•) Compressibilité

$$\text{I)} \quad -i\omega \rho_0 \vec{v} = -\vec{\nabla}p + \eta \nabla^2 \vec{v} + \left(\xi + \frac{1}{3}\eta\right)\vec{\nabla}(\vec{\nabla}\cdot\vec{v})$$

amplitudes

$\Longrightarrow$

$$-i\omega \rho_0 \vec{v} = -\vec{\nabla}p - ikp\,\hat{x} + \eta \nabla^2 \vec{v} + 2i\eta k\,(\partial_x \vec{v}) - \eta k^2 \vec{v}$$

$$+ \left(\xi + \frac{1}{3}\eta\right)\vec{\nabla}(\vec{\nabla}\cdot\vec{v}) + \left(\xi + \frac{1}{3}\eta\right)(ik)(\vec{\nabla}\cdot\vec{v})\,\hat{x}$$

$$+ \left(\xi + \frac{1}{3}\eta\right)(ik)(\vec{\nabla}v_x) - \left(\xi + \frac{1}{3}\eta\right)k^2 v_x\,\hat{x}$$

$$(\text{I}^*)$$

Formulation faible d I*

$$-i\omega \rho_0 \int_V \vec{v}\cdot\vec{u}\,dV - \int_V p(\vec{\nabla}\cdot\vec{u}) + ik\int_V p\,u_x$$

$$+ \eta \int_V (\vec{\nabla}v_x \cdot \vec{\nabla}u_x + \vec{\nabla}v_y \cdot \vec{\nabla}u_y)\,dV + 2i\eta k\int_V (\partial_x \vec{u})\cdot\vec{v}\,dV$$

$$+ \eta k^2 \int_V \vec{v}\cdot\vec{u}\,dV + \left(\xi + \frac{1}{3}\eta\right)\int_V (\vec{\nabla}\cdot\vec{v})(\vec{\nabla}\cdot\vec{u})\,dV$$

$$- i\left(\xi + \frac{1}{3}\eta\right)k\int_V (\vec{\nabla}\cdot\vec{v})u_x\,dV + i\left(\xi + \frac{1}{3}\eta\right)k\int_V (\vec{\nabla}\cdot\vec{u})v_x\,dV$$

$$+ \left(\xi + \frac{1}{3}\eta\right)k^2 \int_V v_x u_x\,dV = 0$$

$$(\text{I - faible})$$

II) $\quad -i\omega \rho_0 \tau = -i\omega \beta_0 T_0 p + \kappa \vec{\nabla}^2 \tau + \beta_0 T_0 q$

$$\begin{cases} q = q_0 \, e^{ikx - i\omega t} \\[6pt] \tau = \tilde{\tau} \, e^{ikx - i\omega t} \\[6pt] q = + \dfrac{\partial \Phi}{\partial t} \\[6pt] \Phi = \underline{\Phi}_0 \, e^{ikx - i\omega t} \end{cases} \Bigg\} \; q_0 = -i\omega \underline{\Phi}_0$$

amplitudes

$\Rightarrow$

$$\boxed{-i\omega \rho_0 \tau = -i\omega \beta_0 T_0 p + \kappa \vec{\nabla}^2 \tau + 2 i \kappa k \, (\partial_x \tau) - k^2 \kappa \tau + \beta_0 T_0 q}$$

$$(\text{II}^\star)$$

Formulation faible de II*

$$\boxed{\begin{aligned} &-i\omega \rho_0 q \int_V \tau \, s \, dV + i\omega \beta_0 T_0 \int_V p \, s \, dV + \kappa \int_V \vec{\nabla}\tau \cdot \vec{\nabla} s \, dV \\[10pt] &+ 2 i k \kappa \int_V \tau \, \partial_x s \, dV + k^2 \kappa \int_V \tau \, s \, dV - \beta_0 T_0 \int_V q_0 \, s \, dV = 0 \end{aligned}}$$

III) $\quad -i\omega b + \vec{\nabla} \cdot \vec{v} = 0$

amplitudes $\Longrightarrow$

$$\boxed{-i\omega b + \vec{\nabla} \cdot \vec{v} + i k \, v_x = 0}$$

$$(\text{III}^\star)$$

Formulation faible de $\overline{III}^{\alpha}$

$$-i\omega \int_V b\ell + \int_V (\vec{\nabla}\cdot\vec{v})\,\ell\,dV + ik\int_V v_x\,\ell = 0$$

$(\overline{III} - faible)$

$\overline{IV})$ $\qquad \gamma \mathcal{X}_0 P = b + \beta_0 \tau$

$(\overline{IV}^{\alpha})$

Formulation faible de $\overline{IV}^{\alpha}$

$$\gamma \mathcal{X}_0 \int_V p\,r\,dV - \int_V b\,r\,dV - \int_V \beta_0 \tau\,r = 0$$

$(\overline{IV} - faible)$