

HW3

2025-1 고체역학(박성훈 교수님)

Problem 2.65, 2.101, 2.103, 2.105, 3.3, 3.11, 3.55

2025-12-27



Problem 2.65

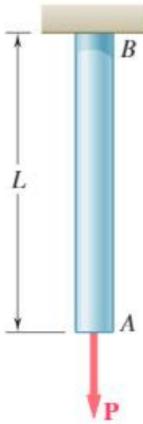
In a standard tensile test, an aluminum rod of 20-mm diameter is subjected to a tension force of $P = 30 \text{ kN}$. Knowing that $\nu = 0.35$ and $E = 70 \text{ GPa}$, determine (a) the elongation of the rod in a 150-mm gage length, (b) the change in diameter of the rod.

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20 \text{ mm})^2 = 100\pi \text{ mm}^2$$

$$\delta = \frac{PL}{EA} = \frac{(30 \times 10^3)(0.15)}{(70 \times 10^9)(100\pi \times 10^{-6})} \text{ m} = 0.205 \text{ mm} \quad \blacktriangleleft \quad (a)$$

$$\Delta d = \varepsilon_x d = -\nu \varepsilon_y d = -\frac{\nu P d}{EA} = -\frac{(0.35)(30 \times 10^3)(0.02)}{(70 \times 10^9)(100\pi \times 10^{-6})} \text{ m} = -9.55 \mu\text{m} \quad \blacktriangleleft \quad (b)$$

Problem 2.101



The 30-mm-square bar AB has a length $L = 2.2 \text{ m}$; it is made of a mild steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_Y = 345 \text{ MPa}$. A force P is applied to the bar until end A has moved down by an amount δ_m . Determine the maximum value of the force P and the permanent set of the bar after, the force has been removed, knowing that (a) $\delta_m = 4.5 \text{ mm}$ (b) $\delta_m = 8 \text{ mm}$.

Fig. P2.101 and P2.102

$$\delta_Y = \frac{\sigma_Y L}{E} = \frac{(345 \times 10^6)(2.2)}{200 \times 10^9} \text{ m} = 3.795 \text{ mm}$$

In part (a),

$$\delta > \delta_Y \Rightarrow (\text{elastic situation}) \quad P_m = P_Y = \sigma_Y A = (345 \text{ MPa})(30 \text{ mm})^2 = 310.5 \text{ kN}$$

$$\delta_p = \delta_m - \delta_Y = 4.5 \text{ mm} - 3.795 \text{ mm} = 0.705 \text{ mm}$$

$$P_m = 310.5 \text{ kN} ; \delta_p = 0.705 \text{ mm} \quad \blacktriangleleft \quad (a)$$

In part (b),

$$\delta > \delta_Y \Rightarrow (\text{elastic situation}) \quad P_m = P_Y = \sigma_Y A = (345 \text{ MPa})(30 \text{ mm})^2 = 310.5 \text{ kN}$$

$$\delta_p = \delta_m - \delta_Y = 8 \text{ mm} - 3.795 \text{ mm} = 4.205 \text{ mm}$$

$$P_m = 310.5 \text{ kN} ; \delta_p = 4.205 \text{ mm} \quad \blacktriangleleft \quad (b)$$

Problem 2.103

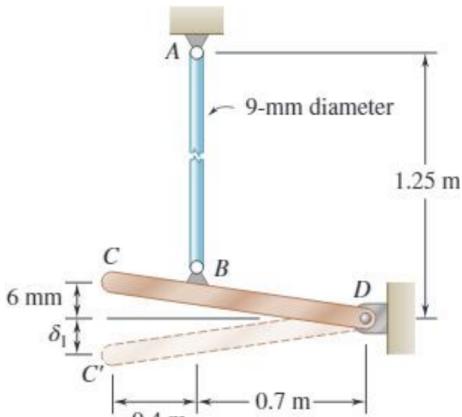


Fig. P2.103

Rod AB is made of a mild steel that is assumed to be elastoplastic with $E = 200 \text{ GPa}$ and $\sigma_Y = 345 \text{ MPa}$. After the rod has been attached to the rigid lever CD , it is found that end C is 6 mm too high. A vertical force Q is then applied at C until this point has moved to position C' . Determine the required magnitude of Q and the deflection δ_1 if the lever is to *snap* back to a horizontal position after Q is removed.

$$\delta_p = \overline{BH} = \frac{0.7 \text{ m}}{1.1 \text{ m}}(6 \text{ mm}) = 3.81818 \text{ mm}, \quad P = \frac{0.7 \text{ m}}{1.1 \text{ m}}Q = \frac{7}{11}Q$$

$$P_m = P_Y = \sigma_Y A = (345 \text{ MPa}) \left\{ \frac{\pi}{4} (9 \text{ mm})^2 \right\} = 21.9480 \text{ kN}$$

$$Q_m = \frac{7}{11}P_m = 13.97 \text{ kN}$$

$$L = \overline{AB} = 1.25 \text{ m} - 3.81818 \times 10^{-3} \text{ m} = 1.246182 \text{ m}$$

$$\delta_Y = \frac{\sigma_Y L}{E} = \frac{(345 \times 10^6)(1.246182)}{200 \times 10^9} \text{ m} = 2.14966 \text{ mm}$$

$$\delta_1 = \frac{11}{7} \cdot \overline{HB'} = \frac{11}{7}(\delta_m - \delta_p) = \frac{11}{7}\delta_Y = 3.38 \text{ mm}$$

$$Q_m = 13.97 \text{ kN}; \quad \delta_1 = 3.38 \text{ mm} \quad \blacktriangleleft$$

Problem 3.3

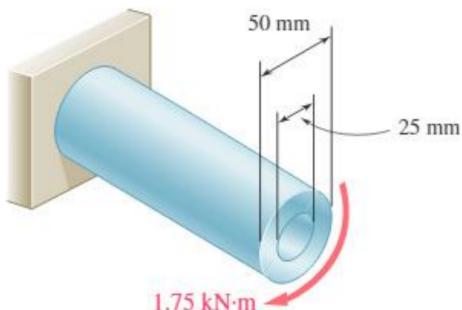


Fig. P3.3

A 1.75-kN·m torque is applied to the solid cylinder shown. Determine (a) the maximum shearing stress, (b) the percent of the torque carried by the inner 25-mm-diameter core.

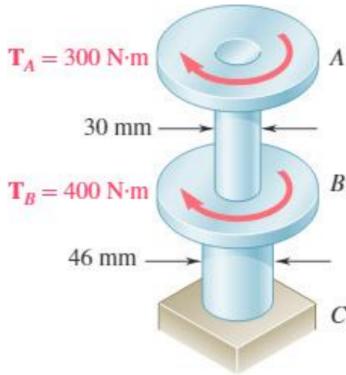
$$J = \frac{\pi}{2} \cdot 0.025^4 \text{ m}^4 = 6.13592 \times 10^{-7} \text{ m}^4$$

$$\tau_m = \frac{Tc}{J} = \frac{(1750)(0.025)}{6.13592 \times 10^{-7}} \text{ N} \cdot \text{m} = 71.3 \text{ MPa} \quad \blacktriangleleft \quad (a)$$

$$J_{in} = \frac{\pi}{2} \cdot 0.0125^4 \text{ m}^4 = \frac{1}{16}J$$

$$T = \frac{G\phi J}{L}, \quad T_{in} = \frac{G\phi J_{in}}{L} = \frac{G\phi J}{16L} = \frac{T}{16}, \quad \frac{T_{in}}{T} = \frac{1}{16} = 6.25\% \quad \blacktriangleleft \quad (b)$$

Problem 3.11



The torques shown are exerted on pulleys *A* and *B*. Knowing that both shafts are solid, determine the maximum shearing stress in (a) shaft *AB*, (b) shaft *BC*.

Fig. P3.11

$$T_{AB} = T_A = 300 \text{ N} \cdot \text{m}, \quad T_{BC} = T_A + T_B = 700 \text{ N} \cdot \text{m}$$

$$J_{AB} = \frac{\pi}{2} \cdot 0.015^4 \text{ m}^4 = 7.95216 \times 10^{-8} \text{ m}^4$$

$$J_{BC} = \frac{\pi}{2} \cdot 0.023^4 \text{ m}^4 = 4.39573 \times 10^{-7} \text{ m}^4$$

$$\tau_{m,AB} = \frac{T_{AB}c_{AB}}{J_{AB}} = \frac{(300)(0.015)}{7.95216 \times 10^{-8}} \text{ Pa} = 56.6 \text{ MPa} \quad \blacktriangleleft \quad (a)$$

$$\tau_{m,BC} = \frac{T_{BC}c_{BC}}{J_{BC}} = \frac{(700)(0.023)}{4.39573 \times 10^{-7}} \text{ Pa} = 36.6 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

Problem 3.55

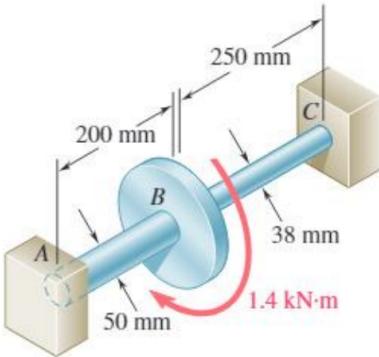


Fig. P3.55

Two solid steel shafts ($G = 77.2 \text{ GPa}$) are connected to a coupling disk B and to fixed supports at A and C . For the loading shown, determine (a) the reaction at each support, (b) the maximum shearing stress in shaft AB , (c) the maximum shearing stress in shaft BC .

$$J_{AB} = \frac{\pi}{2} \cdot 0.025^4 \text{ m}^4 = 6.13592 \times 10^{-7} \text{ m}^4$$

$$J_{BC} = \frac{\pi}{2} \cdot 0.019^4 \text{ m}^4 = 2.04708 \times 10^{-7} \text{ m}^4$$

$$T = T_{AB} + T_{BC}, \quad T_{AB} = \frac{G\phi J_{AB}}{L_{AB}}, \quad T_{BC} = \frac{G\phi J_{BC}}{L_{BC}} \quad \Rightarrow \quad T_{AB} : T_{BC} = \frac{J_{AB}}{L_{AB}} : \frac{J_{BC}}{L_{BC}}$$

$$\left. \begin{aligned} T_{AB} &= T \cdot \frac{\frac{J_{AB}}{L_{AB}}}{\frac{J_{AB}}{L_{AB}} + \frac{J_{BC}}{L_{BC}}} = \frac{T J_{AB}}{J_{AB} + \frac{L_{AB}}{L_{BC}} \cdot J_{BC}} = 1105 \text{ N} \cdot \text{m} \\ T_{BC} &= T - T_{AB} = 1400 \text{ N} \cdot \text{m} - 1105.062 \text{ N} \cdot \text{m} = 295 \text{ N} \cdot \text{m} \end{aligned} \right\} \blacktriangleleft \quad (a)$$

$$\tau_{m,AB} = \frac{T_{AB} c_{AB}}{J_{AB}} = 45.0 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

$$\tau_{m,BC} = \frac{T_{BC} c_{BC}}{J_{BC}} = 27.4 \text{ MPa} \quad \blacktriangleleft \quad (c)$$