HW5

2025-2 구조역학(박성훈 교수님)

Problem 10.12, 10.14, 10.27, 10.31, 10.34, 10.39, 10.40

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Problem 10.12

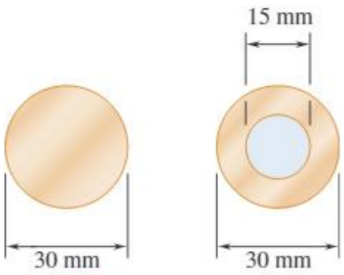

Fig. P10.12

A compression member of 1.5-m effective length consists of a solid 30-mm-diameter brass rod. To reduce the weight of the member by 25%, the solid rod is replaced by a hollow rod of the cross section shown. Determine (*a*) the percent reduction in the critical load, (*b*) the value of the critical load for the hollow rod. Use $E = 200\,\text{GPa}$.

let $\quad r = 15\,\text{mm} = 0.015\,\text{m}$

$$I_{\text{solid}} = \frac{\pi}{4}r^4, \quad I_{\text{hollow}} = \frac{\pi}{4}\left\{r^4 - \left(\frac{r}{2}\right)^4\right\} = \frac{15\pi}{64}r^4$$

$$P_{\text{cr,solid}} = \frac{\pi^2 E I_{\text{solid}}}{L_e^2}, \quad P_{\text{cr,hollow}} = \frac{\pi^2 E I_{\text{hollow}}}{L_e^2}$$

$$\frac{P_{\text{cr,hollow}}}{P_{\text{cr,solid}}} = \frac{I_{\text{hollow}}}{I_{\text{solid}}} = \frac{\frac{15}{64}}{\frac{1}{4}} = \frac{15}{16}, \quad (\text{percent reduction in } P_{\text{cr}}) = \frac{1}{16} = 6.25\% \quad \blacktriangleleft \quad (a)$$

$$P_{\text{cr,hollow}} = \frac{15}{64} \cdot \frac{\pi^3 E r^4}{L_e^2} = \frac{15}{64} \cdot \frac{\pi^3 \cdot 200 \times 10^9 \cdot 0.015^4}{1.5^2}\,\text{N} = 32.7\,\text{kN} \quad \blacktriangleleft \quad (b)$$

Problem 10.14

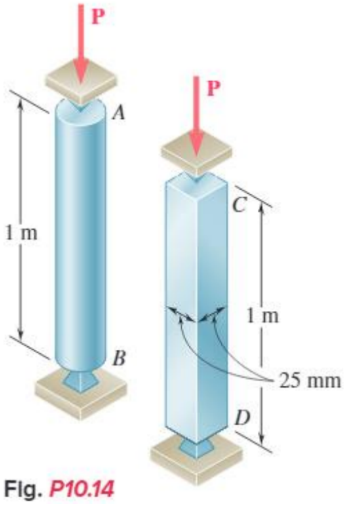

Fig. P10.14

Determine (*a*) the critical load for the square strut, (*b*) the radius of the round strut for which both struts have the same critical load. (*c*) Express the cross-sectional area of the square strut as a percentage of the cross-sectional area of the round strut. Use $E = 200\,\text{GPa}$.

$$\text{let} \quad a = 25\,\text{mm} = 0.025\,\text{m}$$

$$I_{\text{square}} = \frac{a^4}{12} = \frac{10^{-4}}{3072}\,\text{m}^4$$

$$P_{\text{cr}} = \frac{\pi^2 E I_{\text{square}}}{L_e^2} = \frac{\pi^2 \cdot 200 \times 10^9 \cdot \frac{10^{-4}}{3072}}{1^2} = 64.3\,\text{kN} \quad \blacktriangleleft \quad (a)$$

$$P_{\text{cr}} = \frac{\pi^2 E I_{\text{square}}}{L_e^2} = \frac{\pi^2 E I_{\text{circle}}}{L_e^2} \quad \Rightarrow \quad I_{\text{square}} = I_{\text{circle}} \quad \Rightarrow \quad \frac{a^4}{12} = \frac{\pi r^4}{4}$$

$$\Rightarrow \quad r = \frac{a}{\sqrt[4]{3\pi}} = 14.27\,\text{mm} \quad \blacktriangleleft \quad (b)$$

$$\frac{A_{\text{square}}}{A_{\text{circle}}} = \frac{a^2}{\pi r^2} = 97.7\% \quad \blacktriangleleft \quad (c)$$

Problem 10.27

Each of the five struts shown consists of a solid steel rod. (a) Knowing that the strut of Fig.(1) is of a 20-mm diameter, determine the factor of safety with respect to buckling for the loading shown. (b) Determine the diameter of each of the other struts for which the factor of safety is the same as the factor of safety obtained in part a. Use $E = 200\,\text{GPa}$.

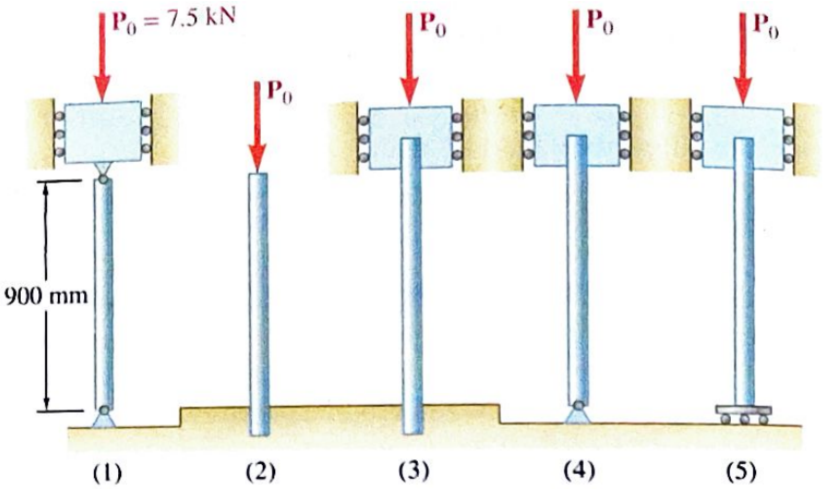

Fig. P10.27

$$\text{let}\quad d = 20\,\text{mm} = 0.02\,\text{m},\qquad L_e = L = 900\,\text{mm} = 0.9\,\text{m},\qquad I = \frac{\pi}{64}d^4$$

In case (1),

$$(F.S.) = \frac{P_{\text{cr}}}{P_0} = \frac{\pi^2 EI}{P_0 L_e^2} = \frac{\pi^3 E d^4}{64 P_0 L^2} = \frac{\pi^3 \cdot 200\times 10^9 \cdot 0.02^4}{64 \cdot 7500 \cdot 0.9^2} = 2.55 \quad \blacktriangleleft \quad (a)$$

In other cases, when $L_e = kL$,

$$(F.S.) = \frac{\pi^3 E d^4}{64 P_0 L^2} = \frac{\pi^3 E d'^4}{64 P_0 k^2 L^2} \quad\Rightarrow\quad d^4 = \frac{d'^4}{k^2} \quad\Rightarrow\quad d' = d\sqrt{k}$$

$$\left.\begin{array}{ll} \text{In case (2),} & k = 2, \quad d_2 = 20\sqrt{2}\,\text{mm} = 28.3\,\text{mm} \\ \text{(3),} & k = 0.5, \quad d_3 = 20\sqrt{0.5}\,\text{mm} = 14.14\,\text{mm} \\ \text{(4),} & k = 0.7, \quad d_4 = 20\sqrt{0.7}\,\text{mm} = 16.73\,\text{mm} \\ \text{(5),} & k = 0.5\times 2 = 1, \quad d_5 = d = 20.0\,\text{mm} \end{array}\right\} \quad \blacktriangleleft \quad (b)$$

Problem 10.31

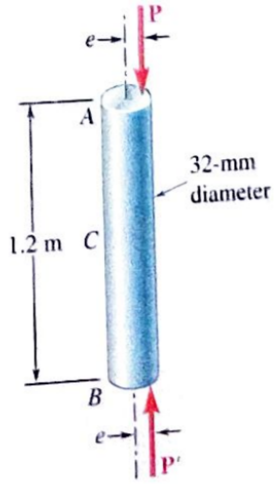

Fig. P10.31

An axial load **P** is applied to the 32-mm-diameter steel rod AB as shown. For $P = 37\,\text{kN}$ and $e = 1.2\,\text{mm}$, determine (a) the deflection at the midpoint C of the rod, (b) the maximum stress in the rod. Use $E = 200\,\text{GPa}$.

$$\text{let} \quad H = \sec\left(\frac{L}{2}\sqrt{\frac{P}{EI}}\right) = \left[\cos\left(\frac{1.2}{2}\sqrt{\frac{37\times10^3}{200\times10^9\cdot\frac{\pi}{64}\cdot 0.032^4}}\right)\right]^{-1} = 2.381730011$$

$$y_{\max} = e(H-1) = 1.2\,\text{mm}(2.381730011-1) = 1.658\,\text{mm} \quad \blacktriangleleft \quad (a)$$

$$r^2 = \frac{I}{A} = \frac{\frac{\pi}{64}d^4}{\frac{\pi}{4}d^2} = \frac{d^2}{16} = 64\,\text{mm}^2$$

$$\frac{ec}{r^2} = \frac{1.2\cdot 16}{64} = 0.3$$

$$\sigma_{\max} = \frac{P}{A}\left(1+\frac{ec}{r^2}\cdot H\right) = \frac{37\times10^3}{\pi\cdot 0.016^2}\,(1+0.3\cdot 2.381730011)\,\text{Pa} = 78.9\,\text{MPa} \quad \blacktriangleleft \quad (b)$$

Problem 10.34

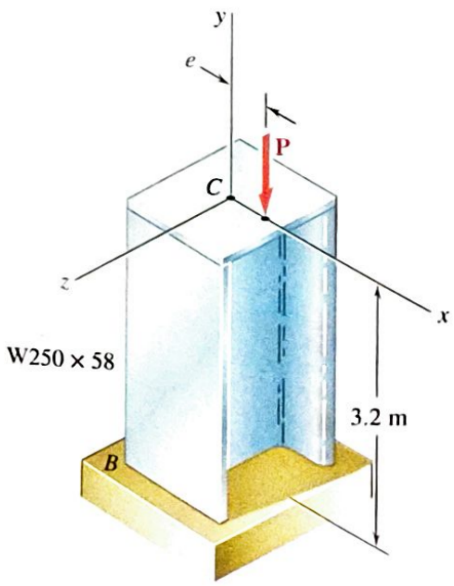

Fig. P10.34

The axial load **P** is applied at a point located on the x axis at a distance e from the geometric axis of the rolled-steel column BC. When $P = 350\,\text{kN}$, the horizontal deflection of the top of the column is 5 mm. Using $E = 200\,\text{GPa}$, determine (a) the eccentricity e of the load, (b) the maximum stress in the column.

$$I = I_Y = 18.7 \times 10^6\,\text{mm}^4, \quad A = 7420\,\text{mm}^2, \quad c = \frac{b_f}{2} = 101.5\,\text{mm} \quad \cdots \quad \text{from appendix E}$$

$$L_e = 2L = 6.4\,\text{m}$$

$$\text{let} \quad H = \sec\left(\frac{L_e}{2}\sqrt{\frac{P}{EI}}\right) = \left[\cos\left(3.2\sqrt{\frac{350 \times 10^3}{200 \times 10^9 \cdot 18.7 \times 10^{-6}}}\right)\right]^{-1} = 1.792380365$$

$$y_{\max} = e(H-1) \quad \Rightarrow \quad e = \frac{y_{\max}}{H-1} = \frac{5\,\text{mm}}{1.792380365 - 1} = 6.31\,\text{mm} \quad \blacktriangleleft \quad (a)$$

$$\frac{ec}{r^2} = \frac{ecA}{I} = \frac{6.31 \cdot 101.5 \cdot 7420}{18.7 \times 10^6} = 0.2541310321$$

$$\sigma_{\max} = \frac{P}{A}\left(1 + \frac{ec}{r^2} \cdot H\right) = \frac{350 \times 10^3}{7420 \times 10^{-6}}(1 + 0.2541310321 \cdot 1.792380365)\,\text{Pa} = 68.7\,\text{MPa}$$

$$\blacktriangle \quad (b)$$

Problem 10.39

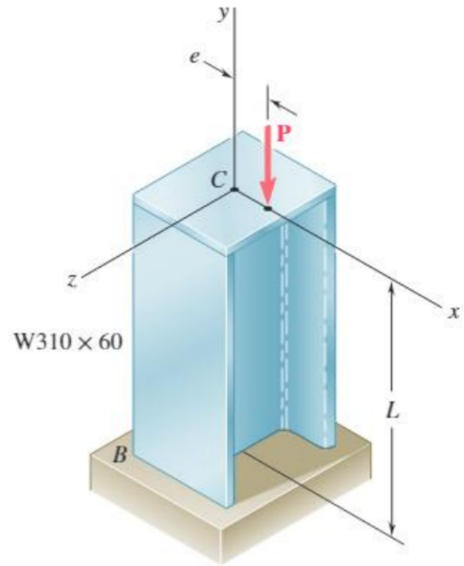

Fig. P10.39

An axial load **P** is applied at a point located on the x axis at a distance $e = 12\,\text{mm}$ from the geometric axis of the W310×60 rolled-steel column BC. Assuming that $L = 3.5\,\text{m}$ and using $E = 200\,\text{GPa}$, determine (a) the load P for which the horizontal deflection of the midpoint C of the column is 15 mm, (b) the corresponding maximum stress in the column.

$$I = I_Y = 18.4 \times 10^6\,\text{mm}^4, \quad A = 7550\,\text{mm}^2, \quad c = \frac{b_f}{2} = 101.5\,\text{mm} \quad \cdots \quad \text{from appendix E}$$

$$L_e = 2L = 7.0\,\text{m}$$

$$y_{\max} = e\left[\sec\left(\frac{L_e}{2}\sqrt{\frac{P}{EI}}\right) - 1\right] \quad \Rightarrow \quad P = \frac{EI}{L^2}\left[\arccos\left(\frac{e}{y_{\max}+e}\right)\right]^2$$

$$P = \frac{200 \times 10^9 \cdot 18.4 \times 10^{-6}}{3.5^2}\left[\arccos\left(\frac{12}{15+12}\right)\right]^2 = 370\,\text{kN} \quad \blacktriangleleft \quad (a)$$

$$\text{let} \quad H = \sec\left(\frac{L_e}{2}\sqrt{\frac{P}{EI}}\right) = \left[\cos\left(3.5\sqrt{\frac{370 \times 10^3}{200 \times 10^9 \cdot 18.4 \times 10^{-6}}}\right)\right]^{-1} = 2.247999207$$

$$\frac{ec}{r^2} = \frac{ecA}{I} = \frac{12 \cdot 101.5 \cdot 7550}{18.4 \times 10^6} = 0.4997771739$$

$$\sigma_{\max} = \frac{P}{A}\left(1 + \frac{ec}{r^2} \cdot H\right) = \frac{370 \times 10^3}{7550 \times 10^{-6}}(1 + 0.4997771739 \cdot 2.247999207)\,\text{Pa} = 104.1\,\text{MPa}$$

$\blacktriangle \quad (b)$

Problem 10.40

Solve Prob. 10.39, assuming that L is 4.5 m.

$$I = I_Y = 18.4 \times 10^6\,\text{mm}^4, \quad A = 7550\,\text{mm}^2, \quad c = \frac{b_f}{2} = 101.5\,\text{mm} \quad \cdots \quad \text{from appendix E}$$

$$L_e = 2L = 9.0\,\text{m}$$

$$y_{\text{max}} = e\left[\sec\left(\frac{L_e}{2}\sqrt{\frac{P}{EI}}\right) - 1\right] \quad \Rightarrow \quad P = \frac{EI}{L^2}\left[\arccos\left(\frac{e}{y_{\text{max}} + e}\right)\right]^2$$

$$P = \frac{200 \times 10^9 \cdot 18.4 \times 10^{-6}}{4.5^2}\left[\arccos\left(\frac{12}{15+12}\right)\right]^2 = 224\,\text{kN} \quad \blacktriangleleft \quad (a)$$

$$\text{let} \quad H = \sec\left(\frac{L_e}{2}\sqrt{\frac{P}{EI}}\right) = \left[\cos\left(4.5\sqrt{\frac{224 \times 10^3}{200 \times 10^9 \cdot 18.4 \times 10^{-6}}}\right)\right]^{-1} = 2.249940064$$

$$\frac{ec}{r^2} = \frac{ecA}{I} = \frac{12 \cdot 101.5 \cdot 7550}{18.4 \times 10^6} = 0.4997771739$$

$$\sigma_{\text{max}} = \frac{P}{A}\left(1 + \frac{ec}{r^2} \cdot H\right) = \frac{224 \times 10^3}{7550 \times 10^{-6}}(1 + 0.4997771739 \cdot 2.249940064)\,\text{Pa} = 63.0\,\text{MPa}$$

$$\blacktriangle \quad (b)$$