HW2

2025-1 고체역학(박성훈 교수님)

Sample Problem 2.1, Problem 2.1, 2.3, 2.9, 2.37, 2.60

2025–12–27

숭실대학교
Soongsil University

Problem 2.1

A 2.2-m-long steel rod must not stretch more than 1.2 mm when it is subjected to an 8.5-kN tension force. Knowing that $E = 200\,\text{GPa}$, determine (a) the smallest diameter rod that should be used, (b) the corresponding normal stress in the rod.

$$P = 8.5\,\text{kN}, \quad L = 2.2\,\text{m}, \quad \delta_{\max} = 1.2\,\text{mm}, \quad A = \frac{\pi}{4}d^2$$

$$\delta = \frac{PL}{EA} \quad \Rightarrow \quad \delta_{\max} = \frac{PL}{EA_{\min}}$$

$$\Rightarrow \quad A_{\min} = \frac{PL}{E\delta_{\max}} = \frac{(8.5\,\text{kN})(2.2\,\text{m})}{(200\,\text{GPa})(1.2\,\text{mm})} = \frac{(8.5\times10^3)(2.2)}{(200\times10^9)(0.0012)}\,\text{m}^2 = 7.79167\times10^{-5}\,\text{m}^2$$

$$A = \frac{\pi}{4}d^2 \quad \Rightarrow \quad d_{\min} = \sqrt{\frac{4A_{\min}}{\pi}} = \sqrt{\frac{4(7.79167\times10^{-5})}{\pi}}\,\text{m} = 9.96025\times10^{-3}\,\text{m} = 9.96\,\text{mm}$$

$$\blacktriangle \quad (a)$$

$$\sigma_{\max} = \frac{P}{A_{\min}} = \frac{8.5\,\text{kN}}{7.79167\times10^{-5}\,\text{m}^2} = \frac{8.5\times10^3}{7.79167\times10^{-5}}\,\text{Pa} = 109.0909\times10^6\,\text{Pa} = 109.1\,\text{MPa}$$

$$\blacktriangle \quad (b)$$

Problem 2.3

A 9-m length of 6-mm-diameter steel wire is to be used in a hanger. It is observed that the wire stretches 18 mm when a tensile force P is applied. Knowing that $E = 200\,\text{GPa}$, determine (a) the magnitude of the force P, (b) the corresponding normal stress in the wire.

$$L = 9\,\text{m}, \quad d = 6\,\text{mm}, \quad \delta = 18\,\text{mm}, \quad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(6\,\text{mm})^2 = 9\pi\,\text{mm}^2$$

$$\delta = \frac{PL}{EA} \quad \Rightarrow \quad P = \frac{\delta EA}{L} = \frac{(18\,\text{mm})(200\,\text{GPa})(9\pi\,\text{mm}^2)}{9\,\text{m}} = \frac{(0.018)(200\times10^9)(9\pi\times10^{-6})}{9}\,\text{N}$$

$$= 11.30973\,\text{N} = 11.31\,\text{kN} \quad \blacktriangleleft \quad (a)$$

$$\sigma = \frac{P}{A} = \frac{11.30973\,\text{N}}{9\pi\,\text{mm}^2} = \frac{11.30973}{9\pi\times10^{-6}}\,\text{Pa} = 400.000\times10^6\,\text{Pa} = 400\,\text{MPa} \quad \blacktriangleleft \quad (b)$$

Problem 2.9

A 9-kN tensile load will be applied to a 50-m length of steel wire with $E = 200\,\text{GPa}$. Determine the smallest diameter wire that can be used, knowing that the normal stress must not exceed 150 MPa and that the increase in length of the wire must not exceed 25 mm.

For maximum δ,

$$\delta_m = \frac{PL}{EA_m} = \frac{4PL}{\pi E d_m^2} \quad \Rightarrow \quad d_m = \sqrt{\frac{4PL}{\pi E\delta_m}} = \sqrt{\frac{4(9000)(50)}{\pi(200\times10^9)(0.025)}}\,\text{m} = 10.70\,\text{mm}$$

For maximum σ,

$$\sigma_m = \frac{P}{A_m} = \frac{4P}{\pi d_m^2} \quad \Rightarrow \quad d_m = \sqrt{\frac{4P}{\pi\sigma_m}} = \sqrt{\frac{4(9000)}{\pi(150\times10^6)}}\,\text{m} = 8.74\,\text{mm}$$

Considering these two conditions,

$$10.70\,\text{mm} > 8.74\,\text{mm} \quad \Rightarrow \quad d_m = 8.74\,\text{mm} \quad \blacktriangleleft$$

Problem 2.37

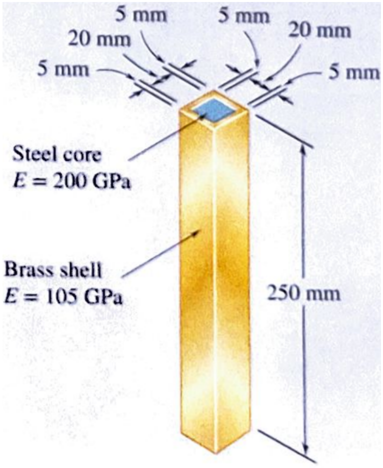

Fig. P2.37 and P2.38

An axial force of 60 kN is applied to the assembly shown by means of rigid end plates. Determine (*a*) the normal stress in the brass shell, (*b*) the corresponding deformation of the assembly.

The word 'rigid end plates' from the text means that δ is the same in both the steel core and the brass shell.

$$A_s = (20\,\text{mm})^2 = 400\,\text{mm}^2, \quad A_b = (30\,\text{mm})^2 - (20\,\text{mm})^2 = 500\,\text{mm}^2$$

$$P = 60\,\text{kN} = P_s + P_b$$

$$\delta = \frac{P_s L}{E_s A_s} = \frac{P_b L}{E_b A_b} \quad \Rightarrow \quad P_s = \frac{\delta E_s A_s}{L}, \quad P_b = \frac{\delta E_b A_b}{L} \quad \Rightarrow \quad P_s : P_b = E_s A_s : E_b A_b$$

$$\Rightarrow \quad P_b = \frac{P E_b A_b}{E_s A_s + E_b A_b} \quad \Rightarrow \quad \sigma_b = \frac{P_b}{A_b} = \frac{P E_b}{E_s A_s + E_b A_b} = \frac{P}{\frac{E_s}{E_b} A_s + A_b}$$

$$= \frac{60 \times 10^3}{\frac{200}{105}(0.0004) + 0.0005}\,\text{Pa} = 47.5472 \times 10^6\,\text{Pa} = 47.5\,\text{MPa} \quad \blacktriangleleft \quad (a)$$

$$\delta = \varepsilon_b L = \frac{\sigma_b L}{E_b} = \frac{(47.5472 \times 10^6)(0.25)}{105 \times 10^9}\,\text{m} = 0.1132\,\text{mm} \quad \blacktriangleleft \quad (b)$$

Problem 2.60

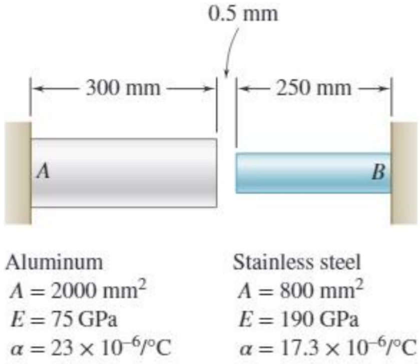

Fig. P2.60

At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature has reached 140°C, determine (*a*) the normal stress in the aluminum rod, (*b*) the change in length of the aluminum rod.

$$\Delta T = 140^\circ\text{C} - 20^\circ\text{C} = 120^\circ\text{C}$$

$$\delta_{T,A} = \alpha_a(\Delta T)L_A = (23\times10^{-6})(120)(300\,\text{mm}) = 0.828\,\text{mm}$$

$$\delta_{T,B} = \alpha_s(\Delta T)L_B = (23\times10^{-6})(120)(300\,\text{mm}) = 0.519\,\text{mm}$$

$$\delta_T = \delta_{T,A} + \delta_{T,B} = 1.347\,\text{mm}$$

$$\delta = \delta_T + \delta_P \quad \Rightarrow \quad \delta_P = \delta - \delta_T = 0.5\,\text{mm} - 1.347\,\text{mm} = -0.847\,\text{mm}$$

$$\delta_P = \delta_{P,A} + \delta_{P,B} = \frac{PL_A}{E_aA_A} + \frac{PL_B}{E_sA_B} \quad \Rightarrow \quad \delta_P = \frac{P}{A_A}\left(\frac{L_A}{E_a} + \frac{L_BA_A}{E_sA_B}\right)$$

$$\sigma_A = \frac{P}{A_A} = \delta_P\left(\frac{L_A}{E_a} + \frac{L_B}{E_s}\cdot\frac{A_A}{A_B}\right)^{-1} = (-0.847\,\text{mm})\left(\frac{300\,\text{mm}}{75\,\text{GPa}} + \frac{250\,\text{mm}}{190\,\text{GPa}}\cdot\frac{2000\,\text{mm}^2}{800\,\text{mm}^2}\right)^{-1}$$

$$= -0.847\left(\frac{300}{75} + \frac{250}{190}\cdot\frac{20}{8}\right)^{-1}\text{GPa} = -0.1161949\,\text{GPa} = -116.2\,\text{MPa} \quad \blacktriangleleft \quad (a)$$

$$\delta_{P,A} = \varepsilon_A L_A = \frac{\sigma_A L_A}{E_a} = \frac{(-0.1161949\,\text{GPa})(300\,\text{mm})}{75\,\text{GPa}} = \frac{(-0.1161949)(300)}{75}\,\text{mm} = -0.465\,\text{mm}$$

$$\delta_A = \delta_{T,A} + \delta_{P,A} = -0.465\,\text{mm} + 0.828\,\text{mm} = 0.363\,\text{mm} \quad \blacktriangleleft \quad (b)$$