

HW1

2025-2 구조역학(박성훈 교수님)

Problem 7.2, 7.7, 7.31, 7.32, 7.36, 7.40, 7.43, 7.63

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Problem 7.2

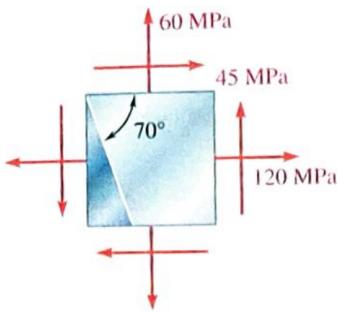


Fig. P7.2

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.1A.

$$\theta = 20^\circ, \quad \sigma_x = 120 \text{ MPa}, \quad \sigma_y = 60 \text{ MPa}, \quad \tau_{xy} = 45 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta = 90 + 30 \cos 40^\circ + 45 \sin 40^\circ = 141.9 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta = -30 \sin 40^\circ + 45 \cos 40^\circ = 15.19 \text{ MPa} \quad \blacktriangleleft$$

Problem 7.7

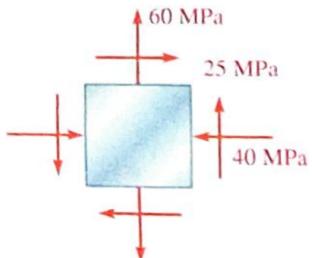


Fig. P7.7 and P7.11

For the given state of stress, determine (a) the principal planes, (b) the principal stresses.

$$\sigma_x = -40 \text{ MPa}, \quad \sigma_y = 60 \text{ MPa}, \quad \tau_{xy} = 25 \text{ MPa}$$

$$\tau_{x'y'}(\theta_p) = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta_p + \tau_{xy} \cos 2\theta_p = 0 \quad \blacktriangleleft \quad (b)$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}, \quad \theta_p = \frac{1}{2} \arctan \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2} \arctan \frac{2(25)}{-40 - 60} = -13.28^\circ \quad \blacktriangleleft \quad (a)$$

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 10 \pm \sqrt{50^2 + 25^2} = 10 \pm 25\sqrt{5} \text{ [MPa]}$$

$$\sigma_{x'} = \sigma_{\min} = 10 - 25\sqrt{5} = -45.9 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

$$\sigma_{y'} = \sigma_{\max} = 10 + 25\sqrt{5} = 65.9 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

Problem 7.31

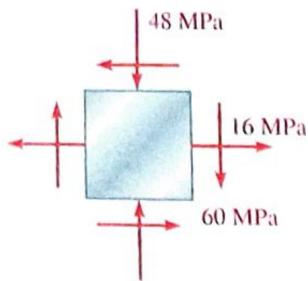
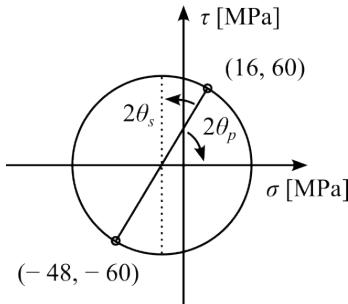


Fig. P7.5 and P7.9

Solve Probs. 7.5 and 7.9, using Mohr's circle.

Prob. 7.5 — For the given state of stress, determine (a) the principal planes, (b) the principal stresses.

Prob. 7.9 — For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.



$$(\sigma_x, -\tau_{xy}) = (16, 60), \quad (\sigma_y, \tau_{xy}) = (-48, -60) \text{ [MPa]} \\ (\text{center of circle}) = (-16, 0)$$

$$\theta_p = -\frac{1}{2} \arctan \frac{60}{32} = -31.0^\circ \quad \blacktriangleleft \quad (a) \text{ of prob. 7.5}$$

$$R = \sqrt{32^2 + 60^2} = 68$$

$$\sigma_{x'} = \sigma_{\max} = -16 + 68 = 52.0 \text{ MPa} \quad \blacktriangleleft \quad (b) \text{ of prob. 7.5}$$

$$\sigma_{y'} = \sigma_{\min} = -16 - 68 = -84.0 \text{ MPa} \quad \blacktriangleleft \quad (b) \text{ of prob. 7.5}$$

$$\tau_{x'y'} = 0 \quad \blacktriangleleft \quad (b) \text{ of prob. 7.5}$$

$$\theta_s = \frac{1}{2} \arctan \frac{32}{60} = 14.04^\circ \quad \blacktriangleleft \quad (a) \text{ of prob. 7.9}$$

$$\tau_{\max} = R = 68.0 \text{ MPa} \quad \blacktriangleleft \quad (b) \text{ of prob. 7.9}$$

$$\sigma_s = -16.00 \text{ MPa} \quad \blacktriangleleft \quad (c) \text{ of prob. 7.9}$$

Problem 7.32

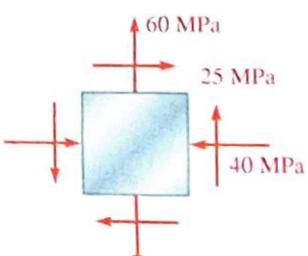
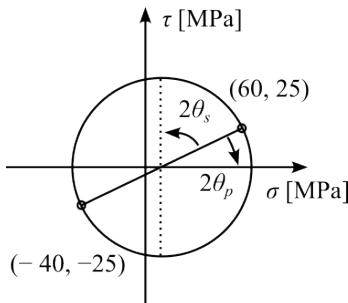


Fig. P7.7 and P7.11

Solve Probs. 7.7 and 7.11, using Mohr's circle.

Prob. 7.7 — For the given state of stress, determine (a) the principal planes, (b) the principal stresses.

Prob. 7.11 — For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.



$$(\sigma_x, -\tau_{xy}) = (-40, -25), \quad (\sigma_y, \tau_{xy}) = (60, 25) \text{ [MPa]} \\ (\text{center of circle}) = (10, 0)$$

$$\theta_p = -\frac{1}{2} \arctan \frac{25}{50} = -13.28^\circ \quad \blacktriangleleft \quad (a) \text{ of prob. 7.7}$$

$$R = \sqrt{25^2 + 50^2} = 55.9$$

$$\sigma_{x'} = \sigma_{\min} = 10 - 55.9 = -45.9 \text{ MPa} \quad \blacktriangleleft \quad (b) \text{ of prob. 7.7}$$

$$\sigma_{y'} = \sigma_{\max} = 10 + 55.9 = 65.9 \text{ MPa} \quad \blacktriangleleft \quad (b) \text{ of prob. 7.7}$$

$$\tau_{x'y'} = 0 \quad \blacktriangleleft \quad (b) \text{ of prob. 7.7}$$

$$\theta_s = \frac{1}{2} \arctan \frac{50}{25} = 31.7^\circ \quad \blacktriangleleft \quad (a) \text{ of prob. 7.11}$$

$$\tau_{\max} = R = 55.9 \text{ MPa} \quad \blacktriangleleft \quad (b) \text{ of prob. 7.11}$$

$$\sigma_s = 10.00 \text{ MPa} \quad \blacktriangleleft \quad (c) \text{ of prob. 7.11}$$

Problem 7.36

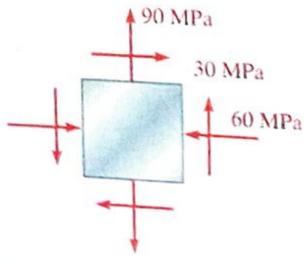
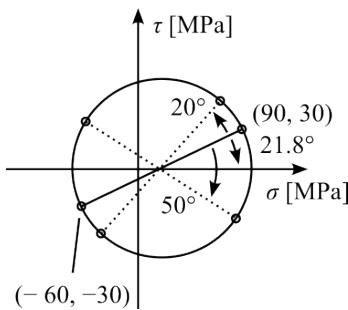


Fig. P7.14

Solve Prob. 7.14, using Mohr's circle.

Prob. 7.14 — For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a) 25° clockwise, (b) 10° counterclockwise.



$$(\sigma_x, -\tau_{xy}) = (-60, -30), \quad (\sigma_y, \tau_{xy}) = (90, 30) \text{ [MPa]} \\ (\text{center of circle}) = (15, 0)$$

$$2\theta_p = -\arctan \frac{30}{75} = -21.80141^\circ$$

$$R = \sqrt{75^2 + 30^2} = 80.77747$$

$$(a) \text{ when } \theta = -25^\circ, \quad \sigma_{x'} = 15 - R \cos(50^\circ - 2\theta_p) = -56.2 \text{ MPa} \quad \blacktriangleleft \quad (a)$$

$$\sigma_{y'} = 15 + R \cos(50^\circ - 2\theta_p) = 86.2 \text{ MPa} \quad \blacktriangleleft \quad (a)$$

$$\tau_{x'y'} = -R \sin(50^\circ - 2\theta_p) = -38.2 \text{ MPa} \quad \blacktriangleleft \quad (a)$$

$$(b) \text{ when } \theta = 10^\circ,$$

$$\sigma_{x'} = 15 - R \cos(20^\circ + 2\theta_p) = -45.2 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

$$\sigma_{y'} = 15 + R \cos(20^\circ + 2\theta_p) = 75.2 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

$$\tau_{x'y'} = R \sin(20^\circ + 2\theta_p) = 53.8 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

Problem 7.40

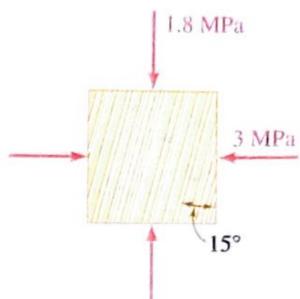
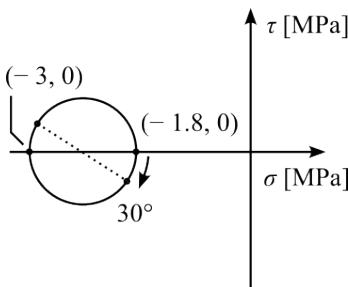


Fig. P7.18

Solve Prob. 7.18, using Mohr's circle.

Prob. 7.18 — The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.



$$(\sigma_x, -\tau_{xy}) = (-3, 0), \quad (\sigma_y, \tau_{xy}) = (-1.8, 0) \text{ [MPa]}$$

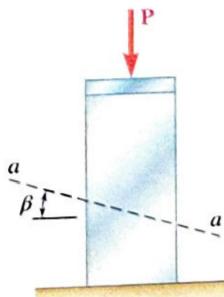
$$\theta = -15^\circ, \quad (\text{center of circle}) = (-2.4, 0)$$

$$R = \frac{(-1.8) - (-3)}{2} = 0.6$$

$$\tau_{x'y'} = -0.6 \sin 30^\circ = -0.300 \text{ MPa} \quad \blacktriangleleft \quad (a)$$

$$\sigma_{x'} = -2.4 - 0.6 \cos 30^\circ = -2.92 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

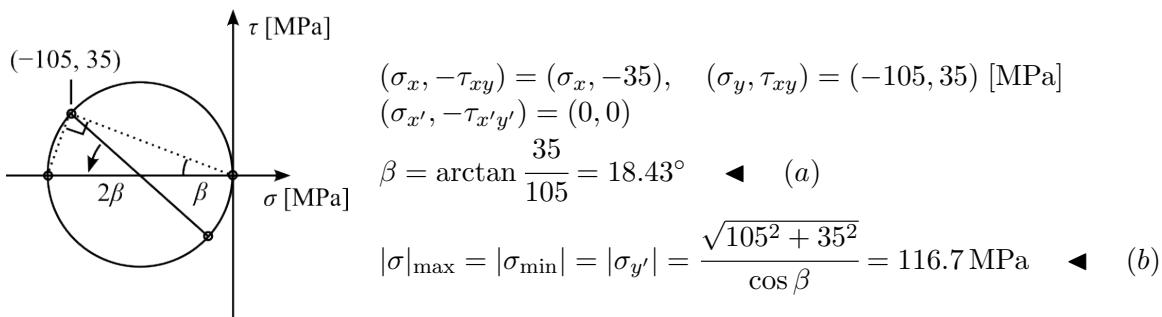
Problem 7.43



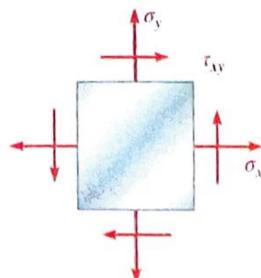
Solve Prob. 7.21, using Mohr's circle.

Prob. 7.21 — The centric force P is applied to a short post as shown. Knowing that the stresses on plane $a-a$ are $\sigma = -105 \text{ MPa}$ and $\tau = 35 \text{ MPa}$, determine (a) the angle β that plane $a-a$ forms with the horizontal, (b) the maximum compressive stress in the post.

Fig. P7.21



Problem 7.63



For the state of stress shown, it is known that the normal and shearing stresses are directed as shown and that $\sigma_x = 98 \text{ MPa}$, $\sigma_y = 63 \text{ MPa}$, and $\sigma_{\min} = 35 \text{ MPa}$. Determine (a) the orientation of the principal planes, (b) the principal stress σ_{\max} , (c) the maximum in-plane shearing stress.

Fig. P7.63

