HW4

2025-2 구조역학(박성훈 교수님)

Problem 9.95, 9.102, 9.103, 9.109, 9.110

2025–12–27

숭실대학교
Soongsil University

Use the moment-area method to solve the following problems.

Problem 9.95

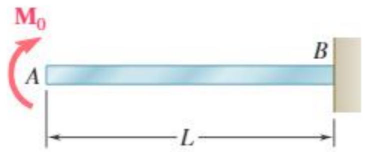
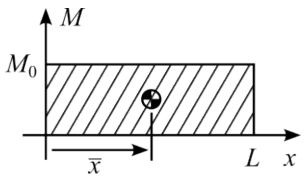


Fig. P9.95

For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.



$$A_m = M_0 L, \quad \bar{x} = \frac{L}{2}$$

$$\theta_{A/B} = \theta_A = -\frac{A_m}{EI} = -\frac{M_0 L}{EI} \quad \blacktriangleleft \quad (a)$$

$$t_{A/B} = y_A = \frac{A_m \bar{x}}{EI} = \frac{M_0 L^2}{2EI} \quad \blacktriangleleft \quad (b)$$

Problem 9.102

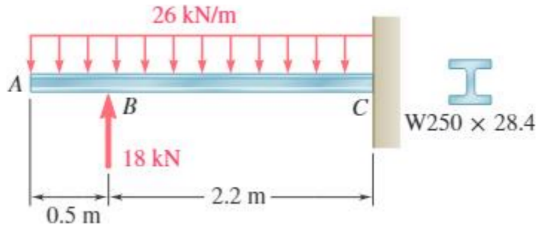
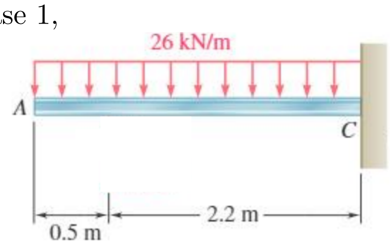


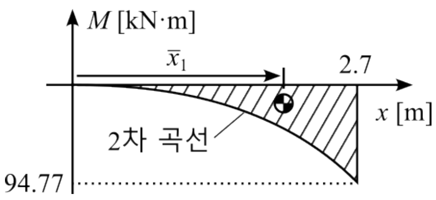
Fig. P9.102

For the cantilever beam and loading shown, determine (*a*) the slope at point A, (*b*) the deflection at point A. Use $E = 200\,\text{GPa}$.

$$I = 40.1 \times 10^6\,\text{mm}^4 = 40.1 \times 10^{-6}\,\text{m}^4$$

Case 1,





$$V_A = 0$$

$$M_C = -(26\,\text{kN/m})(2.7\,\text{m})(1.35\,\text{m}) = 94.77\,\text{kN}\cdot\text{m}$$

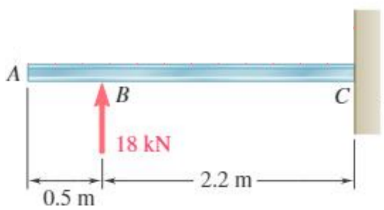
$$A_{m1} = \frac{1}{3}(94.77)(2.7)\,\text{kN}\cdot\text{m}^2 = 85.293\,\text{kN}\cdot\text{m}^2$$

$$\bar{x}_1 = \frac{3}{4}(2.7\,\text{m}) = 2.025\,\text{m}$$

$$\theta_{A/C,1} = \theta_{A1} = \frac{A_{m1}}{EI} = 10.63504 \times 10^{-3}$$

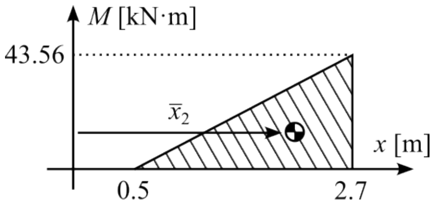
$$t_{A/C,1} = y_{A1} = -\frac{A_{m1}\bar{x}_1}{EI} = -21.5360\,\text{mm}$$

Case 2,



In portion AB, $\quad V = 0$

$$M_C = (18\,\text{kN})(2.2\,\text{m}) = 39.6\,\text{kN}\cdot\text{m}$$



$$A_{m2} = \frac{1}{2}(39.6)(2.2)\,\mathrm{kN}\cdot\mathrm{m}^2 = 43.56\,\mathrm{kN}\cdot\mathrm{m}^2$$

$$\bar{x}_2 = \frac{2}{3}(2.2\,\mathrm{m}) + 0.5\,\mathrm{m} = \frac{59}{30}\,\mathrm{m}$$

$$\theta_{A/C,2} = \theta_{A2} = -\frac{A_{m2}}{EI} = -5.43142 \times 10^{-3}$$

$$t_{A/C,2} = y_{A2} = \frac{A_{m2}\bar{x}_2}{EI} = 10.68180\,\mathrm{mm}$$

$$\theta_A = \theta_{A1} + \theta_{A2} = 5.20 \times 10^{-3} \quad ◀ \quad (a)$$

$$y_A = y_{A1} + y_{A2} = -10.85\,\mathrm{mm} \quad ◀ \quad (b)$$

Problem 9.103

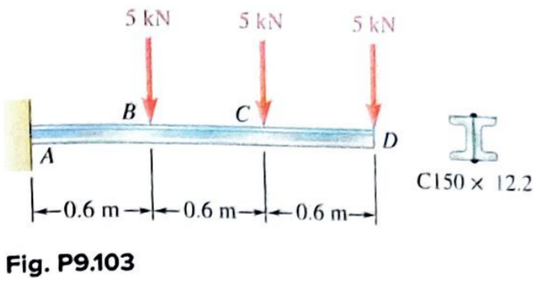
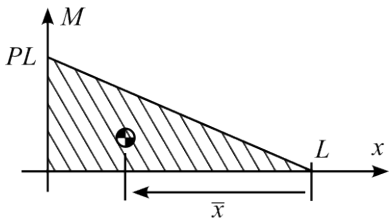


Fig. P9.103

Two C150×12.2 channels are welded back to back and loaded as shown. Knowing that $E = 200\,\mathrm{GPa}$, determine (a) the slope at point D. (b) the deflection at point D.

$$P = 5\,\mathrm{kN}, \quad I = 2 \times 5.45 \times 10^6\,\mathrm{mm}^4 = 10.9 \times 10^{-6}\,\mathrm{m}^4$$

For a single concentrated load P that is applied at $x = L$(point E),



$$A_m = \frac{PL^2}{2}, \quad \bar{x} = \frac{2L}{3}$$

$$\theta_{E/A} = \theta_E(L) = -\frac{A_m}{EI} = -\frac{PL^2}{2EI}$$

$$t_{E/A} = y_E(L) = -\frac{A_m\bar{x}}{EI} = -\frac{PL^3}{3EI}$$

Use superpositions.

$$\theta_D = \theta_E(0.6\,\mathrm{m}) + \theta_E(1.2\,\mathrm{m}) + \theta_E(1.8\,\mathrm{m}) = -5.78 \times 10^{-3} \quad ◀ \quad (a)$$

$$y_D = y_E(0.6\,\mathrm{m}) + 1.2\,\mathrm{m}\cdot\theta_E(0.6\,\mathrm{m}) + y_E(1.2\,\mathrm{m}) + 0.6\,\mathrm{m}\cdot\theta_E(1.2\,\mathrm{m}) + y_E(1.8\,\mathrm{m})$$
$$= -7.43\,\mathrm{mm} \quad ◀ \quad (b)$$

Problem 9.109

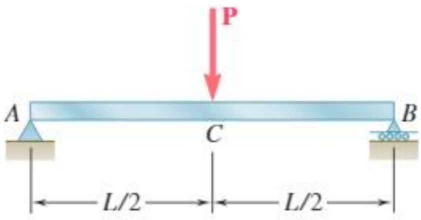
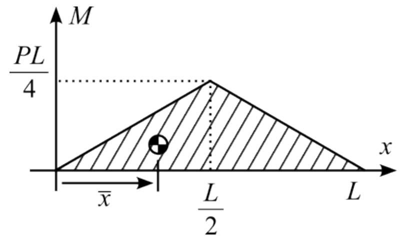


Fig. P9.109

For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.

In portion AC,



$$A_m = \frac{PL^2}{16}, \quad \bar{x} = \frac{2}{3}\left(\frac{L}{2}\right) = \frac{L}{3}$$

$$\theta_{A/C} = \theta_A = -\frac{A_m}{EI} = -\frac{PL^2}{16EI} \quad \blacktriangleleft \quad (a)$$

$$-t_{A/C} = y_C = -\frac{A_m \bar{x}'}{EI} = -\frac{PL^3}{48EI} \quad \blacktriangleleft \quad (b)$$

Problem 9.110

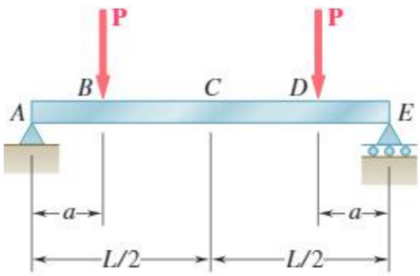
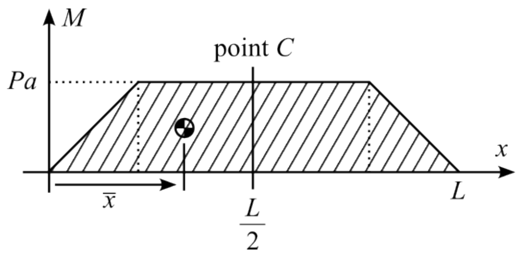


Fig. P9.110

For the prismatic beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.

In portion AC,



$$A_m = \frac{1}{2}(Pa)(a) + Pa\left(\frac{L}{2} - a\right) = \frac{Pa}{2}(L - a)$$

$$\bar{x}' = \frac{L}{2} - \bar{x} = \frac{A_t \bar{x}'_t + A_s \bar{x}'_s}{A_m} = \frac{\frac{Pa^2}{2} \cdot \frac{2a}{3} + Pa\left(\frac{L}{2} - a\right) \cdot \left\{\frac{1}{2}\left(\frac{L}{2} - a\right) + a\right\}}{\frac{Pa}{2}(L - a)} = \frac{3L^2 - 4a^2}{12(L - a)}$$

$$\theta_{A/C} = \theta_A = -\frac{A_m}{EI} = -\frac{Pa(L - a)}{2EI} \quad \blacktriangleleft \quad (a)$$

$$t_{A/C} = y_C = -\frac{A_m \bar{x}'}{EI} = \frac{Pa(4a^2 - 3L^2)}{24EI} \quad \blacktriangleleft \quad (b)$$