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Linear Algebra Programming Project

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① ok..

■ There are two iterative methods to estimate an eigenvalue. One is the power method for estimating a strictly dominant eigenvalue. The other is the ② inverse power method for estimating an eigenvalue λ with roughly estimated eigenvalues. The iterative process of these two methods are described as below.

smallest eigen value를 구하기 못함

THE POWER METHOD FOR ESTIMATING A STRICTLY DOMINANT EIGENVALUE

1. Select an initial vector $\mathbf{x}_0$ whose largest entry is 1.
2. For $k = 0, 1, \dots,$
   a. Compute $A\mathbf{x}_k$.
   b. Let μ_k be an entry in $A\mathbf{x}_k$ whose absolute value is as large as possible.
   c. Compute $\mathbf{x}_{k+1} = (1/\mu_k)A\mathbf{x}_k$.
3. For almost all choices of $\mathbf{x}_0$, the sequence $\{\mu_k\}$ approaches the dominant eigenvalue, and the sequence $\{\mathbf{x}_k\}$ approaches a corresponding eigenvector.

THE INVERSE POWER METHOD FOR ESTIMATING AN EIGENVALUE λ OF A

1. Select an initial estimate α sufficiently close to λ.
2. Select an initial vector $\mathbf{x}_0$ whose largest entry is 1.
3. For $k = 0, 1, \dots,$
   a. Solve $(A - \alpha I)\mathbf{y}_k = \mathbf{x}_k$ for $\mathbf{y}_k$.
   b. Let μ_k be an entry in $\mathbf{y}_k$ whose absolute value is as large as possible.
   c. Compute $\nu_k = \alpha + (1/\mu_k)$.
   d. Compute $\mathbf{x}_{k+1} = (1/\mu_k)\mathbf{y}_k$.
4. For almost all choices of $\mathbf{x}_0$, the sequence $\{\nu_k\}$ approaches the eigenvalue λ of A, and the sequence $\{\mathbf{x}_k\}$ approaches a corresponding eigenvector.

$$\left|\begin{bmatrix} 6-\lambda & 5 \\ 1 & 2-\lambda \end{bmatrix}\right|$$
$$= 12 - 8\lambda + \lambda^2 - 5$$
$$= \lambda^2 - 8\lambda + 7$$
$$(\lambda - 1)(\lambda - 7)$$

strictly dominant eigenvalue is. 7

1. (15pt) Estimate a strictly dominant eigenvalue of a matrix A with initial vector $\mathbf{x}_0$ described as following.

$$A = \begin{bmatrix} 6 & 5 \\ 1 & 2 \end{bmatrix}, \mathbf{x}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Q1.1. What kind of method do you choose? Explain the reason of your selection.

power method → strictly dominant eigen. value를 구하는 것이기 때문이다.

Q1.2. Write a code to estimate a strictly dominant eigenvalue of A with initial vector $\mathbf{x}_0$.

Q1.3. Fill in the blanks in the table below.

Iteration	1	2	3	4	5
$\mathbf{x}_k$	$\begin{bmatrix} 1 \\ 0.4 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0.225 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0.20850877 \end{bmatrix}$	$\begin{bmatrix} 0.99999998 \\ 0.2005 \end{bmatrix}$	$\begin{bmatrix} 0.99999998 \\ 0.2000714 \end{bmatrix}$
$A\mathbf{x}_k$	$\begin{bmatrix} 8 \\ 1.8 \end{bmatrix}$	$\begin{bmatrix} 7.125 \\ 1.45 \end{bmatrix}$	$\begin{bmatrix} 7.01754385 \\ 1.40701754 \end{bmatrix}$	$\begin{bmatrix} 7.00249986 \\ 1.40099997 \end{bmatrix}$	$\begin{bmatrix} 7.00035688 \\ 1.40014278 \end{bmatrix}$
μ_k	8.0.	7.125	7.017544	7.0025	7.000357 ≒ 7

실제 strictly eigenvalue에 수렴

2. (15pt) Estimating an eigenvalue λ, which are the two smallest eigenvalues of A with initial vector $\mathbf{x}_0$ described as following. → 실제 eigen value. 2, 3.3212201246

$$A = \begin{bmatrix} 10 & -8 & -4 \\ -8 & 13 & 4 \\ -4 & 5 & 4 \end{bmatrix}, \mathbf{x}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ with roughly estimated eigenvalues (21, 3.3, and 1.9)

수렴하는지 확인.

Q2.1. What kind of method do you choose? Explain the reason of your selection.

Inverse power method → good. estimated. α of eigenvalue λ is exists. and. Inverse power method provides an approximation for any eigenvalues.

Q2.2. Write a code to estimate the two smallest eigenvalues of A with initial vector $\mathbf{x}_0$.

Q2.3. Draw two tables to estimate the two smallest eigenvalues of A with initial vector $\mathbf{x}_0$.

α = 1.9

Iteration	0	1	2	3	4
$\mathbf{x}_k$	$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0.5139 \\ 0.064649 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0.505364 \\ 0.004453 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 5.003983 \\ 3.128077 \\ 1.000000 \end{bmatrix}$	$\begin{bmatrix} 5.000266 \\ 2.200719 \\ 1.000000 \end{bmatrix}$
$\mathbf{y}_k$	$\begin{bmatrix} 4.450374 \\ 0.50160 \\ 7.758805 \end{bmatrix}$	$\begin{bmatrix} 5.013058 \\ 0.044172 \\ 9.9197 \end{bmatrix}$	$\begin{bmatrix} 5.001247 \\ 3.126492 \\ 9.994931 \end{bmatrix}$	$\begin{bmatrix} 5.000089 \\ 2.20064 \\ 9.999646 \end{bmatrix}$	$\begin{bmatrix} 5.000006 \\ 1.548454 \\ 9.999975 \end{bmatrix}$
μ_k	7.758805	9.9197	9.994931	9.999646	9.999975
v_k	2.028888583	2.0008095	2.00005072	2.00000354	2.00000025 ≒ 2.

α = 3.3.

Iteration	0	1	2	3	4
X_k	$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0.793369. \\ 0.082172 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0.786921 \\ 0.095893 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0.787014 \\ 0.095667 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0.787012 \\ 0.095671 \end{bmatrix}$
Y_k	$\begin{bmatrix} 41.045365 \\ 32.564103 \\ -3.372781 \end{bmatrix}$	$\begin{bmatrix} 47.486662. \\ 37.368252. \\ 4.553654 \end{bmatrix}$	$\begin{bmatrix} 47.119503 \\ 37.083691 \\ 4.507785 \end{bmatrix}$	$\begin{bmatrix} 47.125168 \\ 37.088078 \\ 4.508501 \end{bmatrix}$	$\begin{bmatrix} 47.125075 \\ 37.088006 \\ 4.508489 \end{bmatrix}$
μ_k	41.045365	47.486662	47.119503	47.125168	47.125075
v_k	3.324363329	3.3210585	3.32122263	3.32122008	3.32122013 ≒ 3.3212201246