pushdown Automata.

① NPDA.

② pushdown automata, CFL

③ DPDA, DCFL.

④ Grammars for DCFL.

npda.

$Q \times (\Sigma \cup \{\lambda\}) \times \Gamma \to Q \times \Gamma^*$

$M = (Q, \Sigma, \Gamma, \delta, Q_0, Z, F)$

stack alphabet $Z \in \Gamma$ (start symbol)

$Q \times (\Sigma \cup \{\lambda\}) \times \Gamma \to Q \times \Gamma^*$

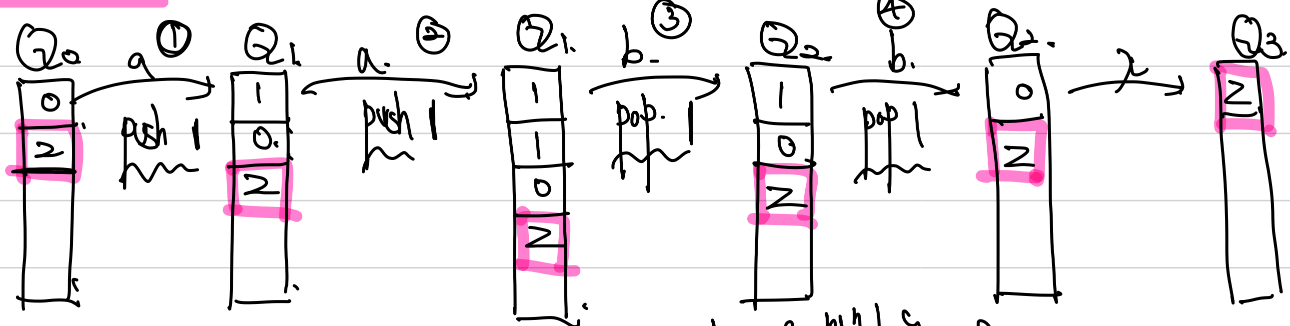
$Q = \{Q_0, Q_1, Q_2, Q_3\}$. $\Sigma = \{a, b\}$. $\Gamma = \{0, 1\}$. $Z = 0$. $F = \{Q_3\}$.

a도 accept

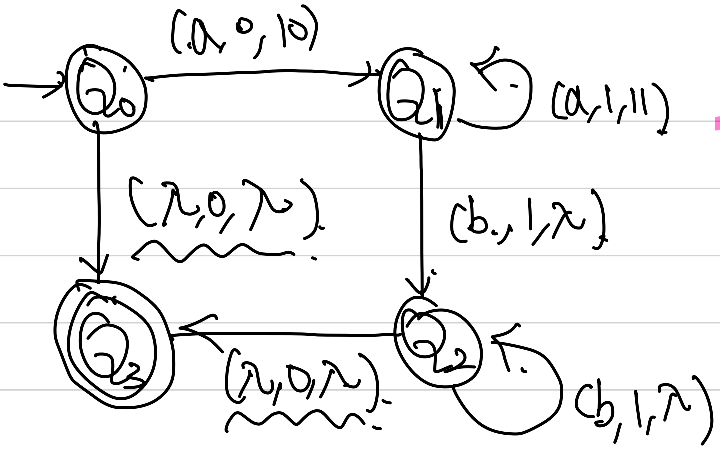
$\delta(Q_0, a, 0) = \{(Q_1, 10), (Q_3, \lambda)\}$ ① $\delta(Q_0, \lambda, 0) = \{(Q_3, \lambda)\}$

$\delta(Q_1, a, 1) = \{(Q_1, 11)\}$ ② $\delta(Q_1, b, 1) = \{(Q_2, \lambda)\}$ ③

$\delta(Q_2, b, 1) = \{(Q_2, \lambda)\}$ ④ $\delta(Q_2, \lambda, 0) = \{(Q_3, \lambda)\}$



$\therefore L = \{a^n b^n \mid n\} \cup \{a\}$.



transition graph.

(Q_i, aw, bx) : current configuration with state Q_i

input : aw, stack string : bx

$(Q_i, aw, bx) \vdash (Q_j, w, yx)$

$\rightarrow \delta(Q_i, a, b) \ni (Q_j, y)$

$(Q_i, w_1 w_2, x) \vdash^* (Q_j, w_2, y)$

$\rightarrow \delta(Q_i, w_1, x) \ni (Q_j, y)$

$L(M) = \{ w \in \Sigma^* \mid (Q_0, w, z) \vdash_M^* (p, \lambda, u), p \in F, \boxed{u \in \Gamma^*} \}$

stack에 놓인건 상관 x

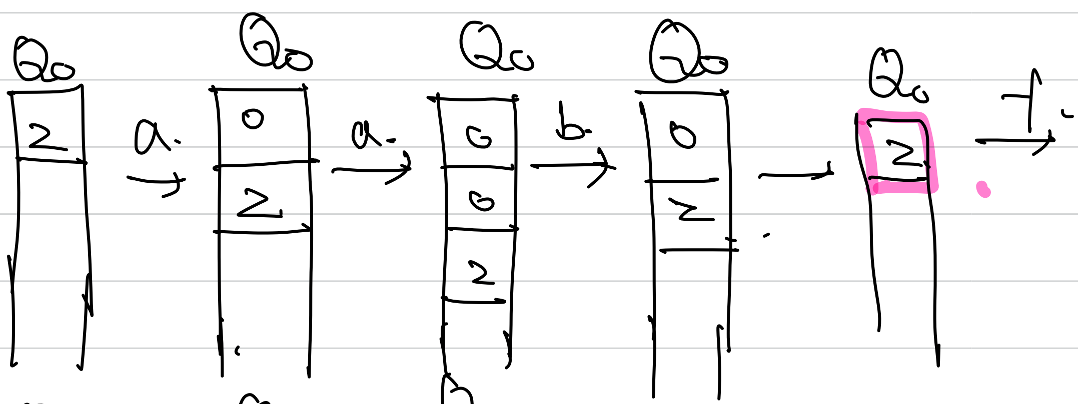
npda. for $L = \{ w \in \{a,b\}^* \mid n_a(w) = n_b(w) \}$.

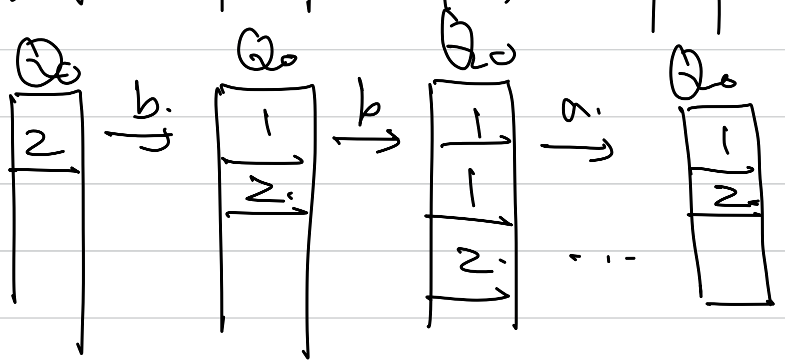
a가 나오면 0을 push. 와 1은 pop.

b가 나오면 0을 pop 1은 push.

aababb:

$M = (\{Q_0, Q_f\}, \{a,b\}, \{0,1,z\}, \delta, Q_0, z, \{Q_f\})$.





$\boxed{\delta(Q_0, a, z) = \{(Q_0, 0z)\}}$ $\delta(Q_0, b, 1) = \{(Q_0, 11)\}$

$\delta(Q_0, a, 0) = \{(Q_0, 00)\}$ $\delta(Q_0, b, 0) = \{(Q_0, \lambda)\}$

$\delta(Q_0, a, 1) = \{(Q_0, \lambda)\}$ $\boxed{\delta(Q_0, \lambda, z) = \{(Q_f, z)\}}$

Final로 가자.

$\boxed{\delta(Q_0, b, z) = \{(Q_1, 1z)\}}$

start 과정.

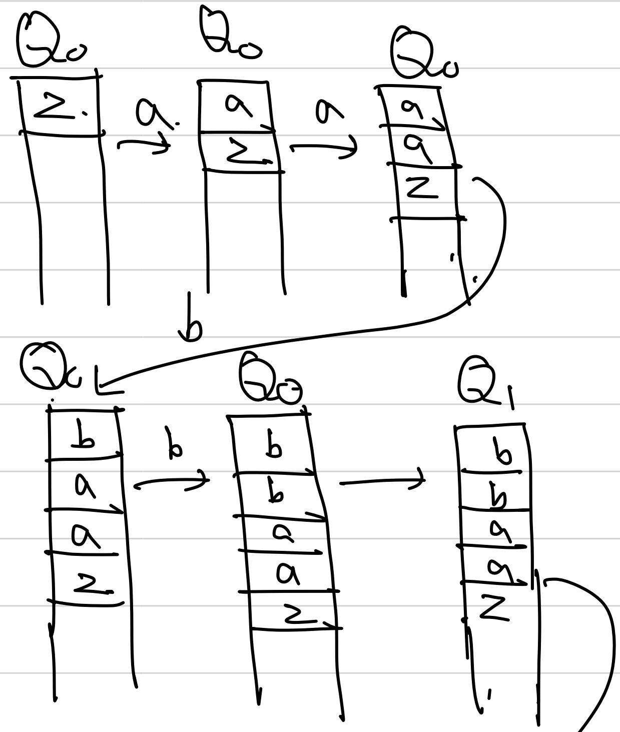
$w = baab$

$(Q_0, baab, z) \vdash (Q_0, aab, bz) \vdash (Q_0, ab, z) \vdash (Q_0, b, az)$

$\vdash (Q_0, \lambda, z) \vdash \boxed{(Q_f, \lambda, z)} \rightarrow$ accept [illegible].

$L = \{ww^R \mid w \in \{a, b\}^+\}$.

$Q_0 \rightarrow$ push w to the stack.



Push w on the stack

$\delta(Q_0, a, z) = \{(Q_0, az)\}$

$\delta(Q_0, a, a) = \{(Q_0, aa)\}$

$\delta(Q_0, a, b) = \{(Q_0, ab)\}$

$\delta(Q_0, b, b) = \{(Q_0, bb)\}$.

$\delta(Q_0, b, z) = \{(Q_0, bz)\}$.

$\delta(Q_0, b, a) = \{(Q_0, ba)\}$.

Match w^R.

$\delta(Q_1, a, a) = \{(Q_1, \lambda)\}$.

$\delta(Q_1, b, b) = \{(Q_1, \lambda)\}$

Guess middle of string

$\delta(Q_0, \lambda, a) = \{(Q_1, a)\}$.

$\delta(Q_1, \lambda, b) = \{(Q_1, b)\}$.

Go to final

$\delta(Q_1, \lambda, z) = \{(Q_f, z)\}$.

Every $\boxed{CFL} \rightarrow \boxed{npda}$ accepting it

CFL

CFG $\rightarrow$ GNF

$S \rightarrow aV \ (a \in T, v \in V^*)$

$S \rightarrow aSbb \mid a$

CFG

$S \rightarrow aSA \mid a$
$A \rightarrow bB$, $B \rightarrow b$

GNF

$[Q_1]$ for $S \rightarrow aSA$, $S \rightarrow a$

$\delta(Q_1, a, S) = \{(Q_1, SA), (Q_1, \lambda)\}$

⋮

$\boxed{\delta(Q_1, \lambda, z) = \{(Q_f z)\}}$ ...

"

$M = (\{Q_0, Q_1, Q_f\}, \{a, b\}, \{S, A, B, z\}, \delta, Q_0, z, \{Q_f\})$ "

$$a_1a_2\cdots a_nA_1A_2\cdots A_n \Rightarrow a_1a_2\cdots a_n bB_1B_2\cdots B_k A_2\cdots A_n$$

$$A_1 \to bB_1B_2\cdots B_k$$

M has $\boxed{(Q_1, B_1B_2\cdots B_k) \in \delta(Q_1, b, A_1)}$

$$(Q_0, w, Z) \vdash (Q_1, w, SZ) \vdash^* (Q_1, \lambda, Z) \vdash (Q_f, \lambda, Z)$$

"production 개수 $\Leftrightarrow$ transition 개수"

$S \to aA$ ①, $A \to aABC$ ② $|\, bB$ ③ $|\, a$ ④, $B \to b$ ⑤, $C \to c$ ⑥

$$M = (\{Q_0, Q_1, Q_f\}, \{a, b, c\}, \{A, B, C, Z\}, \delta, Q_0, Z, \{Q_f\})$$

- First & Last step: $\delta(Q_0, \lambda, Z) = \{(Q_1, SZ)\}$
  $\delta(Q_1, \lambda, Z) = \{(Q_f, Z)\}$

- Other transitions.

① $\delta(Q_1, a, S) = \{(Q_1, A)\}$

②, ④ $\delta(Q_1, a, A) = \{(Q_1, ABC), (Q_1, \lambda)\}$

③ $\delta(Q_1, b, A) = \{(Q_1, B)\}$

⑤ $\delta(Q_1, b, B) = \{(Q_1, \lambda)\}$

⑥ $\delta(Q_1, c, C) = \{(Q_1, \lambda)\}$

w=aaabc

$(Q_0, aaabc, z) \vdash (Q_1, aaabc, Sz) \vdash^{①} (Q_1, aabc, Az) \vdash^{②}$

$(Q_1, abc, ABCz) \vdash^{④} (Q_1, bc, BCz) \vdash^{⑤} (Q_1, c, Cz)$

$\vdash^{⑥} (Q_1, \lambda, z) \vdash (\boxed{Q_f}, \lambda, z)$

accept

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npda → accept 하는 Language 가 CFL → CFG가 존재 한다.

transition → production을 ~~유추해보자~~.

7/8 → final state Q_f is 하나.

→ stack이 empty 된 때만 accept 가능하다.

$\delta(Q_i, a, A) = \{ C_k \mid C_k = (Q_j, x) \text{ or } (Q_j, BC) .. \}$.

transition → production

NPDA → CFL

$$M = (Q, \Sigma, \Gamma, \delta, Q_0, z, \{Q_f\})$$

$$\delta(\boxed{Q_i}, a, A) = (\boxed{Q_j}, \lambda)$$

$[Q_i A Q_j] \to a$

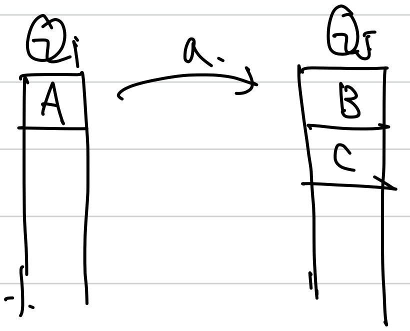
→ state가 변화 했다는 걸 명시 해 줘야 함.

"A"이 처리(변화) 되기 전 시작하는 상태

"A"가 완전히 끝나고 시작는 상태

$$\delta(Q_I, a, A) = (Q_J, BC)$$

$[Q_i A Q_k] \to a\,[Q_j B Q_l]\,[Q_l C Q_k]$



(Q_l, Q_k) : all posible values in Q.

start variable → $Q_0 z Q_f$

① $\delta(Q_0, a, z) = \{(Q_0, Az)\}$

② $\delta(Q_0, a, A) = \{(Q_0, A)\}$ 바꿀 필요가 ㅇ.

③ $\delta(Q_0, b, A) = \{(Q_1, \lambda)\}$

④ $\delta(Q_1, \lambda, z) = \{(Q_2, \lambda)\}$

②' $\delta(Q_0, a, A) = (Q_3, TA)$

②'' $\delta(Q_3, \lambda, T) = (Q_0, \lambda)$

$\boxed{(Q_0 A Q_1) \to b}$

$\boxed{(Q_1 z Q_2) \to \lambda}$

$\boxed{(Q_3 T Q_0) \to \lambda}$

$\boxed{(Q_0 z Q_k) \to a(Q_0 A Q_l)(Q_l z Q_k)}$ $(l, k = 0, 1, 2, 3)$

$2^4 \to 16$개

$\boxed{(Q_0 A Q_k) \to a(Q_3 T Q_l)(Q_l A Q_k)}$ (")

16개 too.

start variable $\boxed{(Q_0 z Q_2)}$

string $\boxed{aab}$

$(Q_0 z Q_2) \Rightarrow a(Q_0 A Q_1)(Q_1 z Q_2)$

$\Rightarrow a\,a\,\underbrace{(Q_3 T Q_0)}_{\lambda}\underbrace{(Q_0 A Q_1)}_{b}\underbrace{(Q_1 z Q_2)}_{\lambda}$

$\Rightarrow^{*}$ aab accept

D.PDA

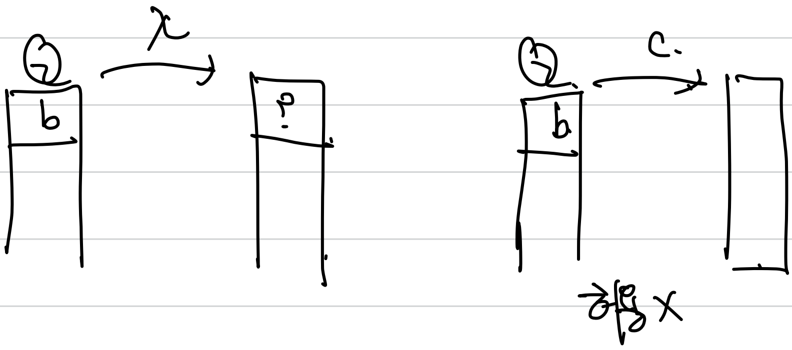
$M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$

★ " $\delta(q, a, b)$ contains $\boxed{\text{at most}}$ one element "

└ 에 주목 된다.

★ $\delta(q, \lambda, b) \neq \phi$ $\quad \delta(q, c, b) = \phi$

λ 읽는 것을 허용 한다



DPDA → DCFL 구별된다.

NPDA → CFL

☆ DPDA and NPDA are not equivalent ☆

NPDA ⇔ CFL

DPDA ⇔ DCFL

$\{a^n b^n\}$. $\{a^n b^{2n}\}$

Reg ⇔ NFA, DFA

$(ab)^*$

DFA · NFA

Reg Lang

$\{a^n b^n\} \cup \{a^n b^{2n}\}$.

$S \to S_1 \mid S_2$
$S_1 \to aS_1b \mid \lambda$
$S_2 \to aS_2b \mid \lambda$

CFL은 만들 수 있다

DCFL → "parsing이 효율적이게 될 수 있다"

DCFG 명확한 구문 X.

"LL(k) grammar"

총 k개 symbol을 보면 production이 결정됨.

$a^n b^n$.

$S \to aSb \mid ab$ "aabb"

$S \Rightarrow aSb \Rightarrow aabb$. LL(2)-grammar.

$\Rightarrow$ ~~ab~~

$S \to SS \mid aSb \mid ab$. "aabbab"

$S \Rightarrow$ 알 수 없다 → 끝까지 다 가봐야 몇 덩어리가 나오는지 나옴

$S \to aSbS \mid \lambda$ $S_0 \to aSbS$ aabbab.

$S_0 \Rightarrow aSbS \Rightarrow aaSbSbS \Rightarrow aabSbS$

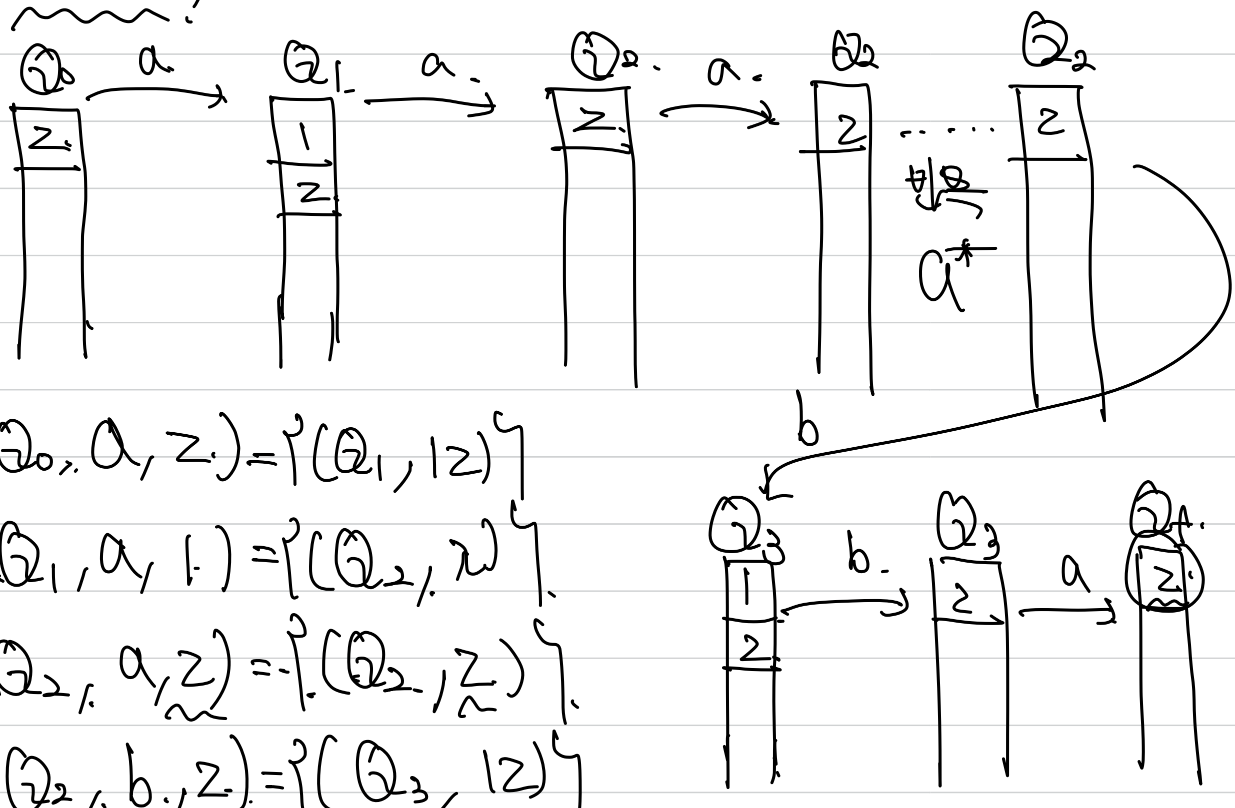
$\Rightarrow aabbS$

$\Rightarrow aabbaSbS$

$\Rightarrow aabbabS$

$\Rightarrow aabbab.$

$L(aaa^*bba)$ →무한개 a 2개



$\delta(q_0, a, Z) = \{(q_1, 1Z)\}$

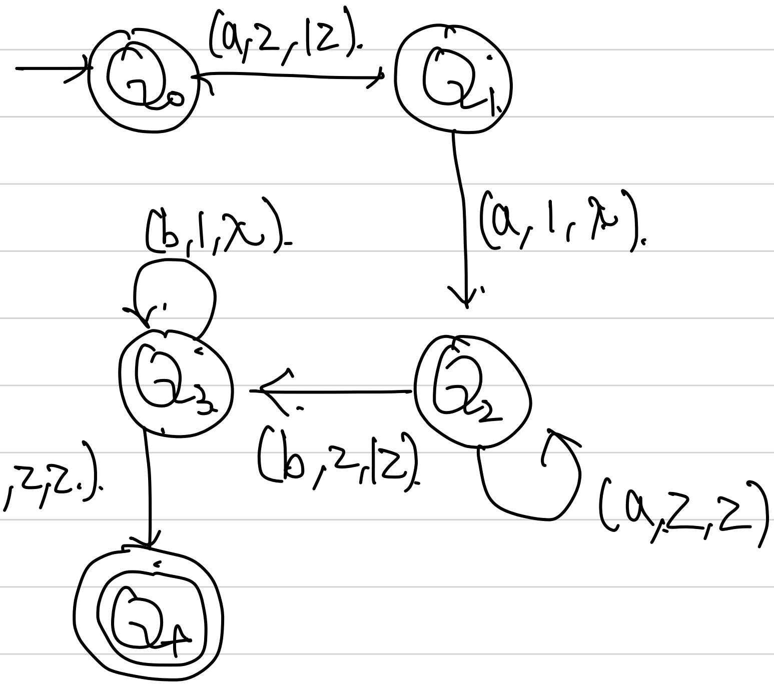
$\delta(q_1, a, 1) = \{(q_2, \lambda)\}$

$\delta(q_2, a, Z) = \{(q_2, Z)\}$

$\delta(q_2, b, Z) = \{(q_3, 1Z)\}$

$\delta(q_3, b, 1) = \{(q_3, \lambda)\}$

$\delta(q_3, a, Z) = \{(q_f, Z)\}$



npda. $S \to bSSS \mid ab$

Q_0

$\delta(Q_0, x, z) = \{(Q_1, Sz)\}$.

\# 3.

$\{a^n b^n \mid n \geq 1\} \cup \{b\}$.