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[Automata 2022 – 1 Homework]

[Automata Homework #1]

1] Grammar G가 다음과 같이 정의되었다. $G=(\{A,S\},\{a,b\},S,P)$ P: $S\rightarrow baA|\lambda$, $A\rightarrow baA|\lambda$

$L(G)$를 찾고, 증명하시오.

$L(G)=\{(ba)^n \mid n\geq 0\}$

① $\forall w \in L$, $S \overset{*}{\Rightarrow} w$.

[basis] $n=0$ $w=b^0a^0=\lambda$ $S\Rightarrow \lambda$ terminal symbol

$n=1$ $w=b^1a^1=ba$. $S\Rightarrow baA \Rightarrow ba$) 둘다 성립.

[assumption] $n=k$ 일때 $w \overset{*}{\Rightarrow} (ba)^kA \Rightarrow (ba)^k$ 일이 성립한다는 가정

[Inductive step] $n=k+1$ 일때 $w=(ba)^{k+1}$ (가정에 의해) $S\overset{*}{\Rightarrow}(ba)^kA \Rightarrow (ba)^k(ba)A$

$\Rightarrow (ba)^{k+1}$

② $\forall w$, if $S\overset{*}{\Rightarrow}w$. then. $w\in L$.

let n: ~~#of~~ derivation step.

[basis] $n=1$ 일때 $S\Rightarrow \lambda=(ba)^0 \in L$ $n=2$ 일때 $S\rightarrow baA \Rightarrow ba \in L$로 성립

[assumption] $n=k$ 일때 $w \overset{k-1}{\Rightarrow} (ba)^{k-1}A \Rightarrow (ba)^{k-1} \in L$ 임을 가정

[Inductive step] $n=k+1$ 일때 ~~가능한~~ derivation들은 다

$S\Rightarrow (ba)^{k-1}A \Rightarrow (ba)^{k-1}(ba)A$

$\Rightarrow (ba)^k \in L$로 성립

2] Find a grammar for $\Sigma=\{a,b\}$ that generates the set of all strings starting and ending b.

b로 시작해서 b로 끝나는

$b\,b$

$b\,a\,b$

$S\rightarrow bAb$

$A\rightarrow aA\,|\,bA\,|\,\lambda$

$G=\{\{A,S\},\{a,b\},S,P\}$, P: $S\rightarrow bAb$, $A\rightarrow aA|bA|\lambda$

3] What language does the grammar with there products generate?

$S\rightarrow Aa,\ A\rightarrow B,\ B\rightarrow Aa|b$

$L(G)=\{ba^n \mid n\geq 1\}$

$S\rightarrow Aa$

$A\rightarrow B$

$B\rightarrow Aa|b$

$S\Rightarrow Aa \Rightarrow Ba \Rightarrow Aaa$

$\Rightarrow ba$ $\Rightarrow Baa \Rightarrow Aaaa$

$\Rightarrow baa$. $\Rightarrow baaa$

terminal symbol

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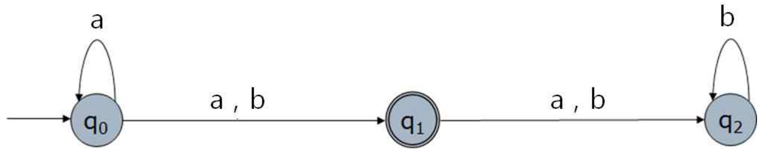
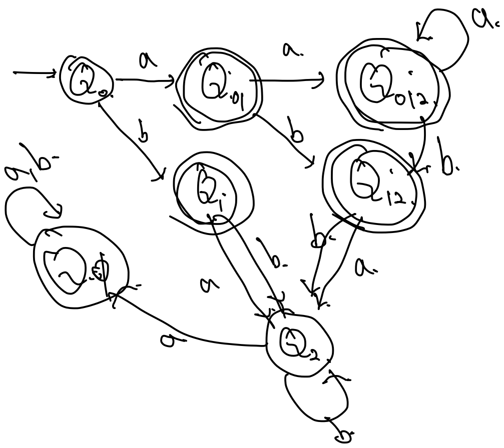
[Automata 2022 – 1 Homework]

[Automata Homework #2] .

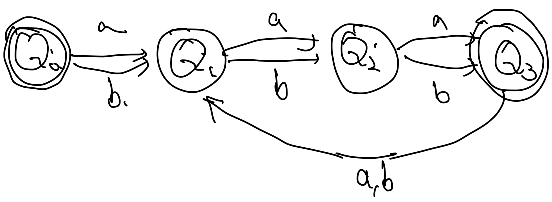
1] Convert nfa to an equivalent dfa.



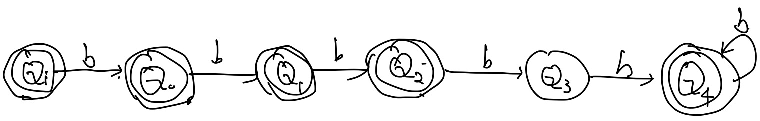


2] Find dfa for the following language on $\Sigma = \{a, b\}$

$L = \{w : |w| \bmod 3 = 0\}$



3] Show that the language $L = \{b^n : n \geq 0, n \neq 4\}$ is regular.



→ DFA

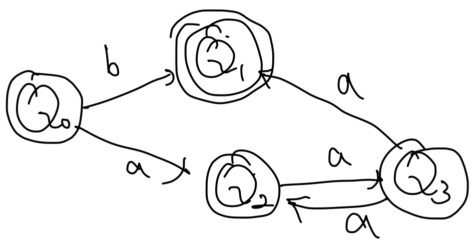
so L is Regular Language.

4] In the graph of Ex2.8 (ppt slide 10 page), find $\delta^*(q_0, 1011)$ and $\delta^*(q_1, 0010)$

$\delta^*(Q_0, 1011) = \{Q_2\}$.

$\delta^*(Q_1, 0010) = \emptyset$

5] Find an nfa without λ-transition and with a single final state that accepts the set $\{b\} \cup \{a^n | n > 2\}$



[Automata 2022 – 1 Homework]

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[Automata Homework #3]

1] Find a regular expression for the language

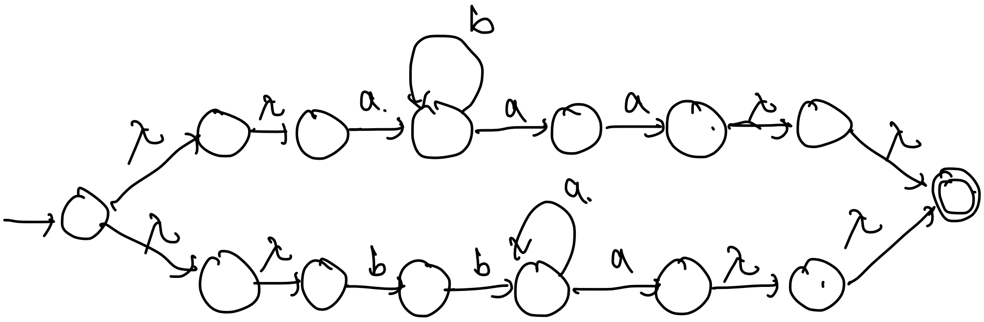
$L = \{w \in \{0,1\}^* : w \text{ has no pair of consecutive 0's}\}$

$r = (1+01)^*(0+\lambda)$

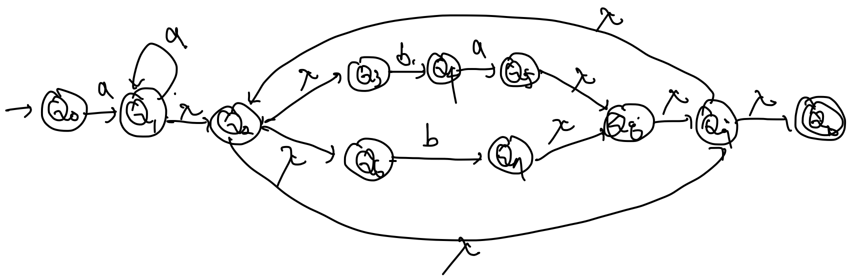
2] Find all strings in $L((10+0)^* 0 (1+10)^*)$ of length less than four.

$L = \{0, 00, 01, 100, 001, 010\}$

3] Use the construction in Theorem 3.1 to find an nfa that accepts the language $L(ab^*aa + bba^*a)$



4] Find a regular grammar that generates the language $L(aa^*(ba+b)^*)$



$P: Q_0 \to aQ_1,\ Q_1 \to aQ_1,\ Q_1 \to \lambda Q_2,\ Q_2 \to \lambda Q_3,\ Q_3 \to bQ_4,\ Q_4 \to aQ_5,\ Q_5 \to \lambda Q_8$

$Q_2 \to \lambda Q_6,\ Q_6 \to bQ_7,\ Q_7 \to \lambda Q_8,\ Q_8 \to \lambda Q_9,\ Q_9 \to \lambda Q_{10}$

$G = (\{Q_0, Q_1, Q_3, Q_4, Q_5, Q_6, Q_7\}, \{a, b\}, Q_0, P)$

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[Automata 2022 – 1 Homework]

[Automata Homework #4]

1] Use pumping lemma to prove that $L=\{a^n b^k c^{n+k} \mid n\ge 0, k\ge 0\}$ is not regular.

Assume L is regular

$\forall m>0$ we choose $w=a^m b^m c^{2m} \in L$ $(|w|\ge m)$

consider $\forall$ possible decomp of $w=xyz$ $(|xy|\le m, |y|\ge 1)$

y has the form of a^k $(1\le k\le m)$

set $i=0$, $xz=a^{m-k} b^m c^{2m} \notin L$ pumping lemma is false

$\therefore$ L is not regular.

2] Use pumping lemma to prove that $L=\{a^n b^l \mid n\ne l\}$ is not regular.

$\forall m>0$, choose $w=a^{m!} b^{m+!}$. $(|w|\ge m)$

consider all possible decomposition of $w=\overbrace{a\cdots a}^{m}\overbrace{b\cdots b}^{m+!}=xyz$ $(|xy|\le m, |y|\ge 1)$

$\Downarrow$ $y=a^k$ $(1\le k\le m)$

$xy^iz=a^{m+(i-1)k}\cdot b^{m+!}$ $\rightarrow$ $(i=\frac{!}{k}+1)$ (i는 음이 아닌 정수) $a^{m+!}\cdot b^{m+!} \notin L$

$\therefore$ L is not regular.

3] Show that $L=\{w \mid n_a(w)=n_b(w)\}$ is not regular. Is L^* regular?

$\forall m>0$, choose $w=a^m b^m$ $(|w|\ge m)$

consider all posible decomposition of $w=\overbrace{a\cdots a}^{m}\overbrace{b\cdots b}^{m}=xyz$ $(|xy|\le m, |y|\ge 1)$

$y=a^k$ $(1\le k\le m)$

set $i=2$, $xy_2z=a^{m+k} b^m \notin L$

L is not regular.

let L's reg expression is r,

L^*'s Reg expression is r^*. (Regular language is closed under $*$.)

so L^* is not regular.

4] Prove that $L=\{w \mid n_a(w)\ne n_b(w)\}$ is not regular.

Assume L is Regular.

$L^c=\{w \mid n_a(w)=n_b(w)\}$ is regular too.

(we know. $L'=\{a^n b^n \mid n\ge 0\}$ is not regular)

if L^c is regular. $L^c \cap L(a^*b^*)=L'$ (L' is regular)

$n_a(w)=n_b(w)$ $\quad$ $a\Rightarrow b$ sequential order.

but L' is not regular so L^c is not regular.

then L is not regular.

$\forall m>0$ $w=a^m b^{m+m!}$ $(|w|\ge m)$

consider all decomposition of $w=\overbrace{a\cdots a}^{m}\overbrace{b\cdots b}^{m+m!}$
$=xyz$ $(|xy|\le m, |y|\ge 1)$

$y=a^k$ $(1\le k\le m)$

$xyz=a^{m+(i-1)k}\cdot b^{m+m!}$ set $i=\frac{m!}{k}+1$ (i not negative Integer)

$xy^{\frac{m!}{k}+1}z=a^{m+m!}b^{m+m!}\notin L$

$\therefore$ L is not regular.

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[Automata 2022 – 1 Homework]

[Automata Homework #5] .

1] Give a derivation tree for $w = abbbaabbaba$ for the grammar in Example 5.2

2] Find context-free grammars for the language $L = \{a^n b^m \mid n \neq 2m\}$

3] Find an s-grammar for $L = (000^* 11 + 11)$

4] Show that the following grammar is ambiguous. $S \rightarrow bSaS \mid aSbS \mid \lambda$

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[Automata 2022 – 1 Homework]

[Automata Homework #6]

1] A grammar is given below.

$S \to aA|aBB,\ A \to aaA|\lambda,\ B \to bB|bbC,\ C \to B$

(1) Remove all unit-productions, all useless productions, and all λ-productions.

① Remove all λ-productions

$V_N = \{A\}$ $S \to aA \mid aBB \mid a,\ A \to aa \mid aaA,\ B \to bB \mid bbC,\ C \to B$

② Remove unit-productions

$C \to B$

S A B C

$C \Rightarrow B$ substitution rule $C \to bB \mid bbC$

$\left(S \to aA \mid aBB \mid a,\ A \to aa \mid aaA,\ B \to bB \mid bbC,\ C \to bB \mid bbC \right)$

③ Remove useless-productions

$V_1 = \{A, S\}$

S A

" $S \to aA \mid a,\ A \to aa \mid aaA$ "

(2) Change to Chomsky Normal Form.

step 1) $S \to V^a A \mid a,\ A \to V^a V^a \mid V^a V^a A,\ V^a \to a$

step 2) $S \to V^a A \mid a,\ A \to V^a V^a \mid V^a D_1,\ V^a \to a,\ D_1 \to V^a A$ "

$S \in V_{15}$ 을 확인 해야 한다

(3) Use CYK algorithm to check the grammar generates a^5 aaaaa.

CNF

$S \to V^a A \mid a$

$A \to V^a V^a \mid V^a D_1$

$V^a \to a,\ D_1 \to V^a A$

$V_{11} = \{S, V^a\}$ $V_{22}\ V_{33}\ V_{44}\ V_{55}$ 동일

$V_{12} = V_{11} V_{22} = \{A\}$ $V_{23}\ V_{34}\ V_{45}$ 동일

$V_{13} = V_{11} V_{23} \cup V_{12} V_{33} = \{S, D_1\}$ $V_{24}\ V_{35}$ 동일

$V_{14} = V_{11} V_{24} \cup V_{12} V_{34} \cup V_{13} V_{44} = \{A\}$ V_{25} 동일

$V_{15} = V_{11} V_{25} \cup V_{12} V_{35} \cup V_{13} V_{45} \cup V_{14} V_{55}$

$\{S\}$

$V_{11} V_{25} = \{S\}$ 에서 해당

$S \in V_{15}$ $\boxed{\therefore a^5 \in L(G)}$

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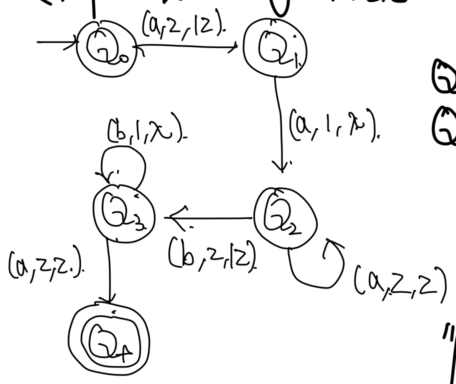
[Automata 2022 – 1 Homework]

[Automata Homework #7]

1] Construct an npda that accepts the regular language $L(aaa^*bba)$ transition.

<Representation by transition graph>.



Q_0 : initial state.
Q_f : final state.

$\delta(Q_0, a, z) = \{(Q_1, 1z)\}$
$\delta(Q_1, a, 1) = \{(Q_2, \lambda)\}$
$\delta(Q_2, a, z) = \{(Q_2, z)\}$
$\delta(Q_2, b, z) = \{(Q_3, 1z)\}$
$\delta(Q_3, b, 1) = \{(Q_3, \lambda)\}$
$\delta(Q_3, a, z) = \{(Q_f, z)\}$.

$M = (\{Q_0, Q_1, Q_2, Q_3, Q_f\}, \{a, b\}, \{1, z\}, Q_0, z, \{Q_f\})$

2] Construct an npda that accepts the language generated by the grammar $S \rightarrow bSSS|ab$

① $S \rightarrow bSSS|ab$ 를 (GNF : $A \rightarrow ax$. ($x \in V^*$, $a \in T$)) 로 바꿈.
를 바꾸면 됨

$S \rightarrow bSSS|aB$
$B \rightarrow b$.

② push S

$\boxed{\delta(Q_0, \lambda, z) = \{(Q_1, Sz)\}}$

③ $S \rightarrow aSSS|aB$

$\boxed{\delta(Q_1, a, S) = \{(Q_1, SSS), (Q_1, B)\}}$

④ $B \rightarrow b$

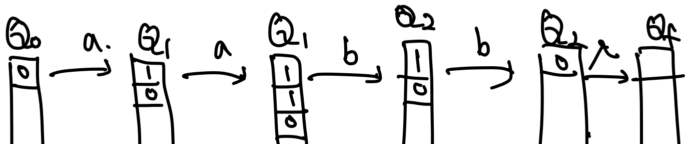
$\boxed{\delta(Q_1, b, B) = \{(Q_1, \lambda)\}}$

⑤. Last for final

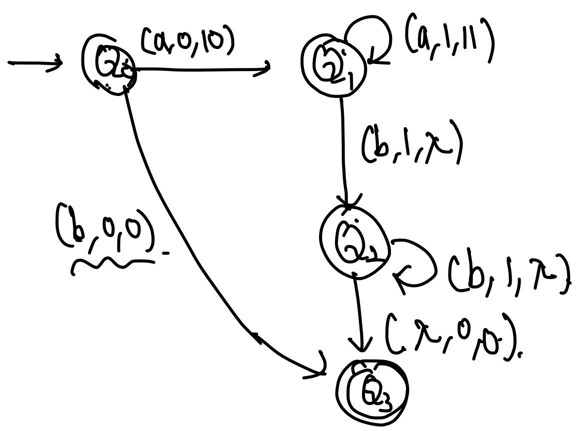
$\boxed{\delta(Q_1, \lambda, z) = \{(Q_f, z)\}}$

$M = (\{Q_0, Q_1, Q_f\}, \{a, b\}, \{S, A, B, z\}, \delta, Q_0, z, \{Q_f\})$

3] Is the language $L = \{a^n b^n : n \geq 1\} \cup \{b\}$ deterministic?



① → start ②



$\delta(Q_0, a, 0) = \{(Q_1, 10)\}$
$\delta(Q_1, a, 1) = \{(Q_1, 11)\}$
$\delta(Q_1, b, 1) = \{(Q_2, \lambda)\}$
$\delta(Q_2, b, 1) = \{(Q_2, \lambda)\}$
$\delta(Q_2, \lambda, 0) = \{(Q_3, 0)\}$
$\delta(Q_0, b, 0) = \{(Q_3, 0)\}$.

DPDA의 정의에 어긋나지 않는다

$\delta(Q, \lambda, b) \neq \emptyset$
$\rightarrow \delta(Q, c, b) = \emptyset$

$\boxed{M = (\{Q_0, Q_1, Q_2, Q_3\}, \{a, b\}, \{0, 1\}, Q_0, z = 0, \{Q_3\})}$

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[Automata 2022 – 1 Homework]

[Automata Homework #8]

1] Prove that $L=\{a^{n!} \mid n\geq 0\}$ is not context free.

Assume L is CFL

$\forall m.$ $w=a^{m!}$ $(|w|\geq m)$

$\overbrace{a \ldots\ldots a}^{m!}$

$\forall$ decomposition of $w=uvxyz$ $(|vxy|\leq m, |vy|\geq 1)$

let $v=a^{k}$, $y=a^{l}$ $(1\leq k+l\leq m)$

$\exists i=0$ $W_0=uxz=a^{m!-(k+l)}$

$m!-(k+l)$ 이 $j!$ 형태여야 함.

$k+l\leq m$ 이고

$(m-1)!$ $<m!-m \leq m!-(k+l)<m!$

$m!-m-(m-1)!$

$(m-1)!(m-1)-m>0$

$(m\geq 3)$ $m\rightarrow$ positive integer.

$(j\geq 3)$ W_0은 $j!$의 형태가 될 수 없다.

$W_0\notin L$

$\therefore$ L is not a CFL

2] Prove that $L=\{a^n b^j \mid n=j^2\}$ is not context free.

Assume L is CFL

$\forall$ positive Integer m. $W=a^{m^2}b^{m}$ $(|w|\geq m)$

$\underbrace{a\ldots\ldots a}_{m^2}\underbrace{b\ldots b}_{m}$ → 3가지 경우

$\forall$ decomposition of $w=uvxyz$ $(|vxy|\leq m, |vy|\geq 1)$

① $vxy=a^{k_1}xa^{k_2}$ $(1\leq k_1+k_2\leq m)$

② $vxy=a^{k_1}xb^{k_2}$ (〃)

③ $vxy=b^{k_1}xb^{k_2}$ (〃)

① $W_0=uxz=a^{m^2-(k_1+k_2)}b^{m}$

$m^2-(k_1+k_2)=m.$

$m^2-(k_1+k_2)\neq m^2$ $(k_1+k_2\geq 1)$

$W_0\notin L.$

② $a^{m^2-k_1}b^{m-k_2}$

$m^2-k_1=(m-k_2)^2$

$=m^2-2mk_2+k_2^2$

$2mk_2=k_2^2+k_1$

$m=\left(\frac{k_2^2+k_1}{2k_2}\right)$

$=\frac{k_2}{2}+\frac{k_1}{2k_2}$

but m = positive integer

$a^{m^2-k_1}b^{m-k_2}\notin L$

$\therefore$ L is not CFL

③ $W_0=a^{m^2}b^{m-(k_1+k_2)}$ $(k_1+k_2\geq 1)$

같은 이유로 $m^2\neq(m-(k_1+k_2))^2$ $m^2-k_1>(m-1)^2$

$W_0\notin L$

3] Determine and Prove whether or not $L=\{a^n b^n c^j \mid n\geq j\}$ is context free.

Assume L is CFL

$\forall$ positive Integer m $W=a^{m+1}b^{m+1}c^{m}$ $(|w|\geq m)$

$a\ldots\ldots ab\ldots\ldots bc\ldots c$

$\forall$ decomposition of $w=uvxyz$ $(|vxy|\leq m, |vy|\geq 1)$

① $vxy=a^{k_1}xa^{k_2}$ $(1\leq k_1+k_2\leq m)$

② $vxy=a^{k_1}xb^{k_2}$ (〃)

③ $vxy=b^{k_1}xb^{k_2}$ (〃)

④ $vxy=b^{k_1}xc^{k_2}$ (〃)

⑤ $vxy=c^{k_1}xc^{k_2}$ (〃)

① $\exists i=0$ $W_0=uxz=a^{m+1-(k_1+k_2)}b^{m+1}c^{m}\notin L.$ $(m+1-(k_1+k_2)\neq m+1)$

② $\exists i=0$ $W_0=uxz=a^{m+1-k_1}b^{m+1-k_2}c^{m}\notin L.$ $(m+1-k_1\neq m+1-k_2)$

③ $\exists i=0$ $W_0=uxz=$ → ① 〃

④ $\exists i=0$ $W_0=uxz=a^{m+1}b^{m+1-k_1}c^{m-k_2}\notin L$ $(m+1\neq m+1-k_1)$

⑤ $\exists i=0$ $W_0=uxz=$ → ① 〃

$\therefore$ L is not CFL

4] Prove that $L=\{a^n b^n a^m b^m \mid n,m\geq 0\}$ is context free but not linear.

$\{a^n b^n \mid n\geq 0\}$. CFL is closed under concatenation → L is CFL

$S\rightarrow aSb \mid \lambda$

$S\rightarrow AA$

$A\rightarrow aAb \mid \lambda$

Assume L is linear language.

$\forall m.$ (positive Integer) $\exists$ $w=a^m b^m a^m b^m$ $(|w|\geq m)$

$\forall$ decomposition of $w=\overbrace{a\ldots a}^{m}\overbrace{b\ldots b}^{m}\overbrace{a\ldots a}^{m}\overbrace{b\ldots b}^{m}\in L$

$=uvxyz$ $(|uvyz|\leq m, |vy|\geq 1)$

w only has the form, $a^{k_1}a^{k_2}x\,b^{k_3}b^{k_4}$ $(k_1+k_2+k_3+k_4\leq m.$ $(k_2+k_3\geq 1))$

$\exists i=0$ $W_0=a^{m-k_2}xb^{m-k_3}$ $(x=a^m b^m)$

$\notin L$ → $(m\neq m-k_3$ or $m\neq m-k_2)$

$\therefore$ L is not linear.

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[Automata 2022 – 1 Homework]

[Automata Homework #9]

1] Construct a Turing machine that will accept the following language on $\{0, 1\}$.

$L = \{w : |w| \bmod 4 = 0\}$

$M = (\{Q_0, Q_1, Q_2, Q_3, Q_4, Q_f\}, \{0,1\}, \{0,1,\square\}, \delta, Q_0, \square, \{Q_f\})$

δ

$\delta(Q_0, 1) = (Q_0, 0, R)$

$\delta(Q_0, 0) = (Q_0, 0, R)$

$\delta(Q_0, \square) = (Q_1, \square, L)$

$\delta(Q_1, 0) = (Q_2, 0, L)$

$\delta(Q_2, 0) = (Q_3, 0, L)$

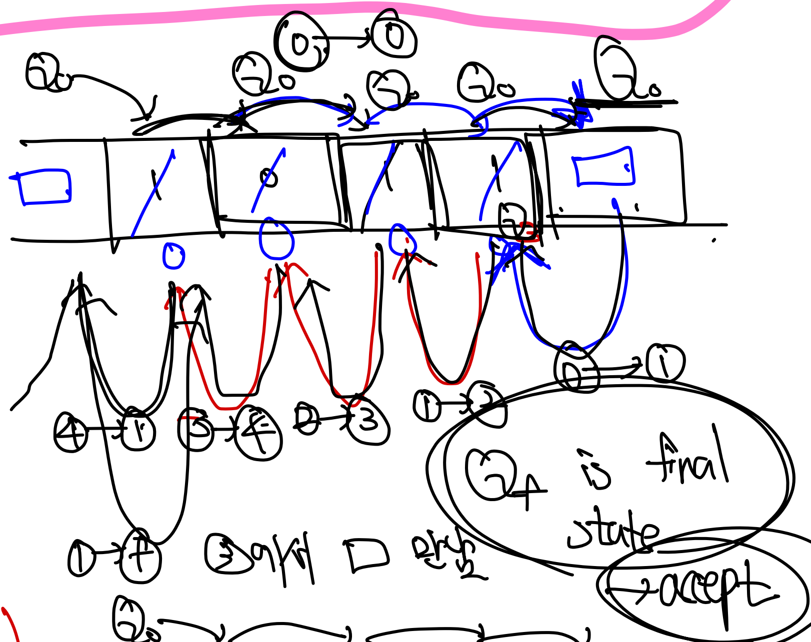
$\delta(Q_3, 0) = (Q_4, 0, L)$

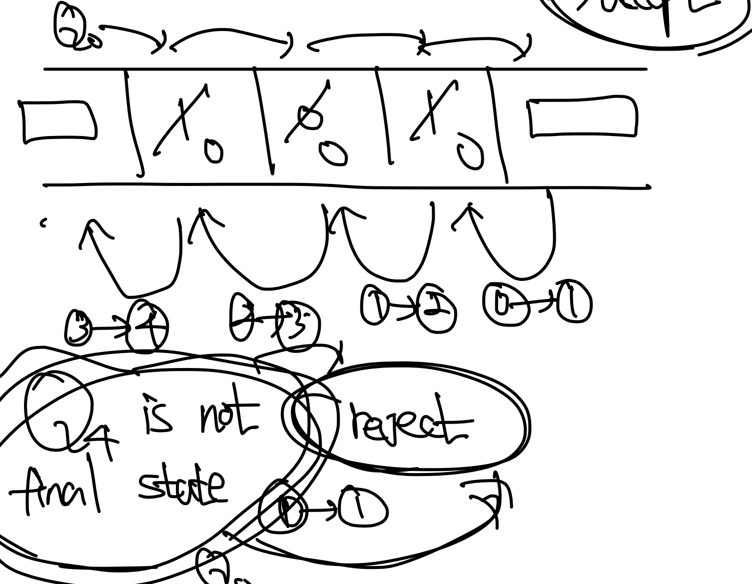
$\delta(Q_4, 0) = (Q_1, 0, L)$

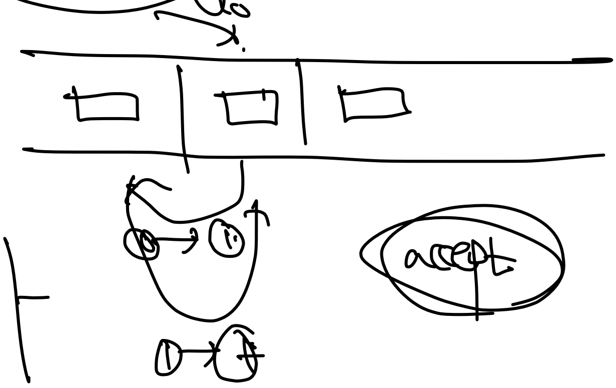
mod 4 계산.

$\delta(Q_1, \square) = (Q_f, \square, R)$

final state로 이동







$Q_0 1011 \vdash 0Q_0 011 \vdash$

$00Q_0 11 \vdash 000Q_0 1 \vdash 0000Q_0 \vdash$

$000Q_1 0 \vdash 00Q_2 00 \vdash 0Q_3 000 \vdash$

$Q_4 0000 \vdash Q_1 \square 0000 \vdash \square Q_f 0000$

accept