#6. (1,2), (2,6), (3,6) $\rightarrow$ least-squares solution.

$mx+b.$

$\cancel{Y=mX}$

$Y=MX.$

$XM=Y$

$X^TXM=X^TY.$

$$Y=\begin{bmatrix}2\\6\\6\end{bmatrix} \quad X=\begin{bmatrix}1&1\\2&1\\3&1\end{bmatrix} \quad \begin{bmatrix}m\\b\end{bmatrix}$$

$$X^TX=\begin{bmatrix}1&2&3\\1&1&1\end{bmatrix}\begin{bmatrix}1&1\\2&1\\3&1\end{bmatrix}=\begin{bmatrix}1+4+9 & 1+2+3\\1+2+3 & 1+1+1\end{bmatrix}$$

$$=\begin{bmatrix}14&6\\6&3\end{bmatrix}$$

$$X^TY=\begin{bmatrix}1&2&3\\1&1&1\end{bmatrix}\begin{bmatrix}2\\6\\6\end{bmatrix}$$

$$=\begin{bmatrix}2+12+18\\2+6+6\end{bmatrix}=\begin{bmatrix}32\\14\end{bmatrix}$$

$$\begin{bmatrix}2\\\frac{2}{3}\end{bmatrix}$$

$\boxed{\therefore y=2x+\frac{2}{3}}$

$$\begin{bmatrix}14&6\\6&3\end{bmatrix}\begin{bmatrix}m\\b\end{bmatrix}=\begin{bmatrix}32\\14\end{bmatrix}$$

$$\begin{bmatrix}\cancel{14}^{7}&\cancel{6}^{3}&\cancel{32}^{16}\\6&3&14\end{bmatrix}$$

$3-3\cdot\frac{6}{7}$

$3(1-\frac{6}{7})$

$14-16\cdot\frac{6}{7}$

$\Downarrow$

$$\begin{bmatrix}7&3&16\\0&\frac{3}{7}&\frac{2}{7}\end{bmatrix}$$

$14-\frac{96}{7}$

$\frac{98-96}{7}$

$\frac{2}{7}.$

$$\begin{bmatrix}7&3&16\\0&3&2\end{bmatrix}$$

$\Downarrow$

$$\begin{bmatrix}7&0&14\\0&3&2\end{bmatrix}$$

$\Downarrow$

$$\left[\begin{array}{cc|c}1&0&2\\0&1&\frac{2}{3}\end{array}\right]$$

$n \times n$ 행렬 A가 n개의 서로 다른 고유값 $\lambda_1, \cdots, \lambda_n$

이들은 $v_1, \cdots, v_n$과 같이 각 고유벡터에 대응

행렬 A의 각 고유벡터들이 선형독립임을 증명하시오

귀류법. 선형종속이라고 가정해보자, 그럼 다음 식이 성립할 것이다.

선형 독립

$$(\boxed{v_1, \cdots, v_p}\ v_{p+1} \cdots v_n)$$

$$v_{p+1} = c_1 v_1 + \cdots + c_p v_p$$

모순이 생긴다

$v_{p+1} \neq 0$

∴ 선형독립이다

$$\lambda_{p+1} v_{p+1} = \lambda_{p+1} c_1 v_1 + \cdots + \lambda_{p+1} c_p v_p$$

$$A v_{p+1} = c_1 A v_1 + \cdots + c_p A v_p$$

$$\lambda_{p+1} v_{p+1} = c_1 \lambda_1 v_1 + \cdots + c_p \lambda_p v_p$$

subtract

선형 독립

$$0 = c_1(\lambda_{p+1} - \lambda_1) v_1 + \cdots + c_p(\lambda_{p+1} - \lambda_p) v_p$$

weight 들이 0이어야함

서로 다른 eigen value.

$\lambda_{p+1} \neq \lambda_i \ (1 \leq i \leq p)$

$\therefore c_1, \cdots, c_p = 0$

$$\frac{1}{5}\begin{bmatrix}3/8 & 0\\ 5/8 & 0\end{bmatrix}\begin{bmatrix}3\\ 2\end{bmatrix} \qquad \begin{bmatrix}3/8 & 0\\ 5/8 & 0\end{bmatrix}\begin{bmatrix}\cancel{0.6} & \frac{3}{5}\\ \cancel{0.4} & \frac{2}{5}\end{bmatrix}$$

$$\begin{bmatrix}\frac{9}{8}\\ \frac{15}{8}\end{bmatrix} \qquad \begin{bmatrix}\frac{9}{40}\\ \frac{3}{8}\end{bmatrix}$$

$$-\frac{1}{8}\begin{bmatrix}3 & 1\\ 5 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}-1 & -1\\ -5 & 3\end{bmatrix}$$

$$\begin{bmatrix}3 & 1\\ 5 & -1\end{bmatrix}\begin{bmatrix}-1 & 0\\ 0 & 0\end{bmatrix}$$

$\underline{PDP^{-1}}$

$$\begin{bmatrix}3 & 1\\ 5 & -1\end{bmatrix}\begin{bmatrix}1^k & 0\\ 0 & \cancel{0.2}^k\end{bmatrix}\left(\begin{bmatrix}3 & 1\\ 5 & -1\end{bmatrix}^{-1}\right) = \begin{bmatrix}-3 & 0\\ -5 & 0\end{bmatrix}$$

$$-\frac{1}{8}\begin{bmatrix}-1 & -1\\ -5 & 3\end{bmatrix}$$

#2.

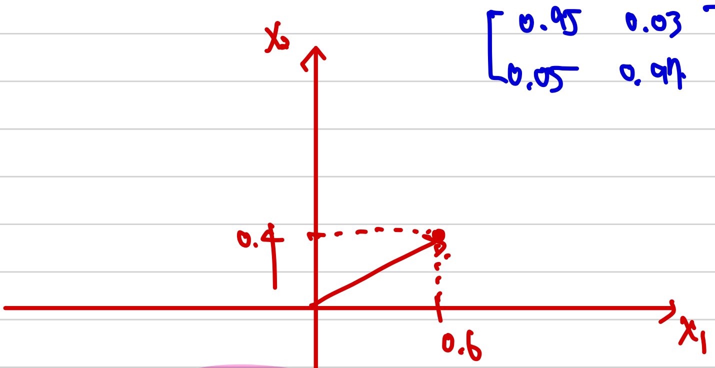
$A = \begin{bmatrix} 0.95 & 0.03 \\ 0.05 & 0.97 \end{bmatrix}$ 초기 벡터의 위치를 $x_0 = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$

시간 k 에서의 벡터 x_k의 위치는 $x_{k+1} = A x_k$ $(k = 0, 1, 2, \cdots)$

행렬 A의 고유값 λ_1, λ_2와 그에 대응하는 v_1, v_2를 구하고 $k \to \infty$ 일 때

$\lim_{k\to\infty} x_{k+1}$ 의 위치를 구하시오.

$\begin{bmatrix} 0.95 & 0.03 \\ 0.05 & 0.97 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$



$A - \lambda I = \begin{bmatrix} 0.95-\lambda & 0.03 \\ 0.05 & 0.97-\lambda \end{bmatrix}$

$= (0.95-\lambda)(0.97-\lambda) - 0.03 \cdot 0.05$

$0.9215 - 1.92\lambda + \lambda^2 - 0.0015$

$\lambda^2 - 1.92\lambda + 0.92$

$\begin{array}{r} 95 \\ 97 \\ \hline 66.5 \\ 855 \\ \hline 9215 \end{array}$

$100(100\lambda^2 - 192\lambda + 92)$

25×23

$4 \quad -4$

25

$= 100 \cdot 92$

$(4\lambda - 4)(25\lambda - 23)$

$\lambda = 1$ or $\lambda = \frac{23}{25}$

92

$\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\lambda = 1$ or $\lambda = 0.92$

$A - \lambda I = \begin{bmatrix} -0.05 & 0.03 \\ 0.05 & -0.03 \end{bmatrix}$

$= \begin{bmatrix} -5 & 3 \\ 5 & -3 \end{bmatrix}$

$= \begin{bmatrix} -5 & 3 \\ 0 & 0 \end{bmatrix}$

$-5x_1 + 3x_2 = 0$

$\begin{pmatrix} \frac{3}{5} x_2 \\ x_2 \end{pmatrix}$

$x_2 \begin{pmatrix} \frac{3}{5} \\ 1 \end{pmatrix}$

$\frac{1}{5} x_2 \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

$$\begin{bmatrix} 0.03 & 0.03 \\ 0.05 & 0.05 \end{bmatrix}$$

$x_1 + x_2 = 0$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 3 \\ 5 & 5 \end{bmatrix}$$

$\mathbb{R}^2$ $\{1\ t\ t^2\}$

P^2 $\mathbb{R}^3$

#3. 선형변환 T는 2차 다항함수의 변환을 나타내며, 2차 다항함수 공간에서의 선형변환 T는 $\begin{bmatrix} 3 & 4 & 0 \\ 0 & 5 & -1 \\ 1 & -2 & 7 \end{bmatrix}$로 표현된다고 가정할 때,

$p(t) = a_0 + a_1 t + a_2 t^2$ 이 T에 의해 변환되는 식

$1 \quad t \quad t^2 \rightarrow$ basis

$$\begin{bmatrix} 3 & 4 & 0 \\ 0 & 5 & -1 \\ 1 & -2 & 7 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \begin{matrix} 1 \\ t \\ t^2 \end{matrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$3a_0 + 4a_1 t + 5a_1 t - a_2 t^2$

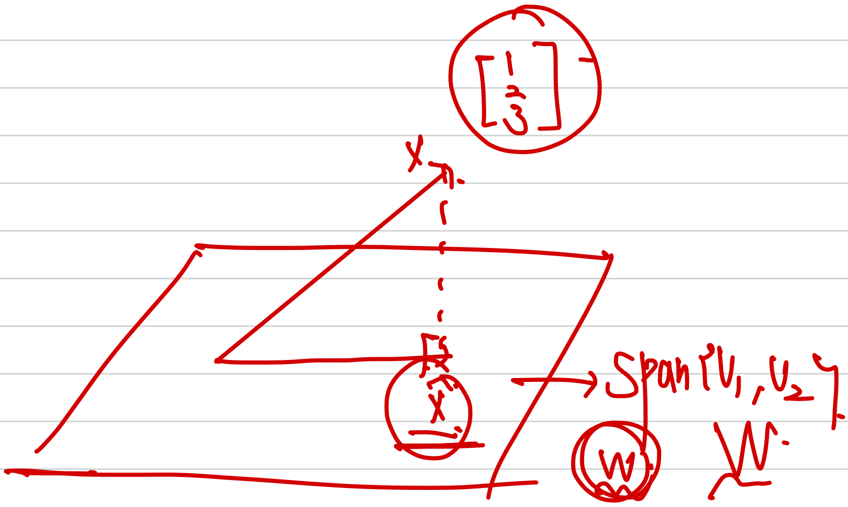
$+ a_0 - 2a_1 t + 7a_2 t^2$

$4a_0 + 7a_1 t + 6a_2 t^2$

$$\begin{bmatrix} 4a_0 & 7a_1 & 6a_2 \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \end{bmatrix}$$

$v_1 = \begin{bmatrix}2\\5\\-1\end{bmatrix}$ $v_2 = \begin{bmatrix}-2\\1\\1\end{bmatrix}$ $\mathbb{R}^3$의 서브공간 $W = \mathrm{Span}\{v_1, v_2\}$.

벡터 $x = \begin{bmatrix}1\\2\\3\end{bmatrix}$



$x \cdot v_1 = 2+10-3 = 9$

$v_1 \cdot v_1 = 4+25+1 = 30$

$x \cdot v_2 = -2+2+3 = 3$

$v_2 \cdot v_2 = 4+1+1 = 6$

$\hat{x} = \mathrm{proj}_W x = \frac{x\cdot v_1}{v_1\cdot v_1} v_1 + \frac{x\cdot v_2}{v_2\cdot v_2} v_2$

$= \frac{9}{30}\begin{bmatrix}2\\5\\-1\end{bmatrix} + \frac{3}{6}\begin{bmatrix}-2\\1\\1\end{bmatrix}$

$= \underbrace{\frac{3}{10}}_{x_1}\begin{bmatrix}2\\5\\-1\end{bmatrix} + \underbrace{\frac{1}{2}}_{x_2}\begin{bmatrix}-2\\1\\1\end{bmatrix}$

$= \begin{bmatrix}\frac{3}{5}\\ \frac{3}{2}\\ -\frac{3}{10}\end{bmatrix} + \begin{bmatrix}-1\\ 1/2\\ 1/2\end{bmatrix}$

$= \begin{bmatrix}\frac{3}{5}-1\\ \frac{3}{2}+\frac{1}{2}\\ -\frac{3}{10}+\frac{1}{2}\end{bmatrix} = \begin{bmatrix}-2/5\\ 2\\ 1/5\end{bmatrix}$

$x = [x]_B P$

$[\hat{x}]_W =$

$[x]_B = P^{-1} x$

$\begin{bmatrix}\frac{3}{10}\\ \frac{1}{2}\end{bmatrix} \begin{bmatrix}2 & 5\end{bmatrix}$

$\frac{2}{10}$

$[x] \quad x_2]$ $\begin{bmatrix}10\\ \frac{1}{2}\end{bmatrix}$

#6.

n개의 0벡터가 아닌 서로 직교하는 벡터 집합 $S=\{v_1,\cdots,v_n\}$

If.

$0 = c_1v_1+\cdots+c_pv_p$ some scalars $\boxed{c_1,\cdots,c_p}$

$0\cdot v_1 = c_1\|v_1\|^2$

$0 = c_1(v_1\cdot v_1)$

$c_1=0$, Similarly $c_2,\cdots,c_p$ must be zero

#7.

$5x_1^2-4x_1x_2+5x_2^2=48$

$\boxed{x=Py}$

$x^T\begin{bmatrix}5 & -2\\ -2 & 5\end{bmatrix}x$

$y^T\begin{bmatrix}3 & 0\\ 0 & 7\end{bmatrix}y$

$\boxed{3y_1^2+7y_2^2 \quad x=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}y}$

$(5-\lambda)(5-\lambda)-4$

$=21-10\lambda+\lambda^2$

$=(\lambda-3)(\lambda-7)$

$\lambda=3$ or $\lambda=7$

char

$\lambda=3 \rightarrow \begin{bmatrix}1\\1\end{bmatrix}$

$\lambda=7 \longrightarrow \begin{bmatrix}1\\-1\end{bmatrix}$

$\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$ $x_1=x_2$ $\begin{pmatrix}x_2\\x_2\end{pmatrix}$

$=$

$\begin{bmatrix}-2 & -2\\ -2 & -2\end{bmatrix}$

#6.

$Q(x) = 3x_1^2 + 2x_2^2 + x_3^2 + 4x_1x_2 + 4x_2x_3$

$$\left(\begin{bmatrix} 3 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 1 \end{bmatrix}\right) \rightarrow \text{eigen value} \rightarrow \boxed{5 \quad 2 \quad -1}$$

$Q(x) = \boxed{5y_1^2 + 2y_2^2 - y_3^2} = 2.$

$\rightarrow$

positive definite 일 수는 없다.