

Solutions

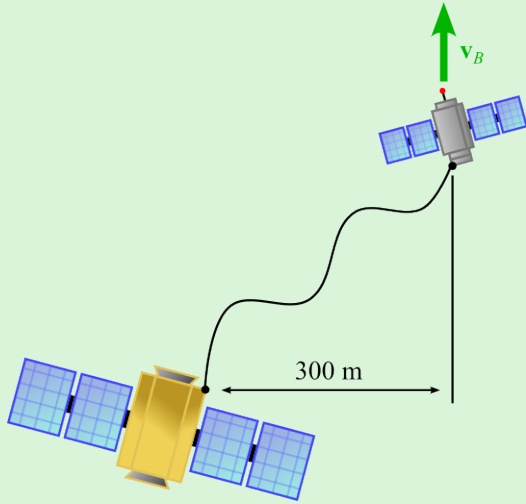
of the Dynamics Final Exam 2023-1

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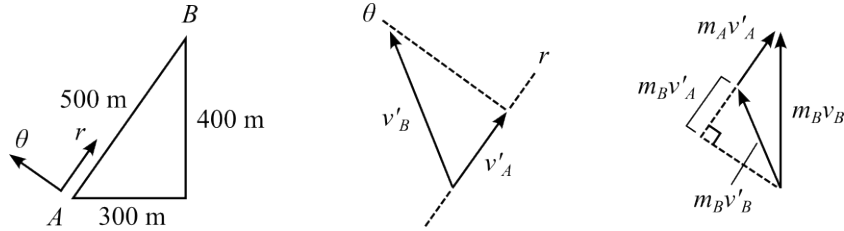
June 25, 2025



Question 1



The 2-kg sub-satellite B has an initial velocity $\mathbf{v}_B = 3 \text{ m/s} \mathbf{j}$. It is connected to the 20-kg base-satellite A by a 500-m space tether. Determine the velocity of the base satellite and sub-satellite immediately after the tether becomes taut (assuming no rebound).



$$\frac{m_B v_B}{5} = \frac{m_A v'_A + m_B v'_B}{4} = \frac{\sqrt{(m_B v'_B)^2 - (m_B v'_A)^2}}{3}$$

$$\begin{cases} 4m_B v_B = 5v'_A(m_A + m_B) \\ 3v_B = 5\sqrt{v'^2_B - v'^2_A} \end{cases}$$

$$v'_A = \frac{4m_B v_B}{5(m_A + m_B)} = \frac{4(2)(3)}{5(20 + 2)} \text{ m/s} = \frac{12}{55} \text{ m/s} = 0.218 \text{ m/s}$$

$$v'_B = \sqrt{\frac{9}{25} v_B^2 + v'^2_A} = \sqrt{\frac{9}{25} (3)^2 + \left(\frac{12}{55}\right)^2} \text{ m/s} = 1.813 \text{ m/s}$$

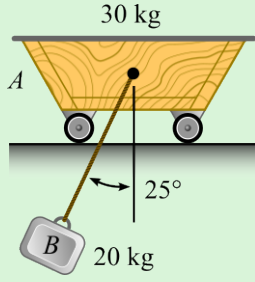
$$\theta_A = \arctan \frac{400 \text{ m}}{300 \text{ m}} = \arctan \frac{4}{3} = 53.1^\circ$$

$$\theta_B = \theta_A + \arccos \frac{v'_A}{v'_B} = 53.1^\circ + \arccos \frac{0.218}{1.813} = 136.2^\circ$$

$$\mathbf{v}_A = 0.218 \text{ m/s} \angle 53.1^\circ \quad \blacktriangleleft$$

$$\mathbf{v}_B = 1.813 \text{ m/s} \angle 136.2^\circ \quad \blacktriangleleft$$

Question 2



A 20-kg block B is suspended from a 2-m cord attached to a 30-kg cart A , which may roll freely on a frictionless, horizontal track. If the system is released from rest in the position shown, determine the velocities of A and B as B passes directly under A .

Let the state 1 be the initial state and the state 2 be when B passes directly under A .

$$\begin{aligned}
 T_1 + V_{g1} &= T_2 + V_{g2} \\
 m_B gl(1 - \cos 25^\circ) &= \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \\
 m_A v_A^2 + m_B v_B^2 &= 2m_B gl(1 - \cos 25^\circ)
 \end{aligned} \tag{1}$$

Impulses by gravity and ground are independent to horizontal component of momentum of A and B .

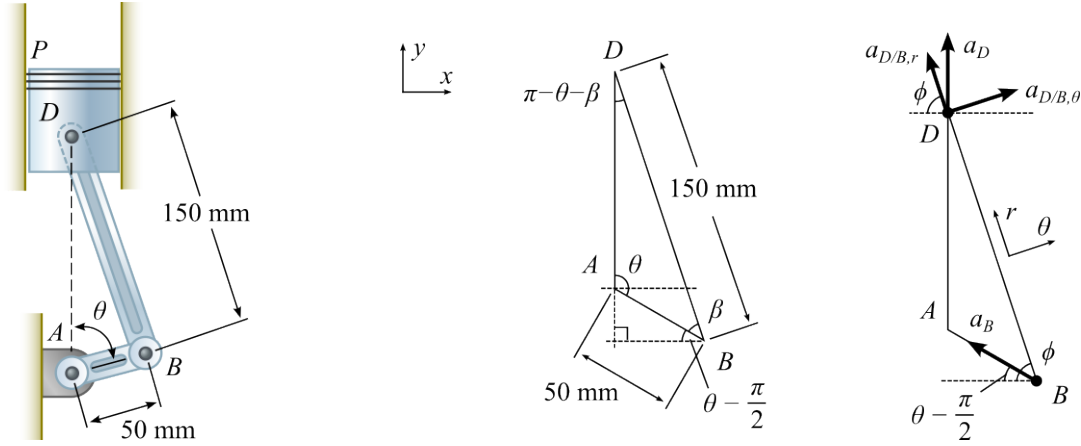
$$\begin{aligned}
 \sum p_{x1} &= \sum p_{x2} \\
 0 &= -m_A v_A + m_B v_B \\
 \begin{cases} m_A v_A^2 + m_B v_B^2 = 2m_B gl(1 - \cos 25^\circ) \\ -m_A v_A + m_B v_B = 0 \end{cases} & \tag{1 \& 2} \\
 m_A v_A &= m_B v_B \\
 m_A m_B v_A^2 + m_A^2 v_A^2 &= 2m_B^2 gl(1 - \cos 25^\circ) \\
 \frac{m_A}{m_B} v_A^2 + \left(\frac{m_A}{m_B} \right)^2 v_A^2 &= 2gl(1 - \cos 25^\circ) \\
 k(k+1)v_A^2 &= 2gl(1 - \cos 25^\circ), \quad k = \frac{m_A}{m_B} = 1.5 \\
 v_A &= \sqrt{\frac{2gl(1 - \cos 25^\circ)}{k(k+1)}} = \sqrt{\frac{2(9.81)(2)(1 - \cos 25^\circ)}{(1.5)(2.5)}} \text{ m/s} = 0.990 \text{ m/s} \\
 v_B &= \frac{m_A}{m_B} v_A = k v_A = (1.5)(0.990) \text{ m/s} = 1.485 \text{ m/s}
 \end{aligned}$$

$$v_A = 0.990 \text{ m/s} \leftarrow \blacktriangleleft$$

$$v_B = 1.485 \text{ m/s} \rightarrow \blacktriangleleft$$

Question 3

A crank AB is rotating clockwise with constant angular speed 900 rpm. Determine the acceleration of piston P when $\theta = 120^\circ$.



$$L = 0.15 \text{ m}, \quad l = 0.05 \text{ m}$$

$$\dot{\theta} = 900 \text{ rpm} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ (rad)}}{1 \text{ rev}} = 30\pi \text{ s}^{-1}$$

$$\frac{\sin(\pi - \theta - \beta)}{50 \text{ mm}} = \frac{\sin \theta}{150 \text{ mm}}$$

$$\sin(\theta + \beta) = \frac{1}{3} \sin \theta$$

$$\phi = \theta + \beta - \frac{\pi}{2}$$

$$\sin\left(\phi + \frac{\pi}{2}\right) = \frac{1}{3} \sin \theta$$

$$3 \cos \phi = \sin \theta$$

$$\phi = \arccos\left(\frac{\sin \theta}{3}\right) = \arccos\left(\frac{\sin 120^\circ}{3}\right) = 73.2213^\circ$$

$$-3\dot{\phi} \sin \phi = \dot{\theta} \cos \theta$$

$$\dot{\phi} = -\frac{\dot{\theta} \cos \theta}{3 \sin \phi} = -\frac{(30\pi \text{ s}^{-1}) \cos 120^\circ}{3 \sin 73.2213^\circ} = 16.40644 \text{ s}^{-1}$$

$$-3\ddot{\phi} \sin \phi - 3\dot{\phi}^2 \cos \phi = -\dot{\theta}^2 \sin \theta \quad (\because \ddot{\theta} = 0)$$

$$\ddot{\phi} = \frac{-\dot{\theta}^2 \sin \theta + 3\dot{\phi}^2 \cos \phi}{-3 \sin \phi} = \frac{-(30\pi \text{ s}^{-1})^2 \sin 120^\circ + 3(16.40644 \text{ s}^{-1})^2 \cos 120^\circ}{-3 \sin 73.2213^\circ} = 2597.06 \text{ s}^{-2}$$

$$a_B = l\dot{\theta}^2$$

$$a_{D/B,r} = -L\dot{\phi}^2$$

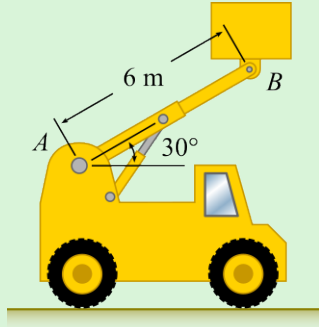
$$a_{D/B,\theta} = L\ddot{\phi}$$

$$\mathbf{a}_D = \mathbf{a}_{D/B} + \mathbf{a}_B$$

$$\begin{aligned} a_D &= a_{D/B,r} \sin \phi + a_{D/B,\theta} \cos \phi + a_B \cos \theta = -L\dot{\phi}^2 \sin \phi + L\ddot{\phi} \cos \phi + l\dot{\theta}^2 \sin\left(\theta - \frac{\pi}{2}\right) \\ &= -(0.15)(16.40644)^2 \sin 73.2213^\circ + (0.15)(2597.06) \cos 73.2213^\circ + (0.05)(30\pi)^2 \sin 30^\circ \\ &= 296 \text{ m/s}^2 \end{aligned}$$

296 m/s² ↑ ◀

Question 4



At the instant shown the length of the boom AB is being increased at the constant rate of 0.2 m/s , the boom is being lowered at the constant rate of 0.08 rad/s , and it is moving forward with a speed of 0.1 m/s and is slowing down at 0.05 m/s^2 . Determine (a) the velocity of Point B , (b) the acceleration of Point B .

$$r = 6 \text{ m}, \quad \dot{r} = 0.2 \text{ m/s}, \quad \ddot{r} = 0,$$

$$V = 0.1 \text{ m/s}, \quad \dot{V} = -0.05 \text{ m/s}^2$$

$$\theta = 30^\circ, \quad \dot{\theta} = -0.08 \text{ s}^{-1}, \quad \ddot{\theta} = 0$$

$$v_{B/A,r} = \dot{r}, \quad v_{B/A,\theta} = r\dot{\theta}$$

$$v_{B,x} = \dot{r} \cos \theta - r\dot{\theta} \sin \theta + V = (0.2) \cos 30^\circ - (6)(-0.08) \sin 30^\circ + 0.1 \text{ [m/s]} = 0.513 \text{ m/s}$$

$$v_{B,y} = \dot{r} \sin \theta + r\dot{\theta} \cos \theta = (0.2) \sin 30^\circ + (6)(-0.08) \cos 30^\circ \text{ [m/s]} = -0.316 \text{ m/s}$$

$$v_B = \sqrt{v_{B,x}^2 + v_{B,y}^2} = 0.603 \text{ m/s}$$

$$\phi_v = \arctan \frac{v_{B,y}}{v_{B,x}} = -31.6^\circ$$

(a) $0.603 \text{ m/s} \angle -31.6^\circ$ ◀

$$a_{B/A,r} = \ddot{r} - r\dot{\theta}^2 = -r\dot{\theta}^2, \quad a_{B/A,\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2\dot{r}\dot{\theta}$$

$$a_{B,x} = -r\dot{\theta}^2 \cos \theta - 2\dot{r}\dot{\theta} \sin \theta + \dot{V} = -(6)(-0.08)^2 \cos 30^\circ - 2(0.2)(-0.08) \sin 30^\circ - 0.05 \text{ [m/s}^2\text{]} \\ = -0.0672554 \text{ m/s}^2$$

$$a_{B,y} = -r\dot{\theta}^2 \sin \theta + 2\dot{r}\dot{\theta} \cos \theta = -(6)(-0.08)^2 \sin 30^\circ + 2(0.2)(-0.08) \cos 30^\circ \text{ [m/s}^2\text{]} \\ = -0.0469128 \text{ m/s}^2$$

$$a_B = \sqrt{a_{B,x}^2 + a_{B,y}^2} = 0.0820 \text{ m/s}^2$$

$$\phi_a = \arctan \frac{a_{B,y}}{a_{B,x}} = 34.9^\circ$$

(b) $0.0820 \text{ m/s}^2 \angle -145.1^\circ$ ◀

Question 4 — Alternative Solution

The system xy on point A . The system XY is fixed framed.

$$\mathbf{r}|_A = \mathbf{r}|_{xy} = 6\mathbf{e}_r \text{ m}, \quad \dot{\mathbf{r}}|_{xy} = 0.2\mathbf{e}_r \text{ m/s} = \text{const.} \Rightarrow \ddot{\mathbf{r}}|_{xy} = \mathbf{0}$$

$$\dot{\mathbf{r}}_A = 0.1\mathbf{i} \text{ m/s}, \quad \ddot{\mathbf{r}}_A = -0.05\mathbf{i} \text{ m/s}^2$$

$$\theta = 30^\circ, \quad \mathbf{e}_r = \cos\theta\mathbf{i} + \sin\theta\mathbf{j}, \quad \boldsymbol{\Omega} = -0.08\mathbf{k} \text{ s}^{-1} = \text{const.} \Rightarrow \dot{\boldsymbol{\Omega}} = \mathbf{0}$$

$$\begin{aligned} \dot{\mathbf{r}}|_{XY} &= \dot{\mathbf{r}}_A + \dot{\mathbf{r}}|_A = \dot{\mathbf{r}}_A + \dot{\mathbf{r}}|_{xy} + \boldsymbol{\Omega} \times \mathbf{r}|_A \\ &= (0.1)(\rightarrow) + (0.2)\angle 30^\circ + (0.08 \cdot 6)\angle (-60^\circ) \text{ [m/s]} \\ &= \{0.1 + 0.2 \cos 30^\circ + (0.08)(6) \cos(-60^\circ)\}\mathbf{i} + \{0.2 \sin 30^\circ + (0.08)(6) \sin(-60^\circ)\}\mathbf{j} \text{ [m/s]} \\ &= (0.513205\mathbf{i} - 0.315692\mathbf{j}) \text{ m/s} \end{aligned}$$

$$|\dot{\mathbf{r}}|_{XY}| = \sqrt{0.513205^2 + 0.315692^2} \text{ m/s} = 0.603 \text{ m/s}$$

$$\theta_v = -\arctan \frac{0.315692}{0.513205} = -31.6^\circ$$

$$(a) \quad 0.603 \text{ m/s} \angle -31.6^\circ \quad \blacktriangleleft$$

$$\begin{aligned} \ddot{\mathbf{r}}|_{XY} &= \ddot{\mathbf{r}}_A + \ddot{\mathbf{r}}|_{xy} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}|_A) + \dot{\boldsymbol{\Omega}} \times \mathbf{r}|_A + 2\boldsymbol{\Omega} \times \dot{\mathbf{r}}|_{xy} \\ &= (0.05)(\leftarrow) + (6)(0.08)^2 \angle (-150^\circ) + 2(0.08)(0.2) \angle (-60^\circ) \text{ [m/s}^2\text{]} \\ &= \{-0.05 + (6)(0.08)^2 \cos(-150^\circ) + 2(0.08)(0.2) \cos(-60^\circ)\}\mathbf{i} \\ &\quad + \{(6)(0.08)^2 \sin(-150^\circ) + 2(0.08)(0.2) \sin(-60^\circ)\}\mathbf{j} \text{ [m/s}^2\text{]} \\ &= (-0.0672554\mathbf{i} - 0.0469128\mathbf{j}) \text{ m/s}^2 \end{aligned}$$

$$|\ddot{\mathbf{r}}|_{XY}| = \sqrt{0.0672554^2 + 0.0469128^2} \text{ m/s}^2 = 0.0820 \text{ m/s}^2$$

$$\theta_a = \arctan \frac{0.0469128}{0.0672554} - 180^\circ = -145.1^\circ$$

$$(b) \quad 0.0820 \text{ m/s}^2 \angle -145.1^\circ \quad \blacktriangleleft$$