

HW7

2025-1 고체역학(박성훈 교수님)

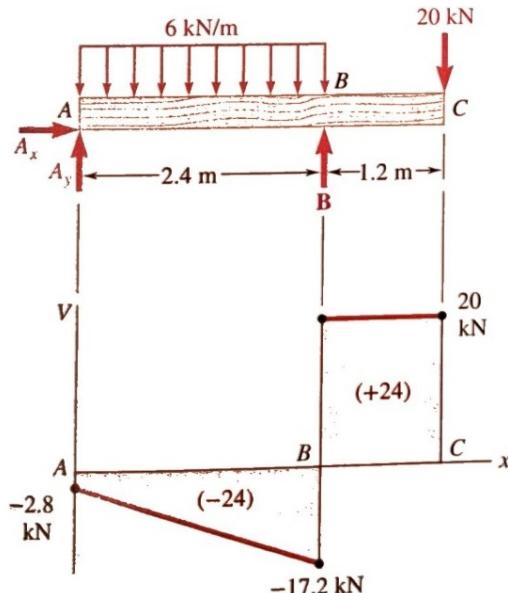
Concept Application 6.2, Problem 6.18, 6.19

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2025-12-23



Concept Application 6.2



Knowing that the allowable shearing stress for the timber beam Sample Prob. 5.7 is $\tau_{\text{all}} = 1.75 \text{ MPa}$, check that the design is acceptable from the point of view of the shearing stresses.

Recall from the shear diagram of Sample Prob. 5.7 that $V_{\max} = 20 \text{ kN}$. The actual width of the beam was given $b = 90 \text{ mm}$, and the value obtained for its depth was $h = 366 \text{ mm}$.

$$\tau_{\max} = \frac{V_{\max} Q_{\text{half}}}{bI} = \frac{V_{\max} \cdot \frac{h}{4} \cdot \frac{bh}{2}}{b \left(\frac{1}{12} bh^3 \right)} = \frac{3V_{\max}}{2bh} = \frac{3(20 \text{ kN})}{2(90 \text{ mm})(366 \text{ mm})} = 0.911 \text{ MPa}$$

$\tau_{\max} < \tau_{\text{all}}$ \Rightarrow Acceptable ◀

Problem 6.18

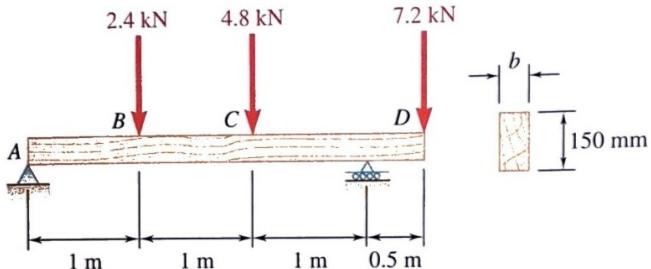


Fig. P6.18

For the beam and loading shown, determine the minimum required width b , knowing that for the grade of timber used, $\sigma_{\text{all}} = 12 \text{ MPa}$ and $\tau_{\text{all}} = 825 \text{ kPa}$.

$$+ \circlearrowleft \sum M|_A = -(2.4 \text{ kN})(1 \text{ m}) - (4.8 \text{ kN})(2 \text{ m}) + R_D(3 \text{ m}) - (7.2 \text{ kN})(3.5 \text{ m}) = 0$$

$$\Rightarrow R_D = 12.4 \text{ kN}$$

$$+ \uparrow \sum F_y = R_A + R_D - 2.4 \text{ kN} - 4.8 \text{ kN} - 7.2 \text{ kN} = 0$$

$$\Rightarrow R_A = 2 \text{ kN}$$

$$V(x) = \begin{cases} 2 \text{ kN}, & 0 \text{ m} \leq x < 1 \text{ m} \\ -0.4 \text{ kN}, & 1 \text{ m} \leq x < 2 \text{ m} \\ -5.2 \text{ kN}, & 2 \text{ m} \leq x < 3 \text{ m} \\ 7.2 \text{ kN}, & 3 \text{ m} \leq x < 3.5 \text{ m} \end{cases} \Rightarrow |V|_{\max} = 7.2 \text{ kN}$$

$$M(1 \text{ m}) = (2 \text{ kN})(1 \text{ m}) = 2 \text{ kN} \cdot \text{m}$$

$$M(3 \text{ m}) = -(7.2 \text{ kN})(0.5 \text{ m}) = -3.6 \text{ kN} \cdot \text{m}$$

$$\Rightarrow |M|_{\max} = 3.6 \text{ kN} \cdot \text{m}$$

$$\sigma_{\max} = \frac{|M|_{\max} c}{I} = \frac{3|M|_{\max}}{2bc^2} \leq \sigma_{\text{all}}$$

$$b \geq \frac{3|M|_{\max}}{2\sigma_{\text{all}}c^2} = \frac{3(3.6 \text{ kN} \cdot \text{m})}{2(12 \text{ MPa})(75 \text{ mm})^2} = \frac{3(3.6)}{2(12)(75)^2} \times 10^6 \text{ mm} = 80.0 \text{ mm}$$

$$\tau_{\max, \text{avg}} = \frac{|V|_{\max} Q}{tI} = \frac{3|V|_{\max}}{2bh} \leq \tau_{\text{all}}$$

$$b \geq \frac{|V|_{\max} Q}{tI} = \frac{3|V|_{\max}}{2\tau_{\text{all}}h} = \frac{3(7.2 \text{ kN})}{2(0.825 \text{ MPa})(150 \text{ mm})} = \frac{3(7.2)}{2(0.825)(150)} \times 10^3 \text{ mm} = 87.3 \text{ mm}$$

$$\Rightarrow b_{\min} = 87.3 \text{ mm}, \quad \text{Use } b = 88 \text{ mm} \quad \blacktriangleleft$$

Problem 6.19

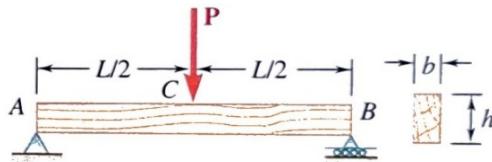


Fig. P6.19

A timber beam AB of length L and rectangular cross section carries a single concentrated load P at its midpoint C . (a) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $h/2L$, where h and L are, respectively, the depth and the length of the beam. (b) Determine the depth h and the width b of the beam, knowing that $L = 2\text{ m}$, $P = 40\text{ kN}$, $\tau_m = 960\text{ kPa}$, and $\sigma_m = 12\text{ MPa}$.

$$R_A = R_B = \frac{P}{2}$$

$$V(x) = \begin{cases} \frac{P}{2} & \left(0 \leq x < \frac{L}{2}\right) \\ -\frac{P}{2} & \left(\frac{L}{2} \leq x < L\right) \end{cases} \Rightarrow |V|_{\max} = V_m = \frac{P}{2}, \quad |M|_{\max} = M(L/2) = M_m = \frac{PL}{4}$$

$$\tau_m = \frac{3V_m}{2bh} = \frac{3P}{8bc}, \quad \sigma_m = \frac{M_m c}{I} = \frac{3M_m}{2bc^2} = \frac{3PL}{8bc^2}$$

$$\frac{\tau_m}{\sigma_m} = \frac{c}{L} = \frac{h}{2L} \quad \blacktriangleleft \quad (a)$$

$$h = \frac{\tau_m}{\sigma_m} \cdot 2L = \frac{0.96 \text{ MPa}}{12 \text{ MPa}} \cdot 2(2 \text{ m}) = 0.32 \text{ m} = 320 \text{ mm}$$

$$b = \frac{3P}{4h\tau_m} = \frac{3(40 \text{ kN})}{4(320 \text{ mm})(0.96 \text{ MPa})} = \frac{3(40)}{4(320)(0.96)} \times 10^3 \text{ mm} = 97.7 \text{ mm} \quad \blacktriangleleft \quad (b)$$