

$L = \{a^n b^n c^n \mid n \geq 0\}$ not CFL.

L is CFL

$\rightarrow \exists$ positive integer m.

$\forall w \in L$ with $|w| \geq m$.

$$\frac{A \xrightarrow{*} vAy}{A \xrightarrow{*} \alpha}$$

$$\exists \boxed{w = vxyz}$$

$$S \Rightarrow SS \Rightarrow SSS \Rightarrow aAbSS$$

$$\boxed{a^n b^n a^m b^m} a \underbrace{a^n b^n}_{\text{pumping}} \dots b$$

$$\boxed{a^n b^n}$$

$$a \dots a b \dots b$$

CFL \rightarrow union, concatenation, star-closure.

$$S_3 \rightarrow (S_1 | S_2)$$

$$S_4 \rightarrow S_1 S_2$$

$$S_5 \rightarrow S_1 S_2^*$$

intersection.

$$\frac{a^n b^n \vee c^m}{a^n \vee b^m c^m} \text{ CFL}$$

$$\Rightarrow a^n b^n c^n \rightarrow \text{not CFL}$$

complementation



L

$\cup/\cap \rightarrow$ closed for CFL.

$$(L_1 \vee L_2)' \rightarrow \text{CFL}$$

$$\underline{L_1 \cap L_2} \rightarrow \text{not CFL}$$

6.5.

regular intersection

$$M_1 = (Q, \Sigma, \Gamma, \delta, Q_0, z, f_1) \quad \text{npda.} \quad L(M_1) = L_1$$

$$M_2 = (P, \Sigma, \delta_2, p_0, f_2) \quad \text{dfa.} \quad L(M_2) = L_2.$$

$$M' = (Q \times P, \Sigma, \Gamma, \delta', (Q_0, p_0), f_1 \times f_2)$$

$$\delta(Q_i, a, b) = \gamma(\underbrace{Q_j, x}_?)$$

$$\delta_2(\underline{p_i}, a) = \underline{p_j}$$

$$((Q_i, p_i), a, b) = ((Q_j, p_j), x)$$

$L = \{a^n b^n \mid n \geq 0, n \neq 100\}$ is CFL.

$L_2 = \{a^n b^n\}$ is context-free.

$L_1 \cap L_2 = \{a^{100} b^{100}\}$ is CFL.
 $\underbrace{L_1}_{\text{CFL}} \cap \underbrace{L_2}_{\text{reg}} = \underbrace{\{a^{100} b^{100}\}}_{\text{reg}}$

$L = \{w \mid n_a(w) = n_b(w) = n_c(w)\}$

Assume $L = \{a^n b^n c^n \mid n \geq 0\}$ is CFL.
 $\underbrace{L}_{\text{CFL}} \cap \underbrace{\{a^n b^n c^n \mid n \geq 0\}}_{\text{reg}} = \underbrace{\{a^n b^n c^n \mid n \geq 0\}}_{\text{not CFL}}$

\rightarrow not CFL.

