

HW4

2025-1 고체역학(박성훈 교수님)

Problem 3.67, 3.68, 3.71, 3.74, 3.92, 3.95, 3.98, 3.116

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Problem 3.67

Determine the maximum shearing stress in a solid shaft of 18-mm diameter as it transmits 3.4 kW at a frequency of (a) 25 Hz, (b) 50 Hz.

$$c = 9 \text{ mm}, \quad T = \frac{P}{2\pi f}, \quad J = \frac{\pi}{2} c^4$$
$$\tau_{\max} = \frac{Tc}{J} = \frac{P}{\pi^2 f c^3}$$

When (a) $f = 25 \text{ Hz}$,

$$\tau_{\max} = \frac{3.4 \text{ kW}}{\pi^2 (25 \text{ Hz})(9 \text{ mm})^3} = \frac{3400}{\pi^2 (25)(0.009)^3} \text{ Pa} = 18.90 \text{ MPa} \quad \blacktriangleleft \quad (a)$$

When (b) $f = 50 \text{ Hz}$,

$$\tau_{\max} = \frac{3.4 \text{ kW}}{\pi^2 (50 \text{ Hz})(9 \text{ mm})^3} = \frac{3400}{\pi^2 (50)(0.009)^3} \text{ Pa} = 9.45 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

Problem 3.68

While a steel shaft of the cross section shown rotates at 120 rpm, a stroboscopic measurement indicates that the angle of twist is 2° in a 4-m length. Using $G = 77.2 \text{ GPa}$, determine the power being transmitted.

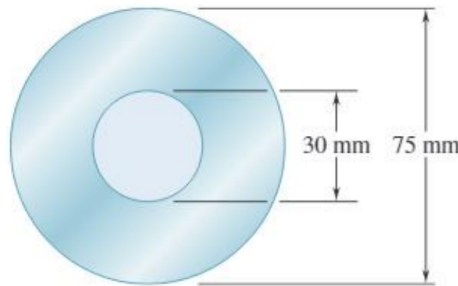


Fig. P3.68

$$f = 120 \text{ rpm} = 2 \text{ Hz}, \quad \phi = 2^\circ = \frac{\pi}{90} \text{ (rad)}$$

$$J = \frac{\pi}{2} \left\{ \left(\frac{75}{2} \text{ mm} \right)^4 - \left(\frac{30}{2} \text{ mm} \right)^4 \right\} = 3.026789531 \times 10^{-6} \text{ m}^4$$

$$T = \frac{G\phi J}{L}, \quad T = \frac{P}{2\pi f}$$

$$\frac{P}{2\pi f} = \frac{G\phi J}{L}$$

$$P = \frac{2\pi f G \phi J}{L} = \frac{2\pi (2 \text{ Hz})(77.2 \text{ GPa}) \left(\frac{\pi}{90} \right) (3.026789531 \times 10^{-6} \text{ m}^4)}{4 \text{ m}}$$
$$= \frac{2\pi (2)(77.2 \times 10^9) \left(\frac{\pi}{90} \right) (3.026789531 \times 10^{-6})}{4} \text{ W} = 25.6 \text{ kW} \quad \blacktriangleleft$$

Problem 3.71

The hollow steel shaft shown ($G = 77.2 \text{ GPa}$, $\tau_{\text{all}} = 50 \text{ MPa}$) rotates at 240 rpm. Determine (a) the maximum power that can be transmitted, (b) the corresponding angle of twist of the shaft.

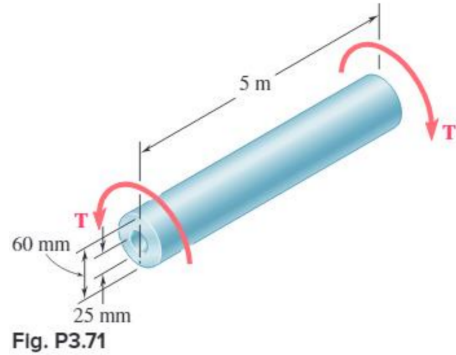


Fig. P3.71

$$f = 240 \text{ rpm} = 4 \text{ Hz}$$

$$J = \frac{\pi}{2} \left\{ \left(\frac{60}{2} \text{ mm} \right)^4 - \left(\frac{25}{2} \text{ mm} \right)^4 \right\} = 1.233995505 \times 10^{-6} \text{ m}^4$$

$$T_{\text{max}} = \frac{\tau_{\text{all}} J}{c}, \quad P_{\text{max}} = 2\pi f T_{\text{max}}$$

$$P_{\text{max}} = \frac{2\pi f \tau_{\text{all}} J}{c} = \frac{2\pi(4 \text{ Hz})(50 \text{ MPa})(1.233995505 \times 10^{-6} \text{ m}^4)}{30 \text{ mm}} \\ = \frac{2\pi(4)(50 \times 10^6)(1.233995505 \times 10^{-6})}{0.03} \text{ W} = 51.7 \text{ kW} \quad \blacktriangleleft \quad (a)$$

$$\phi_{\text{max}} = \frac{\gamma_{\text{all}} L}{c} = \frac{\tau_{\text{all}} L}{Gc} = \frac{(50 \text{ MPa})(5 \text{ m})}{(77.2 \text{ GPa})(30 \text{ mm})} = \frac{(50 \times 10^6)(5)}{(77.2 \times 10^9)(0.03)} = \frac{125}{1158} = 6.18^\circ \quad \blacktriangleleft \quad (b)$$

Problem 3.74

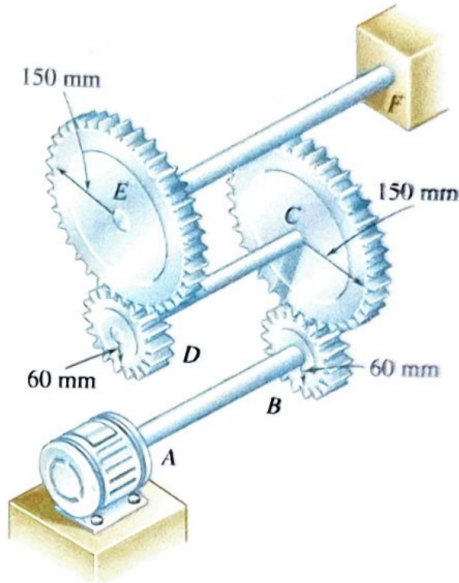


Fig. P3.74 and P3.75

Three shafts and four gears are used to form a gear train that will transmit power from the motor at *A* to a machine tool at *F*. (Bearings for the shafts are omitted in the sketch.) The diameter of each shaft is as follows: $d_{AB} = 16$ mm, $d_{CD} = 20$ mm, $d_{EF} = 28$ mm. Knowing that the frequency of the motor is 24 Hz and that the allowable shearing stress for each shaft is 75 MPa, determine the maximum power that can be transmitted.

$$T_{\max} = \frac{\tau_{\text{all}} J}{c}, \quad P_{\max} = 2\pi f T_{\max} = \frac{2\pi f \tau_{\text{all}} J}{c}$$

$$f_{AB} = 24 \text{ Hz}, \quad f_{CD} = \frac{60 \text{ mm}}{150 \text{ mm}} f_{AB} = 9.6 \text{ Hz}, \quad f_{EF} = \frac{60 \text{ mm}}{150 \text{ mm}} f_{CD} = 3.84 \text{ Hz}$$

$$J_{AB} = \frac{\pi}{2} \left(\frac{d_{AB}}{2} \right)^4 = \frac{\pi}{2} (8 \text{ mm})^4 = 2.048\pi \times 10^{-9} \text{ m}^4$$

$$P_{\max, AB} = \frac{2\pi(24 \text{ Hz})(75 \text{ MPa})(2.048\pi \times 10^{-9} \text{ m}^4)}{8 \text{ mm}} = \frac{2\pi(24)(75 \times 10^6)(2.048\pi \times 10^{-9})}{0.008} \text{ W}$$

$$= 9.10 \text{ kW}$$

$$J_{CD} = \frac{\pi}{2} \left(\frac{d_{CD}}{2} \right)^4 = \frac{\pi}{2} (10 \text{ mm})^4 = 5\pi \times 10^{-9} \text{ m}^4$$

$$P_{\max, CD} = \frac{2\pi(9.6 \text{ Hz})(75 \text{ MPa})(5\pi \times 10^{-9} \text{ m}^4)}{10 \text{ mm}} = \frac{2\pi(9.6)(75 \times 10^6)(5\pi \times 10^{-9})}{0.01} \text{ W}$$

$$= 7.11 \text{ kW}$$

$$J_{EF} = \frac{\pi}{2} \left(\frac{d_{EF}}{2} \right)^4 = \frac{\pi}{2} (14 \text{ mm})^4 = 1.9208\pi \times 10^{-9} \text{ m}^4$$

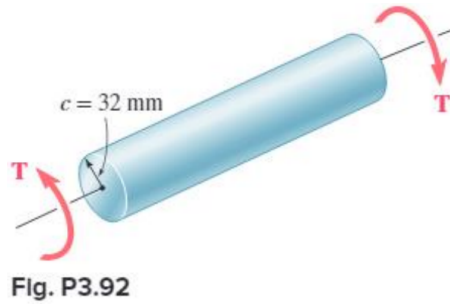
$$P_{\max, EF} = \frac{2\pi(3.84 \text{ Hz})(75 \text{ MPa})(1.9208\pi \times 10^{-9} \text{ m}^4)}{14 \text{ mm}} = \frac{2\pi(3.84)(75 \times 10^6)(1.9208\pi \times 10^{-9})}{0.014} \text{ W}$$

$$= 7.80 \text{ kW}$$

$$9.10 \text{ kW} > 7.80 \text{ kW} > 7.11 \text{ kW} \quad \Rightarrow \quad P_{\max} = 7.11 \text{ kW} \quad \blacktriangleleft$$

Problem 3.92

The solid circular shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145 \text{ MPa}$. Determine the magnitude T of the applied torques when the plastic zone is (a) 16 mm deep, (b) 24 mm deep.



$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (32 \text{ mm})^4 = 5.24288\pi \times 10^{-7} \text{ m}^4$$

$$T_Y = \frac{\tau_Y J}{c} = \frac{(145 \text{ MPa})(5.24288\pi \times 10^{-7} \text{ m}^4)}{32 \text{ mm}} = \frac{(145 \times 10^6)(5.24288\pi \times 10^{-7})}{0.032} \text{ N} \cdot \text{m}$$

$$= 7.463418835 \text{ kN} \cdot \text{m}$$

When $\rho_Y = 16 \text{ mm}$,

$$\frac{\rho_Y}{c} = \frac{16 \text{ mm}}{32 \text{ mm}} = \frac{1}{2}$$

$$T = \frac{4}{3} T_Y \left\{ 1 - \frac{1}{4} \left(\frac{\rho_Y}{c} \right)^3 \right\} = \frac{4}{3} T_Y \left\{ 1 - \frac{1}{4} \left(\frac{1}{2} \right)^3 \right\} = \frac{31}{24} T_Y = \frac{31}{24} (7.463418835 \text{ kN} \cdot \text{m})$$

$$= 9.64 \text{ kN} \cdot \text{m} \quad \blacktriangleleft \quad (a)$$

When $\rho_Y = 32 \text{ mm} - 24 \text{ mm} = 8 \text{ mm}$,

$$\frac{\rho_Y}{c} = \frac{8 \text{ mm}}{32 \text{ mm}} = \frac{1}{4}$$

$$T = \frac{4}{3} T_Y \left\{ 1 - \frac{1}{4} \left(\frac{\rho_Y}{c} \right)^3 \right\} = \frac{4}{3} T_Y \left\{ 1 - \frac{1}{4} \left(\frac{1}{4} \right)^3 \right\} = \frac{85}{64} T_Y = \frac{85}{64} (7.463418835 \text{ kN} \cdot \text{m})$$

$$= 9.91 \text{ kN} \cdot \text{m} \quad \blacktriangleleft \quad (b)$$

Problem 3.95

The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 77.2 \text{ GPa}$ and $\tau_Y = 145 \text{ MPa}$. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (a) $T = 600 \text{ N}\cdot\text{m}$, (b) $T = 1000 \text{ N}\cdot\text{m}$.

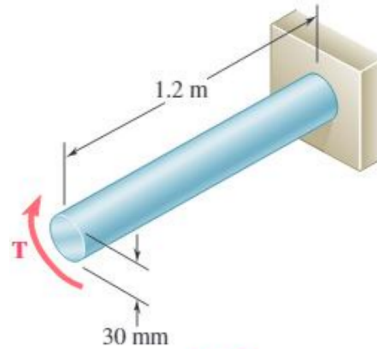


Fig. P3.95 and P3.96

$$c = \frac{30 \text{ mm}}{2} = 15 \text{ mm}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (15 \text{ mm})^4 = 2.53125\pi \times 10^{-8} \text{ m}^4$$

$$T_Y = \frac{\tau_Y J}{c} = \frac{(145 \text{ MPa})(2.53125\pi \times 10^{-8} \text{ m}^4)}{15 \text{ mm}} = \frac{(145 \times 10^6)(2.53125\pi \times 10^{-8})}{0.015} \text{ N}\cdot\text{m}$$

$$= 768.7084524 \text{ N}\cdot\text{m}$$

When (a) $T = 600 \text{ N}\cdot\text{m} < T_Y$,

$$\tau_{\max} = \frac{Tc}{J} = \frac{(600 \text{ N}\cdot\text{m})(15 \text{ mm})}{2.53125\pi \times 10^{-8} \text{ m}^4} = \frac{(600)(0.015)}{2.53125\pi \times 10^{-8}} \text{ Pa} = 113.2 \text{ MPa}$$

$$\rho_Y = c = 15 \text{ mm}$$

113.2 MPa; 15.00 mm ◀ (a)

When (b) $T = 1000 \text{ N}\cdot\text{m} > T_Y$,

$$\tau_{\max} = \tau_Y = 145 \text{ MPa}$$

$$T = \frac{4}{3} T_Y \left\{ 1 - \frac{1}{4} \left(\frac{\rho_Y}{c} \right)^3 \right\}$$

$$\rho_Y = c \sqrt[3]{4 - \frac{3T}{T_Y}} = (15 \text{ mm}) \sqrt[3]{4 - \frac{3(1000 \text{ N}\cdot\text{m})}{768.7084524 \text{ N}\cdot\text{m}}} = 6.90 \text{ mm}$$

145.0 MPa; 6.90 mm ◀ (b)

Problem 3.98

The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 77.2 \text{ GPa}$ and $\tau_Y = 145 \text{ MPa}$. Determine the angle of twist caused by the application of a torque of magnitude (a) $T = 600 \text{ N}\cdot\text{m}$, (b) $T = 1000 \text{ N}\cdot\text{m}$.

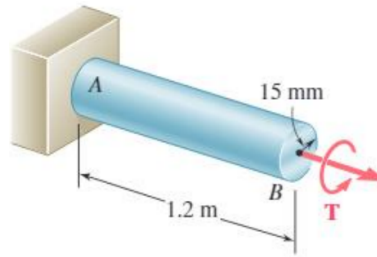


Fig. P3.98

From the solution of Prob.3.95,

$$J = 2.53125\pi \times 10^{-8} \text{ m}^4$$

$$T_Y = 768.7084524 \text{ N}\cdot\text{m}$$

When (a) $T = 600 \text{ N}\cdot\text{m} < T_Y$,

$$\phi = \frac{TL}{GJ} = \frac{(600 \text{ N}\cdot\text{m})(1.2 \text{ m})}{(77.2 \text{ GPa})(2.53125\pi \times 10^{-8} \text{ m}^4)} = \frac{(600)(1.2)}{(77.2 \times 10^9)(2.53125\pi \times 10^{-8})} = 6.72^\circ \quad \blacktriangle (a)$$

When (b) $T = 1000 \text{ N}\cdot\text{m} > T_Y$,

$$\phi_Y = \frac{\gamma_Y L}{c} = \frac{\tau_Y L}{Gc} = \frac{(145 \text{ MPa})(1.2 \text{ m})}{(77.2 \text{ GPa})(15 \text{ mm})} = \frac{(145 \times 10^6)(1.2)}{(77.2 \times 10^9)(0.015)} = \frac{29}{193}$$

$$T = \frac{4}{3}T_Y \left\{ 1 - \frac{1}{4} \left(\frac{\phi_Y}{\phi} \right)^3 \right\}$$

$$\phi = \phi_Y \left(4 - \frac{3T}{T_Y} \right)^{-\frac{1}{3}} = \frac{29}{193} \left(4 - \frac{3(1000 \text{ N}\cdot\text{m})}{768.7084524 \text{ N}\cdot\text{m}} \right)^{-\frac{1}{3}} = 18.71^\circ \quad \blacktriangleleft (b)$$

Problem 3.116

The solid shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145 \text{ MPa}$ and $G = 77.2 \text{ GPa}$. The torque is increased in magnitude until the shaft has been twisted through 6° ; the torque is then removed. Determine (a) the magnitude and location of the maximum residual shearing stress, (b) the permanent angle of twist.

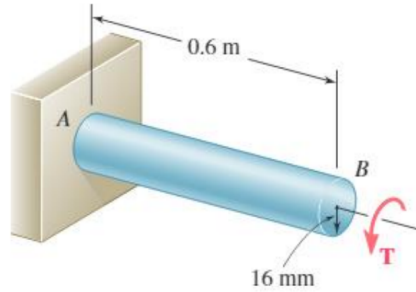


Fig. P3.116

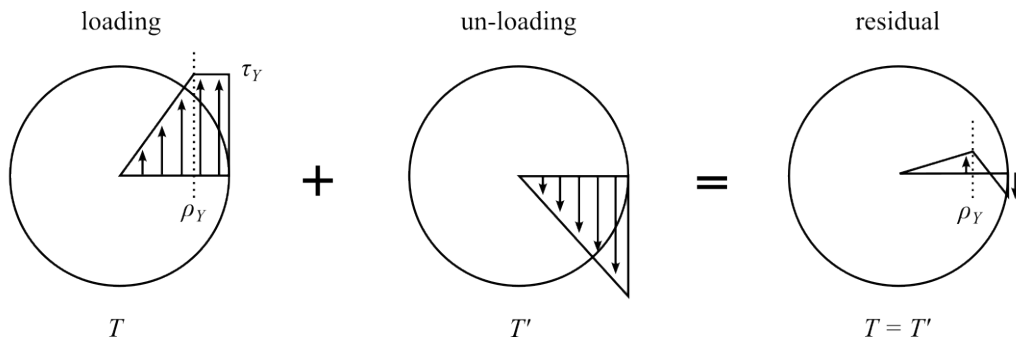
$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} (16 \text{ mm})^4 = 3.2768\pi \times 10^{-8} \text{ m}^4$$

$$T_Y = \frac{\tau_Y J}{c} = \frac{(145 \text{ MPa})(3.2768\pi \times 10^{-8} \text{ m}^4)}{16 \text{ mm}} = \frac{(145 \times 10^6)(3.2768\pi \times 10^{-8})}{0.016} \text{ N} \cdot \text{m}$$

$$= 932.9273544 \text{ N} \cdot \text{m}$$

$$\phi_Y = \frac{\tau_Y L}{Gc} = \frac{(145 \text{ MPa})(0.6 \text{ m})}{(77.2 \text{ GPa})(16 \text{ mm})} = \frac{(145 \times 10^6)(0.6)}{(77.2 \times 10^9)(0.016)} = \frac{435}{6176} = 4.035567372^\circ$$

$$R = \frac{\rho_Y}{c} = \frac{\phi_Y}{\phi} = \frac{4.035567372^\circ}{6^\circ} = 0.672594562$$



$$\text{loading : } T = \frac{4}{3} T_Y \left(1 - \frac{1}{4} R^3 \right) = \frac{4}{3} (932.9273544 \text{ N} \cdot \text{m}) \left\{ 1 - \frac{1}{4} (0.672594562)^3 \right\}$$

$$= 1149.282337 \text{ N} \cdot \text{m}$$

$$\text{un-loading : } T' = \frac{\tau'_{\max} J}{c}$$

$$T = T', \quad T = \frac{\tau'_{\max} J}{c}$$

$$\tau'_{\max} = \frac{Tc}{J} = \frac{(1149.282337 \text{ N} \cdot \text{m})(16 \text{ mm})}{3.2768\pi \times 10^{-8} \text{ m}^4} = \frac{(1149.282337)(0.016)}{3.2768\pi \times 10^{-8}} \text{ Pa}$$

$$= 178.6269189 \text{ MPa}$$

$$\tau_{\text{res}}(\rho) = |\tau(\rho) - \tau'(\rho)|$$

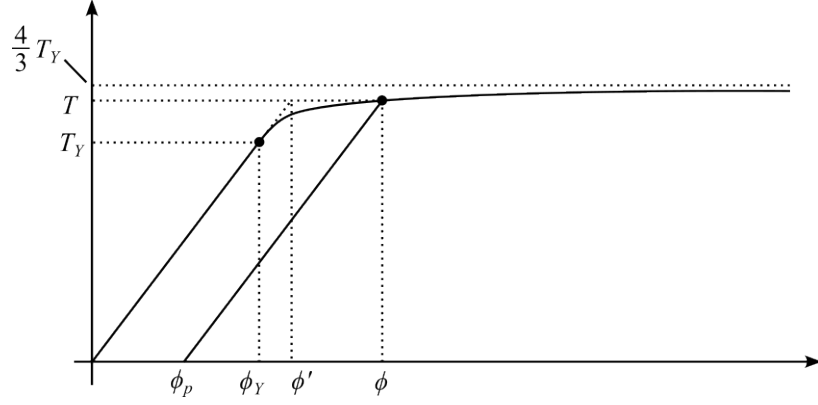
$$\tau_{\text{res}}(\rho_Y) = |\tau(\rho_Y) - \tau'(\rho_Y)| = |\tau_Y - R\tau'_{\max}|$$

$$= |145 \text{ MPa} - (0.672594562)(178.6269189 \text{ MPa})| = 24.9 \text{ MPa}$$

$$\begin{aligned}\tau_{\text{res}}(c) &= |\tau(c) - \tau'(c)| = |\tau_Y - \tau'_c| = |145 \text{ MPa} - 178.6269189 \text{ MPa}| \\ &= 33.6 \text{ MPa}\end{aligned}$$

$$\left. \begin{array}{l} \tau_{\text{res}}(\rho_Y) = 24.9 \text{ MPa} \\ \tau_{\text{res}}(c) = 33.6 \text{ MPa} \end{array} \right\} \Rightarrow \tau_{\text{res},\text{max}} = 33.6 \text{ MPa}$$

$$33.6 \text{ MPa} \quad \text{at} \quad \rho = 16.00 \text{ mm} \quad \blacktriangleleft \quad (a)$$



$$\phi_p = \phi - \phi' = \phi - T \cdot \frac{\phi_Y}{T_Y} = 6^\circ - \frac{1149.282337 \text{ N} \cdot \text{m}}{932.9273544 \text{ N} \cdot \text{m}}(4.035567372^\circ) = 1.028^\circ \quad \blacktriangleleft \quad (b)$$