해설

고체역학(박성훈 교수님) 2025-1 중간고사

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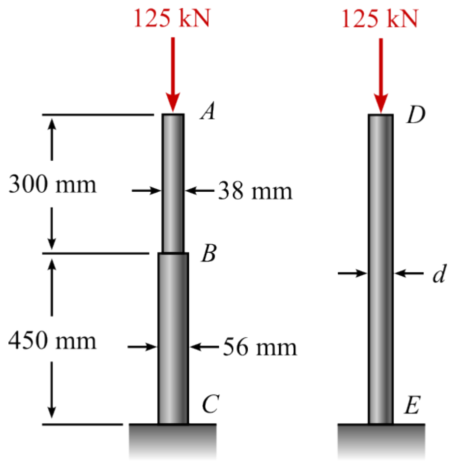
Question 1 — prob.2.125



The aluminum rod ABC ($E = 70\,\text{GPa}$), which consists of two cylindrical portions AB and BC, is to be replaced with a cylindrical steel rod DE ($E = 200\,\text{GPa}$) of the same overall length. Determine the minimum required diameter d of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 165 MPa.

$$\delta_{AC} = \frac{PL_{AB}}{A_{AC}E_{\text{Al}}} + \frac{PL_{BC}}{A_{BC}E_{\text{Al}}} = \frac{P}{E_{\text{Al}}}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$$

$$= \frac{125\,\text{kN}}{70\,\text{GPa}}\left(\frac{300\,\text{mm}}{\frac{\pi}{4}(38\,\text{mm})^2} + \frac{450\,\text{mm}}{\frac{\pi}{4}(56\,\text{mm})^2}\right) = 0.7986193387\,\text{mm}$$

$$\delta_{DE} = \frac{PL_{AB}}{\frac{\pi}{4}d^2E_{\text{st}}} \leq \delta_{AC} \qquad \left(\because A_{DE} = \frac{\pi}{4}d^2\right)$$

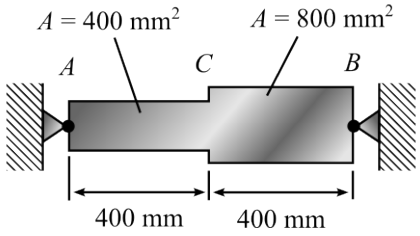
$$d \geq \sqrt{\frac{4PL_{AB}}{\pi E_{\text{st}}\delta_{AC}}} = \sqrt{\frac{4(125\,\text{kN})(750\,\text{mm})}{\pi(200\,\text{GPa})(0.7986193387\,\text{mm})}} = 27.34\,\text{mm}$$

$$\sigma_{DE} = \frac{P}{\frac{\pi}{4}d^2} \leq \sigma_{\text{all}}$$

$$d \geq \sqrt{\frac{4P}{\pi\sigma_{\text{all}}}} = \sqrt{\frac{4(125\,\text{kN})}{\pi(165\,\text{MPa})}} = \sqrt{\frac{4(125\times10^3)}{\pi(165)}}\,\text{mm} = 31.06\,\text{mm}$$

$$\left.\begin{array}{l} d \geq 27.34\,\text{mm} \\ d \geq 31.06\,\text{mm} \end{array}\right\} \quad \Rightarrow \quad d_{\text{max}} = 31.06\,\text{mm} \quad \blacktriangleleft$$

Question 2 — variation of concept application 2.6



Determine the values of the stress in portions AC and CB of the steel bar shown (Fig. 2.28a) when the temperature of the bar is $-50°\mathrm{C}$, knowing that a close fit exists at both of the rigid supports when the temperature is $20°\mathrm{C}$. Use the values $E = 200\,\mathrm{GPa}$ and $\alpha = 11.7 \times 10^{-6}/°\mathrm{C}$ for steel.

$$\Delta T = -50°\mathrm{C} - 20°\mathrm{C} = -70°\mathrm{C}$$

$$\delta_T = \alpha \Delta T L_{AC} + \alpha \Delta T L_{CB} = 2(11.7 \times 10^{-6})(-70°\mathrm{C})(400\,\mathrm{mm}) = -0.6552\,\mathrm{mm}$$

$$\delta_P = \frac{PL_{AC}}{A_{AC}E_{\mathrm{st}}} + \frac{PL_{CB}}{A_{CB}E_{\mathrm{st}}} = \frac{PL}{E_{\mathrm{st}}}\left(\frac{1}{A_{AC}} + \frac{1}{A_{CB}}\right) = P\left(\frac{400\,\mathrm{mm}}{200\,\mathrm{GPa}}\right)\left(\frac{1}{400\,\mathrm{mm}^2} + \frac{1}{800\,\mathrm{mm}^2}\right)$$

$$= 7.5 \times 10^{-3}\,\mathrm{mm/kN}$$

$$\delta_T + \delta_P = 0 \quad \Rightarrow \quad -0.6552\,\mathrm{mm} + 7.5 \times 10^{-3}\,\mathrm{mm/kN} = 0$$

$$P = \frac{0.6552\,\mathrm{mm}}{7.5 \times 10^{-3}\,\mathrm{mm}}\,\mathrm{kN} = 87.36\,\mathrm{kN}$$

$$\sigma_{AC} = \frac{P}{A_{AC}} = \frac{87.36\,\mathrm{kN}}{400\,\mathrm{mm}^2} = 218.40\,\mathrm{MPa} \quad \blacktriangleleft$$

$$\sigma_{CB} = \frac{P}{A_{CB}} = \frac{87.36\,\mathrm{kN}}{800\,\mathrm{mm}^2} = 109.20\,\mathrm{MPa} \quad \blacktriangleleft$$

Question 3

Drive the general Hook's law about normal stress and shearing stress. Also, describe the components of the normal stress that generate each term.

For one component of normal stress,

$$
\begin{aligned}
&\text{by } \sigma_x: \quad \varepsilon_x = \frac{\sigma_x}{E}, \quad \varepsilon_y = -\frac{\nu\sigma_x}{E}, \quad \varepsilon_z = -\frac{\nu\sigma_x}{E} \\
&\text{by } \sigma_y: \quad \varepsilon_x = -\frac{\nu\sigma_y}{E}, \quad \varepsilon_y = \frac{\sigma_y}{E}, \quad \varepsilon_z = -\frac{\nu\sigma_y}{E} \\
&\text{by } \sigma_y: \quad \varepsilon_x = -\frac{\nu\sigma_z}{E}, \quad \varepsilon_y = -\frac{\nu\sigma_z}{E}, \quad \varepsilon_z = \frac{\sigma_z}{E}
\end{aligned}
$$

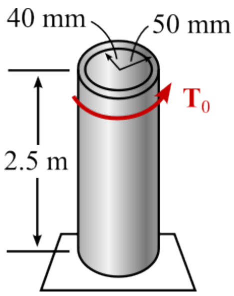
For three components of normal stress,

$$
\begin{array}{ccccc}
\varepsilon_x & = & +\dfrac{\sigma_x}{E} & -\dfrac{\nu\sigma_y}{E} & -\dfrac{\nu\sigma_z}{E} \\
\varepsilon_y & = & -\dfrac{\nu\sigma_x}{E} & +\dfrac{\sigma_y}{E} & -\dfrac{\nu\sigma_z}{E} \\
\varepsilon_z & = & -\dfrac{\nu\sigma_x}{E} & -\dfrac{\nu\sigma_y}{E} & +\dfrac{\sigma_z}{E} \\
 & & \uparrow & \uparrow & \uparrow \\
 & & \text{by } \sigma_x & \text{by } \sigma_y & \text{by } \sigma_z
\end{array}
$$

For three components of shearing stress, they do not affect each other. Therefore,

$$
\gamma_{xy} = \frac{\tau_{xy}}{G}, \quad \gamma_{yz} = \frac{\tau_{yz}}{G}, \quad \gamma_{zx} = \frac{\tau_{zx}}{G}
$$

Question 4 — variation of prob.3.34



For the aluminum pipe shown ($G = 27\,\text{GPa}$), determine (*a*) the torque $\mathbf{T}_0$ causing an angle of twist of $5°$. Determine (*b*) the angle of twist if the same torque $\mathbf{T}_0$ is applied to a solid cylindrical shaft of the same length and cross-section.

$$\phi = 5° = 5 \times \frac{\pi}{180} = \frac{\pi}{36}\ (\text{rad})$$

$$\phi = \frac{TL}{GJ}, \quad T = \frac{G\phi J}{L}$$

$$J = \frac{\pi}{2}\left\{(50\,\text{mm})^4 - (40\,\text{mm})^4\right\} = 5.796238446 \times 10^{-6}\,\text{m}^4$$

$$T_o = \frac{(27\,\text{GPa})\left(\frac{\pi}{36}\right)(5.796238446 \times 10^{-6}\,\text{m}^4)}{2.5\,\text{m}} = 5.46\,\text{kN} \quad ◀ \quad (a)$$

$$T_0 = 5.462826036\,\text{kN}\cdot\text{m}$$

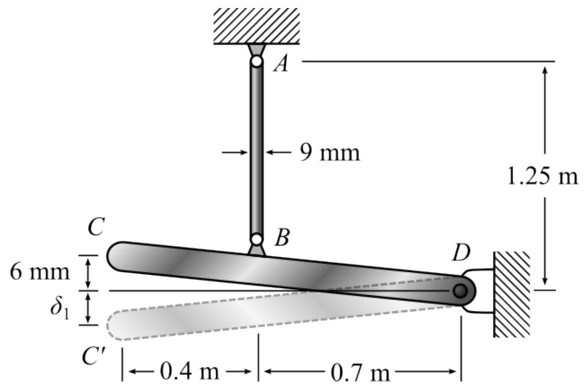
$$A = \pi\left(50^2 - 40^2\right)\text{mm}^2 = \pi r^2$$

$$r = \sqrt{50^2 - 40^2}\,\text{mm} = 30\,\text{mm}$$

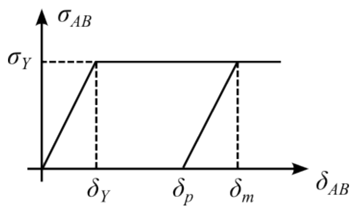
$$J' = \frac{\pi}{2}(30\,\text{mm})^4 = 1.272345025 \times 10^{-6}\,\text{m}^4$$

$$\phi' = \frac{T_0 L}{GJ'} = \frac{(5.462826036\,\text{kN}\cdot\text{m})(2.5\,\text{m})}{(27\,\text{GPa})(1.272345025 \times 10^{-6}\,\text{m}^4)} = 0.3975472185\ \text{rad} = 22.78° \quad ◀ \quad (b)$$

Question 5 — variation of prob.2.104

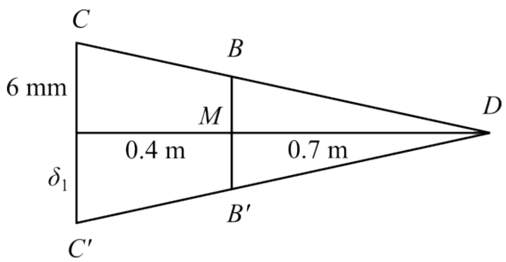


Rod AB is made of a mild steel that is assumed to be elastoplastic with $E = 200\,\text{GPa}$ and $\sigma_Y = 250\,\text{MPa}$. After the rod has been attached to the rigid lever CD, it is found that end C is 6 mm too high. A vertical force Q is then applied at C until this point has moved to position C'. Determine the required magnitude of Q and the deflection δ_1 if the lever is to *snap* back to a horizontal position after Q is removed.



$$\delta_p = \delta_m - \delta_Y \quad \Rightarrow \quad \delta_Y = \delta_m - \delta_p$$

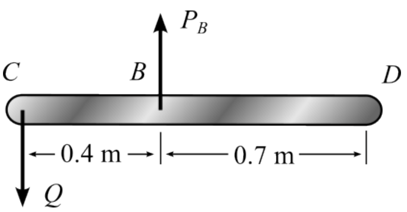
$$\delta_m = \overline{BB'}, \quad \delta_p = \overline{BM}$$



$$\delta_Y = \overline{MB'} = \frac{0.7\,\text{m}}{1.1\,\text{m}}\delta_1 = \frac{7}{11}\delta_1$$

$$\delta_Y = \frac{\sigma_Y L}{E} = \frac{(250\times 10^6)(1.25)}{200\times 10^9}\,\text{m} = 1.5625\,\text{mm}$$

$$1.5625\,\text{mm} = \frac{7}{11}\delta_1 \quad \Rightarrow \quad \delta_1 = 2.46\,\text{mm} \quad \blacktriangleleft$$



$$+\circlearrowleft \sum M|_D = -P(0.7\,\text{m}) + Q(1.1\,\text{m}) = 0$$

$$\Rightarrow \quad Q = \frac{0.7\,\text{m}}{1.1\,\text{m}}P = \frac{7}{11}P$$

$$P_m = P_Y = \sigma_Y A$$

$$Q_m = \frac{7}{11}P_m = \frac{7}{11}\sigma_Y A = \frac{7}{11}(250\,\text{MPa})\left\{\frac{\pi}{4}(9\,\text{mm})^2\right\} = 10.12\,\text{kN} \quad \blacktriangleleft$$