

해설

고체역학(박성훈 교수님) 2025-1 기말고사

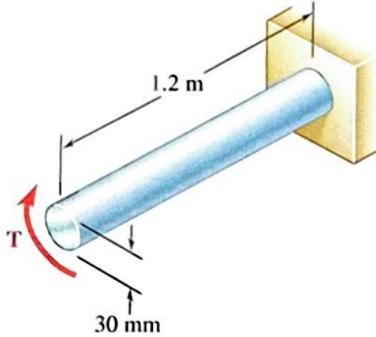
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Question 1 — variation of prob. 3.95 in textbook



The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 77.2 \text{ GPa}$ and $\tau_Y = 145 \text{ MPa}$. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (a) $T = 500 \text{ N} \cdot \text{m}$, (b) $T = 1200 \text{ N} \cdot \text{m}$.

$$J = \frac{\pi}{2} r^4 = \frac{\pi}{2} (15 \text{ mm})^4 = 25312.5\pi \text{ mm}^4$$

$$T_Y = \frac{\tau_Y J}{c} = \frac{(145 \text{ MPa})(25312.5\pi \text{ mm}^4)}{15 \text{ mm}} = \frac{(145)(25312.5\pi)}{15} \times 10^{-3} \text{ N} \cdot \text{m} = 768.7084524 \text{ N} \cdot \text{m}$$

$$T_p = \frac{4}{3} T_Y = 1024.944603 \text{ N} \cdot \text{m}$$

(a) When $T = 500 \text{ N} \cdot \text{m}$,

$$T < T_Y \Rightarrow \rho_Y = 15.00 \text{ mm}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{(500 \text{ N} \cdot \text{m})(15 \text{ mm})}{25312.5\pi \text{ mm}^4} = \frac{(500)(15)}{25312.5\pi} \times 10^3 \text{ MPa} = 94.31 \text{ MPa}$$

$$(a) \quad \tau_{\max} = 94.31 \text{ MPa}; \rho_Y = 15.00 \text{ mm} \quad \blacktriangleleft$$

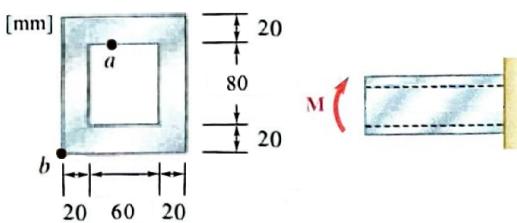
(b) When $T = 1200 \text{ N} \cdot \text{m}$,

$$T > T_p \Rightarrow \rho_Y = 0.00 \text{ mm}$$

$$\tau_{\max} = \tau_Y = 145.00 \text{ MPa}$$

$$(b) \quad \tau_{\max} = 145.00 \text{ MPa}; \rho_Y = 0.00 \text{ mm} \quad \blacktriangleleft$$

Question 2



An aluminum beam has the cross section shown. Knowing that the magnitude of couple applying on the beam is $20 \text{ kN} \cdot \text{m}$, determine the stress at (a) point a, (b) point b.

*이 문제에서 주어진 수치와 답안은 실제 실시된 시험과 차이가 있을 수 있습니다.

$$I = \frac{1}{12}(100)(120)^3 - \frac{1}{12}(60)(80)^3 [\text{mm}^4] = 11840000 \text{ mm}^4$$

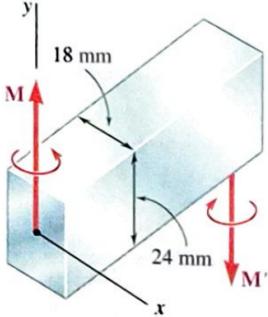
$$\sigma_a = -\frac{My_a}{I} = -\frac{(20 \text{ kN} \cdot \text{m})(40 \text{ mm})}{11840000 \text{ mm}^4} = \frac{(20)(40)}{11840000} \times 10^6 \text{ MPa} = 67.57 \text{ MPa}$$

$$\sigma_b = -\frac{My_a}{I} = -\frac{(20 \text{ kN} \cdot \text{m})(60 \text{ mm})}{11840000 \text{ mm}^4} = \frac{(20)(60)}{11840000} \times 10^6 \text{ MPa} = 101.35 \text{ MPa}$$

$$(a) \quad 67.57 \text{ MPa} \quad \blacktriangleleft$$

$$(b) \quad 101.35 \text{ MPa} \quad \blacktriangleleft$$

Question 3 — prob. 4.71 in textbook



The prismatic rod shown is made of a steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_Y = 280 \text{ MPa}$. Knowing that couples M and M' of moment 525 N·m are applied and maintained about axes parallel to the y axis, determine (a) the thickness of the elastic core, (b) the radius of curvature of the bar.

$$I_y = \frac{1}{12}(24)(18)^3 \text{ mm}^4 = 11664 \text{ mm}^4$$

$$M_Y = \frac{\sigma_Y I_y}{c} = \frac{(280 \text{ MPa})(11664 \text{ mm}^4)}{9 \text{ mm}} = \frac{(280)(11664)}{9} \times 10^3 \text{ N} \cdot \text{m} = 362.88 \text{ N} \cdot \text{m}$$

$$M = \frac{3}{2}M_Y \left(1 - \frac{x_Y^2}{3c^2} \right)$$

$$x_Y = c \sqrt{2 - \frac{3M}{M_Y}} = (9 \text{ mm}) \sqrt{3 - \frac{2(525 \text{ N} \cdot \text{m})}{362.88 \text{ N} \cdot \text{m}}} = 2.936835031 \text{ mm}$$

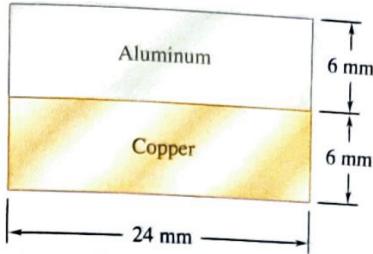
$$(\text{thickness of elastic core}) = 2x_Y = 5.87 \text{ mm}$$

(a) 5.87 mm ◀

$$\rho = -\frac{x_Y}{\varepsilon_Y} = -\frac{Ex_Y}{\sigma_Y} = -\frac{(200 \text{ GPa})(-2.936835031 \text{ mm})}{280 \text{ MPa}} = 2.10 \text{ m}$$

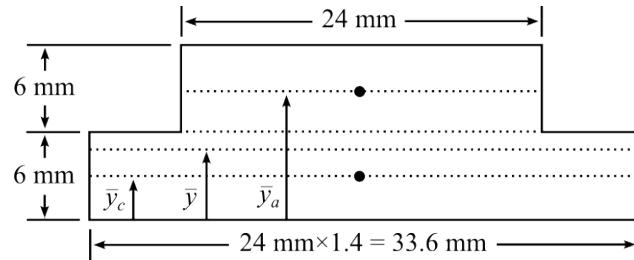
(b) 2.10 m ◀

Question 4 — prob. 4.39 in textbook



A copper strip ($E_c = 105 \text{ GPa}$) and an aluminum strip ($E_a = 75 \text{ GPa}$) are bonded together to form the composite beam shown. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 35 \text{ N} \cdot \text{m}$, determine the maximum stress in (a) the aluminum strip, (b) the copper strip.

$$\text{let. } E_c = nE_a = nE, \quad n = 1.4, \quad E = 75 \text{ GPa}$$



$$\bar{y}_c = 3 \text{ mm}, \quad \bar{y}_a = 9 \text{ mm}$$

$$A_c = (33.6 \text{ mm})(6 \text{ mm}) = 201.6 \text{ mm}^2, \quad A_a = (24 \text{ mm})(6 \text{ mm}) = 144 \text{ mm}^2$$

$$\bar{y} = \frac{A_c \bar{y}_c + A_a \bar{y}_a}{A_c + A_a} = \frac{(201.6)(3) + (144)(9)}{201.6 + 144} \text{ mm} = 5.50 \text{ mm}$$

$$I = \frac{1}{12}(33.6)(6)^3 + (201.6)(2.5)^2 + \frac{1}{12}(24)(6)^3 + (144)(3.5)^2 [\text{mm}^4] = 4060.8 \text{ mm}^4$$

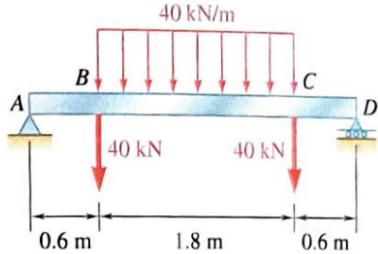
$$|\sigma_a|_{\max} = \frac{My_{\max}}{I} = \frac{(35 \text{ N} \cdot \text{m})(6.5 \text{ mm})}{4060.8 \text{ mm}^4} = 56.02 \text{ MPa}$$

(a) 56.02 MPa ◀

$$|\sigma_c|_{\max} = n \cdot \frac{M|y_{\min}|}{I} = (1.4) \frac{(35 \text{ N} \cdot \text{m})(5.5 \text{ mm})}{4060.8 \text{ mm}^4} = 66.37 \text{ MPa}$$

(b) 66.37 MPa ◀

Question 5 — variation of prob. 5.108 in textbook



Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value of (a) the shear, (b) the bending moment.

Let the centroid of the beam be point E .

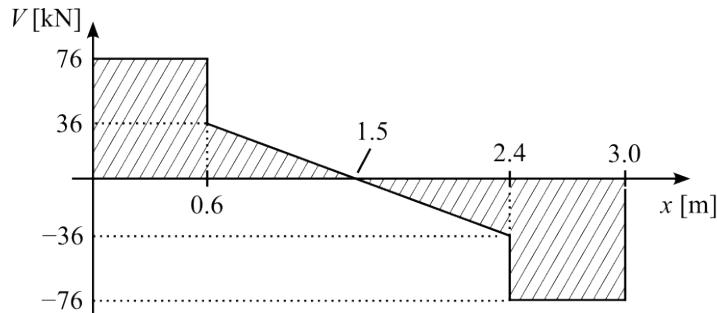
$$+\circlearrowleft \sum M|_E = -R_A(1.5 \text{ m}) + R_D(1.5 \text{ m}) = 0 \Rightarrow R_A = R_D = R$$

$$+\uparrow \sum F_y = 2R - 40 \text{ kN} - (40 \text{ kN/m})(1.8 \text{ m}) - 40 \text{ kN} = 0 \Rightarrow R = 76 \text{ kN}$$

$$0.0 \text{ m to } 0.6 \text{ m} : V = 76 \text{ kN}$$

$$0.6 \text{ m to } 2.4 \text{ m} : V = 36 \text{ kN} - (40 \text{ kN/m})(x - 0.6 \text{ m})$$

$$2.4 \text{ m to } 3.0 \text{ m} : V = -76 \text{ kN}$$



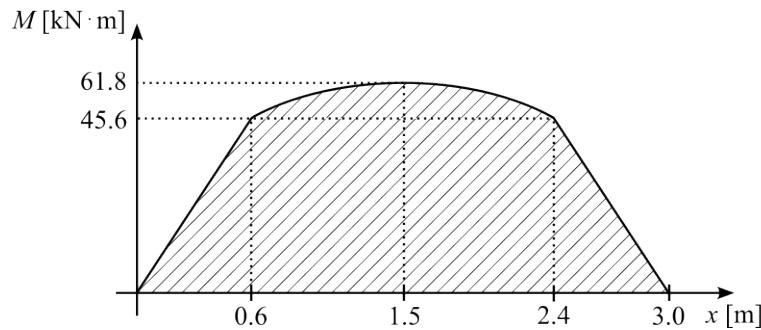
(a) 76 kN ◀

$$\text{at } 0.0 \text{ m and } 3.0 \text{ m} : M = 0$$

$$\text{at } 0.6 \text{ m} : M = (76 \text{ kN})(0.6 \text{ m}) = 45.6 \text{ kN} \cdot \text{m}$$

$$\text{at } 1.5 \text{ m} : M = (76 \text{ kN})(1.5 \text{ m}) - (40 \text{ kN})(0.9 \text{ m}) - (40 \text{ kN/m})(0.45 \text{ m}) = 61.8 \text{ kN} \cdot \text{m}$$

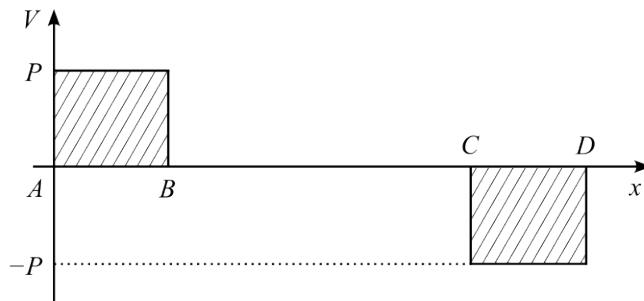
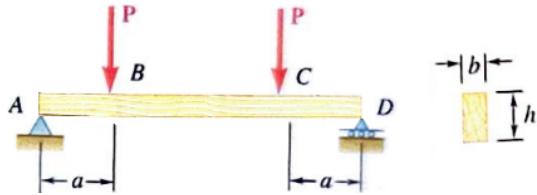
$$\text{at } 2.4 \text{ m} : M = 45.6 \text{ kN} \cdot \text{m}$$



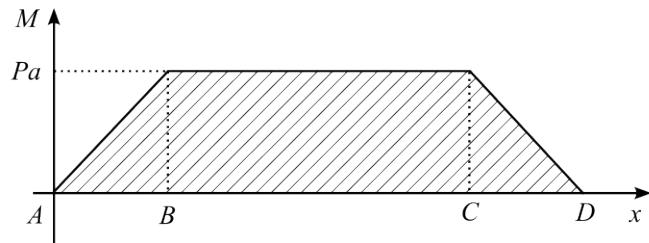
(b) 61.8 kN · m ◀

Question 6 — variation of prob. 5.5 in textbook

A timber beam AD of rectangular cross section whose width b is 100 mm and depth h is 150 mm carries loadings shown. Knowing that a is 0.5 m and the allowable normal stress and shear stress are $\sigma_{\text{all}} = 11 \text{ MPa}$ and $\tau_{\text{all}} = 1.2 \text{ MPa}$, determine the maximum permissible load P_{max} .



Because $V = \frac{dM}{dx}$ and $M = 0$ at point A and B ,



$$|V|_{\text{max}} = P, \quad |M|_{\text{max}} = Pa$$

$$|\sigma|_{\text{max}} = \frac{|M|_{\text{max}}}{S} = \frac{6|M|_{\text{max}}}{bh^2} = \frac{6Pa}{bh^2} \leq \sigma_{\text{all}}$$

$$P \leq \frac{\sigma_{\text{all}}bh^2}{6a} = \frac{(11 \text{ MPa})(100 \text{ mm})(150 \text{ mm})^2}{6(0.5 \text{ m})} = 8.25 \text{ kN}$$

$$|\tau|_{\text{max}} = \frac{|V|_{\text{max}}Q_{\text{half}}}{It} = \frac{3|V|_{\text{max}}}{2bh} = \frac{3P}{2bh} \leq \tau_{\text{all}}$$

$$P \leq \frac{2\tau_{\text{all}}bh}{3} = \frac{2(1.2 \text{ MPa})(100 \text{ mm})(150 \text{ mm})}{3} = 12 \text{ kN}$$

$$P_{\text{max}} = 8.25 \text{ kN}$$

8.25 kN ◀