해설

구조역학(박성훈 교수님) 2025-2 기말고사

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2025–12–27

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Question 1 — Prob. 9.100

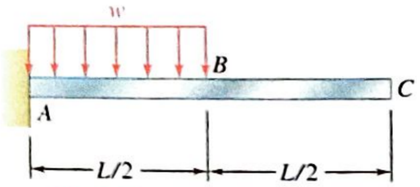
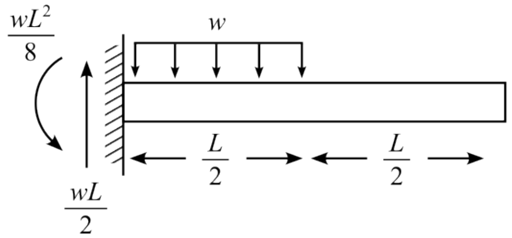
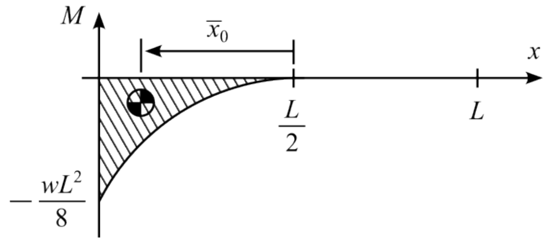


Fig. *P9.100*

For the uniform cantilever beam and loading shown, determine the slope and deflection at (a) point B, (b) point C. Use the momnet-area method to solve this problem. (부호 또는 화살표를 통해 처짐과 처짐각의 방향을 명확히 표시하시오.)





In portion AB,

$$A_m = \frac{1}{3}\cdot\frac{wL^2}{8}\cdot\frac{L}{2} = \frac{wL^3}{48}, \quad \bar{x} = \bar{x}_0 = \frac{3}{4}\cdot\frac{L}{2} = \frac{3L}{8}$$

$$\theta_{B/A} = \theta_B = -\frac{A_m}{EI} = -\frac{wL^3}{48EI}$$

$$t_{B/A} = y_B = -\frac{A_m\bar{x}}{EI} = -\frac{wL^4}{128EI}$$

$$\theta_B = -\frac{wL^3}{48EI}\,;\; y_B = -\frac{wL^4}{128EI} \quad \blacktriangleleft \quad (a)$$

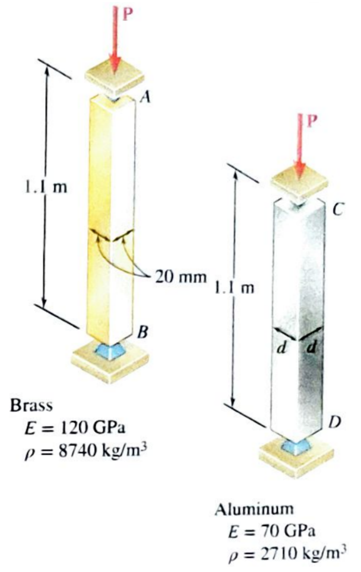
In portion AC,

$$\theta_{C/A} = \theta_C = -\frac{A_m}{EI} = -\frac{wL^3}{48EI}$$

$$y_C = y_B + \theta_B \times \frac{L}{2} = -\frac{7wL^4}{384EI}$$

$$\theta_C = -\frac{wL^3}{48EI}\,;\; y_C = -\frac{7wL^4}{384EI} \quad \blacktriangleleft \quad (b)$$

Question 2 — Prob. 10.13



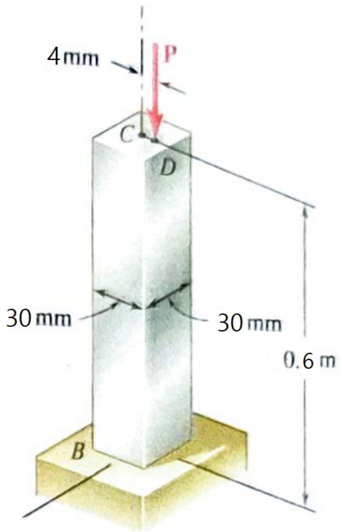
Determine (*a*) the critical load for the brass strut, (*b*) the dimension d for which the aluminum strut will have the same critical load, (*c*) the weight of the aluminum strut as a percent of the weight of the brass strut.

$$P_{\text{cr}} = \frac{\pi^2 E_b I_1}{L^2} = \frac{\pi^2 (120 \times 10^9)(\frac{1}{12} \cdot 0.02^4)}{1.1^2}\,\text{N} = 13050.71656\,\text{N} = 13.05\,\text{kN} \quad \blacktriangleleft \quad (a)$$

$$P_{\text{cr}} = \frac{\pi^2 E_a I_2}{L^2} = \frac{\pi^2 E_a d^4}{12 L^2} \quad \Rightarrow \quad d = \left(\frac{12 P_{\text{cr}} L^2}{\pi^2 E_a}\right)^{\frac{1}{4}} = 0.0228849937\,\text{m} = 22.88\,\text{mm} \quad \blacktriangleleft \quad (b)$$

$$\frac{m_a}{m_b} = \frac{\rho_a d^2 L}{\rho_b (0.02\,\text{m})^2 L} = \frac{\rho_a d^2}{\rho_b (0.02\,\text{m})^2} = \frac{(2710)(0.228849937)^2}{(8740)(0.02)^2} = 40.60\% \quad \blacktriangleleft \quad (c)$$

Question 3 — variation of Prob. 10.35



An axial load $P = 15\,\text{kN}$ is applied at point D that is 6 mm from the geomatric axis of the square aluminum bar BC. Using 70 GPa. determine (a) the horizontal deflection of end C, (b) the corresponding maximum stress in the column.

$$I = \frac{1}{12}(0.03)^4\,\text{m}^4 = 6.75 \times 10^{-8}\,\text{m}^4, \quad L_e = 2L = 1.2\,\text{m}$$

$$\text{let} \quad H = \sec\left(\frac{L_e}{2}\sqrt{\frac{P}{EI}}\right) = \sec\left(\frac{1.2}{2}\sqrt{\frac{15 \times 10^3}{(70 \times 10^9)(6.75 \times 10^{-8})}}\right) = 2.079167354$$

$$y_{\max} = e(H-1) = 4.32\,\text{mm} \quad \blacktriangleleft \quad (a)$$

$$\frac{ec}{r^2} = \frac{ecA}{I} = \frac{(0.004)(0.015)(0.03^2)}{6.75 \times 10^{-8}} = 0.8$$

$$\sigma_{\max} = \frac{P}{A}\left(1 + \frac{ec}{r^2} \cdot H\right) = 44.39\,\text{MPa} \quad \blacktriangleleft \quad (b)$$

Question 4 — variation of Prob. 3.36

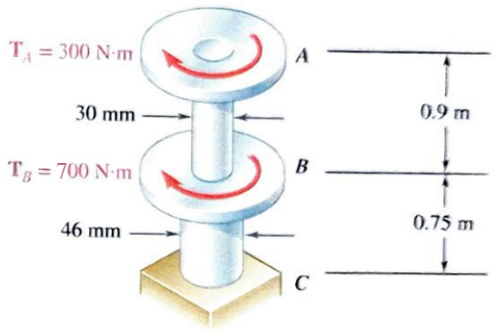


Fig. P3.36

The torques shown are exerted on pulleys A and B. Knowing that the shafts are solid and made of steel ($G = 73\,\text{GPa}$), determine the strain energy of this system.

$$T_{AB} = T_A = 300\,\text{N}\cdot\text{m}, \quad T_{BC} = T_A + T_B = 1000\,\text{N}\cdot\text{m}$$

$$J_{AB} = \frac{\pi}{2}(0.015^4)\,\text{m}^4 = 7.952156404 \times 10^{-8}\,\text{m}^4$$

$$J_{BC} = \frac{\pi}{2}(0.023^4)\,\text{m}^4 = 4.395732149 \times 10^{-7}\,\text{m}^4$$

$$U_{\text{tot}} = \frac{T_{AB}^2 L_{AB}}{2GJ_{AB}} + \frac{T_{BC}^2 L_{BC}}{2GJ_{BC}} = 18.66\,\text{J} \quad \blacktriangleleft$$

Question 5 — variation of Prob. 11.45

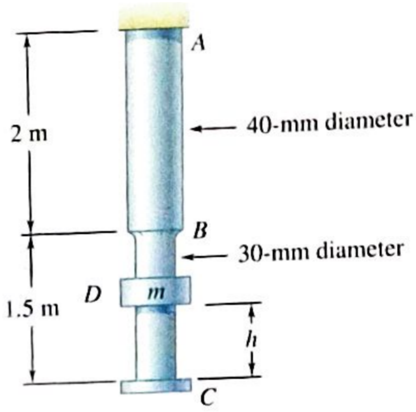


Fig. P11.126

The 50-kg collar D is released from rest in the position shown and is stopped by a plate attached at end C of the vertical rod ABC. Knowing that $E = 200\,\text{GPa}$ and the maximum normal stress is $250\,\text{MPa}$ for both portions of the rod, determine the maximum height h_m.

$$P_m = \sigma_m A_{BC} = (250\,\text{MPa})(\pi \cdot 15^2\,\text{mm}^2) = 176714.5868\,\text{N}$$

$$U_m = \frac{P_m^2 L_{AB}}{2EA_{AB}} + \frac{P_m^2 L_{BC}}{2EA_{BC}} = 289.9223689\,\text{J}$$

$$U_m = \frac{1}{2}P_m\delta_m \quad \Rightarrow \quad \delta_m = \frac{2U_m}{P_m} = 3.28125\,\text{mm}$$

$$U_m = mg(h_m + \delta_m) \quad \Rightarrow \quad h_m = \frac{U_m}{mg} - \delta_m = 0.587793916\,\text{m} = 587.79\,\text{mm} \quad ◀$$