

# **HW6**

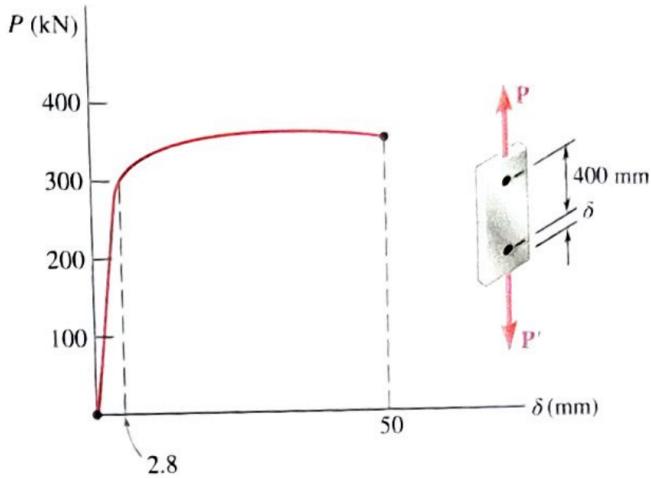
2025-2 구조역학(박성훈 교수님)

Problem 11.7, 11.15, 11.30, 11.34, 11.42, 11.45, 11.50, 11.59

2025-12-27



### Problem 11.7



**Fig. P11.7**

The load-deformation diagram shown has been drawn from data obtained during a tensile test of a specimen of an aluminum alloy. Knowing that the cross-sectional area of the specimen was  $600 \text{ mm}^2$  and that the deformation was measured using a 400-mm gage length, determine by approximate means (a) the modulus of resilience of the alloy, (b) the modulus of toughness of the alloy.

$$\sigma_Y = \frac{P_Y}{A} = \frac{300 \times 10^3}{600 \times 10^{-6}} \text{ Pa} = 500 \text{ MPa}$$

$$\varepsilon_Y = \frac{\delta_Y}{L} = \frac{2.8}{400} = 0.007$$

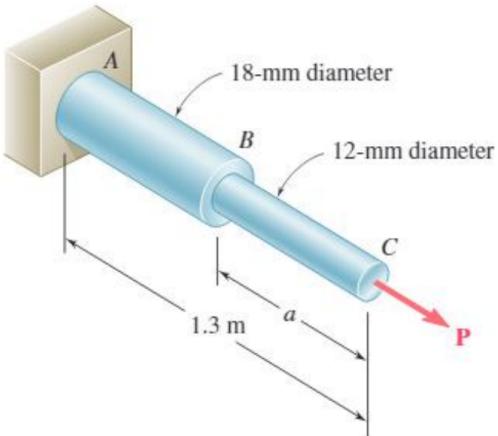
$$(\text{modulus of resilience}) = \frac{1}{2} \sigma_Y \varepsilon_Y = \frac{1}{2} (500 \times 10^6) (0.007) \text{ J/m}^3 = 1.750 \text{ MJ/m}^3 \quad \blacktriangleleft \quad (a)$$

$$\varepsilon_R = \frac{\delta_R}{L} = \frac{50}{400} = 0.125$$

$$P_{\text{avg}} \approx 350 \text{ kN} \quad \Rightarrow \quad \sigma_{\text{avg}} \approx \frac{350 \times 10^3}{600 \times 10^{-6}} \text{ Pa} = 583.333 \text{ MPa}$$

$$(\text{modulus of resilience}) = \sigma_{\text{avg}} \varepsilon_R \approx (583.333 \times 10^6) (0.125) \text{ J/m}^3 = 72.9 \text{ MJ/m}^3 \quad \blacktriangleleft \quad (b)$$

### Problem 11.15



Rod  $ABC$  is made of a steel for which the yield strength is  $\sigma_Y = 450 \text{ MPa}$  and the modulus of elasticity is  $E = 200 \text{ GPa}$ . Knowing that a strain energy of  $11.2 \text{ J}$  must be acquired by the rod as the axial load  $P$  is applied, determine the factor of safety of the rod with respect to permanent deformation when  $a = 0.5 \text{ m}$ .

Fig. P11.15

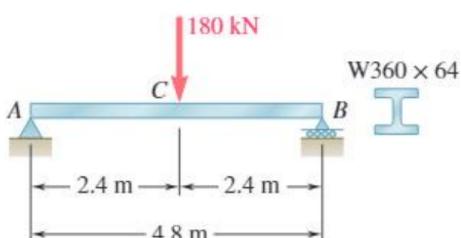
$$A_{AB} = \pi(0.009)^2 \text{ m}^2 = 2.54469 \times 10^{-4} \text{ m}^2, \quad A_{BC} = \pi(0.006)^2 \text{ m} = 1.130973 \times 10^{-4} \text{ m}^2$$

$$P_Y = \sigma_Y A_{BC} = (450 \times 10^6)(1.130973 \times 10^{-4}) = 50.8938 \text{ kN}$$

$$\begin{aligned} U_Y &= \frac{P_Y^2 L_{AB}}{2EA_{AB}} + \frac{P_Y^2 L_{BC}}{2EA_{BC}} = \frac{P_Y^2}{2E} \left( \frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}} \right) \\ &= \frac{50893.8^2}{2(200 \times 10^9)} \left( \frac{0.8}{2.54469 \times 10^{-4}} + \frac{0.5}{1.130973 \times 10^{-4}} \right) \text{ J} = 48.9853 \text{ J} \end{aligned}$$

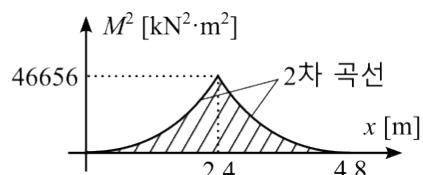
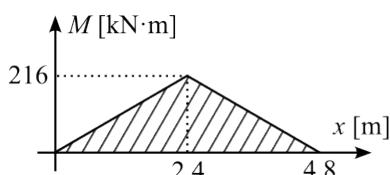
$$(F.S.) = \frac{U_Y}{U} = \frac{48.9853 \text{ J}}{11.2 \text{ J}} = 4.37 \quad \blacktriangleleft$$

### Problem 11.30



Using  $E = 200 \text{ GPa}$ , determine the strain energy due to bending for the steel beam and loading shown. (Neglect the effect of shearing stresses.)

Fig. P11.30



$$I = 178 \times 10^6 \text{ mm}^4 = 178 \times 10^{-6} \text{ m}^4$$

$$\int_0^L M^2 dx = A_{m^2} = \frac{1}{3}(46656)(4.8) \text{ kN}^2 \cdot \text{m}^3 = 74649.6 \text{ kN}^2 \cdot \text{m}^3$$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{A_{m^2}}{2EI} = \frac{74649.6 \times 10^6}{2(200 \times 10^9)(178 \times 10^{-6})} \text{ J} = 1048 \text{ J} \quad \blacktriangleleft$$

### Problem 11.34

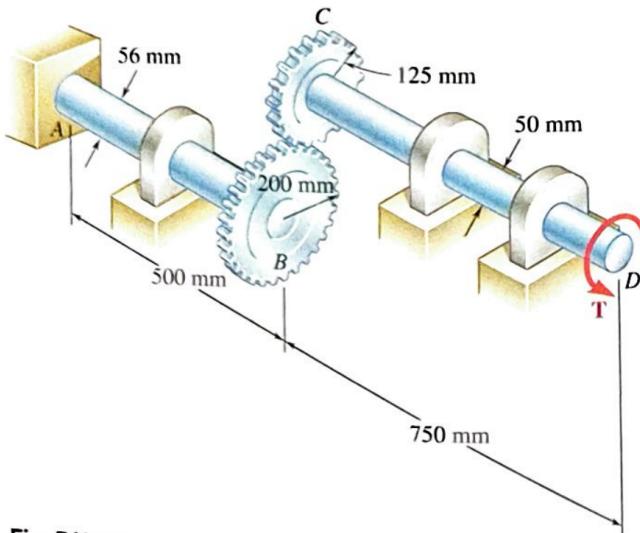


Fig. P11.34

Two solid shafts are connected by the gears shown. Using  $G = 77 \text{ GPa}$ , determine the strain energy of each shaft when a 2.7-kN-m. torque  $T$  is applied at  $D$ . (Ignore the strain energy due to bending of the shafts.)

$$T_{AB} = \frac{200 \text{ mm}}{125 \text{ mm}} (2.7 \text{ kN} \cdot \text{m}) = 4.32 \text{ kN} \cdot \text{m}, \quad T_{CD} = 2.7 \text{ kN} \cdot \text{m}$$

$$J_{AB} = \frac{\pi}{2} (0.025)^4 \text{ m}^4 = 9.65499 \times 10^{-7} \text{ m}^4$$

$$J_{CD} = \frac{\pi}{2} (0.025)^4 \text{ m}^4 = 6.13592 \times 10^{-7} \text{ m}^4$$

$$U = \frac{T_{AB}^2 L_{AB}}{2GJ_{AB}} + \frac{T_{CD}^2 L_{CD}}{2GJ_{CD}} = 120.6 \text{ J} \quad \blacktriangleleft$$

### Problem 11.42

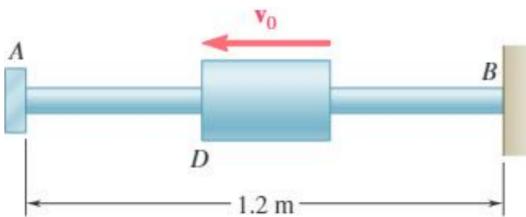


Fig. P11.42

A 5-kg collar  $D$  moves along the uniform rod  $AB$  and has a speed  $V = 6 \text{ m/s}$  when it strikes a small plate attached to end  $A$  of the rod. Using  $E = 200 \text{ GPa}$  and knowing that the allowable stress in the rod is  $250 \text{ MPa}$ , determine the smallest diameter that can be used for the rod.

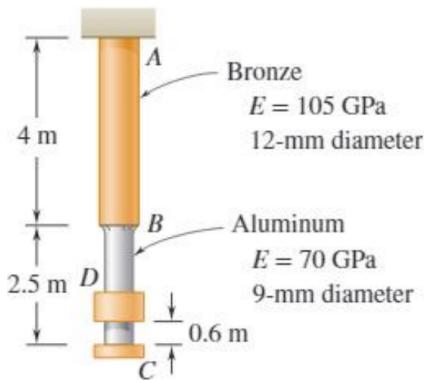
$$U = \frac{1}{2}mv_0^2 = \frac{1}{2}(5)(6^2) \text{ J} = 90 \text{ J}$$

$$P_{\text{all}} = \sigma_{\text{all}}A, \quad A = \frac{\pi}{4}d^2$$

$$U \leq U_{\text{all}} = \frac{P_{\text{all}}^2 L}{2EA} = \frac{\sigma_{\text{all}}^2 AL}{2E} = \frac{\pi\sigma_{\text{all}}^2 d^2 L}{8E}$$

$$d \geq \sqrt{\frac{8UE}{\pi\sigma_{\text{all}}^2 L}} = \sqrt{\frac{8(90)(200 \times 10^9)}{\pi(250 \times 10^6)^2(1.2)}} \text{ m} = 24.7 \text{ mm} \quad \blacktriangleleft$$

**Problem 11.45**



Collar  $D$  is released from rest in the position shown and is stopped by a small plate attached at end  $C$  of the vertical rod  $ABC$ . Determine the mass of the collar for which the maximum normal stress in portion  $BC$  is 125 MPa.

**Fig. P11.45**

$$A_{AB} = \pi(0.006)^2 \text{ m}^2 = 1.130973 \times 10^{-4} \text{ m}^2$$

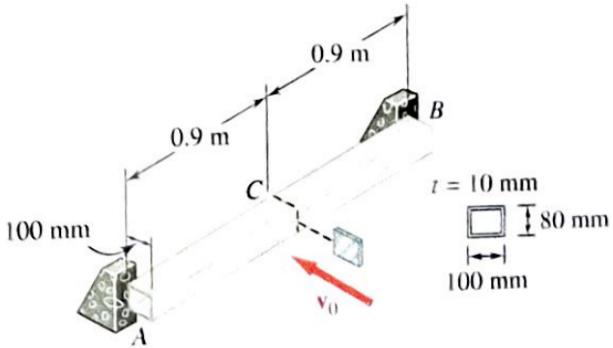
$$A_{BC} = \pi(0.0045)^2 \text{ m}^2 = 6.36173 \times 10^{-5} \text{ m}^2$$

$$P_m = \sigma_m A_{BC} = 7952.16 \text{ N} \quad (\because A_{\min} = A_{BC})$$

$$U_m = \frac{1}{2} P_m \delta_m = \frac{P_m^2 L_{AB}}{2E_b A_{AB}} + \frac{P_m^2 L_{BC}}{2E_a A_{BC}} \Rightarrow \delta_m = P_m \left( \frac{L_{AB}}{E_b A_{AB}} + \frac{L_{BC}}{E_a A_{BC}} \right) = \frac{1}{140} \text{ m}$$

$$U \leq U_m \Rightarrow mg(h + \delta_m) \leq \frac{1}{2} P_m \delta_m \Rightarrow m \leq \frac{P_m \delta_m}{2g(h + \delta_m)} = 4.77 \text{ kg} \quad \blacktriangleleft$$

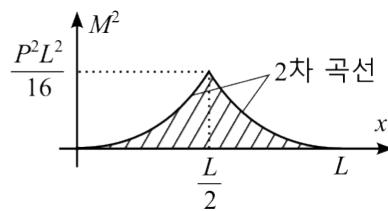
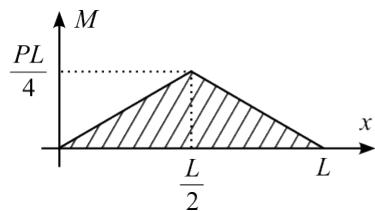
### Problem 11.50



An aluminum tube having the cross section shown is struck squarely in its midsection by a 6-kg block moving horizontally with a speed of 2 m/s. Using  $E = 70 \text{ GPa}$ , determine (a) the equivalent static load, (b) the maximum stress in the beam, (c) the maximum deflection at the midpoint  $C$  of the beam.

**Fig. P11.50**

$$U = \frac{1}{2}mv_0^2 = \frac{1}{2}(6)(2^2) \text{ J} = 12 \text{ J}$$



$$L = 1.8 \text{ m}, \quad M_C = \frac{P}{2} \cdot \frac{L}{2} = \frac{PL}{4}$$

$$\int_0^L M^2 dx = A_{m^2} = \frac{1}{3} \left( \frac{P^2 L^2}{16} \right) L = \frac{P^2 L^3}{48}$$

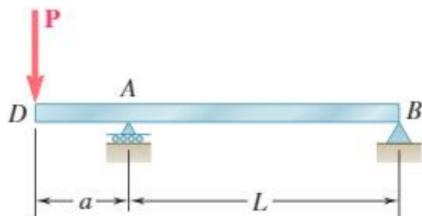
$$I = \frac{1}{12} \{ (0.08)(0.1)^3 - (0.06)(0.08)^3 \} \text{ m}^4 = 4.10667 \times 10^{-6} \text{ m}^4$$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{A_{m^2}}{2EI} = \frac{P^2 L^3}{96EI} \Rightarrow P = \sqrt{\frac{96UEI}{L^3}} = 7535.49 \text{ N} = 7.54 \text{ kN} \quad \blacktriangleleft \quad (a)$$

$$\sigma_m = \frac{|M|_{mc}}{I} = \frac{PLc}{4I} = 41.3 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

$$U = \frac{1}{2}Py_C \Rightarrow y_C = \frac{2U}{P} = 3.18 \text{ mm} \quad \blacktriangleleft \quad (c)$$

**Problem 11.59**

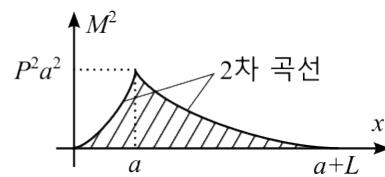
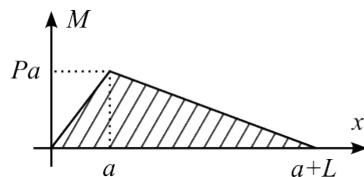


Using the method of work and energy, determine the deflection at point D caused by the load  $\mathbf{P}$ .

Fig. P11.59

$$+ \circ \sum M|_A = Pa - R_B L = 0, \quad R_B = \frac{Pa}{L}$$

$$+ \uparrow \sum F_y = -P - R_B + R_A = 0, \quad R_A = P \left(1 + \frac{a}{L}\right)$$



$$\int_0^L M^2 dx = A_{m^2} = \frac{1}{3} P^2 a^2 (a + L)$$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{A_{m^2}}{2EI} = \frac{P^2 a^2 (a + L)}{6EI} = \frac{1}{2} P |y_D| \quad \Rightarrow \quad y_D = -\frac{P a^2 (a + L)}{3EI}$$