HW1

2025-1 고체역학(박성훈 교수님)

Sample Problem 1.1, 1.2, Problem 1.1, 1.3, 1.7, 1.18, 1.29, 1.38, 1.43

2025-12-27

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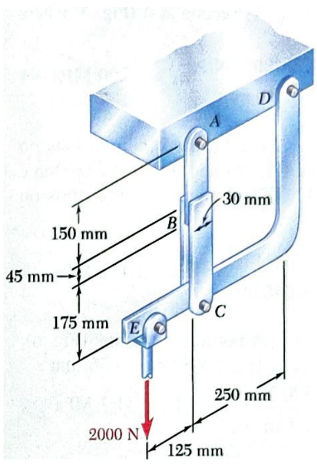
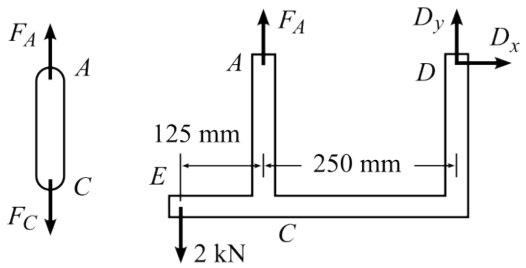
Sample Problem 1.1



In the hanger shown, the upper portion of link ABC is 10 mm. thick and the lower portions are each 6.0 mm. thick. Epoxy resin is used to bond the upper and lower portions together at B. The pin at A has a 10 mm. diameter, while a 6.0-mm diameter pin is used at C. Determine (a) the shearing stress in pin A, (b) the shearing stress in pin C, (c) the largest normal stress in link ABC, (d) the average shearing stress on the bonded surfaces at B, and (e) the bearing stress in the link at C.

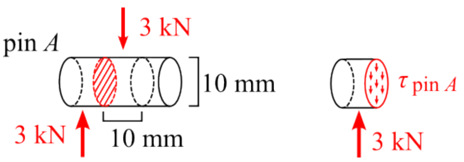


For the entire structure,

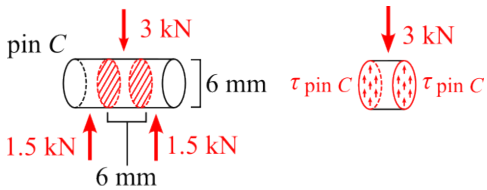
$$\sum F_x = +D_x = 0$$
$$\sum F_y = -2\,\text{kN} + F_A + D_y = 0$$
$$\sum M|_A = (2\,\text{kN})(125\,\text{mm}) + D_y(250\,\text{mm}) = 0$$

$$\Rightarrow \; D_x = 0,\; D_y = -1\,\text{kN},\; F_A = 3\,\text{kN}$$

For the link AC, $\quad F_C = F_A = 3\,\text{kN}$

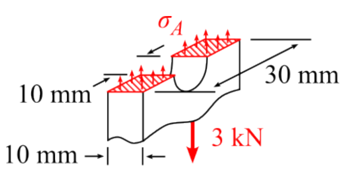


$$\tau_A = \frac{3\,\text{kN}}{\frac{\pi}{4}\,(10\,\text{mm})^2} = 38.2\,\text{MPa} \quad ◀ \quad (a)$$

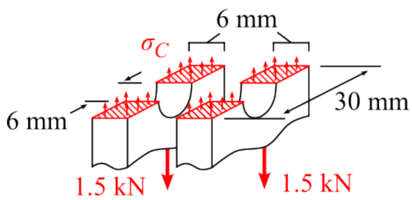


$$\tau_C = \frac{3\,\text{kN}}{2 \cdot \frac{\pi}{4}\,(6\,\text{mm})^2} = 53.1\,\text{MPa} \quad ◀ \quad (b)$$

Because a tensile force is applying at link ABC,

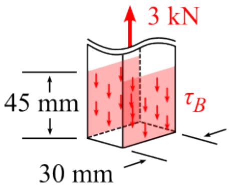


$$\sigma_A = \frac{3\,\text{kN}}{(30\,\text{mm} - 10\,\text{mm})(10\,\text{mm})} = 15.00\,\text{MPa}$$

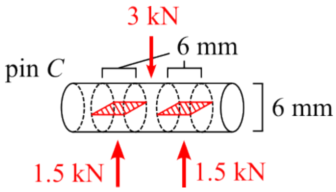


$$\sigma_C = \frac{1.5\,\text{kN}}{(30\,\text{mm} - 6\,\text{mm})(6\,\text{mm})} = 10.42\,\text{MPa}$$

$$\sigma_A > \sigma_C \quad \Rightarrow \quad \sigma_{\max} = \sigma_A = 15.00\,\text{MPa} \quad ◀ \quad (c)$$



$$\tau_B = \frac{3\,\text{kN}}{2(45\,\text{mm})(30\,\text{mm})} = 1.111\,\text{MPa} \quad \blacktriangleleft \quad (d)$$



$$\sigma_{b,C} = \frac{1.5\,\text{kN}}{(6\,\text{mm})(6\,\text{mm})} = 41.7\,\text{MPa} \quad \blacktriangleleft \quad (e)$$

Sample Problem 1.2

The steel tie bar shown is to be designed to carry a tension force of magnitude $P = 120\,\text{kN}$ when bolted between double brackets at A and B. The bar will be fabricated from 20-mm-thick plate stock. For the grade of steel to be used, the maximum allowable stresses are $\sigma = 175\,\text{MPa}$, $\tau = 100\,\text{MPa}$, and $\sigma_b = 350\,\text{MPa}$. Design the tie bar by determining the required values of (a) the diameter d of the bolt, (b) the dimension b at each end of the bar, and (c) the dimension h of the bar.

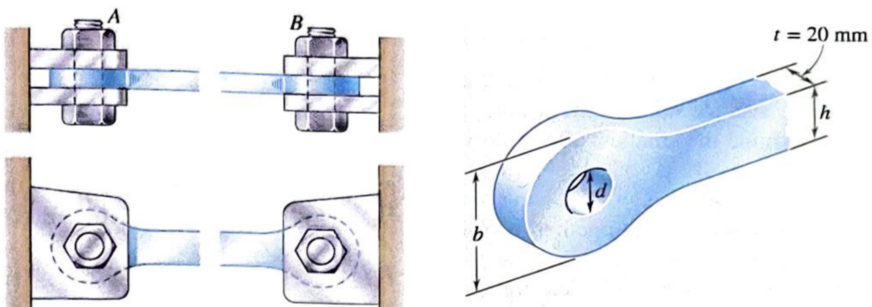
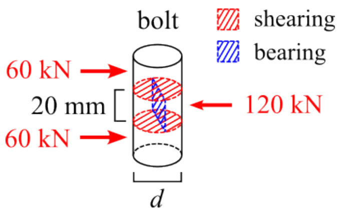


Fig. 2 Tie bar geometry.

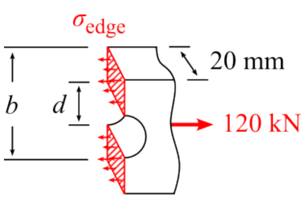


$$\tau = \frac{60\,\text{kN}}{\frac{\pi}{4}d^2} \le \tau_{\text{all}} = 100\,\text{MPa} \quad \Rightarrow \quad d \ge 27.6\,\text{mm}$$

$$\sigma_{b,\text{bolt}} = \frac{120\,\text{kN}}{d(20\,\text{mm})} \le \sigma_{b,\text{all}} = 350\,\text{MPa}, \quad d \ge 17.14\,\text{mm}$$

$$d_{\text{min}} = 27.6\,\text{mm} \quad \blacktriangleleft \quad (a)$$

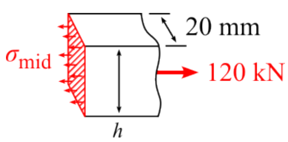
Because it is a tensile force,



$$\sigma_{\text{edge}} = \frac{120\,\text{kN}}{(b-d)(20\,\text{mm})} \le \sigma_{\text{all}} = 175\,\text{MPa}$$

$$b \ge \frac{120\,\text{kN}}{(175\,\text{MPa})(20\,\text{mm})} + d$$

$$b_{\text{min}} = \frac{120\,\text{kN}}{(175\,\text{MPa})(20\,\text{mm})} + d_{\text{min}} = 61.9\,\text{mm} \quad \blacktriangleleft \quad (b)$$



$$\sigma_{\text{mid}} = \frac{120\,\text{kN}}{h(20\,\text{mm})} \le \sigma_{\text{all}} = 175\,\text{MPa}$$

$$h_{\text{min}} = \frac{120\,\text{kN}}{(175\,\text{MPa})(20\,\text{mm})} = 34.3\,\text{mm} \quad \blacktriangleleft \quad (c)$$

Problem 1.1

Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that $d_1 = 30\,\text{mm}$ and $d_2 = 50\,\text{mm}$, find the average normal stress at the midsection of (a) rod AB, (b) rod BC.

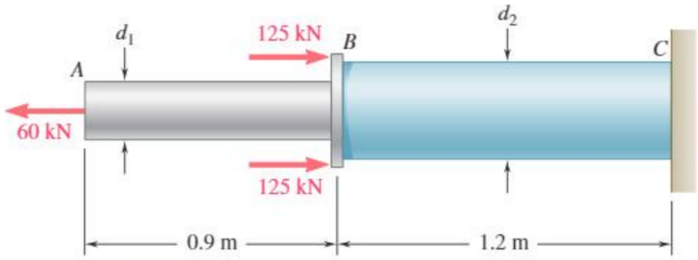
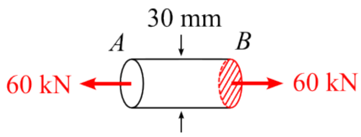
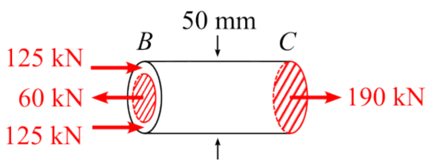


Fig. P1.1 and P1.2

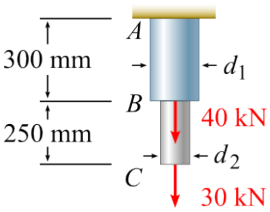


$$\sigma_{AB} = \frac{60\,\text{kN}}{\frac{\pi}{4}(30\,\text{mm})^2} = 84.9\,\text{MPa} \quad ◀ \quad (a)$$

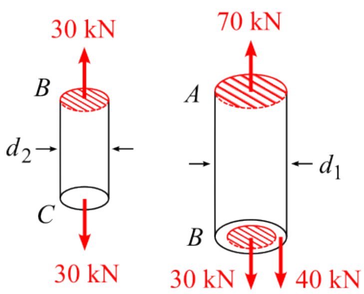


$$\sigma_{BC} = \frac{-190\,\text{kN}}{\frac{\pi}{4}(50\,\text{mm})^2} = -96.8\,\text{MPa} \quad ◀ \quad (b)$$

Problem 1.3



Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that the average normal stress must not exceed 175 MPa in rod AB and 150 MPa in rod BC, determine the smallest allowable values of d_1 and d_2.



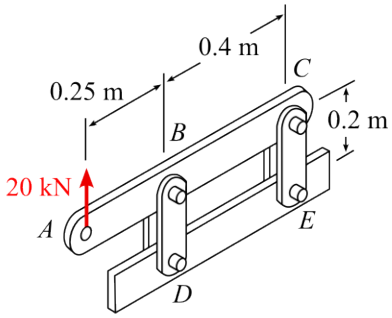
$$\sigma_{AB} = \frac{70\,\text{kN}}{\frac{\pi}{4}d_1^2} \leq \sigma_{\text{all},AB} = 175\,\text{MPa}$$

$$\sigma_{BC} = \frac{30\,\text{kN}}{\frac{\pi}{4}d_2^2} \leq \sigma_{\text{all},BC} = 150\,\text{MPa}$$

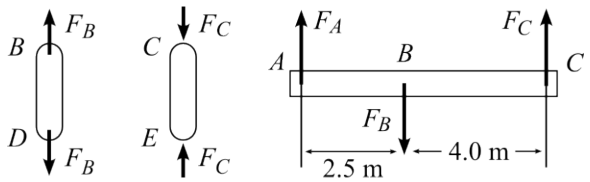
$$d_{1,\text{min}} = \sqrt{\frac{4}{\pi} \cdot \frac{70\,\text{kN}}{175\,\text{MPa}}} = 22.6\,\text{mm} \quad ◀$$

$$d_{2,\text{min}} = \sqrt{\frac{4}{\pi} \cdot \frac{30\,\text{kN}}{150\,\text{MPa}}} = 15.96\,\text{mm} \quad ◀$$

Problem 1.7



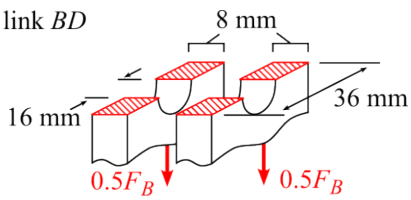
Each of the four vertical links has an 8×36-mm uniform rectangular cross section, and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points B and D, (*b*) points C and E.



$$\sum M|_B = -0.25\,\mathrm{m} \cdot 20\,\mathrm{kN} + 0.4\,\mathrm{m} \cdot F_C = 0$$

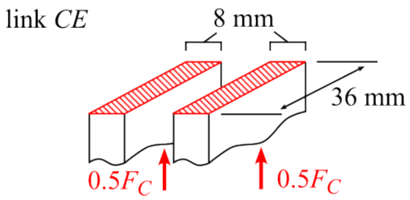
$$\sum F_y = 20\,\mathrm{kN} - F_B + F_C = 0$$

$$\Rightarrow \quad F_C = 12.5\,\mathrm{kN}, \quad F_B = 32.5\,\mathrm{kN}$$



$$A_{BD} = 8\,\mathrm{mm}(36\,\mathrm{mm} - 16\,\mathrm{mm}) = 160\,\mathrm{mm}^2$$

$$\sigma_{BD} = \frac{\frac{1}{2}F_B}{A_{BD}} = \frac{16.25\,\mathrm{kN}}{160\,\mathrm{mm}^2} = 101.6\,\mathrm{MPa} \quad \blacktriangleleft \quad (a)$$



$$A_{CE} = (8\,\mathrm{mm})(36\,\mathrm{mm}) = 288\,\mathrm{mm}^2$$

$$\sigma_{CE} = \frac{-\frac{1}{2}F_C}{A_{CE}} = \frac{-6.25\,\mathrm{kN}}{288\,\mathrm{mm}^2} = -21.7\,\mathrm{MPa} \quad \blacktriangleleft \quad (b)$$

Problem 1.18

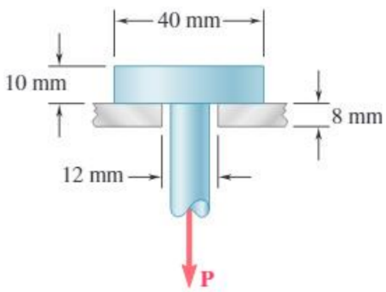
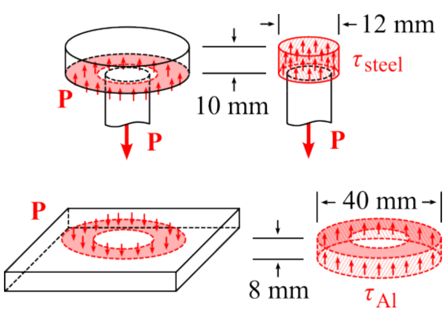


Fig. P1.18

A load P is applied to a steel rod supported as shown by an aluminum plate into which a 12-mm-diameter hole has been drilled. Knowing that the shearing stress must not exceed 180 MPa in the steel rod and 70 MPa in the aluminum plate, determine the largest load P that can be applied to the rod.



$$\tau_{\mathrm{steel}} = \frac{P}{\pi(12\,\mathrm{mm})(10\,\mathrm{mm})} \leq 180\,\mathrm{MPa}$$

$$\Rightarrow \quad P \leq 67.9\,\mathrm{kN}$$

$$\tau_{\mathrm{Al}} = \frac{P}{\pi(40\,\mathrm{mm})(8\,\mathrm{mm})} \leq 70\,\mathrm{MPa}$$

$$\Rightarrow \quad P \leq 70.4\,\mathrm{kN}$$

$$\therefore \quad P_{\max} = 67.9\,\mathrm{kN} \quad \blacktriangleleft$$

Problem 1.29

Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 11\,\text{kN}$, determine the normal and shearing stresses in the glued splice.

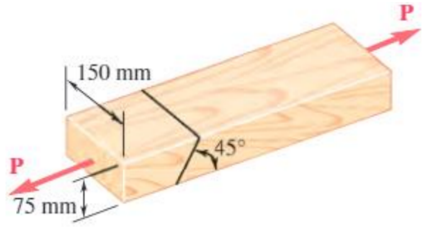
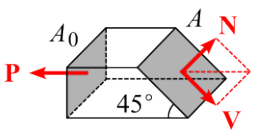


Fig. P1.29 and P1.30



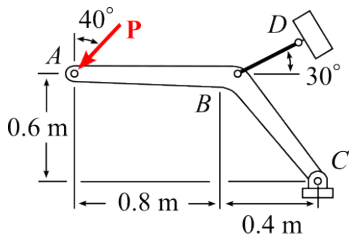
$$A_0 = (150\,\text{mm})(75\,\text{mm}) = 11250\,\text{mm}^2, \quad A = \frac{A_0}{\sin 45°} = A_0\sqrt{2}$$

$$N = P\sin 45° = \frac{P}{\sqrt{2}}, \quad V = P\cos 45° = \frac{P}{\sqrt{2}}$$

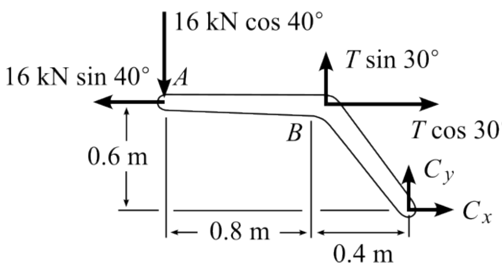
$$\sigma = \frac{N}{A} = \frac{\frac{P}{\sqrt{2}}}{A_0\sqrt{2}} = \frac{P}{2A_0} = \frac{11\,\text{kN}}{2(11250\,\text{mm}^2)} = 489\,\text{kPa} \quad \blacktriangleleft$$

$$\tau = \frac{V}{A} = \frac{\frac{P}{\sqrt{2}}}{A_0\sqrt{2}} = \frac{P}{2A_0} = \frac{11\,\text{kN}}{2(11250\,\text{mm}^2)} = 489\,\text{kPa} \quad \blacktriangleleft$$

Problem 1.38



Member ABC, which is supported by a pin and bracket at C and a cable BD, was designed to support the 16 kN load P as shown. Knowing that the ultimate load for cable BD is 100 kN, determine the factor of safety with respect to cable failure.



$$\sum M|_C = (16\,\text{kN}\sin 40°)0.6\,\text{m} + (16\,\text{kN}\cos 40°)1.2\,\text{m} - (T\sin 30°)0.4\,\text{m} - (T\cos 30°)0.6\,\text{m} = 0$$

$$\Rightarrow \quad T = 29.01385787\,\text{kN}, \quad F.S. = \frac{100\,\text{kN}}{T} = 3.45 \quad \blacktriangleleft$$

Problem 1.43

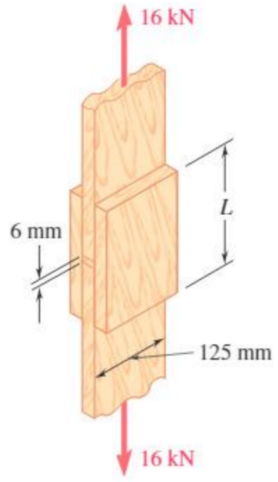
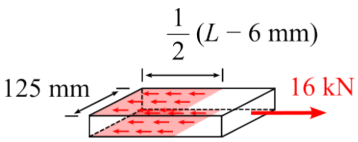


Fig. *P1.43*

Two wooden members are joined by plywood splice plates that are fully glued on the contact surfaces. Knowing that the clearance between the ends of the members is 6 mm and that the ultimate shearing stress in the glued joint is 2.5 MPa, determine the length L for which the factor of safety is 2.75 for the loading shown.



$$\tau = \frac{\tau_U}{F.S.} = \frac{2.5\,\text{MPa}}{2.75} = \frac{10}{11}\,\text{MPa}$$

$$A = 2\left\{\frac{1}{2}(L - 6\,\text{mm})\right\}(125\,\text{mm}) = (125\,\text{mm})(L - 6\,\text{mm})$$

$$\tau = \frac{16\,\text{kN}}{(125\,\text{mm})(L - 6\,\text{mm})} = \frac{10}{11}\,\text{MPa}$$

$$L = \left(\frac{11 \cdot 16 \times 10^3}{10 \cdot 125} + 6\right)\text{mm} = 146.8\,\text{mm} \quad \blacktriangleleft$$