# HW7

2025-1 고체역학(박성훈 교수님)

Concept Aplication 6.2, Problem 6.18, 6.19

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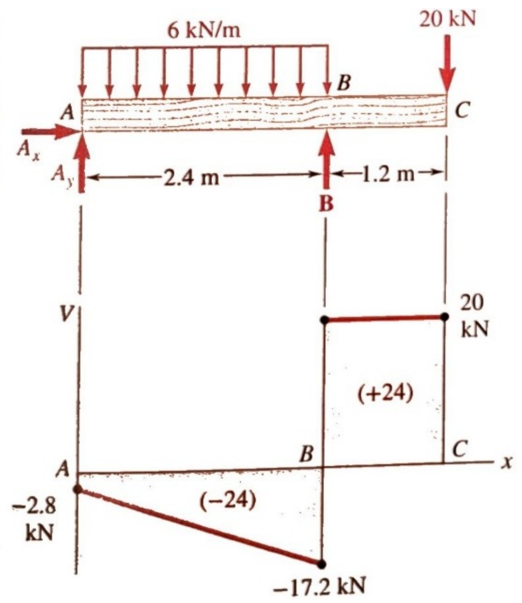
## Concept Aplication 6.2




Knowing that the allowable shearing stress for the timber beam Sample Prob. 5.7 is $\tau_{\text{all}} = 1.75\,\text{MPa}$, check that the design is acceptable from the point of view of the shearing stresses.

Recall from the shear diagram of Sample Prob. 5.7 that $V_{\text{max}} = 20\,\text{kN}$. The actual width of the beam was given $b = 90\,\text{mm}$, and the value obtained for its depth was $h = 366\,\text{mm}$.

$$\tau_{\text{max}} = \frac{V_{\text{max}} Q_{\text{half}}}{bI} = \frac{V_{\text{max}} \cdot \frac{h}{4} \cdot \frac{bh}{2}}{b\left(\frac{1}{12}bh^3\right)} = \frac{3V_{\text{max}}}{2bh} = \frac{3(20\,\text{kN})}{2(90\,\text{mm})(366\,\text{mm})} = 0.911\,\text{MPa}$$

$$\tau_{\text{max}} < \tau_{\text{all}} \quad \Rightarrow \quad \text{Acceptable} \quad \blacktriangleleft$$

## Problem 6.18

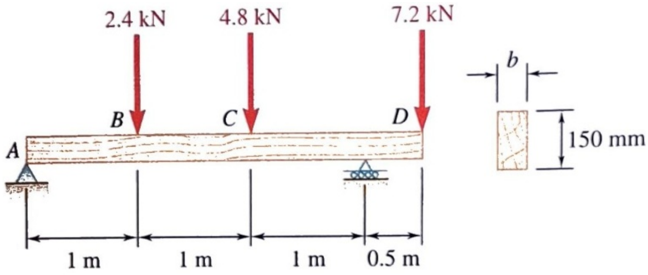


Fig. P6.18

For the beam and loading shown, determine the minimum required width $b$, knowing that for the grade of timber used, $\sigma_{\text{all}} = 12\,\text{MPa}$ and $\tau_{\text{all}} = 825\,\text{kPa}$.

$$+\circlearrowleft \sum M|_A = -(2.4\,\text{kN})(1\,\text{m}) - (4.8\,\text{kN})(2\,\text{m}) + R_D(3\,\text{m}) - (7.2\,\text{kN})(3.5\,\text{m}) = 0$$

$$\Rightarrow \quad R_D = 12.4\,\text{kN}$$

$$+\uparrow \sum F_y = R_A + R_D - 2.4\,\text{kN} - 4.8\,\text{kN} - 7.2\,\text{kN} = 0$$

$$\Rightarrow \quad R_A = 2\,\text{kN}$$

$$V(x) = \begin{cases} 2\,\text{kN}, & 0\,\text{m} \le x < 1\,\text{m} \\ -0.4\,\text{kN}, & 1\,\text{m} \le x < 2\,\text{m} \\ -5.2\,\text{kN}, & 2\,\text{m} \le x < 3\,\text{m} \\ 7.2\,\text{kN}, & 3\,\text{m} \le x < 3.5\,\text{m} \end{cases} \quad \Rightarrow \quad |V|_{\text{max}} = 7.2\,\text{kN}$$

$$M(1\,\text{m}) = (2\,\text{kN})(1\,\text{m}) = 2\,\text{kN}\cdot\text{m}$$

$$M(3\,\text{m}) = -(7.2\,\text{kN})(0.5\,\text{m}) = -3.6\,\text{kN}\cdot\text{m}$$

$$\Rightarrow \quad |M|_{\text{max}} = 3.6\,\text{kN}\cdot\text{m}$$

$$\sigma_{\text{max}} = \frac{|M|_{\text{max}}c}{I} = \frac{3|M|_{\text{max}}}{2bc^2} \le \sigma_{\text{all}}$$

$$b \ge \frac{3|M|_{\text{max}}}{2\sigma_{\text{all}}c^2} = \frac{3(3.6\,\text{kN}\cdot\text{m})}{2(12\,\text{MPa})(75\,\text{mm})^2} = \frac{3(3.6)}{2(12)(75)^2} \times 10^6\,\text{mm} = 80.0\,\text{mm}$$

$$\tau_{\text{max,avg}} = \frac{|V|_{\text{max}}Q}{tI} = \frac{3|V|_{\text{max}}}{2bh} \le \tau_{\text{all}}$$

$$b \ge \frac{|V|_{\text{max}}Q}{tI} = \frac{3|V|_{\text{max}}}{2\tau_{\text{all}}h} = \frac{3(7.2\,\text{kN})}{2(0.825\,\text{MPa})(150\,\text{mm})} = \frac{3(7.2)}{2(0.825)(150)} \times 10^3\,\text{mm} = 87.3\,\text{mm}$$

$$\Rightarrow \quad b_{\text{min}} = 87.3\,\text{mm}, \quad \text{Use} \quad b = 88\,\text{mm} \quad \blacktriangleleft$$

## Problem 6.19

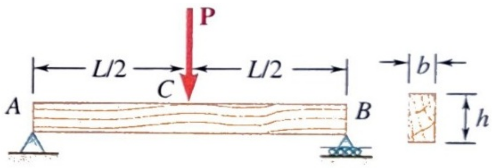


Fig. P6.19

A timber beam $AB$ of length $L$ and rectangular cross section carries a single concentrated load $P$ at its midpoint $C$. ($a$) Show that the ratio $\tau_m/\sigma_m$ of the maximum values of the shearing and normal stresses in the beam is equal to $h/2L$, where $h$ and $L$ are, respectively, the depth and the length of the beam. ($b$) Determine the depth $h$ and the width $b$ of the beam, knowing that $L = 2\,\text{m}$, $P = 40\,\text{kN}$, $\tau_m = 960\,\text{kPa}$, and $\sigma_m = 12\,\text{MPa}$.

$$R_A = R_B = \frac{P}{2}$$

$$V(x) = \begin{cases} \dfrac{P}{2} & \left(0 \le x < \dfrac{L}{2}\right) \\ -\dfrac{P}{2} & \left(\dfrac{L}{2} \le x < L\right) \end{cases} \quad \Rightarrow \quad |V|_{\max} = V_m = \frac{P}{2}, \quad |M|_{\max} = M(L/2) = M_m = \frac{PL}{4}$$

$$\tau_m = \frac{3V_m}{2bh} = \frac{3P}{8bc}, \quad \sigma_m = \frac{M_m c}{I} = \frac{3M_m}{2bc^2} = \frac{3PL}{8bc^2}$$

$$\frac{\tau_m}{\sigma_m} = \frac{c}{L} = \frac{h}{2L} \quad \blacktriangleleft \quad (a)$$

$$\left.\begin{aligned} h &= \frac{\tau_m}{\sigma_m} \cdot 2L = \frac{0.96\,\text{MPa}}{12\,\text{MPa}} \cdot 2(2\,\text{m}) = 0.32\,\text{m} = 320\,\text{mm} \\ b &= \frac{3P}{4h\tau_m} = \frac{3(40\,\text{kN})}{4(320\,\text{mm})(0.96\,\text{MPa})} = \frac{3(40)}{4(320)(0.96)} \times 10^3\,\text{mm} = 97.7\,\text{mm} \end{aligned}\right\} \quad \blacktriangleleft \quad (b)$$