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[Automata 2012 – 2 Homework]

[Automata Homework #4]

Example 4.12] Use pumping lemma to prove that $L=\{a^n b^k c^{n+k} \mid n \ge 0, k \ge 0\}$ is not regular.

$\forall m>0$, choose $w = a^m b^m c^{2m}$ $(|w| \ge m)$

Consider all possible decomposition of $w = \overbrace{a\cdots a}^{m}\overbrace{b\cdots b}^{m}\overbrace{c\cdots c}^{2m}$

$= xyz$ $(|xy| \le m, |y| \ge 1)$

$y = a^k$ $(1 \le k \le m)$

Set $i=0$, $xz = \underbrace{a^{m-k} b^m}_{m-k+m = 2m-k \ne 2m} c^{2m} \notin L$

$\therefore$ L is not regular.

Example 4.13] Use pumping lemma to prove that $L=\{a^n b^l \mid n \ne l\}$ is not regular.

$\forall m>0$, choose $w = a^m b^{m+m!}$ $(|w| \ge m)$

Consider all possible decomposition of $w = \overbrace{a\cdots a}^{m}\overbrace{b\cdots b}^{m+m!}$

$= xyz$ $(|xy| \le m, |y| \ge 1)$

위첨자 $y = a^k$ $(1 \le k \le m)$

$xy_iz = a^{m+(i-1)k} b^{m+m!}$, $m+(i-1)k = m+m!$, $(i-1)k = m!$, $i = \frac{m!}{k}+1$ (i는 정수)

위첨자 Set $i = \frac{m!}{k}+1$, $xy_{\frac{m!}{k}+1}z = a^{m+m!} b^{m+m!} \notin L$

$\therefore$ L is not regular.

Exercises 4.3.3] Use pumping lemma to prove that $L=\{w \mid n_a(w) = n_b(w)\}$ is not regular.

$\forall m>0$, choose $w = a^m b^m$ $(|w| \ge m)$

Consider all possible decomposition of $w = \overbrace{a\cdots a}^{m}\overbrace{b\cdots b}^{m}$

$= xyz$ $(|xy| \le m, |y| \ge 1)$

$y = a^k$ $(1 \le k \le m)$

Set $i=2$, $xy_2z = a^{m+k} b^m \notin L$

$n_a(w) = m+k \ne n_b(w) = m$

$\therefore$ L is not regular.

Exercises 4.3.4] Prove that $L=\{w \mid n_a(w) \ne n_b(w)\}$ is not regular.

i) We know $L = \{a^n b^n \mid n \ge 0\}$ is not regular.

Assume L is regular.

$L^c = \{w \mid n_a(w) = n_b(w)\}$

L^c and $L(a^*b^*)$ is regular

but $L^c \cap L(a^*b^*)$

$= \{a^n b^n \mid n \ge 0\} \leftarrow \boxed{\text{not regular}}$

$\rightarrow \boxed{\text{contradiction}} \leftarrow$ 이유명시해야함.

$\therefore$ L is not regular.

ii) $\forall m>0$, choose $w = a^m b^{m+m!}$ $(|w| \ge m)$

consider all possible decomposition of $w = \overbrace{a\cdots a}^{m}\overbrace{b\cdots b}^{m+m!}$

$= xyz$ $(|xy| \le m, |y| \ge 1)$

$y = a^k$ $(1 \le k \le m)$

$xy_iz = a^{m+(i-1)k} b^{m+m!}$, $m+(i-1)k = m+m!$, $(i-1)k = m!$, $i = \frac{m!}{k}+1$

Set $i = \frac{m!}{k}+1$, $xy_{\frac{m!}{k}+1}z = a^{m+m!} a^{m+m!} \notin L$

$n_a(w) = n_b(w)$

$\therefore$ L is not regular.

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[Automata 2012 – 2 Homework]

[Automata Homework #4]

Example 4.12] Use pumping lemma to prove that $L = \{a^n b^k c^{n+k} \mid n \geq 0, k \geq 0\}$ is not regular.

good!

Assume L is regular

$\forall m > 0$, we choose $w = a^m b^m c^{m+m}$

Consider all possible decomposition of $w = \underbrace{a \cdots a}_{xy}\overbrace{b \cdots b}^{m}\overbrace{c \cdots c}^{2m} = xyz$

$(|xy| \leq m,\ |y| \geq 1)$,

$xy = a^m \quad y = a^l \ (1 \leq l \leq m)$

Set $i = 0$ $\quad w_0 = xy^0z = a^{m-l} b^m c^{2m} \notin L$ ($\because m-l+m \neq 2m$) → contradiction!

가정이 잘못되었다. ⇒ ∴ L is not regular.

good!

Example 4.13] Use pumping lemma to prove that $L = \{a^n b^l \mid n \neq l\}$ is not regular.

Assume L is regular

$\forall m > 0$, we choose $w = a^{m!} b^{(m+1)!}$

Consider all possible decomposition of $w = xyz = \underbrace{a \cdot}_{xy} \mid \underbrace{\cdot ab \cdots b}_{z}$

$(|xy| \leq m,\ |y| \geq 1)$ $\quad y = a^k \ (1 \leq k \leq m)$

$w_i = xy^iz$

w_i에서 a의 개수: $m! + (i-1)k$

→ So. $m! + (i-1)k = (m+1)!$

$i = \dfrac{(m+1)! - m!}{k} + 1 = \dfrac{m\, m!}{k} + 1$ $\quad k \leq m$

정수

we know $a^n b^n$ is not regular.

∴ L is not regular. contradiction! ⇐ $\boxed{i\text{가 정수가 되게하는 } m, k \text{ 가 존재}}$ → $w_i \notin L$.

잘못된 표현. "임의의 m, k에 대해 $w_i \notin L$ 이 되는 정수 i가 존재"

Exercises 4.3.3] Use pumping lemma to prove that $L = \{w \mid n_a(w) = n_b(w)\}$ is not regular.

Assume L is regular

$\forall m > 0$, we choose $w = a^m b^m$

Consider all possible decomposition of $w = xyz$

$(|xy| \leq m,\ |y| \geq 1)$ $\quad w = \underbrace{\overbrace{a \cdots a}^{m}}_{xy} \mid \underbrace{\overbrace{b \cdots b}^{m}}_{z}$

→ $y = a^k \ (1 \leq k \leq m)$

For any y, set $i = 0$, $w_0 = xz = a^{m-k} b \notin L$

∴ L is not regular.

Exercises 4.3.4] Prove that $L = \{w \mid n_a(w) \neq n_b(w)\}$ is not regular.

Assume L is regular

$\forall m > 0$, we choose $w = a^m b^{m+1}$

Consider all possible decomposition of w

$w = \underbrace{\overbrace{a \cdots a}^{m}}_{xy} \mid \underbrace{\overbrace{b \cdots b}^{m+1}}_{z} = xyz$

$y = a^k \ (1 \leq k \leq m)$

$w = \underbrace{a^{m-k}}_{x} \underbrace{a^k}_{y} \underbrace{b^{m+1}}_{z}$

$w_i = xy^iz$

For any y, set $i = 3$.

$w_3 = xyyyz = a^{m-k} a^k a^k a^k b^m$

$= a^{m+2k} b^m \notin L$. 오류!

$|xy| \leq m$ —— contradiction! ($\because |xy| = m+2k > m$)

이것은 만족하므로 $a^{m+2k} b^m \in L$.

⇒ ∴ L is regular.