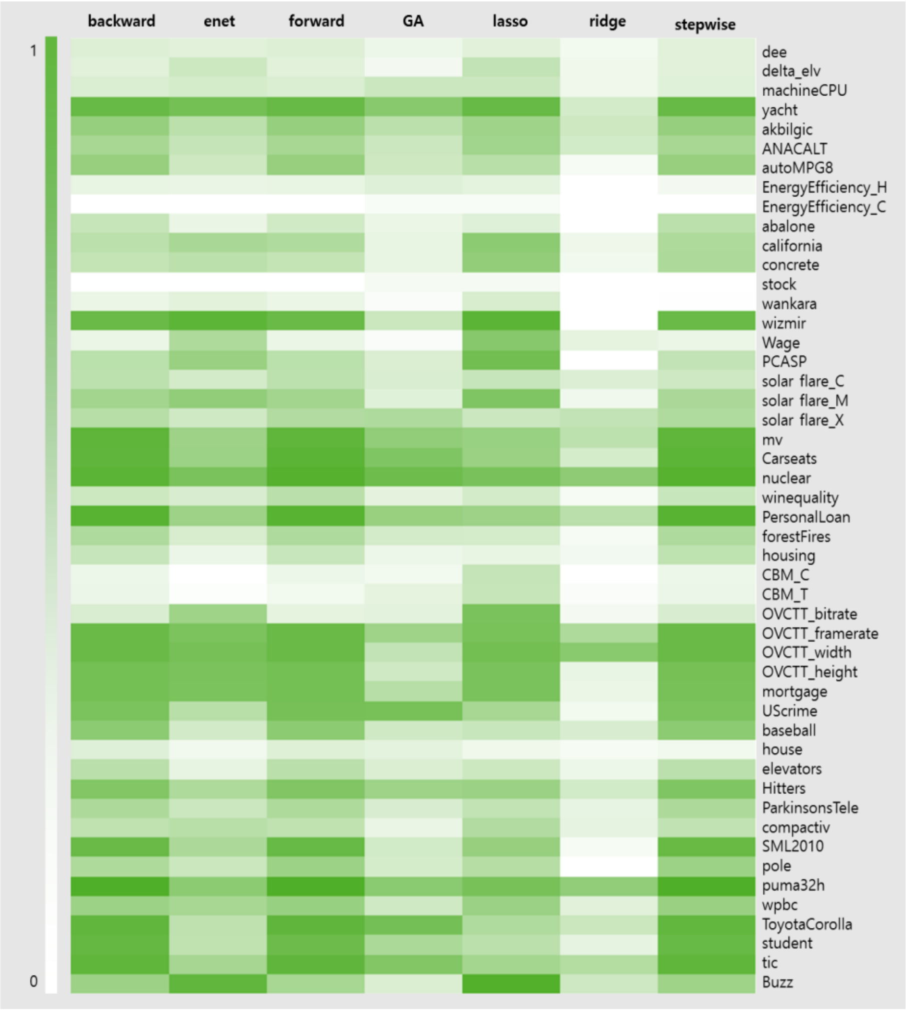
Dataset	source	records	variables	Dataset	source	records	variables
abalone	KEEL	4,177	9	OVCTT_bitrate	UCI	68,784	16
akbilgic	UCI	536	8	OVCTT_framerate	UCI	68,784	16
ANACALT	KEEL	4,052	8	OVCTT_height	UCI	68,784	16
autoMPG8	KEEL	392	8	OVCTT_width	UCI	68,784	16
baseball	KEEL	336	17	ParkinsonsTele	UCI	5,875	21
Buzz	UCI	28,179	95	PersonalLoan	etc.	2,500	13
california	KEEL	20,640	9	PCASP	UCI	45,730	10
Carseats	R	400	11	pole	KEEL	14,998	27
CBM_C	UCI	11,934	15	puma32h	KEEL	4,124	33
CBM_T	UCI	11,934	15	SML2010	UCI	4,137	24
compactiv	KEEL	8,192	22	solar flare_C	UCI	323	11
concrete	KEEL	1,030	9	solar flare_M	UCI	323	11
dee	KEEL	365	7	solar flare_X	UCI	323	11
delta_elv	KEEL	9,517	7	stock	KEEL	950	10
elevators	KEEL	16,599	19	student	UCI	382	51
EnergyEfficiency_C	UCI	768	9	tic	KEEL	9,822	86
EnergyEfficiency_H	UCI	768	9	ToyotaCorolla	etc.	1,436	34
forestFires	KEEL	517	13	UScrime	R	47	16
Hitters	R	263	20	Wage	R	3,000	10
house	KEEL	22,784	17	wankara	KEEL	1,609	10
housing	UCI	506	14	winequality	UCI	6,497	12
machineCPU	KEEL	209	7	wizmir	KEEL	1,461	10
mortgage	KEEL	1,049	16	wpbc	UCI	194	34
mv	KEEL	40,768	11	yacht	UCI	308	7
nuclear	R	32	11				

Empirical Study

Error Rate Improvement

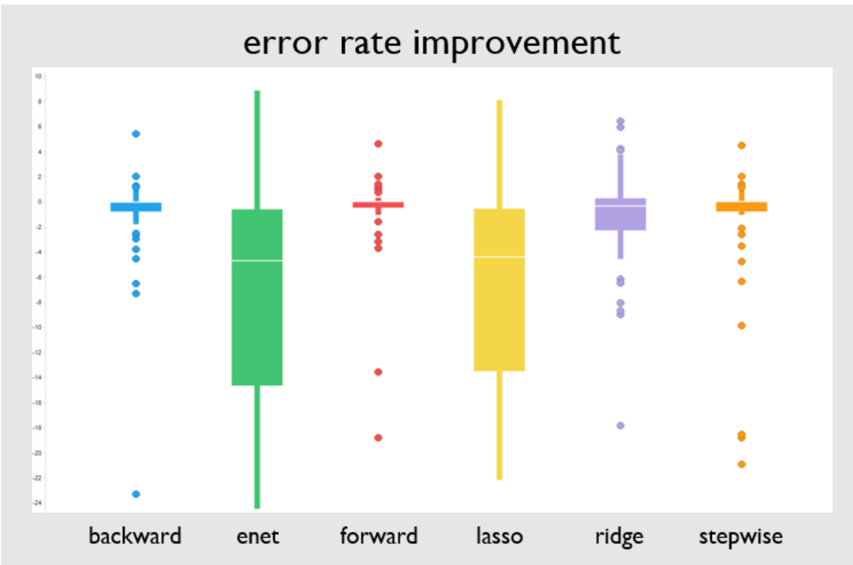


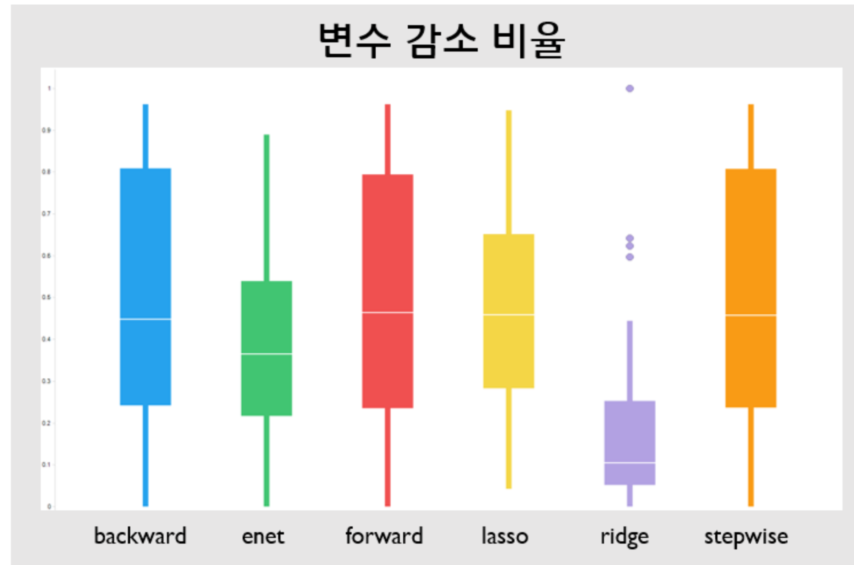
Variable Reduction Ratio



Empirical Study

- Performance comparison





변수선택 방법	예측 정확도	변수 감소율	계산 효율성
Forward	4	4	1
Backward	3	3	2
Stepwise	2	2	6
Ridge	1	6	5
Lasso	6	1	3
Elastic Net	5	5	4

AGENDA

R Exercise: Data Set

- Personal Loan
  - ✓ Purpose: identify future customer who will use the personal loan service based on his/her demographic information and banking service history

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	ID	Age	Experienc	Income	ZIP Code	Family	CCAvg	Education	Mortgage	Personal L	Securities	CD Accou	Online	CreditCard
2	1	25	1	49	91107	4	1.6	1	0	0	1	0	0	0
3	2	45	19	34	90089	3	1.5	1	0	0	1	0	0	0
4	3	39	15	11	94720	1	1	1	0	0	0	0	0	0
5	4	35	9	100	94112	1	2.7	2	0	0	0	0	0	0
6	5	35	8	45	91330	4	1	2	0	0	0	0	0	1
7	6	37	13	29	92121	4	0.4	2	155	0	0	0	1	0
8	7	53	27	72	91711	2	1.5	2	0	0	0	0	1	0
9	8	50	24	22	93943	1	0.3	3	0	0	0	0	0	1
10	9	35	10	81	90089	3	0.6	2	104	0	0	0	1	0

  - ▪ A total of 14 variables (columns)
  - ▪ ID, ZIP Code: irrelevant column (remove)
  - ▪ Personal loan: target variable

R Exercise: Install packages

- Install packages and prepare them to be used

```
# Install necessary packages
# glmnet: Ridge, Lasso, Elastic Net Logistic Regression
# GA: genetic algorithm
install.packages("glmnet")
install.packages("GA")
library(glmnet)
library(GA)
```

R Exercise: Performance Evaluation Function

- Performance Evaluation Function

```
# Performance Evaluation Function -----------------------------------------
perf_eval <- function(cm){
    # True positive rate: TPR (Recall)
    TPR <- cm[2,2]/sum(cm[2,])
    # Precision
    PRE <- cm[2,2]/sum(cm[,2])
    # True negative rate: TNR
    TNR <- cm[1,1]/sum(cm[1,])
    # Simple Accuracy
    ACC <- (cm[1,1]+cm[2,2])/sum(cm)
    # Balanced Correction Rate
    BCR <- sqrt(TPR*TNR)
    # F1-Measure
    F1 <- 2*TPR*PRE/(TPR+PRE)
    return(c(TPR, PRE, TNR, ACC, BCR, F1))
}
```

✓ Function name: perf_eval

- Argument: confusion matrix
- Outputs: six classification performance metrics

R Exercise: Performance Evaluation Function

- Performance Evaluation Function

```
Perf_Table <- matrix(0, nrow = 8, ncol = 6)
rownames(Perf_Table) <- c("All", "Forward", "Backward", "Stepwise", "GA", "Ridge",
                          "Lasso", "Elastic Net")
colnames(Perf_Table) <- c("TPR", "Precision", "TNR", "Accuracy", "BCR",
                          "F1-Measure")
```

✓ Initialize the performance comparison matrix

✓ A total of 8 logistic regression models are compared

- All: All variables are used
- Forward/Backward/Stepwise: Variables selected by Forward Selection, Backward Elimination, and Stepwise selection are used
- GA: Variables selected by genetic algorithm are used
- Ridge/Lasso/Elastic Net

R Exercise: Data Load and Preprocessing

- Data Load and Preprocessing

```
# Load the data & Preprocessing
Ploan <- read.csv("Personal Loan.csv")

Ploan_input <- Ploan[,-c(1,5,10)]
Ploan_input_scaled <- scale(Ploan_input, center = TRUE, scale = TRUE)
Ploan_target <- Ploan$Personal.Loan
Ploan_data_scaled <- data.frame(Ploan_input_scaled, Ploan_target)

trn_idx <- 1:1500
tst_idx <- 1501:2500

Ploan_trn <- Ploan_data_scaled[trn_idx,]
Ploan_tst <- Ploan_data_scaled[tst_idx,]
```

✓ Remove 1st, 5th, and 10th columns from input variables

✓ Perform input variable normalization

✓ Use the first 1,500 rows for training and the other 1,000 rows for test

R Exercise: Use All Variables

- Logistic Regression 1: All variables

```
# Variable selection method 0: Logistic Regression with all variables
full_model <- glm(Ploan_target ~ ., family=binomial, Ploan_trn)
summary(full_model)
full_model_coeff <- as.matrix(full_model$coefficients, 12, 1)
```

✓ glm( ) function: provide logistic regression model

- Arg 1: Formula, "Target ~ Input" form, period(.) for input means all variables except the target variable are used as input variables
- Arg 2: dataset for training

✓ summary( ): provide summarized information of the trained model

✓ Store the regression coefficients for further comparison

R Exercise: Use All Variables

- Logistic Regression 1: All variables
  - ✓ Insignificant variables (alpha = 0.05)
    - Age
    - Experience
    - Mortgage
    - Online

```
> summary(full_model)

Call:
glm(formula = Ploan_target ~ ., family = binomial, data = Ploan_trn)

Deviance Residuals:
    Min       1Q    Median      3Q      Max
-2.1781  -0.2189  -0.0906  -0.0365   3.5345

Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
(Intercept)        -4.445483   0.272085 -16.339  < 2e-16 ***
Age                -0.776031   1.276510  -0.608 0.543233
Experience          0.910984   1.272791   0.716 0.474154
Income              2.374701   0.206877  11.479  < 2e-16 ***
Family              0.739703   0.153237   4.827 1.38e-06 ***
CCAvg               0.264244   0.120946   2.185 0.028902 *
Education           1.328860   0.170227   7.806 5.88e-15 ***
Mortgage            0.009294   0.100392   0.093 0.926239
Securities.Account -0.501459   0.183861  -2.727 0.006384 **
CD.Account          0.982082   0.151231   6.494 8.36e-11 ***
Online             -0.182069   0.139815  -1.302 0.192843
CreditCard         -0.617374   0.181094  -3.409 0.000652 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 984.01  on 1499  degrees of freedom
Residual deviance: 401.11  on 1488  degrees of freedom
AIC: 425.11

Number of Fisher Scoring iterations: 7
```

R Exercise: Use All Variables

- Logistic Regression 1: All variables

```
# Make prediction
full_model_prob <- predict(full_model, type = "response", newdata = Ploan_tst)
full_model_prey <- rep(0, nrow(Ploan_tst))
full_model_prey[which(full_model_prob >= 0.5)] <- 1
full_model_cm <- table(Ploan_tst$Ploan_target, full_model_prey)
full_model_cm

# Peformance evaluation
Perf_Table[1,] <- perf_eval(full_model_cm)
Perf_Table
```

✓ type = "response" option for predict( ) function gives the probability of belonging to class 1

✓ Use 0.5 as the cut-off

```
> full_model_cm
   full_model_prey
      0   1
  0 881  15
  1  36  68
```

R Exercise: Use All Variables

- Logistic Regression 1: All variables

```
> Perf_Table
                  TPR Precision       TNR Accuracy       BCR F1-Measure
All         0.6538462 0.8192771 0.9832589    0.949 0.8018105  0.7272727
Forward     0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Backward    0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Stepwise    0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
GA          0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Ridge       0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Lasso       0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Elastic Net 0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
```

R Exercise: Forward Selection

- Logistic Regression 2: Forward Selection

```
# Variable selection method 1: Forward selection
tmp_x <- paste(colnames(Ploan_trn)[-12], collapse=" + ")
tmp_xy <- paste("Ploan_target ~ ", tmp_x, collapse = "")
as.formula(tmp_xy)

forward_model <- step(glm(Ploan_target ~ 1, data = Ploan_trn),
                      scope = list(upper = as.formula(tmp_xy),
                      lower = Ploan_target ~ 1), direction="forward", trace = 1)

summary(forward_model)
forward_model_coeff <- as.matrix(forward_model$coefficients, 12, 1)
forward_model_coeff
```

✓ step( ) function: perform forward selection/backward elimination/stepwise selection

- Arg 1: Initial model, forward selection model begins with the model with no input variable
- Arg 2: Range of selected variables
  - Upper: the largest set of selected variables (all variables in this experiment)
  - Lower: the smallest set of selected variables (0 in this experiment)
- Arg 3: direction = “forward” (perform forward selection)

R Exercise: Forward Selection

- Logistic Regression 2: Forward Selection
  - ✓ Mortgage and Online are not selected
    - Age
    - Experience
    - ~~Mortgage~~
    - ~~Online~~

```
Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)          0.101881   0.006029  16.899  < 2e-16 ***
Income               0.149121   0.008154  18.288  < 2e-16 ***
CD.Account           0.083880   0.006887  12.180  < 2e-16 ***
Education            0.068442   0.006366  10.750  < 2e-16 ***
Family               0.039866   0.006121   6.513 1.00e-10 ***
CreditCard          -0.024395   0.006331  -3.853 0.000122 ***
Securities.Account -0.025492   0.006506  -3.918 9.32e-05 ***
CCAvg                0.019494   0.007808   2.497 0.012642 *
Experience           0.107328   0.058000   1.850 0.064444 .
Age                 -0.098853   0.058078  -1.702 0.088950 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

R Exercise: Forward Selection

- Logistic Regression 2: Forward Selection

```
# Make prediction
forward_model_prob <- predict(forward_model, type = "response",
                              newdata = Ploan_tst)
forward_model_prey <- rep(0, nrow(Ploan_tst))
forward_model_prey[which(forward_model_prob >= 0.5)] <- 1
forward_model_cm <- table(Ploan_tst$Ploan_target, forward_model_prey)
forward_model_cm

# Peformance evaluation Perf_Table[2,] <- perf_eval(forward_model_cm)
Perf_Table
```

✓ type = "response" option for predict( ) function gives the probability of belonging to class 1

✓ Use 0.5 as the cut-off

```
> forward_model_cm
   forward_model_prey
      0   1
  0 893   3
  1  57  47
```

R Exercise: Forward Selection

- Logistic Regression 2: Forward Selection

```
> Perf_Table
                  TPR Precision       TNR Accuracy       BCR F1-Measure
All         0.6538462 0.8192771 0.9832589    0.949 0.8018105  0.7272727
Forward     0.4519231 0.9400000 0.9966518    0.940 0.6711259  0.6103896
Backward    0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Stepwise    0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
GA          0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Ridge       0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Lasso       0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Elastic Net 0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
```

R Exercise: Backward Elimination

- Logistic Regression 3: Backward Elimination

```
# Variable selection method 2: Backward elimination
backward_model <- step(full_model, scope = list(upper = as.formula(tmp_xy),
             lower = Ploan_target ~ 1), direction = "backward",
                      trace = 1)

summary(backward_model)
backward_model_coeff <- as.matrix(backward_model$coefficients, 12, 1)
backward_model_coeff
```

✓ step( ) function: perform forward selection/backward elimination/stepwise selection

- Arg 1: Initial model, forward selection model begins with the model with no input variable
- Arg 2: Range of selected variables
  - Upper: the largest set of selected variables (all variables in this experiment)
  - Lower: the smallest set of selected variables (0 in this experiment)
- Arg 3: direction = “backward” (perform backward elimination)

R Exercise: Backward Elimination

- Logistic Regression 3: Backward Elimination
  - ✓ Age, Experiment, Mortgage, and Online are not selected
    - ~~Age~~
    - ~~Experience~~
    - ~~Mortgage~~
    - ~~Online~~

```
Coefficients:
                     Estimate Std. Error z value Pr(>|z|)    
(Intercept)           -4.4217     0.2685 -16.467  < 2e-16 ***
Income                 2.3871     0.2021  11.814  < 2e-16 ***
Family                 0.7319     0.1507   4.858 1.18e-06 ***
CCAvg                  0.2482     0.1197   2.073 0.038149 *  
Education              1.3033     0.1661   7.849 4.20e-15 ***
Securities.Account    -0.4675     0.1809  -2.585 0.009751 ** 
CD.Account             0.9373     0.1426   6.572 4.95e-11 ***
CreditCard            -0.5889     0.1769  -3.329 0.000871 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

R Exercise: Backward Elimination

- Logistic Regression 3: Backward Elimination

```
# Make prediction
backward_model_prob <- predict(backward_model, type = "response",
                               newdata = Ploan_tst)
backward_model_prey <- rep(0, nrow(Ploan_tst))
backward_model_prey[which(backward_model_prob >= 0.5)] <- 1
backward_model_cm <- table(Ploan_tst$Ploan_target, backward_model_prey)
backward_model_cm

# Peformance evaluation
Perf_Table[3,] <- perf_eval(backward_model_cm)
Perf_Table
```

✓ type = “response” option for predict( ) function gives the probability of belonging to class 1

✓ Use 0.5 as the cut-off

```
> backward_model_cm
   backward_model_prey
      0    1
  0 881   15
  1  37   67
```

R Exercise: Backward Elimination

- Logistic Regression 3: Backward Elimination

```
> Perf_Table
                  TPR Precision       TNR Accuracy       BCR F1-Measure
All         0.6538462 0.8192771 0.9832589    0.949 0.8018105  0.7272727
Forward     0.4519231 0.9400000 0.9966518    0.940 0.6711259  0.6103896
Backward    0.6442308 0.8170732 0.9832589    0.948 0.7958930  0.7204301
Stepwise    0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
GA          0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Ridge       0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Lasso       0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Elastic Net 0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
```

R Exercise: Stepwise Selection

- Logistic Regression 4: Stepwise Selection

```
# Variable selection method 3: Stepwise selection
tmp_x <- paste(colnames(Ploan_trn)[-12], collapse=" + ")
tmp_xy <- paste("Ploan_target ~ ", tmp_x, collapse = "")
as.formula(tmp_xy)

stepwise_model <- step(glm(Ploan_target ~ 1, data = Ploan_trn),
                       scope = list(upper = as.formula(tmp_xy),
                       lower = Ploan_target ~ 1), direction="both", trace = 1)
summary(stepwise_model)
stepwise_model_coeff <- as.matrix(stepwise_model$coefficients, 12, 1)
stepwise_model_coeff
```

✓ step( ) function: perform forward selection/backward elimination/stepwise selection

- Arg 1: Initial model, forward selection model begins with the model with no input variable
- Arg 2: Range of selected variables
  - Upper: the largest set of selected variables (all variables in this experiment)
  - Lower: the smallest set of selected variables (0 in this experiment)
- Arg 3: direction = “backward” (perform backward elimination)

R Exercise: Stepwise Selection

- Logistic Regression 4: Stepwise Selection
  - ✓ Mortgage and Online are not selected (same result with the forward selection)
    - Age
    - Experience
    - ~~Mortgage~~
    - ~~Online~~

```
Coefficients:
                    Estimate Std. Error t value Pr(>|t|)    
(Intercept)         0.101881   0.006029  16.899  < 2e-16 ***
Income              0.149121   0.008154  18.288  < 2e-16 ***
CD.Account          0.083880   0.006887  12.180  < 2e-16 ***
Education           0.068442   0.006366  10.750  < 2e-16 ***
Family              0.039866   0.006121   6.513 1.00e-10 ***
CreditCard         -0.024395   0.006331  -3.853 0.000122 ***
Securities.Account -0.025492   0.006506  -3.918 9.32e-05 ***
CCAvg               0.019494   0.007808   2.497 0.012642 *  
Experience          0.107328   0.058000   1.850 0.064444 .  
Age                -0.098853   0.058078  -1.702 0.088950 .  
```

R Exercise: Stepwise Selection

- Logistic Regression 4: Stepwise Selection

```
# Make prediction
stepwise_model_prob <- predict(stepwise_model, type = "response",
                               newdata = Ploan_tst)
stepwise_model_prey <- rep(0, nrow(Ploan_tst))
stepwise_model_prey[which(stepwise_model_prob >= 0.5)] <- 1
stepwise_model_cm <- table(Ploan_tst$Ploan_target, stepwise_model_prey)
stepwise_model_cm

# Peformance evaluation
Perf_Table[4,] <- perf_eval(stepwise_model_cm)
Perf_Table
```

✓ type = "response" option for predict( ) function gives the probability of belonging to class 1

✓ Use 0.5 as the cut-off

```
> stepwise_model_cm
   stepwise_model_prey
      0   1
  0 893   3
  1  57  47
```

R Exercise: Stepwise Selection

- Logistic Regression 4: Stepwise Selection

```
> Perf_Table
                  TPR Precision       TNR Accuracy       BCR F1-Measure
All         0.6538462 0.8192771 0.9832589    0.949 0.8018105  0.7272727
Forward     0.4519231 0.9400000 0.9966518    0.940 0.6711259  0.6103896
Backward    0.6442308 0.8170732 0.9832589    0.948 0.7958930  0.7204301
Stepwise    0.4519231 0.9400000 0.9966518    0.940 0.6711259  0.6103896
GA          0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Ridge       0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Lasso       0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Elastic Net 0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
```

R Exercise: Genetic Algorithm

- Logistic Regression 5: Genetic Algorithm

```
# Variable selection method 4: Genetic Algorithm
# Fitness function: F1 for the training dataset
fit_F1 <- function(string){
    sel_var_idx <- which(string == 1)
    # Use variables whose gene value is 1
    sel_x <- x[, sel_var_idx]
    xy <- data.frame(sel_x, y)
    # Training the model
    GA_lr <- glm(y ~ ., family = binomial, data = xy)
    GA_lr_prob <- predict(GA_lr, type = "response", newdata = xy)
    GA_lr_prey <- rep(0, length(y))
    GA_lr_prey[which(GA_lr_prob >= 0.5)] <- 1
    GA_lr_cm <- matrix(0, nrow = 2, ncol = 2)
    GA_lr_cm[1,1] <- length(which(y == 0 & GA_lr_prey == 0))
    GA_lr_cm[1,2] <- length(which(y == 0 & GA_lr_prey == 1))
    GA_lr_cm[2,1] <- length(which(y == 1 & GA_lr_prey == 0))
    GA_lr_cm[2,2] <- length(which(y == 1 & GA_lr_prey == 1))
    GA_perf <- perf_eval(GA_lr_cm)
    return(GA_perf[6])
}
```

R Exercise: Genetic Algorithm

- Logistic Regression 5: Genetic Algorithm
  - ✓ fit_F1( ) function
    - Input: chromosome (binary vector whose length is the same as the number of variables)
      - 1: use the corresponding variable in the current model
      - 0: do not use the corresponding variable in the current model
      - Example

1	1	0	0	0	1	0	1	0	0	1

    - Output: fitness function in terms of F1 measure (cut-off = 0.5)

R Exercise: Genetic Algorithm

- Logistic Regression 5: Genetic Algorithm

```
x <- as.matrix(Ploan_trn[,-12])
y <- Ploan_trn[,12]

# Variable Selection via Genetic Algorithm
start_time <- proc.time()
GA_F1 <- ga(type = "binary", fitness = fit_F1, nBits = ncol(x),
            names = colnames(x), popSize = 50, pcrossover = 0.5, pmutation = 0.01,
            maxiter = 100, elitism = 2, seed = 123)
end_time <- proc.time()
end_time - start_time
```

✓ ga( ) function: variable selected via genetic algorithm

- Arg 1: type of chromosome, if it is "binary", each gene has either 0 or 1 value
- Arg 2: fitness function
- Arg 3 & 4: number of variables and variable names
- Arg 5 & 6 & 7: number of chromosomes, crossover rate, mutation rate
- Arg 8 & 9: Maximum number of iterations, number of chromosomes to preserve

R Exercise: Genetic Algorithm

- Logistic Regression 5: Genetic Algorithm

```
best_var_idx <- which(GA_F1@solution == 1)

# Model training based on the best variable subset
GA_trn_data <- Ploan_trn[,c(best_var_idx, 12)]
GA_tst_data <- Ploan_tst[,c(best_var_idx, 12)]
GA_model <- glm(Ploan_target ~ ., family=binomial, GA_trn_data)

summary(GA_model)
GA_model_coeff <- as.matrix(GA_model$coefficients, 12, 1)
GA_model_coeff
```

✓ best_var_idx: index of best variable subset selected by GA

```
> best_var_idx
[1]  1  3  4  5  6  7  8  9 11
```

R Exercise: Genetic Algorithm

- Logistic Regression 5: Genetic Algorithm
  - ✓ Unselected variables
    - Experience
    - Online

```
Coefficients:
                   Estimate Std. Error z value Pr(>|z|)    
(Intercept)         -4.4333     0.2705 -16.391  < 2e-16 ***
Age                  0.1292     0.1353   0.954  0.33985    
Income               2.3956     0.2063  11.611  < 2e-16 ***
Family               0.7508     0.1527   4.918 8.76e-07 ***
CCAvg                0.2572     0.1202   2.140  0.03237 *  
Education            1.3081     0.1670   7.833 4.75e-15 ***
Mortgage             0.0135     0.1003   0.135  0.89298    
Securities.Account  -0.4726     0.1822  -2.595  0.00947 ** 
CD.Account           0.9276     0.1429   6.489 8.63e-11 ***
CreditCard          -0.5733     0.1770  -3.240  0.00120 ** 
```

R Exercise: Genetic Algorithm

- Logistic Regression 5: Genetic Algorithm

```
# Make prediction
GA_model_prob <- predict(GA_model, type = "response", newdata = GA_tst_data)
GA_model_prey <- rep(0, nrow(Ploan_tst))
GA_model_prey[which(GA_model_prob >= 0.5)] <- 1
GA_model_cm <- table(GA_tst_data$Ploan_target, GA_model_prey)
GA_model_cm

# Peformance evaluation
Perf_Table[5,] <- perf_eval(GA_model_cm)
Perf_Table
```

✓ type = “response” option for predict( ) function gives the probability of belonging to class 1

✓ Use 0.5 as the cut-off

```
> GA_model_cm
  GA_model_prey
      0   1
  0 883  13
  1  38  66
```

R Exercise: Genetic Algorithm

- Logistic Regression 5: Genetic Algorithm

```
> Perf_Table
                   TPR Precision       TNR Accuracy       BCR F1-Measure
All          0.6538462 0.8192771 0.9832589    0.949 0.8018105  0.7272727
Forward      0.4519231 0.9400000 0.9966518    0.940 0.6711259  0.6103896
Backward     0.6442308 0.8170732 0.9832589    0.948 0.7958930  0.7204301
Stepwise     0.4519231 0.9400000 0.9966518    0.940 0.6711259  0.6103896
GA           0.6346154 0.8354430 0.9854911    0.949 0.7908273  0.7213115
Ridge        0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Lasso        0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Elastic Net  0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
```

R Exercise: Ridge Regression

- Logistic Regression 6: Ridge Regression

```
# Shrinkage method 1: Ridge logistic regression
Ploan_trn_X <- as.matrix(Ploan_trn[,-12])
Ploan_trn_y <- as.factor(Ploan_trn[,12])
Ploan_tst_X <- as.matrix(Ploan_tst[,-12])
Ploan_tst_y <- as.factor(Ploan_tst[,12])

Ridge_model <- glmnet(Ploan_trn_X, Ploan_trn_y, family = "binomial", alpha = 0)
plot(Ridge_model, xvar = "lambda")
```

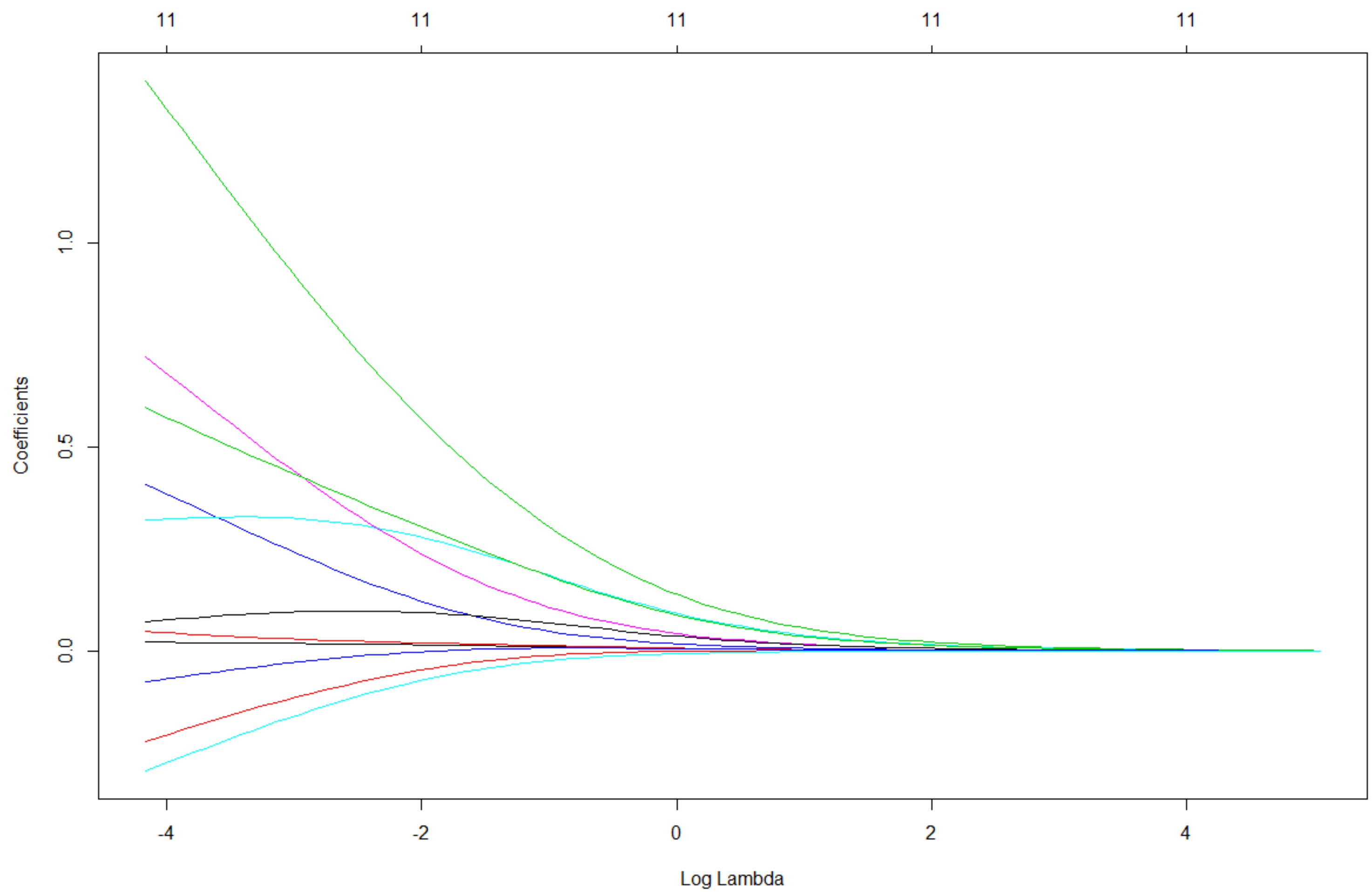
✓ glmnet( ) function: can learn shrinkage methods

- Arg 1: Input variables (matrix form)
- Arg 2: Target variable (factor form)
- Arg 3: family = “binomial” (binary classification = logistic regression)
- Arg 4: Weight for L1 and L2 norm (alpha = 0 → Ridge regression)

$$(1-\alpha) \times \lambda_1 \sum_{j=1}^{d} |\hat{\beta}_j| + \alpha \times \lambda_2 \sum_{j=1}^{d} \hat{\beta}_j^2$$

R Exercise: Ridge Regression

- Logistic Regression 6: Ridge Regression



R Exercise: Ridge Regression

- Logistic Regression 6: Ridge Regression

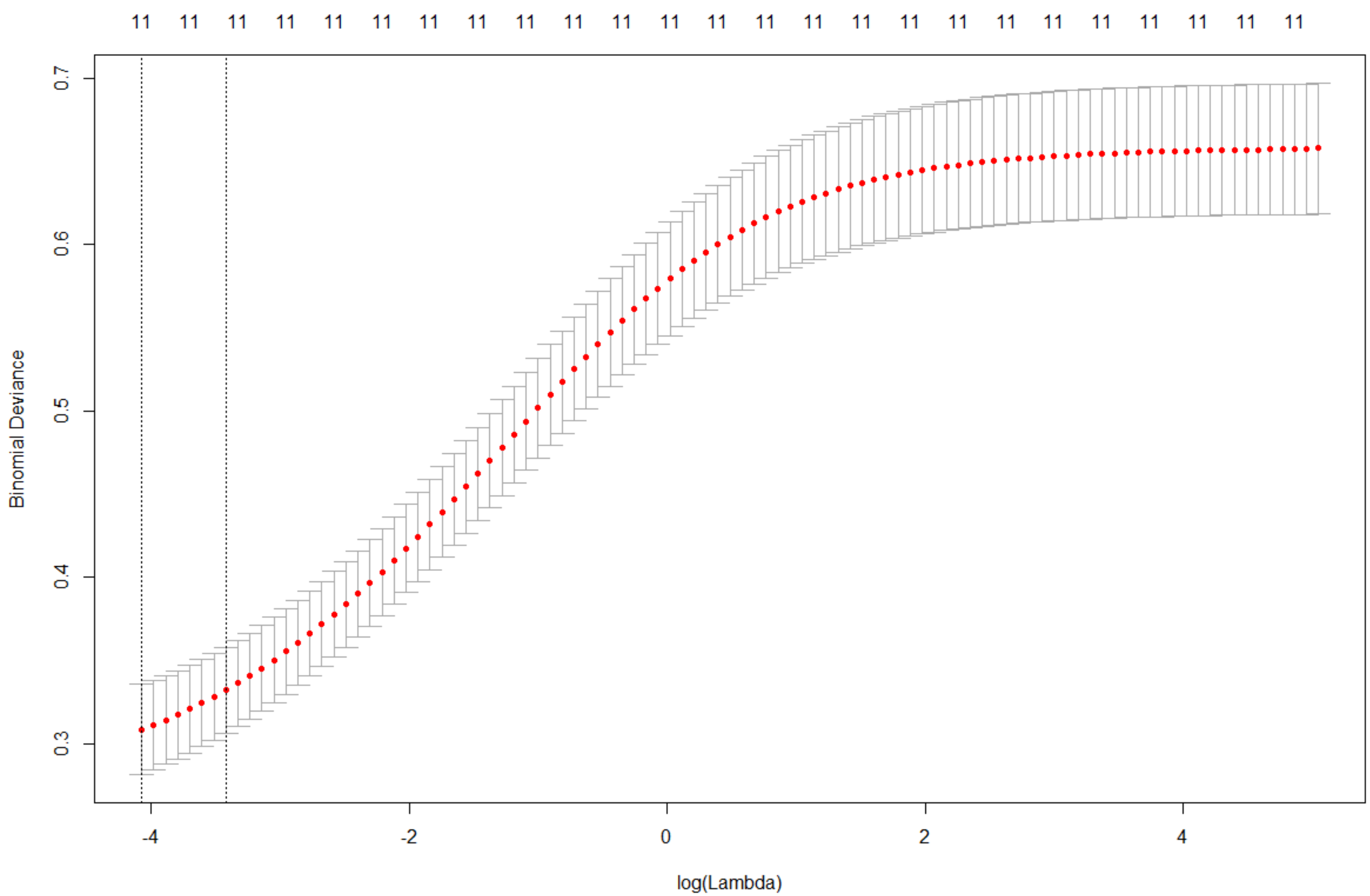
```
# Find the best lambda based in 5-fold cross validation
CV_Ridge <- cv.glmnet(Ploan_trn_X, Ploan_trn_y, family = "binomial", alpha = 0)
plot(CV_Ridge)
best_lambda <- CV_Ridge$lambda.min
```

✓ cv.glmnet( ): function for 5-fold cross-validation

- Arg 1: Input variables (matrix form)
- Arg 2: Target variable (factor form)
- Arg 3: family = "binomial" (binary classification = logistic regression)
- Arg 4: Weight for L1 and L2 norm (alpha = 0 → Ridge regression)

R Exercise: Ridge Regression

- Logistic Regression 6: Ridge Regression



R Exercise: Ridge Regression

- Logistic Regression 6: Ridge Regression

```r
# Check the coefficients
Ridge_model_coeff <- predict(Ridge_model, s = best_lambda,
                             newx = Ploan_tst_X, type = "coefficient")
Ridge_model_coeff

# Make predictions
Ridge_model_prey <- predict(Ridge_model, s = best_lambda,
                            newx = Ploan_tst_X, type = "class")

Ridge_model_prey <- as.factor(Ridge_model_prey)
Ridge_model_cm <- table(Ploan_tst_y, Ridge_model_prey)
Ridge_model_cm
```

✓ predict( )

- Arg 1: trained model
- Arg 2: Lambda
- Arg 3: Input variables of test dataset
- Arg 4: Type of the output (estimated coefficients or predicted class)

R Exercise: Ridge Regression

- Logistic Regression 6: Ridge Regression

```
> Perf_Table
                  TPR Precision       TNR Accuracy       BCR F1-Measure
All         0.6538462 0.8192771 0.9832589    0.949 0.8018105  0.7272727
Forward     0.4519231 0.9400000 0.9966518    0.940 0.6711259  0.6103896
Backward    0.6442308 0.8170732 0.9832589    0.948 0.7958930  0.7204301
Stepwise    0.4519231 0.9400000 0.9966518    0.940 0.6711259  0.6103896
GA          0.6346154 0.8354430 0.9854911    0.949 0.7908273  0.7213115
Ridge       0.5384615 0.8888889 0.9921875    0.945 0.7309274  0.6706587
Lasso       0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
Elastic Net 0.0000000 0.0000000 0.0000000    0.000 0.0000000  0.0000000
```

R Exercise: Lasso Regression

- Logistic Regression 7: Lasso Regression

```
# Shrinkage method 2: Lasso regression
Lasso_model <- glmnet(Ploan_trn_X, Ploan_trn_y, family = "binomial", alpha = 1)
plot(Lasso_model, xvar = "lambda")

# Find the best lambda based in 5-fold cross validation
CV_Lasso <- cv.glmnet(Ploan_trn_X, Ploan_trn_y, family = "binomial", alpha = 1)
plot(CV_Lasso)
best_lambda <- CV_Lasso$lambda.min
```

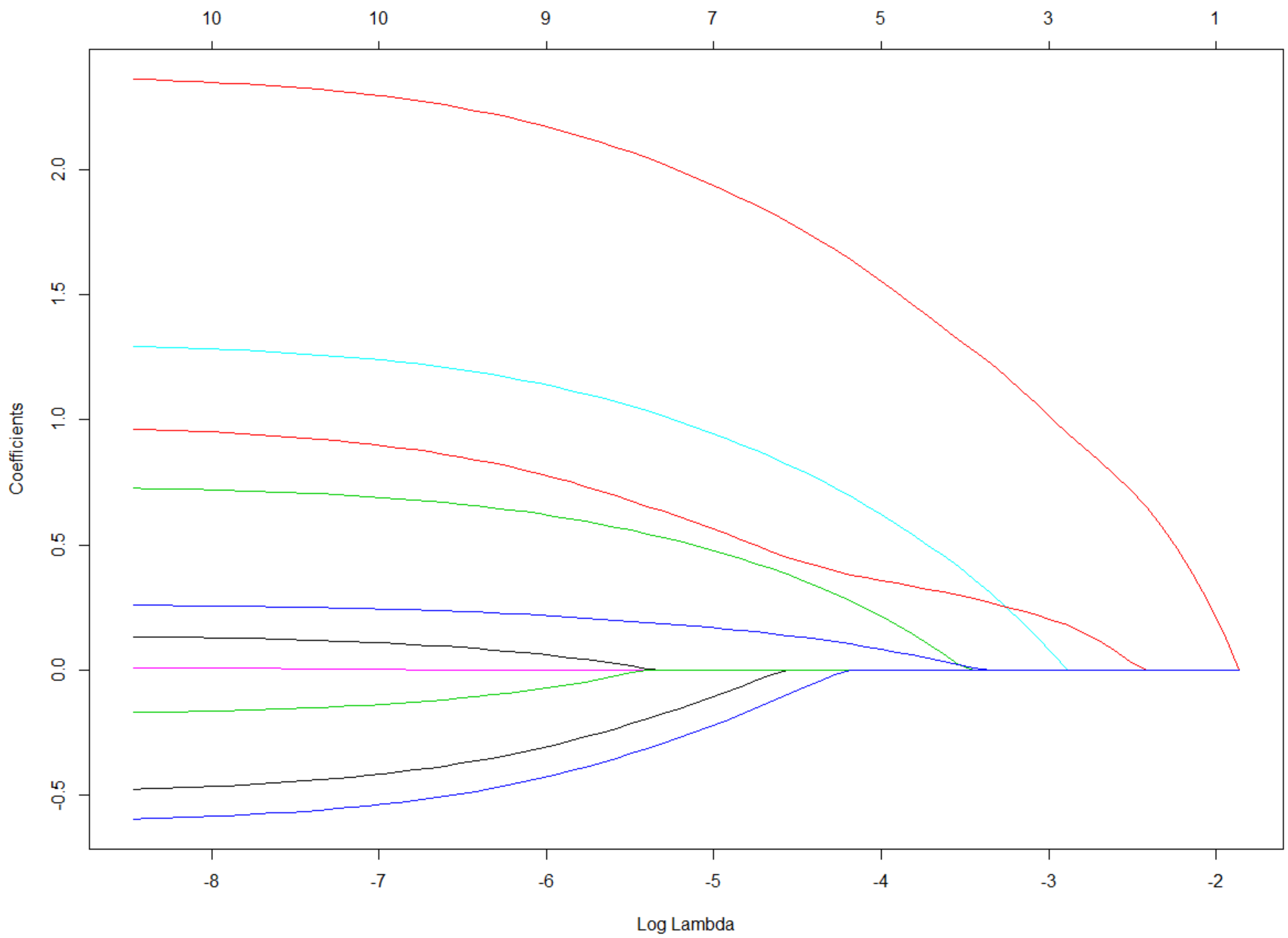
✓ glmnet( ) function: can learn shrinkage methods

- Arg 1: Input variables (matrix form)
- Arg 2: Target variable (factor form)
- Arg 3: family = "binomial" (binary classification = logistic regression)
- Arg 4: Weight for L1 and L2 norm (alpha = 1 → Lasso regression)

$$(1-\alpha) \times \lambda_1 \sum_{j=1}^{d} |\hat{\beta}_j| + \alpha \times \lambda_2 \sum_{j=1}^{d} \hat{\beta}_j^2$$

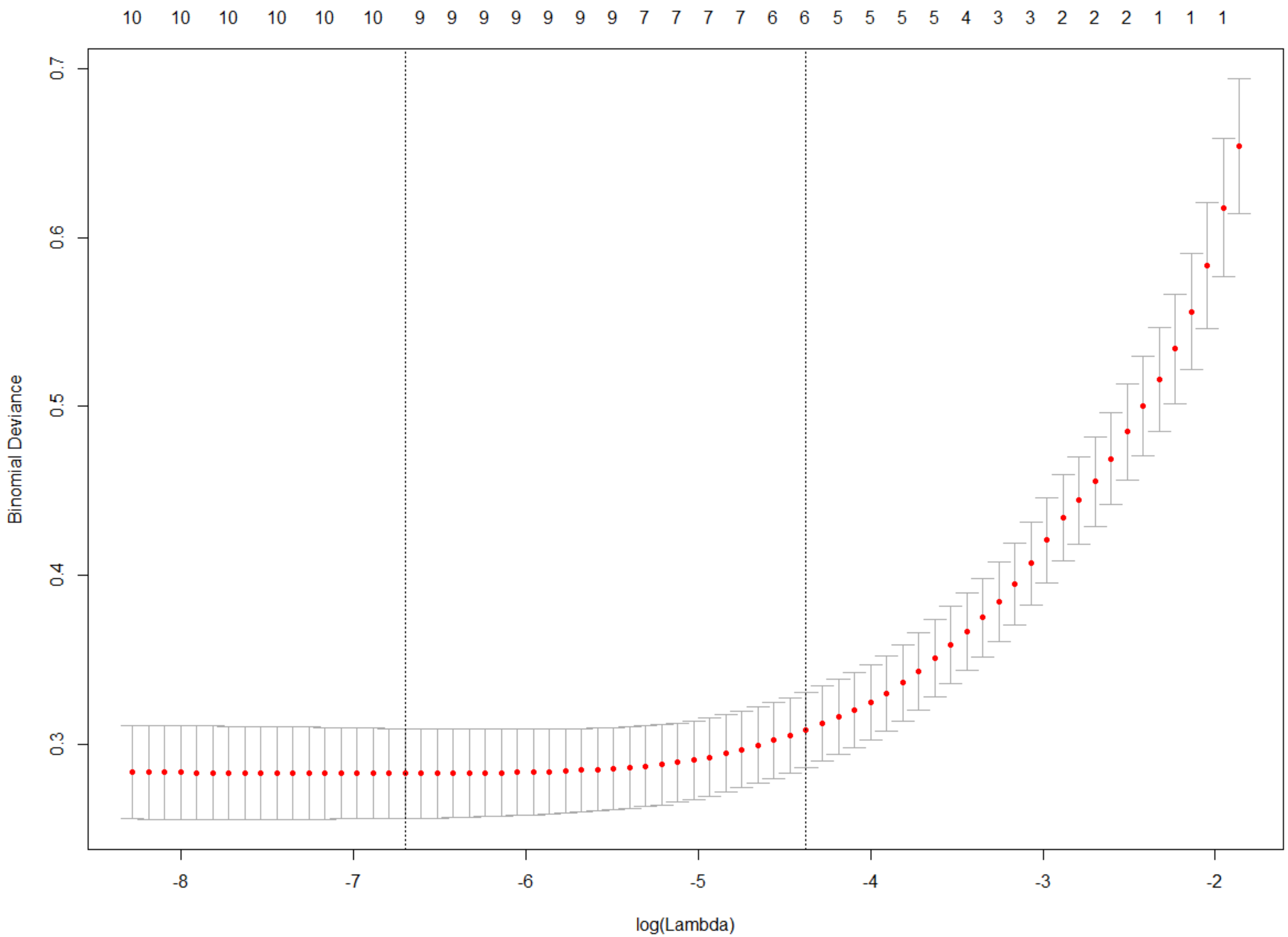
R Exercise: Lasso Regression

- Logistic Regression 7: Lasso Regression



R Exercise: Lasso Regression

- Logistic Regression 7: Lasso Regression



R Exercise: Lasso Regression

- Logistic Regression 7: Lasso Regression

```
# Check the coefficients
Lasso_model_coeff <- predict(Lasso_model, s = best_lambda,
                            newx = Ploan_tst_X, type = "coefficient")
Lasso_model_coeff

# Make predictions
Lasso_model_prey <- predict(Lasso_model, s = best_lambda,
                           newx = Ploan_tst_X, type = "class")
Lasso_model_prey <- as.factor(Lasso_model_prey)
Lasso_model_cm <- table(Ploan_tst_y, Lasso_model_prey)
Lasso_model_cm
```

✓ predict( )

- Arg 1: Trained model
- Arg 2: Lambda
- Arg 3: Input variable of test dataset
- Arg 4: Output type (regression coefficients or predicted class)

R Exercise: Lasso Regression

- Logistic Regression 7: Lasso Regression

```
> Perf_Table
```

	TPR	Precision	TNR	Accuracy	BCR	F1-Measure
All	0.6538462	0.8192771	0.9832589	0.949	0.8018105	0.7272727
Forward	0.4519231	0.9400000	0.9966518	0.940	0.6711259	0.6103896
Backward	0.6442308	0.8170732	0.9832589	0.948	0.7958930	0.7204301
Stepwise	0.4519231	0.9400000	0.9966518	0.940	0.6711259	0.6103896
GA	0.6346154	0.8354430	0.9854911	0.949	0.7908273	0.7213115
Ridge	0.5384615	0.8888889	0.9921875	0.945	0.7309274	0.6706587
Lasso	0.6250000	0.8227848	0.9843750	0.947	0.7843688	0.7103825
Elastic Net	0.0000000	0.0000000	0.0000000	0.000	0.0000000	0.0000000

R Exercise: Elastic Net

- Logistic Regression 8: Elastic Net

```
# Shrinkage method 3: Elastic net regression
Elastic_model <- glmnet(Ploan_trn_X, Ploan_trn_y, family = "binomial", alpha = 0.5)
plot(Elastic_model, xvar = "lambda")

# Find the best lambda based in 5-fold cross validation
CV_Elastic <- cv.glmnet(Ploan_trn_X, Ploan_trn_y, family = "binomial", alpha = 0.5)
plot(CV_Elastic)
best_lambda <- CV_Elastic$lambda.min
```

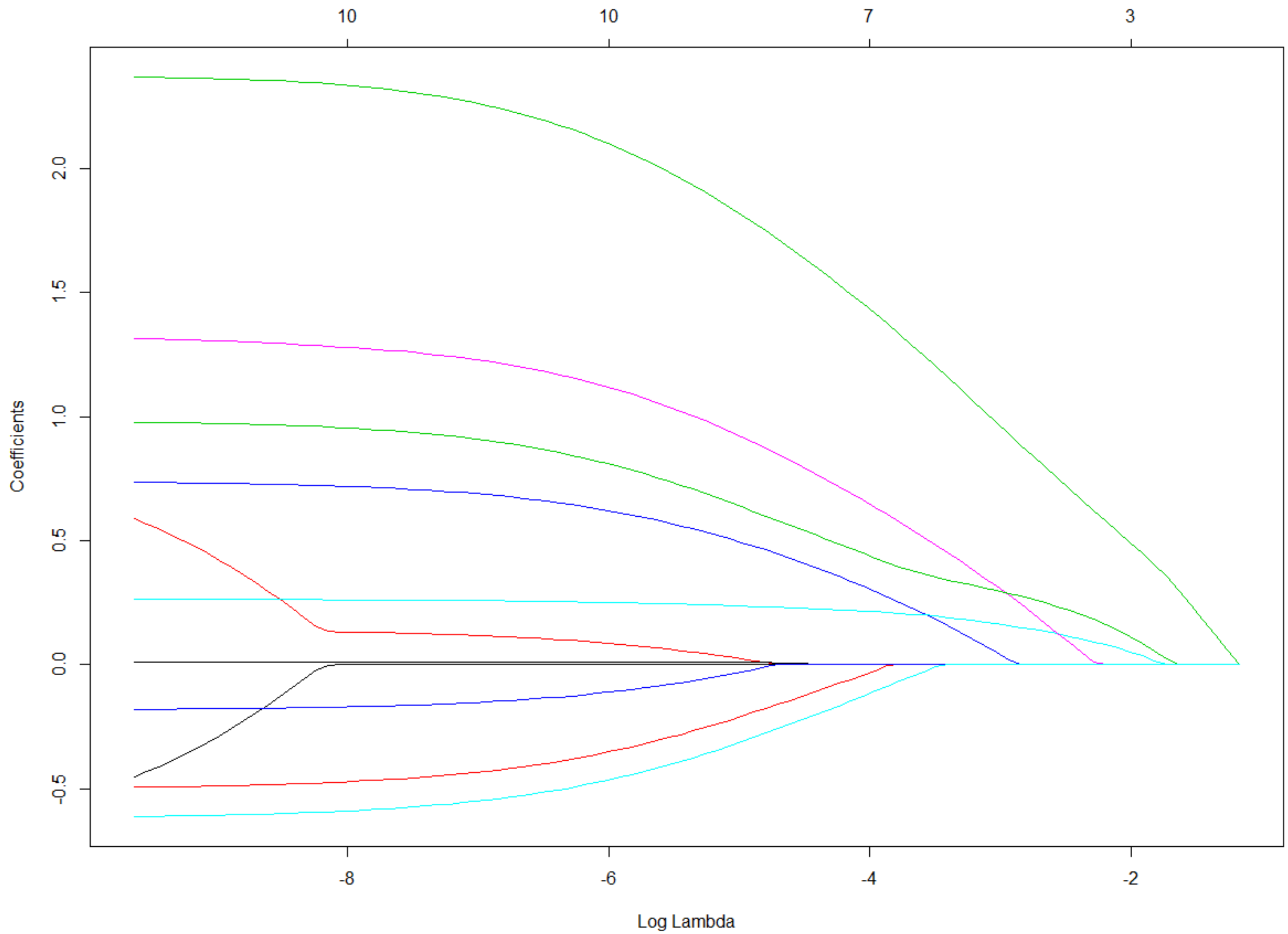
✓ glmnet( ) function: can learn shrinkage methods

- Arg 1: Input variables (matrix form)
- Arg 2: Target variable (factor form)
- Arg 3: family = "binomial" (binary classification = logistic regression)
- Arg 4: Weight for L1 and L2 norm (alpha = 0.5 → Elastic Net)

$$(1-\alpha) \times \lambda_1 \sum_{j=1}^{d} |\hat{\beta}_j| + \alpha \times \lambda_2 \sum_{j=1}^{d} \hat{\beta}_j^2$$

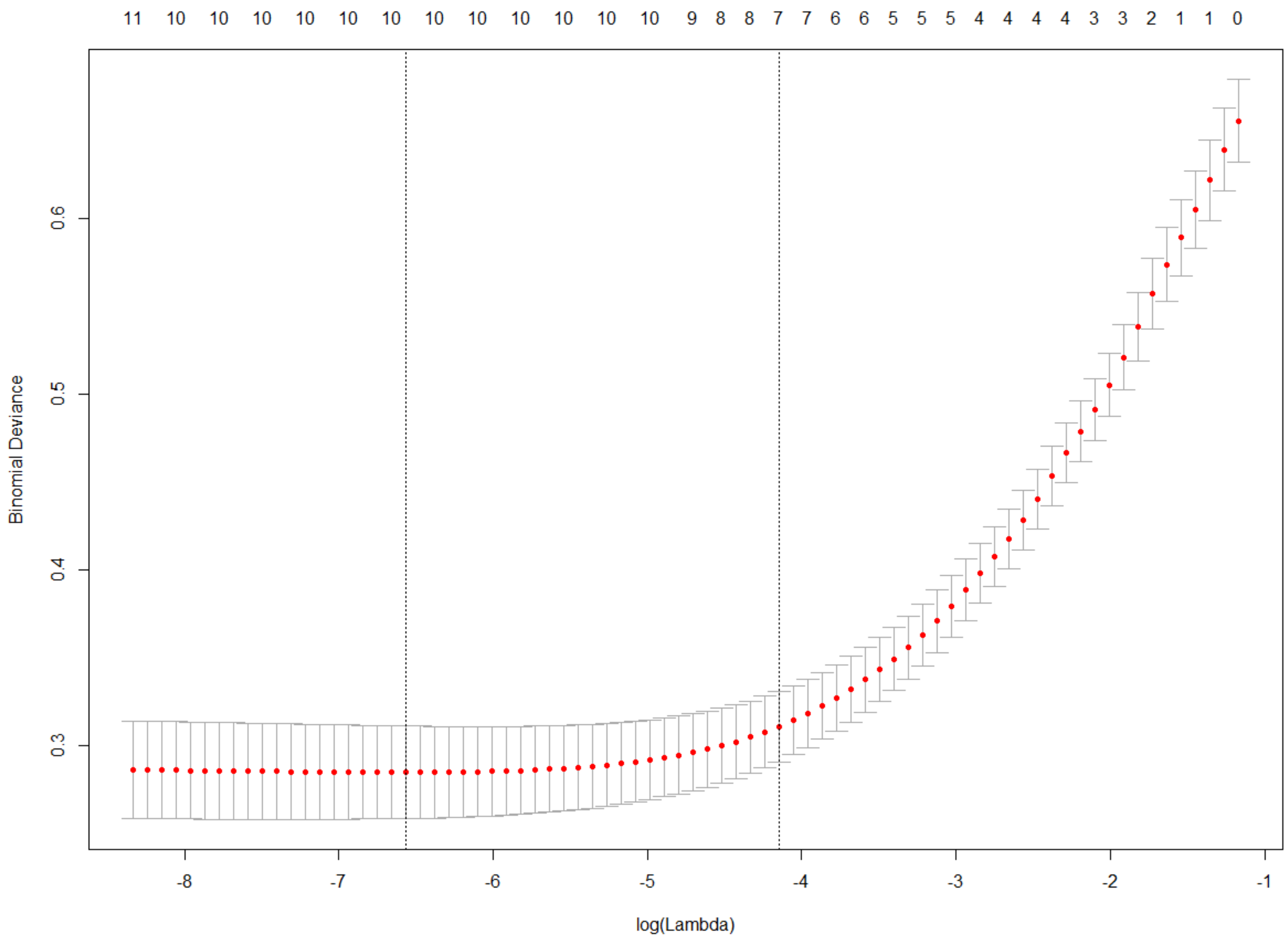
R Exercise: Elastic Net

- Logistic Regression 8: Elastic Net



R Exercise: Elastic Net

- Logistic Regression 8: Elastic Net



R Exercise: Elastic Net

- Logistic Regression 8: Elastic Net

```
# Check the coefficients
Elastic_model_coeff <- predict(Elastic_model, s = best_lambda,
                              newx = Ploan_tst_X, type = "coefficient")
Elastic_model_coeff

# Make predictions
Elastic_model_prey <- predict(Elastic_model, s = best_lambda,
                             newx = Ploan_tst_X, type = "class")
Elastic_model_prey <- as.factor(Elastic_model_prey)
Elastic_model_cm <- table(Ploan_tst_y, Elastic_model_prey)
Elastic_model_cm
```

✓ predict( )

- Arg 1: Trained model
- Arg 2: Lambda
- Arg 3: Input variable of test dataset
- Arg 4: Output type (regression coefficients or predicted class)

R Exercise: Elastic Net

- Logistic Regression 8: Elastic Net

```
> Perf_Table
                  TPR Precision       TNR Accuracy       BCR F1-Measure
All         0.6538462 0.8192771 0.9832589    0.949 0.8018105  0.7272727
Forward     0.4519231 0.9400000 0.9966518    0.940 0.6711259  0.6103896
Backward    0.6442308 0.8170732 0.9832589    0.948 0.7958930  0.7204301
Stepwise    0.4519231 0.9400000 0.9966518    0.940 0.6711259  0.6103896
GA          0.6346154 0.8354430 0.9854911    0.949 0.7908273  0.7213115
Ridge       0.5384615 0.8888889 0.9921875    0.945 0.7309274  0.6706587
Lasso       0.6250000 0.8227848 0.9843750    0.947 0.7843688  0.7103825
Elastic Net 0.6250000 0.8227848 0.9843750    0.947 0.7843688  0.7103825
```

R Exercise: Summary

- Logistic Regression Coefficients

```
> full_model_coeff
                         [,1]
(Intercept)      -4.445483193
Age              -0.776030543
Experience        0.910983523
Income            2.374700527
Family            0.739703391
CCAvg             0.264244401
Education         1.328860208
Mortgage          0.009294095
Securities.Account -0.501459121
CD.Account        0.982081783
Online           -0.182069399
CreditCard       -0.617373701
```

```
> forward_model_coeff
                        [,1]
(Intercept)       0.10188147
Income            0.14912124
CD.Account        0.08388042
Education         0.06844227
Family            0.03986637
CreditCard       -0.02439480
Securities.Account -0.02549191
CCAvg             0.01949386
Experience        0.10732764
Age              -0.09885277
```

```
> backward_model_coeff
                       [,1]
(Intercept)      -4.4217370
Income            2.3871278
Family            0.7319393
CCAvg             0.2481614
Education         1.3033304
Securities.Account -0.4674934
CD.Account        0.9372616
CreditCard       -0.5889305
```

```
> stepwise_model_coeff
                        [,1]
(Intercept)       0.10188147
Income            0.14912124
CD.Account        0.08388042
Education         0.06844227
Family            0.03986637
CreditCard       -0.02439480
Securities.Account -0.02549191
CCAvg             0.01949386
Experience        0.10732764
Age              -0.09885277
```

```
> GA_model_coeff
                        [,1]
(Intercept)      -4.43332725
Age               0.12916530
Income            2.39554937
Family            0.75075280
CCAvg             0.25721313
Education         1.30812212
Mortgage          0.01350098
Securities.Account -0.47264049
CD.Account        0.92755663
CreditCard       -0.57332418
```

```
> Ridge_model_coeff
12 x 1 sparse Matrix of class
                            1
(Intercept)      -3.25969793
Age               0.02036741
Experience        0.04528453
Income            1.35739246
Family            0.39426207
CCAvg             0.32094559
Education         0.69815256
Mortgage          0.07387779
Securities.Account -0.21344808
CD.Account        0.58220572
Online           -0.07193090
CreditCard       -0.28325806
```

```
> Lasso_model_coeff
12 x 1 sparse Matrix of class "dg
                               1
(Intercept)      -4.2408732055
Age               .
Experience        0.0986384046
Income            2.2682994528
Family            0.6748085859
CCAvg             0.2384203451
Education         1.2171062156
Mortgage          0.0006765386
Securities.Account -0.3918108753
CD.Account        0.8714276903
Online           -0.1234223620
CreditCard       -0.5131301095
```

```
> Elastic_model_coeff
12 x 1 sparse Matrix of class
                            1
(Intercept)      -4.21928953
Age               .
Experience        0.11110504
Income            2.22361220
Family            0.67302703
CCAvg             0.25676164
Education         1.20324138
Mortgage          0.01181456
Securities.Account -0.41388674
CD.Account        0.88543356
Online           -0.14219408
CreditCard       -0.52966366
```


ANY
questions?