HW1

2025-2 구조역학(박성훈 교수님)

Problem 7.2, 7.7, 7.31, 7.32, 7.36, 7.40, 7.43, 7.63

2025–12–27

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Problem 7.2

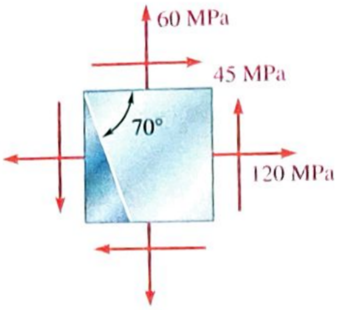


Fig. P7.2

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.1A.

$$\theta = 20^\circ, \quad \sigma_x = 120\,\text{MPa}, \quad \sigma_y = 60\,\text{MPa}, \quad \tau_{xy} = 45\,\text{MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta = 90 + 30\cos 40^\circ + 45\sin 40^\circ = 141.9\,\text{MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta = -30\sin 40^\circ + 45\cos 40^\circ = 15.19\,\text{MPa} \quad \blacktriangleleft$$

Problem 7.7

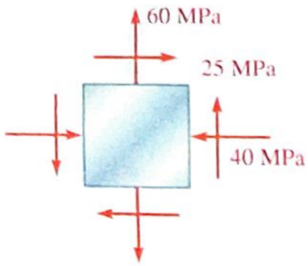


Fig. P7.7 and P7.11

For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

$$\sigma_x = -40\,\text{MPa}, \quad \sigma_y = 60\,\text{MPa}, \quad \tau_{xy} = 25\,\text{MPa}$$

$$\tau_{x'y'}(\theta_p) = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta_p + \tau_{xy}\cos 2\theta_p = 0 \quad \blacktriangleleft \quad (b)$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}, \quad \theta_p = \frac{1}{2}\arctan\frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2}\arctan\frac{2(25)}{-40-60} = -13.28^\circ \quad \blacktriangleleft \quad (a)$$

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 10 \pm \sqrt{50^2 + 25^2} = 10 \pm 25\sqrt{5} \;\; [\text{MPa}]$$

$$\sigma_{x'} = \sigma_{\min} = 10 - 25\sqrt{5} = -45.9\,\text{MPa} \quad \blacktriangleleft \quad (b)$$

$$\sigma_{y'} = \sigma_{\max} = 10 + 25\sqrt{5} = 65.9\,\text{MPa} \quad \blacktriangleleft \quad (b)$$

Problem 7.31

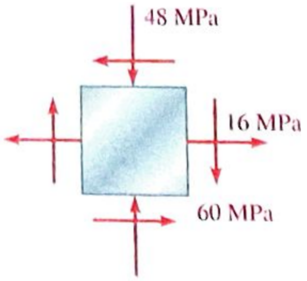
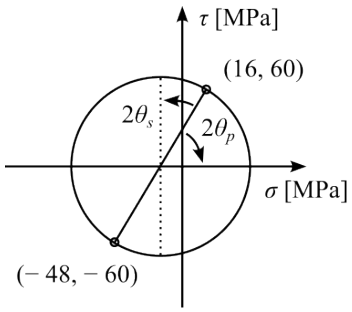


Fig. P7.5 and P7.9

Solve Probs. 7.5 and 7.9, using Mohr's circle.

Prob. 7.5 — For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

Prob. 7.9 — For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.



$(\sigma_x, -\tau_{xy}) = (16, 60), \quad (\sigma_y, \tau_{xy}) = (-48, -60)$ [MPa]

(center of circle) $= (-16, 0)$

$\theta_p = -\dfrac{1}{2}\arctan\dfrac{60}{32} = -31.0°$ ◀ (*a*) of prob. 7.5

$R = \sqrt{32^2 + 60^2} = 68$

$\sigma_{x'} = \sigma_{\max} = -16 + 68 = 52.0\,\text{MPa}$ ◀ (*b*) of prob. 7.5

$\sigma_{y'} = \sigma_{\min} = -16 - 68 = -84.0\,\text{MPa}$ ◀ (*b*) of prob. 7.5

$\tau_{x'y'} = 0$ ◀ (*b*) of prob. 7.5

$\theta_s = \dfrac{1}{2}\arctan\dfrac{32}{60} = 14.04°$ ◀ (*a*) of prob. 7.9

$\tau_{\max} = R = 68.0\,\text{MPa}$ ◀ (*b*) of prob. 7.9

$\sigma_s = -16.00\,\text{MPa}$ ◀ (*c*) of prob. 7.9

Problem 7.32

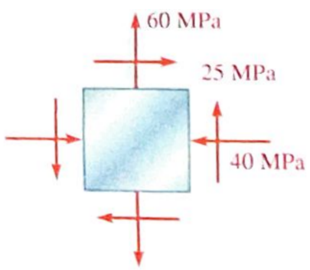
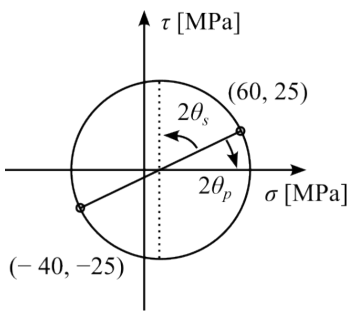


Fig. P7.7 and P7.11

Solve Probs. 7.7 and 7.11, using Mohr's circle.

Prob. 7.7 — For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

Prob. 7.11 — For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.



$(\sigma_x, -\tau_{xy}) = (-40, -25), \quad (\sigma_y, \tau_{xy}) = (60, 25)$ [MPa]

(center of circle) $= (10, 0)$

$\theta_p = -\dfrac{1}{2}\arctan\dfrac{25}{50} = -13.28°$ ◀ (*a*) of prob. 7.7

$R = \sqrt{25^2 + 50^2} = 55.9$

$\sigma_{x'} = \sigma_{\min} = 10 - 55.9 = -45.9\,\text{MPa}$ ◀ (*b*) of prob. 7.7

$\sigma_{y'} = \sigma_{\max} = 10 + 55.9 = 65.9\,\text{MPa}$ ◀ (*b*) of prob. 7.7

$\tau_{x'y'} = 0$ ◀ (*b*) of prob. 7.7

$\theta_s = \dfrac{1}{2}\arctan\dfrac{50}{25} = 31.7°$ ◀ (*a*) of prob. 7.11

$\tau_{\max} = R = 55.9\,\text{MPa}$ ◀ (*b*) of prob. 7.11

$\sigma_s = 10.00\,\text{MPa}$ ◀ (*c*) of prob. 7.11

Problem 7.36

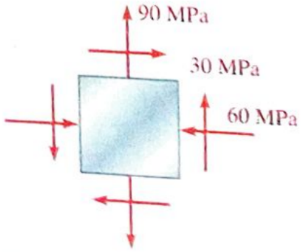
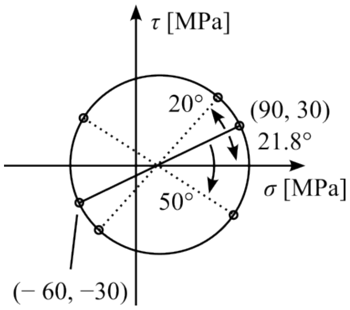


Fig. P7.14

Solve Prob. 7.14, using Mohr's circle.

Prob. 7.14 — For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



$(\sigma_x, -\tau_{xy}) = (-60, -30), \quad (\sigma_y, \tau_{xy}) = (90, 30)$ [MPa]
(center of circle) $= (15, 0)$

$$2\theta_p = -\arctan\frac{30}{75} = -21.80141°$$

$$R = \sqrt{75^2 + 30^2} = 80.77747$$

(*a*) when $\theta = -25°$,

$\sigma_{x'} = 15 - R\cos(50° - 2\theta_p) = -56.2\,\text{MPa}$ ◄ (*a*)

$\sigma_{y'} = 15 + R\cos(50° - 2\theta_p) = 86.2\,\text{MPa}$ ◄ (*a*)

$\tau_{x'y'} = -R\sin(50° - 2\theta_p) = -38.2\,\text{MPa}$ ◄ (*a*)

(*b*) when $\theta = 10°$,

$\sigma_{x'} = 15 - R\cos(20° + 2\theta_p) = -45.2\,\text{MPa}$ ◄ (*b*)

$\sigma_{y'} = 15 + R\cos(20° + 2\theta_p) = 75.2\,\text{MPa}$ ◄ (*b*)

$\tau_{x'y'} = R\sin(20° + 2\theta_p) = 53.8\,\text{MPa}$ ◄ (*b*)

Problem 7.40

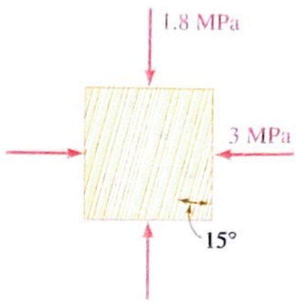
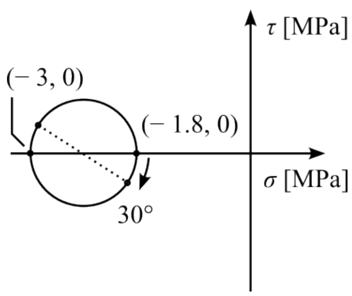


Fig. P7.18

Solve Prob. 7.18, using Mohr's circle.

Prob. 7.18 — The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.



$(\sigma_x, -\tau_{xy}) = (-3, 0), \quad (\sigma_y, \tau_{xy}) = (-1.8, 0)$ [MPa]
$\theta = -15°, \quad$ (center of circle) $= (-2.4, 0)$

$$R = \frac{(-1.8) - (-3)}{2} = 0.6$$

$\tau_{x'y'} = -0.6\sin 30° = -0.300\,\text{MPa}$ ◄ (*a*)

$\sigma_{x'} = -2.4 - 0.6\cos 30° = -2.92\,\text{MPa}$ ◄ (*b*)

Problem 7.43

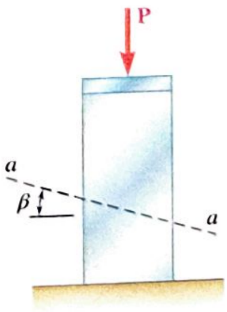
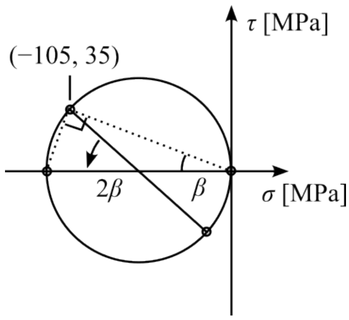


Fig. P7.21

Solve Prob. 7.21, using Mohr's circle.

Prob. 7.21 — The centric force P is applied to a short post as shown. Knowing that the stresses on plane *a-a* are $\sigma = -105\,\text{MPa}$ and $\tau = 35\,\text{MPa}$, determine (*a*) the angle β that plane *a-a* forms with the horizontal, (*b*) the maximum compressive stress in the post.



$(\sigma_x, -\tau_{xy}) = (\sigma_x, -35), \quad (\sigma_y, \tau_{xy}) = (-105, 35)$ [MPa]
$(\sigma_{x'}, -\tau_{x'y'}) = (0, 0)$

$$\beta = \arctan\frac{35}{105} = 18.43^\circ \quad \blacktriangleleft \quad (a)$$

$$|\sigma|_{\max} = |\sigma_{\min}| = |\sigma_{y'}| = \frac{\sqrt{105^2 + 35^2}}{\cos\beta} = 116.7\,\text{MPa} \quad \blacktriangleleft \quad (b)$$

Problem 7.63

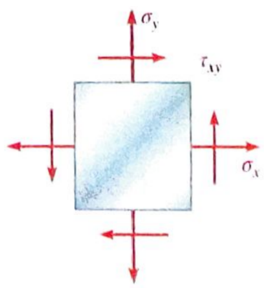
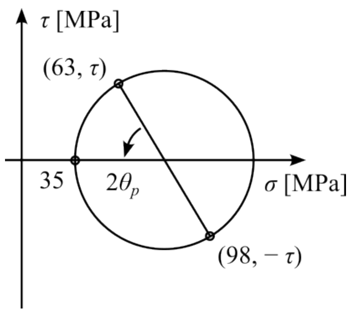


Fig. P7.63

For the state of stress shown, it is known that the normal and shearing stresses are directed as shown and that $\sigma_x = 98\,\text{MPa}$, $\sigma_y = 63\,\text{MPa}$, and $\sigma_{\min} = 35\,\text{MPa}$. Determine (*a*) the orientation of the principal planes, (*b*) the principal stress $\sigma_{\max}$, (*c*) the maximum in-plane shearing stress.



$(\sigma_x, -\tau_{xy}) = (-98, -\tau), \quad (\sigma_y, \tau_{xy}) = (63, \tau)$ [MPa] $\quad (\tau > 0)$
(center of circle) $= (80.5, 0)$
$R = 80.5 - 35 = 45.5$

$$\theta_p = \frac{1}{2}\arccos\frac{80.5 - 63}{45.5} = 33.7^\circ \quad \blacktriangleleft \quad (a)$$

$$\sigma_{\max} = 80.5 + 45.5 = 126.0\,\text{MPa} \quad \blacktriangleleft \quad (b)$$
$$\tau_{\max} = R = 45.5\,\text{MPa} \quad \blacktriangleleft \quad (c)$$