useless production

$S \to aSb \mid \lambda \mid A \qquad A \to aA$

↳ $S \Rightarrow A \Rightarrow aA \Rightarrow aaA$ (useless variables 들이 존재)

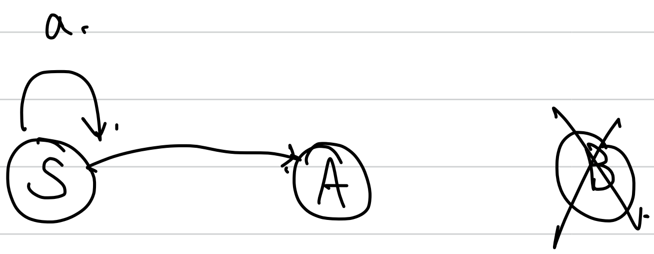
$S \to A, \quad A \to aA \mid \lambda, \quad B \to bA$

S에서 B로 갈 수 x → useless

ex:)

$S \to aS \mid A \mid C, \quad A \to a, \quad B \to aa, \quad C \to aCb$

$G_1 = (\{S, A, B\}, \{a, b\}, S, P_1) \qquad P_1 : S \to aS \mid A, \quad A \to a, \quad \cancel{B \to aa}$



$G' = (\{S, A\}, \{a, b\}, S, P')$

$P' = S \to aS \mid A, \quad A \to a$

Find $G_1 = (V_1, T_2, S, P_1)$ having only variables A for

which $A \overset{*}{\Rightarrow} w \in T^*$

(a) set $V_1 = \{\ \}$

(b) V_1에 추가 안될 때까지 $A \to x_1 x_2 \cdots x_n$ $(x_i \in V_1 \cup T)$,

A를 V_1에 추가 한다

$S \to aS \mid A \mid C$, $A \to a$, $B \to aa$, $C \to aCb$

$\{A, B, S\}$

λ production

$A \to \lambda$

$S \to \lambda$
$S \to A$) $S \to A \Rightarrow \lambda$
nullable.

$S \to aS_1b$, $S_1 \to aS_1b \mid \lambda$

" $L(G) = \{a^n b^n \mid n > 0\}$. "

$S \Rightarrow aS_1b \Rightarrow aaS_1bb$
$\Rightarrow aabb$

$$\boxed{S \to aS_1b \mid ab}$$

→ substitude rull 을 적용 시켜 준다.

⇓.

CFG with $\lambda \in L(G)$

(Then → exist an equivalent grammar G'
without λ-productions.).

V_n. of all nullable variables of G

$A \to \lambda$, put A into $V_n - \{A\}$

Repeat: until no more variables are added to V_n

For. all $B \to A_1A_2 \ldots A_n$ with all $A_i \in V_n$. put B into V_n

$S \to ABaC$, $A \to BC$, $B \to b \mid \lambda$, $C \to D \mid \lambda$, $D \to d$

$V_N = \{B, C, A\}$

find P'

① $S \to ABaC \mid a \mid BaC \mid AaC \mid ABa \mid aC \mid Aa \mid Ba$

② $A \to BC \mid B \mid C \mid \lambda$ → delete

③ $B \to b$

④ $C \to D$

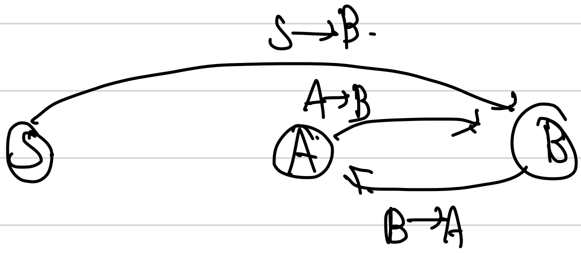
⑤ $D \to d$

a CFG without λ-productions. $\quad \underbrace{A \to B}_{\text{unit-production}}$ $(A, B \in V)$

→ there exist an equivalent grammar G' without unit-productions

$S \to Aa \mid B$, $B \to A \mid bb$, $A \to a \mid bc \mid B$.

without λ-production 이라 가정



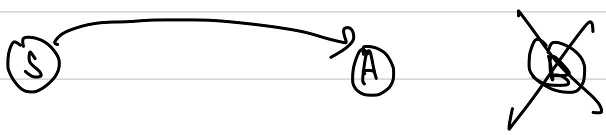
put non-unit product into P'

$S \to Aa$, $B \to bb$, $A \to a \mid bc$.

$S \Rightarrow B$ $\quad$ $S \to bb$ $\quad$ $A \Rightarrow B$ $\quad$ $A \to bb$

$B \Rightarrow A$ $\quad$ $B \to a \mid bc$ $\quad$ $S \overset{*}{\Rightarrow} A$ $\quad$ $S \to a \mid bc$

$S \to Aa \mid bb \mid a \mid bc$, $B \to \cancel{bb \mid a \mid bc}$, $A \to a \mid bc \mid bb$.



λ productoin

↓

unit production

↓

useless productions

Chomsky Normal form.

$A \to BC$, or. $A \to a$ $(A, B, C \in V,\ a \in T)$

$S \to AS \mid a$. , $A \to SA \mid b$ → CNF

$S \to AS \mid AAS$, $A \to SA \mid aa$ → CNF ×

$S \to ABa$, $A \to aab$, $B \to Ac$

중간 정리

step 1)

$$\left(\begin{array}{l} ① \ S \to ABV^a, \ V^a \to a \\ ② \ A \to V^a V^a V^b, \ V^b \to b \\ ③ \ B \to AV^c, \ V^c \to c \end{array} \right)$$

$B \to AV^c$, $V^a \to a$, $V^b \to b$, $V^c \to c$

step 2)

$S \to AD_1$, $D_1 \to BV^a$

$A \to V^a D_2$, $D_2 \to V^a V^b$

Greibach Normal Form

CFG with. → $A \to ax$. (x in V^*, a in T)

모든 CFG → Greibach Normal Form

✗→ S-grammar.

$S \to AB$, $A \to aA \mid bB \mid b$, $B \to b$

$S \to aAB \mid bBB \mid bB$

$w_{1n} \in L(G) \Leftrightarrow S \overset{*}{\Rightarrow} w_{1n}$ ($V_{ij} = \{A \in V \mid A \overset{*}{\Rightarrow} w_{ij}\}$)

$V_{11} = \{A \mid A \to a_i\}$ $V_{11} \cup V_{22}$

$V_{12} = \{A \mid A \overset{*}{\Rightarrow} w_{12} = a_1 a_2\}$

$A \Downarrow$

$BC \overset{*}{\Rightarrow} a_1 a_2$

$V_{13} = \{A \mid A \overset{*}{\Rightarrow} w_{13} = a_1 a_2 a_3\}$

$A \Downarrow BC$, $B \Uparrow V_{11}$, $C \Uparrow V_{23}$

$= \{A \mid A \to BC, B \in V_{11}, C \in V_{23}\} \cup \{A \mid A \to BC, B \in V_{12}, C \in V_{33}\}$

$V_{ij} = \bigcup_{k \in \{i, i+1, \cdots, j-1\}} \{A \mid A \to BC,\ B \in V_{ik},\ C \in V_{k+1\,j}\}$

$w = aabbb$

$S \to AB$, $A \to BB \mid a$, $B \to AB \mid b$

$V_{11} = \boxed{\{A\}}$ $V_{22} = \boxed{\{A\}}$ $V_{33} = \boxed{\{B\}}$ $V_{44} = \boxed{\{B\}}$ $V_{55} = \boxed{\{B\}}$

$\to AA$

$V_{12} = V_{11} V_{12} = \boxed{\emptyset}$ $\to AB$ $V_{23} = V_{22} V_{33} = \boxed{\{S, B\}}$ $V_{34} = V_{33} V_{44} = \boxed{\{A\}}$

$V_{45} = \boxed{\{A\}}$

$\to AS$
$\to AB$

$V_{13} = \underset{\{A\}\,\{S,B\}}{V_{11} V_{23}} \cup \underset{\emptyset\ \ \{B\}}{V_{12} V_{33}} = \boxed{\{S, B\}}$ $V_{24} = \underset{\{A\}\ \ \{A\}}{V_{22} V_{34}} \cup \underset{\{S,B\}\ \{B\}}{V_{23} V_{44}} = \boxed{\{A\}}$ $\to SB, BB$

$V_{35} = \boxed{\{S, B\}}$

$V_{14} = \boxed{\{A\}}$ $V_{25} = \boxed{\{S, B\}}$

$V_{15} = V_{11} V_{25} \cup \cdots \to \{S, B\} \cdots \in L(G)$

S로부터
derivation이
가능함

#(1)

$S \to aA \mid aBB$, $A \to aaA \mid \lambda$, $B \to bB \mid bbC$, $C \to B$.

(1)

① Remove all λ-productions.

$V_N = \{A\}$.

$S \to aA \mid aBB \mid a$, $A \to aa \mid aaA$, $B \to bB \mid bbC$, $C \to B$

② Remove unit-productions

$C \Rightarrow B$.

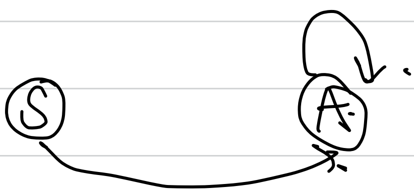
$C \to bB \mid bbC$

$S \to aA \mid aBB \mid a$, $A \to aa \mid aaA$, $B \to bB \mid bbC$, $C \to bB \mid bbC$.

③ Remove (useless -productions).

$V_1 = \{A, S\}$.

$\boxed{S \to aA \mid a ,\ A \to aa \mid aaA}$



(2)

change to Chomsky Normal Form

$$S \to aA \mid a, \quad A \to aa \mid aaA.$$

step 1)

$$S \to V^a A \mid a, \quad A \to V^a V^a \mid V^a V^a A, \quad V^a \to a.$$

step 2)

$$P' \quad S \to V^a A \mid a, \quad A \to V^a V^a \mid V^a D_1, \quad V^a \to a, \quad D_1 \to V^a A.$$

#3.

$V_{11} = \{S, V^a\}$

$V_{22} = \{S, V^a\}$

$V_{12} = V_{11} V_{22}$

$S\ V^a$

$S\ V^a$

$\cancel{S\ V^a}$

$\cancel{V^a\ S}$ $\rightarrow$

$\cancel{S\ S}$

$V^a\ V^a = \{A\}$

$S\ V^a$

A

$S\ \ A$

$V^a\ \ A$

$\{S, D_1\}$

A

$S\ V^a$

$\cancel{A\ S}$

$\cancel{A\ V^a}$

$S\ V^a$

$S\ D_1$

$\cancel{S\ S}$

$\cancel{S\ D_1}$

$\cancel{V^a\ S}$

$V^a\ D_1$

$\cancel{A}$

A

$\{A\}$

$S\ D_1$

$S\ V^a$

$\cancel{S\ V^a}$

$\cancel{S\ S}$

$\cancel{D_1\ V^a}$

$\cancel{D_1\ S}$

S r^{a}.

A.

SA

$r^{a}A$.