Engineering Mathematics Assignment #1

1. (5pt) An object having a temperature of 32.2 degrees Celsius is placed in an environment maintained at 15.5 degrees. Ten minutes later the object has cooled to 31.1 degrees. What will the temperature of the object be after it has been in this environment for 20 minutes? How long will it take for the object to cool to 18 degrees?

2. (5pt) Evaluate $\int_0^\infty e^{-t^2-\left(\frac{9}{t}\right)^2}\,dt$.

3. (5pt) This problem explores the logistic model of population growth. In 1837, the Dutch biologist Pierre Francois Verhulst developed a differential equation to model change in a population(he was studying fish populations in the Adriatic Sea). Verhulst reasoned that the rate of change of a population $P(t)$ with respect to time should be influenced by growth factors(such as the current population) and also factors tending to retard growth(such as limitations on food and space). He formed a model by postulating that growth factors can be incorporated into a term $aP(t)$, and retarding factors into a term $-bP(t)^2$, with a and b positive numbers chosen to fit the particular population. This led to his logistic equation $P^{'}(t) = aP(t) - bP(t)^2$. When $b = 0$, this is an exponential model, which is unrealistic for most populations. Solve the logistic equation, subject to the condition $P(0) = p_0$, to obtain $P(t) = \frac{ap_0}{a-bp_0+bp_o e^{at}} e^{at}$.

This is the logistic model of population growth. Show that $P(t)$ determined by this equation is bounded above and asymptotically approaches the limit $\frac{a}{b}$ as $t \to \infty$. This is dramatically different from an exponential model, in which the population grows increasingly and without bound as time increases.

4. (5pt) Continuing Problem #1, a 1920 study by Pearl and Reed in the Proceedings of the National Academy of Sciences suggested the values $a = 0.03134, b = 1.5887E^{-10}$ for the United States. Table in the next slide gives the census data for the United States in ten years intervals from 1790 through 1980. Taking 1790 as year zero to determine p_0, show that the logistic model for the United States population is $P(t) = \frac{123{,}141.5667}{0.03072+0.00062e^{0.03134t}} e^{0.03134t}$. Use this data to fill in the $P(t)$ column of the table. Calculate the percentage error to fill in this column. You should observe that the numbers predicted by the logistic model are quite accurate for a fairly long period of time, then diverge increasingly from the actual numbers. Show that the limit of the population in this model is about 197,300,000,

which the United States passed by 1970.

Next try an exponential model $Q'(t) = kQ(t)$, using the 1790 population to determine k. You will find that this model diverge fairly quickly from the census data. Exponential model are not sophisticated enough to model the complexities of human populations.

Year	Population	P(t)	Percent Error	Q(t)	Percent Error
1790	3,929,213				
1800	5,308,483				
1810	7,239,881				
1820	9,638,453				
1830	12,886,020				
1840	17,169,453				
1850	23,191,876				
1860	31,443,321				
1870	38,558,371				
1880	50,189,209				
1890	62,979,766				
1900	76,212,168				
1910	92,228,496				
1920	106,021,537				
1930	123,202,624				
1940	132,164,569				
1950	151,325,798				
1960	179,323,175				
1970	203,302,031				
1980	226,547,042				

Where percent error can be measured as following:

$$\frac{|\mathrm{P(t)} - \mathrm{Population}|}{\mathrm{Population}} \times 100(\%)$$

You should answer the following questions:

1. Limit of population
2. Fill the above census prediction table for $P(t)$ and percentage error
3. Find out the constant k for exponential model $Q'(t) = kQ(t)$
4. Fill the above census prediction table for $Q(t)$ and percentage error

5. (5pt) Find all functions with the property that the y-intercept of the tangent to the graph at (x, y) is $2x^2$

6. (5pt) A 2000 liter cylindrical tank initially containers 200 liters of brine solution in which 12.5 kilograms of salt have been dissolved. Beginning at time zero, brine containing 0.25 kilogram of salt per liter is added at the rate of 12liters per minute, and the mixture is poured out of the tank at the rate of 8 liters per minute. How much salt is in the tank when it contains 400 liters of brines?

7. (10pt) Two tanks are connected as in Figure 1.6. Tank 1 initially contains 10 kilograms of salt dissolved in 400 liters of brine. Tank 2 initially contains 600 liters of brine in which 45 kilograms of salt have been dissolved. At time zero, a brine solution containing 1/16 kilogram of salt per liter is added to tank 1 at the rate of 20 liters per minute. Tank 1 has an output that discharges brine into tank 2 at the rate of 20 liters per minute, and tank 2 also has a discharge of 20 liters per minute. Determine that amount of salt in each tank at any time $t > 0$. Also determine when the concentration of salt in tank 2 is a minimum and how much salt is in this tank at that time.

Hint: solve for the amount of salt in tank 1 at time t and use this solution to determine the amount in tank 2.