

# HW1

2025-2 구조역학(박성훈 교수님)

Problem 7.2, 7.7, 7.31, 7.32, 7.36, 7.40, 7.43, 7.63

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## Problem 7.2

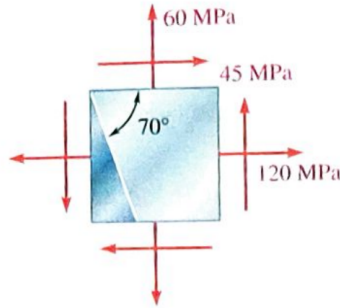


Fig. P7.2

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.1A.

$$\theta = 20^\circ, \quad \sigma_x = 120 \text{ MPa}, \quad \sigma_y = 60 \text{ MPa}, \quad \tau_{xy} = 45 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta = 90 + 30 \cos 40^\circ + 45 \sin 40^\circ = 141.9 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta = -30 \sin 40^\circ + 45 \cos 40^\circ = 15.19 \text{ MPa} \quad \blacktriangleleft$$

## Problem 7.7

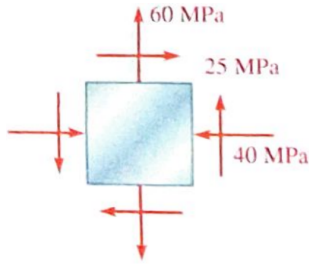


Fig. P7.7 and P7.11

For the given state of stress, determine (a) the principal planes, (b) the principal stresses.

$$\sigma_x = -40 \text{ MPa}, \quad \sigma_y = 60 \text{ MPa}, \quad \tau_{xy} = 25 \text{ MPa}$$

$$\tau_{x'y'}(\theta_p) = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta_p + \tau_{xy} \cos 2\theta_p = 0 \quad \blacktriangleleft \quad (b)$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}, \quad \theta_p = \frac{1}{2} \arctan \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{1}{2} \arctan \frac{2(25)}{-40 - 60} = -13.28^\circ \quad \blacktriangleleft \quad (a)$$

$$\sigma_{\max, \min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 10 \pm \sqrt{50^2 + 25^2} = 10 \pm 25\sqrt{5} \text{ [MPa]}$$

$$\sigma_{x'} = \sigma_{\min} = 10 - 25\sqrt{5} = -45.9 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

$$\sigma_{y'} = \sigma_{\max} = 10 + 25\sqrt{5} = 65.9 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

### Problem 7.31

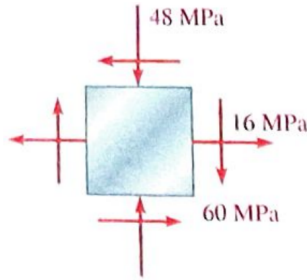
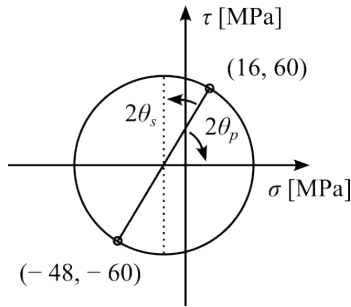


Fig. P7.5 and P7.9

Solve Probs. 7.5 and 7.9, using Mohr's circle.

**Prob. 7.5** — For the given state of stress, determine (a) the principal planes, (b) the principal stresses.

**Prob. 7.9** — For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.



$$(\sigma_x, -\tau_{xy}) = (16, 60), \quad (\sigma_y, \tau_{xy}) = (-48, -60) \text{ [MPa]}$$

$$(\text{center of circle}) = (-16, 0)$$

$$\theta_p = -\frac{1}{2} \arctan \frac{60}{32} = -31.0^\circ \quad \blacktriangleleft \quad (a) \text{ of prob. 7.5}$$

$$R = \sqrt{32^2 + 60^2} = 68$$

$$\sigma_{x'} = \sigma_{\max} = -16 + 68 = 52.0 \text{ MPa} \quad \blacktriangleleft \quad (b) \text{ of prob. 7.5}$$

$$\sigma_{y'} = \sigma_{\min} = -16 - 68 = -84.0 \text{ MPa} \quad \blacktriangleleft \quad (b) \text{ of prob. 7.5}$$

$$\tau_{x'y'} = 0 \quad \blacktriangleleft \quad (b) \text{ of prob. 7.5}$$

$$\theta_s = \frac{1}{2} \arctan \frac{32}{60} = 14.04^\circ \quad \blacktriangleleft \quad (a) \text{ of prob. 7.9}$$

$$\tau_{\max} = R = 68.0 \text{ MPa} \quad \blacktriangleleft \quad (b) \text{ of prob. 7.9}$$

$$\sigma_s = -16.00 \text{ MPa} \quad \blacktriangleleft \quad (c) \text{ of prob. 7.9}$$

### Problem 7.32

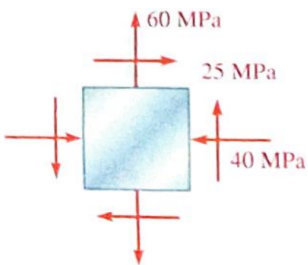
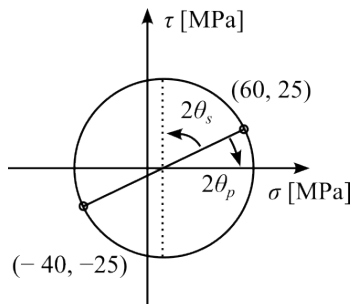


Fig. P7.7 and P7.11

Solve Probs. 7.7 and 7.11, using Mohr's circle.

**Prob. 7.7** — For the given state of stress, determine (a) the principal planes, (b) the principal stresses.

**Prob. 7.11** — For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress.



$$(\sigma_x, -\tau_{xy}) = (-40, -25), \quad (\sigma_y, \tau_{xy}) = (60, 25) \text{ [MPa]}$$

$$(\text{center of circle}) = (10, 0)$$

$$\theta_p = -\frac{1}{2} \arctan \frac{25}{50} = -13.28^\circ \quad \blacktriangleleft \quad (a) \text{ of prob. 7.7}$$

$$R = \sqrt{25^2 + 50^2} = 55.9$$

$$\sigma_{x'} = \sigma_{\min} = 10 - 55.9 = -45.9 \text{ MPa} \quad \blacktriangleleft \quad (b) \text{ of prob. 7.7}$$

$$\sigma_{y'} = \sigma_{\max} = 10 + 55.9 = 65.9 \text{ MPa} \quad \blacktriangleleft \quad (b) \text{ of prob. 7.7}$$

$$\tau_{x'y'} = 0 \quad \blacktriangleleft \quad (b) \text{ of prob. 7.7}$$

$$\theta_s = \frac{1}{2} \arctan \frac{50}{25} = 31.7^\circ \quad \blacktriangleleft \quad (a) \text{ of prob. 7.11}$$

$$\tau_{\max} = R = 55.9 \text{ MPa} \quad \blacktriangleleft \quad (b) \text{ of prob. 7.11}$$

$$\sigma_s = 10.00 \text{ MPa} \quad \blacktriangleleft \quad (c) \text{ of prob. 7.11}$$

### Problem 7.36

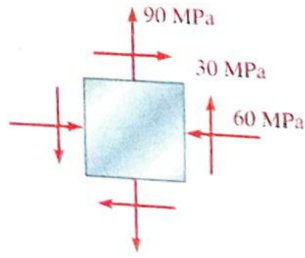
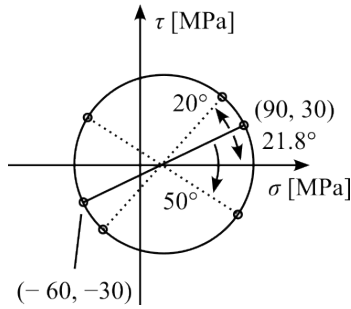


Fig. P7.14

Solve Prob. 7.14, using Mohr's circle.

**Prob. 7.14** — For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (a)  $25^\circ$  clockwise, (b)  $10^\circ$  counterclockwise.



$$(\sigma_x, -\tau_{xy}) = (-60, -30), \quad (\sigma_y, \tau_{xy}) = (90, 30) \text{ [MPa]}$$

$$(\text{center of circle}) = (15, 0)$$

$$2\theta_p = -\arctan \frac{30}{75} = -21.80141^\circ$$

$$R = \sqrt{75^2 + 30^2} = 80.77747$$

$$(a) \text{ when } \theta = -25^\circ,$$

$$\sigma_{x'} = 15 - R \cos(50^\circ - 2\theta_p) = -56.2 \text{ MPa} \quad \blacktriangleleft \quad (a)$$

$$\sigma_{y'} = 15 + R \cos(50^\circ - 2\theta_p) = 86.2 \text{ MPa} \quad \blacktriangleleft \quad (a)$$

$$\tau_{x'y'} = -R \sin(50^\circ - 2\theta_p) = -38.2 \text{ MPa} \quad \blacktriangleleft \quad (a)$$

$$(b) \text{ when } \theta = 10^\circ,$$

$$\sigma_{x'} = 15 - R \cos(20^\circ + 2\theta_p) = -45.2 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

$$\sigma_{y'} = 15 + R \cos(20^\circ + 2\theta_p) = 75.2 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

$$\tau_{x'y'} = R \sin(20^\circ + 2\theta_p) = 53.8 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

### Problem 7.40

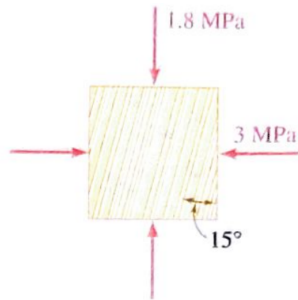
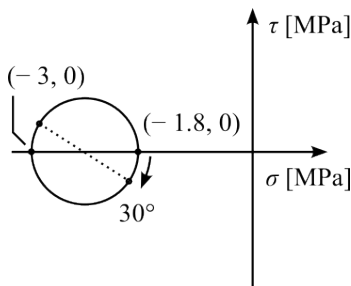


Fig. P7.18

Solve Prob. 7.18, using Mohr's circle.

**Prob. 7.18** — The grain of a wooden member forms an angle of  $15^\circ$  with the vertical. For the state of stress shown, determine (a) the in-plane shearing stress parallel to the grain, (b) the normal stress perpendicular to the grain.



$$(\sigma_x, -\tau_{xy}) = (-3, 0), \quad (\sigma_y, \tau_{xy}) = (-1.8, 0) \text{ [MPa]}$$

$$\theta = -15^\circ, \quad (\text{center of circle}) = (-2.4, 0)$$

$$R = \frac{(-1.8) - (-3)}{2} = 0.6$$

$$\tau_{x'y'} = -0.6 \sin 30^\circ = -0.300 \text{ MPa} \quad \blacktriangleleft \quad (a)$$

$$\sigma_{x'} = -2.4 - 0.6 \cos 30^\circ = -2.92 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

### Problem 7.43

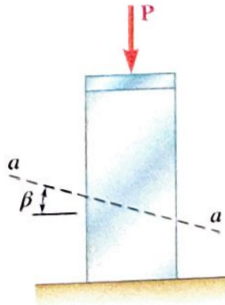
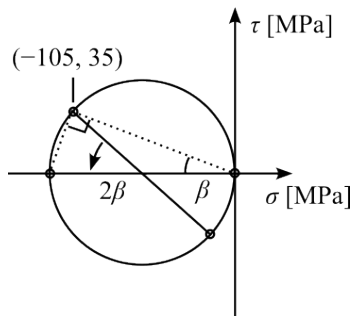


Fig. P7.21

Solve Prob. 7.21, using Mohr's circle.

**Prob. 7.21** — The centric force  $P$  is applied to a short post as shown. Knowing that the stresses on plane  $a-a$  are  $\sigma = -105$  MPa and  $\tau = 35$  MPa, determine (a) the angle  $\beta$  that plane  $a-a$  forms with the horizontal, (b) the maximum compressive stress in the post.



$$(\sigma_x, -\tau_{xy}) = (\sigma_x, -35), \quad (\sigma_y, \tau_{xy}) = (-105, 35) \text{ [MPa]}$$

$$(\sigma_{x'}, -\tau_{x'y'}) = (0, 0)$$

$$\beta = \arctan \frac{35}{105} = 18.43^\circ \quad \blacktriangleleft \quad (a)$$

$$|\sigma|_{\max} = |\sigma_{\min}| = |\sigma_{y'}| = \frac{\sqrt{105^2 + 35^2}}{\cos \beta} = 116.7 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

### Problem 7.63

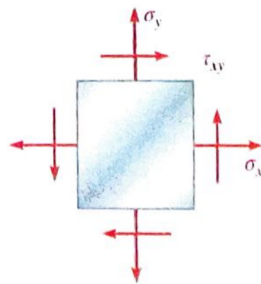
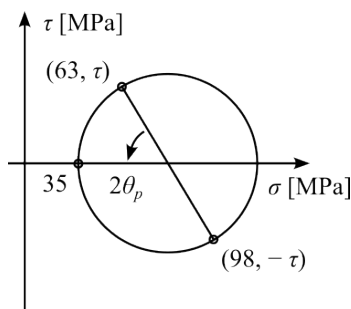


Fig. P7.63

For the state of stress shown, it is known that the normal and shearing stresses are directed as shown and that  $\sigma_x = 98$  MPa,  $\sigma_y = 63$  MPa, and  $\sigma_{\min} = 35$  MPa. Determine (a) the orientation of the principal planes, (b) the principal stress  $\sigma_{\max}$ , (c) the maximum in-plane shearing stress.



$$(\sigma_x, -\tau_{xy}) = (-98, -\tau), \quad (\sigma_y, \tau_{xy}) = (63, \tau) \text{ [MPa]} \quad (\tau > 0)$$

$$(\text{center of circle}) = (80.5, 0)$$

$$R = 80.5 - 35 = 45.5$$

$$\theta_p = \frac{1}{2} \arccos \frac{80.5 - 63}{45.5} = 33.7^\circ \quad \blacktriangleleft \quad (a)$$

$$\sigma_{\max} = 80.5 + 45.5 = 126.0 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

$$\tau_{\max} = R = 45.5 \text{ MPa} \quad \blacktriangleleft \quad (c)$$