

# HW2

2025-2 구조역학(박성훈 교수님)

Problem 7.132, 7.135, 8.4, 8.13, 문제1, 문제2

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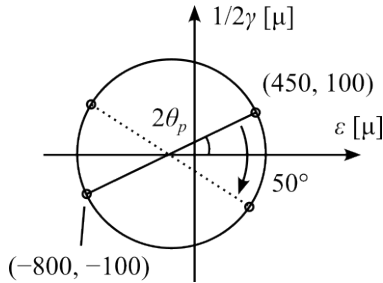
2025-12-25



### Problem 7.132

For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes  $x'$  and  $y'$  rotated through the given angle  $\theta$ .

$$\varepsilon_x = -800 \mu, \quad \varepsilon_y = +450 \mu, \quad \gamma_{xy} = +200 \mu, \quad \theta = 25^\circ \curvearrowright$$



$$c = \frac{-800 + 450}{2} \mu = -175 \mu$$

$$2\theta_p = \arctan \frac{100}{450 + 175} = 9.09028^\circ$$

$$R = \sqrt{(450 + 175)^2 + 100^2} \mu = 632.94945 \mu$$

$$\varepsilon_{x'} = c - R \cos(50^\circ - 2\theta_p) = -653 \mu \quad \blacktriangleleft$$

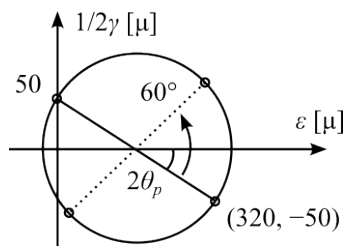
$$\varepsilon_{y'} = c + R \cos(50^\circ - 2\theta_p) = 303 \mu \quad \blacktriangleleft$$

$$\gamma_{x'y'} = -2R \sin(50^\circ - 2\theta_p) = -829 \mu \quad \blacktriangleleft$$

### Problem 7.135

For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes  $x'$  and  $y'$  rotated through the given angle  $\theta$ .

$$\varepsilon_x = 0, \quad \varepsilon_y = +320 \mu, \quad \gamma_{xy} = -100 \mu, \quad \theta = 30^\circ \curvearrowright$$



$$c = \frac{320}{2} \mu = 160 \mu$$

$$2\theta_p = \arctan \frac{50}{320 - 160} = 17.35402^\circ$$

$$R = \sqrt{(320 - 160)^2 + 50^2} \mu = 167.63055 \mu$$

$$\varepsilon_{x'} = c - R \cos(60^\circ - 2\theta_p) = 36.7 \mu \quad \blacktriangleleft$$

$$\varepsilon_{y'} = c + R \cos(60^\circ - 2\theta_p) = 283 \mu \quad \blacktriangleleft$$

$$\gamma_{x'y'} = 2R \sin(60^\circ - 2\theta_p) = 227 \mu \quad \blacktriangleleft$$

## Problem 8.4

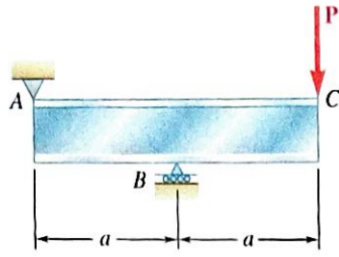
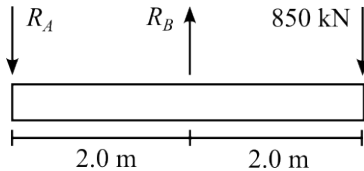


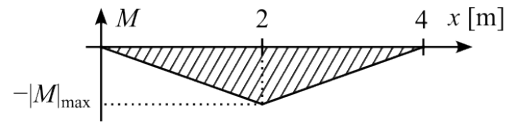
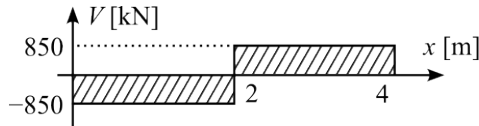
Fig. P8.3

Solve Prob. 8.3, assuming that  $P = 850 \text{ kN}$  and  $a = 2.0 \text{ m}$ .

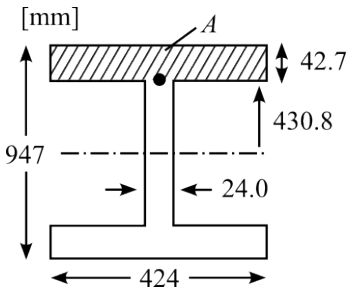
**Prob. 8.3** — An overhanging W920×449 rolled-steel beam supports a load  $P$  as shown. Knowing that  $P = 700 \text{ kN}$ ,  $a = 2.5 \text{ m}$ , and  $\sigma_{\text{all}} = 100 \text{ MPa}$ , determine (a) the maximum value of the normal stress  $\sigma_m$ , in the beam, (b) the maximum value of the principal stress  $\sigma_{\text{max}}$  at the junction of the flange and web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.



$$\begin{aligned}
 + \circlearrowleft \sum M|_B &= R_A(2 \text{ m}) - (850 \text{ kN})(2 \text{ m}) = 0 \\
 \Rightarrow R_A &= 850 \text{ kN} \\
 + \uparrow \sum F_y &= -R_A + R_B - 850 \text{ kN} = 0 \\
 \Rightarrow R_B &= 1700 \text{ kN}
 \end{aligned}$$



$$|M|_{\text{max}} = (850 \text{ kN})(2 \text{ m}) = 1700 \text{ kN} \cdot \text{m}$$



$$\begin{aligned}
 I &= 8780 \times 10^6 \text{ mm}^4 \\
 S &= 18500 \times 10^3 \text{ mm}^3 \\
 y_{\text{junction}} &= \left( \frac{947}{2} - 42.7 \right) \text{ mm} = 430.8 \text{ mm}
 \end{aligned}$$

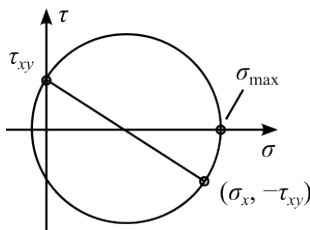
$$\begin{aligned}
 Q_A &= A\bar{y}_A = (424)(42.7) \left( 430.8 + \frac{42.7}{2} \right) \text{ mm}^3 \\
 &= 8186085.32 \text{ mm}^3
 \end{aligned}$$

$$\sigma_m = \frac{|M|_{\text{max}}}{S} = \frac{1700 \text{ kN} \cdot \text{m}}{18500 \times 10^3 \text{ mm}^3} = 91.9 \text{ MPa} \quad \blacktriangleleft \quad (a)$$

$$\sigma_{\text{junction}} = \frac{|M|_{\text{max}} y_{\text{junction}}}{I} = \frac{(1700 \text{ kN})(430.8 \text{ mm})}{8780 \times 10^6 \text{ mm}^4} = 83.41230068 \text{ MPa}$$

$$|\tau_{\text{junction}}| = \frac{VQ}{It} = \frac{(850 \text{ kN})(8186085.32 \text{ mm}^3)}{(8780 \times 10^6 \text{ mm}^4)(24 \text{ mm})} = 33.02094021 \text{ MPa}$$

$$\text{let. } \sigma_{\text{junction}} = \sigma_x, \quad |\tau_{\text{junction}}| = \tau_{xy}$$



$$\begin{aligned}
 c &= \frac{\sigma_x}{2} \\
 R &= \sqrt{c^2 + \tau_{xy}^2} = 53.19572797 \text{ MPa} \\
 \sigma_{\text{max}} &= c + R = 94.9 \text{ MPa} \quad \blacktriangleleft \quad (b)
 \end{aligned}$$

$$\sigma_{\text{all}} \Rightarrow \sigma_m < \sigma_{\text{max}} < \sigma_{\text{all}} \Rightarrow \text{The specified shape is acceptable.} \quad \blacktriangleleft \quad (c)$$

### Problem 8.13

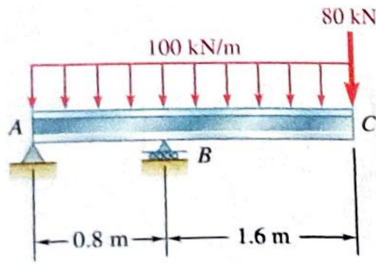
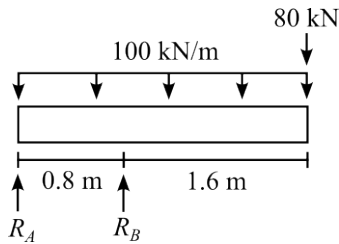


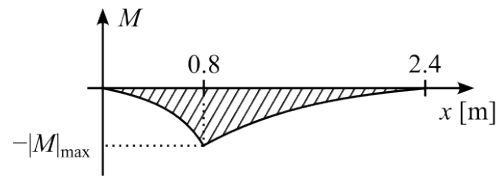
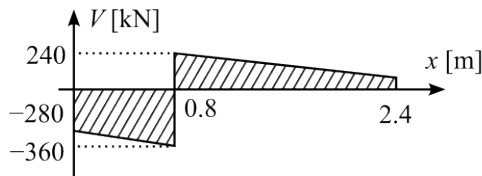
Fig. P5.77

Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement  $\sigma_m \leq \sigma_{all}$ . For the selected design, determine (a) the actual value of  $\sigma_m$  in the beam. (b) the maximum value of the principal stress  $\sigma_{max}$  at the junction of a flange and the web.

Loading of Prob. 5.77 and selected S510×98.2 shape

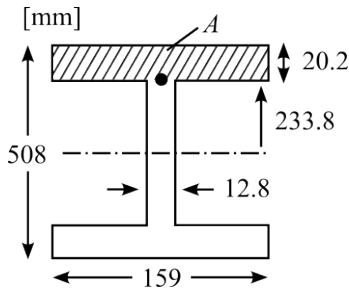


$$\begin{aligned}
 + \circlearrowleft \sum M|_A &= R_B(0.8 \text{ m}) - (80 \text{ kN})(2.4 \text{ m}) - (100 \text{ kN/m})(2.4 \text{ m})(1.2 \text{ m}) = 0 \\
 \Rightarrow R_B &= 600 \text{ kN} \\
 + \uparrow \sum F_y &= R_A + R_B - (100 \text{ kN/m})(2.4 \text{ m}) - 80 \text{ kN} = 0 \\
 \Rightarrow R_A &= -280 \text{ kN}
 \end{aligned}$$



$$|V|_m = -360 \text{ kN}$$

$$|M|_m = \frac{1}{2}(280 + 360)(0.8) \text{ kN} \cdot \text{m} = 256 \text{ kN} \cdot \text{m}$$



$$I = 495 \times 10^6 \text{ mm}^4$$

$$S = 1950 \times 10^3 \text{ mm}^3$$

$$y_j = \left( \frac{508}{2} - 20.2 \right) \text{ mm} = 233.8 \text{ mm}$$

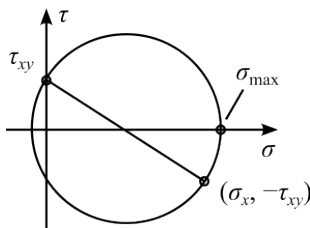
$$\begin{aligned}
 Q_A &= A\bar{y}_A = (159)(20.2) \left( 233.8 + \frac{20.2}{2} \right) \text{ mm}^3 \\
 &= 783358.02 \text{ mm}^3
 \end{aligned}$$

$$\sigma_m = \frac{|M|_m}{S} = \frac{256 \text{ kN} \cdot \text{m}}{1950 \times 10^3 \text{ mm}^3} = 131.3 \text{ MPa} \quad \blacktriangleleft \quad (a)$$

$$\sigma_j = \frac{|M|_m y_j}{I} = \frac{(256 \text{ kN})(233.8 \text{ mm})}{495 \times 10^6 \text{ mm}^4} = 120.9147475 \text{ MPa}$$

$$|\tau_j| = \frac{|V|_m Q}{It} = \frac{(360 \text{ kN})(783358.02 \text{ mm}^3)}{(495 \times 10^6 \text{ mm}^4)(12.8 \text{ mm})} = 44.50897841 \text{ MPa}$$

$$\text{let. } \sigma_j = \sigma_x, \quad |\tau_j| = \tau_{xy}$$

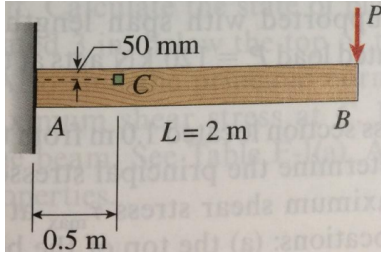


$$c = \frac{\sigma_x}{2}$$

$$R = \sqrt{c^2 + \tau_{xy}^2} = 75.07425123 \text{ MPa}$$

$$\sigma_{max} = c + R = 135.5 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

## 문제 1



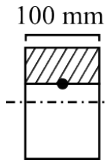
A cantilever beam with a width  $b = 100 \text{ mm}$  and depth  $h = 150 \text{ mm}$  has a length  $L = 2 \text{ m}$  and is subjected to a point load  $P = 500 \text{ N}$  at  $B$ . Calculate the state of plane stress at point  $C$  located  $50 \text{ mm}$  below the top of the beam and  $0.5 \text{ m}$  to the right of point  $A$ . Also find the principal stresses and the maximum shear stress at  $C$ . Neglect the weight of the beam.

$$R_A = 500 \text{ N } (\uparrow), \quad M_A = (500 \text{ N})(2 \text{ m}) = 1000 \text{ N} \cdot \text{m } (\circlearrowleft)$$

$$\text{at } C : \quad V = 500 \text{ N}, \quad M = -1000 \text{ N} \cdot \text{m} + (500 \text{ N})(0.5 \text{ m}) = -750 \text{ N} \cdot \text{m}$$

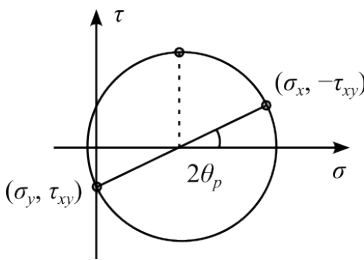
$$I = \frac{bh^3}{12} = \frac{(0.1)(0.15)^3}{12} \text{ m}^4 = 2.8125 \times 10^{-5} \text{ m}^4$$

$$\sigma_x = -\frac{My}{I} = -\frac{(-750 \text{ N} \cdot \text{m})(0.05 \text{ m})}{2.8125 \times 10^{-5} \text{ m}^4} = \frac{2000}{3} \text{ kPa} = 667 \text{ kPa} \quad \blacktriangleleft$$



$$Q = A\bar{y} = (0.1)(0.05)(0.05) \text{ m}^3 = 2.5 \times 10^{-4} \text{ m}^3$$

$$\tau_{xy} = -\frac{VQ}{It} = -\frac{(500 \text{ N})(2.5 \times 10^{-4} \text{ m}^3)}{(2.8125 \times 10^{-5} \text{ m}^4)(0.1 \text{ m})} = -\frac{400}{9} \text{ kPa} = -44.4 \text{ kPa} \quad \blacktriangleleft$$



$$c = \frac{\sigma_x}{2} = \frac{1}{2} \cdot \frac{2000}{3} \text{ kPa} = \frac{1000}{3} \text{ kPa}$$

$$\theta_p = -\frac{1}{2} \arctan \frac{-\tau_{xy}}{\sigma_x - c} = -\frac{1}{2} \arctan \frac{(\frac{400}{9})}{(\frac{1000}{3})} = -3.80^\circ \quad \blacktriangleleft$$

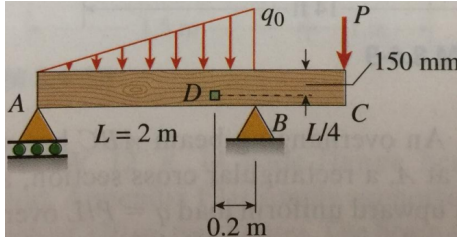
$$R = \sqrt{c^2 + \tau_{xy}^2} = 336.283 \text{ kPa}$$

$$\sigma_{\max} = c + R = 670 \text{ kPa} \quad \blacktriangleleft$$

$$\sigma_{\min} = c - R = -2.95 \text{ kPa} \quad \blacktriangleleft$$

$$\tau_{\max} = R = 336 \text{ kPa} \quad \blacktriangleleft$$

## 문제 2



Beam  $ABC$  with an overhang  $BC$  is subjected to a linearly varying distributed load on span  $AB$  with peak intensity  $q_0 = 2500 \text{ N/m}$  and a point load  $P = 1250 \text{ N}$  applied at  $C$ . The beam has a width  $b = 100 \text{ mm}$  and depth  $h = 200 \text{ mm}$ . Find the state of plane stress at point  $D$  located  $150 \text{ mm}$  below the top of the beam and  $0.2 \text{ m}$  to the left of point  $B$ . Also find the principal stresses at  $D$ . Neglect the weight of the beam.

$$+\circlearrowleft \sum M|_A = -\frac{1}{2}(2500 \text{ N/m})(2.0 \text{ m})\left(\frac{4}{3} \text{ m}\right) + R_B(2.0 \text{ m}) - (1250 \text{ N})(2.5 \text{ m}) = 0$$

$$\Rightarrow R_B = \frac{19375}{6} \text{ N}$$

$$+\uparrow \sum F_y = R_A + R_B - \frac{1}{2}(2500 \text{ N/m})(2.0 \text{ m}) - 1250 \text{ N} = 0$$

$$\Rightarrow R_A = \frac{3125}{6} \text{ N}$$

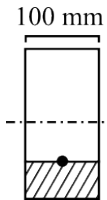
at point  $D$ ,

$$V = \frac{3125}{6} \text{ N} - \frac{1}{2}\left(\frac{1.8}{2.0} \times 2500 \text{ N/m}\right)(1.8 \text{ m}) = -\frac{9025}{6} \text{ N}$$

$$M = -\frac{1}{2}\left(\frac{1.8}{2.0} \times 2500 \text{ N/m}\right)(1.8 \text{ m})\left(\frac{1}{3} \times 1.8 \text{ m}\right) + \left(\frac{3125}{6} \text{ N}\right)(1.8 \text{ m}) = -\frac{555}{2} \text{ N} \cdot \text{m}$$

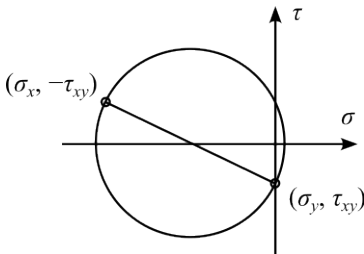
$$I = \frac{bh^3}{12} = \frac{(0.1)(0.2)^3}{12} \text{ m}^4 = \frac{2}{3} \times 10^{-4} \text{ m}^4$$

$$\sigma_x = -\frac{My}{I} = -\frac{\left(-\frac{555}{2} \text{ N} \cdot \text{m}\right)(-0.05 \text{ m})}{\frac{2}{3} \times 10^{-4} \text{ m}^4} = -\frac{1665}{8} \text{ kPa} = -208 \text{ kPa} \quad \blacktriangleleft$$



$$Q = A\bar{y} = (0.1)(0.05)(-0.075) \text{ m}^3 = 375 \times 10^{-6} \text{ m}^3$$

$$\tau_{xy} = -\frac{VQ}{It} = -\frac{\left(-\frac{9025}{6} \text{ N}\right)(375 \times 10^{-6} \text{ m}^3)}{\left(\frac{2}{3} \times 10^{-4} \text{ m}^4\right)(0.1 \text{ m})} = -\frac{5415}{64} \text{ kPa} = -84.6 \text{ kPa} \quad \blacktriangleleft$$



$$c = \frac{\sigma_x}{2} = \frac{1}{2} \cdot \left(-\frac{1665}{8} \text{ kPa}\right) = -\frac{1665}{16} \text{ kPa}$$

$$R = \sqrt{c^2 + \tau_{xy}^2} = 134.1184187 \text{ kPa}$$

$$\sigma_{\max} = c + R = 30.1 \text{ kPa} \quad \blacktriangleleft$$

$$\sigma_{\min} = c - R = -238 \text{ kPa} \quad \blacktriangleleft$$