해설

구조역학(박성훈 교수님) 2025-2 중간고사

시험 실시 : 2025-10-20 16:30-19:00(150분)

2025–12–27

숭실대학교
Soongsil University

Question 1 — Prob. 7.63

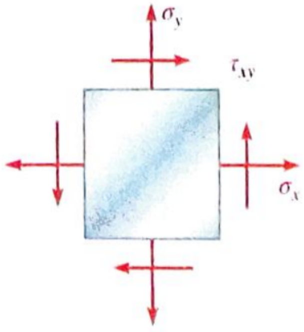
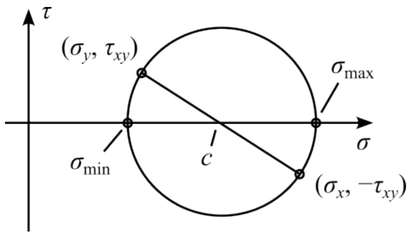


Fig. P7.63

For the state of stress shown, it is known that the normal and shearing stresses are directed as shown and that $\sigma_x = 98\,\text{MPa}$, $\sigma_y = 63\,\text{MPa}$, and $\sigma_{\min} = 35\,\text{MPa}$. Determine (a) the orientation of the principal planes, (b) the principal stress $\sigma_{\max}$, (c) the maximum in-plane shearing stress.



$$c = \frac{\sigma_x + \sigma_y}{2} = \frac{98 + 63}{2}\,\text{MPa} = 80.5\,\text{MPa}$$

$$R = \sqrt{(\sigma_x - c)^2 + \tau_{xy}^2}$$

$$\sigma_{\min} = c - R = c - \sqrt{(\sigma_x - c)^2 + \tau_{xy}^2}$$

$$(\sigma_x - c)^2 + \tau_{xy}^2 = (c - \sigma_{\min})^2$$

$$\tau_{xy}^2 = (c - \sigma_{\min})^2 - (\sigma_x - c)^2$$

$$\tau_{xy}^2 = (\sigma_x - \sigma_{\min})(2c - \sigma_{\min} - \sigma_x)$$

$$\tau_{xy}^2 = (\sigma_x - \sigma_{\min})(\sigma_y - \sigma_{\min})$$

$$\tau_{xy} = \sqrt{(98 - 35)(63 - 35)}\,\text{MPa} = 42\,\text{MPa}$$

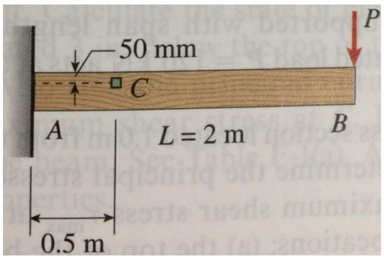
$$\theta_p = \frac{1}{2}\arctan\frac{\tau_{xy}}{\sigma_x - c} = \frac{1}{2}\arctan\frac{42}{98 - 80.5} = 33.69^\circ \quad \blacktriangleleft \quad (a)$$

$$R = \sqrt{(98 - 80.5)^2 + 42^2} = 45.5\,\text{MPa}$$

$$\sigma_{\max} = c + R = 126.00\,\text{MPa} \quad \blacktriangleleft \quad (b)$$

$$\tau_{\max} = R = 45.50\,\text{MPa} \quad \blacktriangleleft \quad (c)$$

Question 2 — variation of 문제1 from HW2



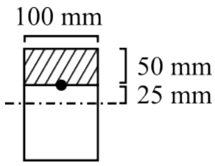
A cantilever beam with a width $b = 100\,\text{mm}$ and depth $h = 150\,\text{mm}$ has a length $L = 2\,\text{m}$ and is subjected to a point load $P = 800\,\text{N}$ at B. Calculate the state of plane stress at point C located $50\,\text{mm}$ below the top of the beam and $0.5\,\text{m}$ to the right of point A. Also find the principal stresses and the maximum shear stress at C. Neglect the weight of the beam.

$$R_A = 800\,\text{N}\ (\uparrow), \quad M_A = (800\,\text{N})(2\,\text{m}) = 1600\,\text{N}\cdot\text{m}\ (\circlearrowleft)$$

$$\text{at } C: \quad V = 500\,\text{N}, \quad M = -1000\,\text{N}\cdot\text{m} + (500\,\text{N})(0.5\,\text{m}) = -1200\,\text{kN}\cdot\text{m}$$

$$I = \frac{bh^3}{12} = \frac{(0.1)(0.15)^3}{12}\,\text{m}^4 = 2.8125\times10^{-5}\,\text{m}^4$$

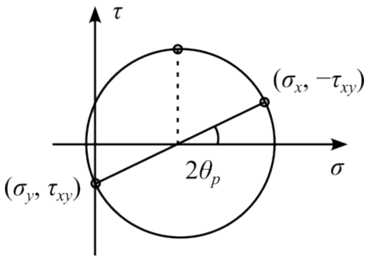
$$\sigma_x = -\frac{My}{I} = -\frac{(-1200\,\text{N}\cdot\text{m})(0.05\,\text{m})}{2.8125\times10^{-5}\,\text{m}^4} = \frac{3200}{3}\,\text{kPa} = 1066.67\,\text{kPa}$$



$$Q = A\bar{y} = (0.1)(0.05)(0.05)\,\text{m}^3 = 2.5\times10^{-4}\,\text{m}^3$$

$$\tau_{xy} = -\frac{VQ}{It} = -\frac{(800\,\text{N})(2.5\times10^{-4}\,\text{m}^3)}{(2.8125\times10^{-5}\,\text{m}^4)(0.1\,\text{m})} = -\frac{640}{9}\,\text{kPa} = -71.11\,\text{kPa}$$

$$\sigma_x = 1066.67\,\text{kPa}\ ;\ \sigma_y = 0\ ;\ \tau_{xy} = -71.11\,\text{kPa} \quad \blacktriangleleft$$



$$c = \frac{\sigma_x}{2} = \frac{1600}{3}\,\text{kPa}$$

$$R = \sqrt{c^2 + \tau_{xy}^2} = 538.0531893\,\text{kPa}$$

$$\sigma_{\text{max}} = c + R = 1071.39\,\text{kPa}$$

$$\sigma_{\text{min}} = c - R = -4.72\,\text{kPa}$$

$$\tau_{\text{max}} = R = 538.05\,\text{kPa}$$

$$\sigma_{\text{max}} = 1071.39\,\text{kPa}\ ;\ \sigma_{\text{min}} = -4.72\,\text{kPa}\ ;\ \tau_{\text{max}} = 538.05\,\text{kPa} \quad \blacktriangleleft$$

Question 3 — Prob. 8.1

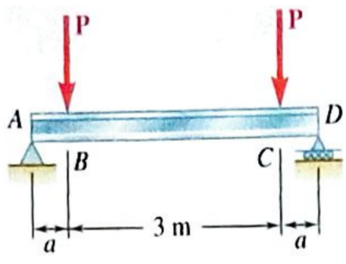
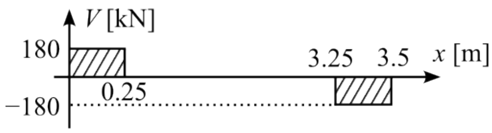


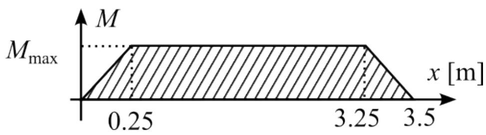
Fig. P8.1

A W250×101 rolled-steel beam supports a load P as shown. Knowing that $P = 180\,\text{kN}$, $a = 0.25\,\text{m}$, and $\sigma_{\text{all}} = 126\,\text{MPa}$, determine ($a$) the maximum value of the normal stress σ_m, in the beam, (b) the maximum value of the principal stress $\sigma_{\max}$ at the junction of the flange and web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.(이유를 자세히 쓸 것)

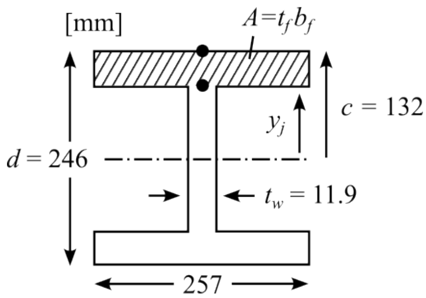
*Appendix E를 참고할 것

$$R_A = R_D = P = 180\,\text{kN}\ (\uparrow)$$



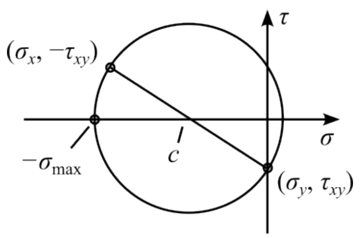


$$M_{\max} = (180\,\text{kN})(0.25\,\text{m}) = 45\,\text{kN}\cdot\text{m}$$



$$c = \frac{d}{2} = 132\,\text{mm}$$
$$y_j = c - t_f = (132 - 19.6)\,\text{mm} = 112.4\,\text{mm}$$
$$A = t_f b_f = (257)(19.6)\,\text{mm}^2 = 5037.2\,\text{mm}^2$$
$$I = 164 \times 10^6\,\text{mm}^4 = 164 \times 10^{-6}\,\text{m}^4$$
$$\bar{y} = c - \frac{t_f}{2} = \left(132 - \frac{19.6}{2}\right)\text{mm} = 122.2\,\text{mm}$$
$$Q = A\bar{y} = (5037.2)(122.2)\,\text{mm}^3 = 6.1554584 \times 10^{-4}\,\text{m}^3$$

$$\sigma_m = \frac{M_{\max} c}{I} = \frac{(45 \times 10^3)(0.132)}{164 \times 10^{-6}}\,\text{Pa} = 36.22\,\text{MPa} \quad \blacktriangleleft \quad (a)$$
$$\sigma_x = -\frac{M_{\max} y_j}{I} = -\frac{(45 \times 10^3)(0.1124)}{164 \times 10^{-6}}\,\text{Pa} = -\frac{2529}{82}\,\text{MPa} = -30.84146341\,\text{MPa}$$
$$\tau_{xy} = -\frac{VQ}{It_w} = -\frac{(180 \times 10^3)(6.1554584 \times 10^{-4})}{(164 \times 10^{-6})(0.0119)}\,\text{Pa} = -56.773033\,\text{MPa}$$



$$c = \frac{\sigma_x}{2}$$
$$R = \sqrt{c^2 + \tau_{xy}^2}$$
$$\sigma_{\max} = |c - R| = 74.25\,\text{MPa} \quad \blacktriangleleft \quad (b)$$
$$\sigma_m < \sigma_{\max} < \sigma_{\text{all}} \quad \Rightarrow \quad \text{Acceptable.} \quad \blacktriangleleft \quad (c)$$

Question 4 — Prob. 9.68

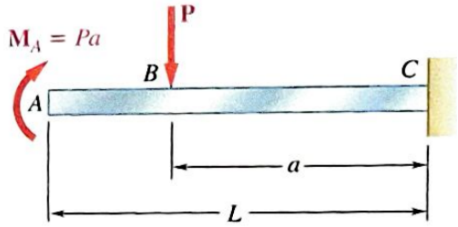
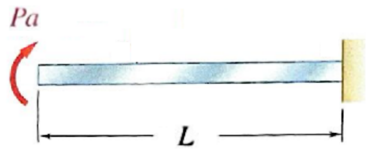


Fig. P9.68

For the cantilever beam and loading shown, determine the slope and deflection at the free end.

In the case (1), x is zero at the free end.

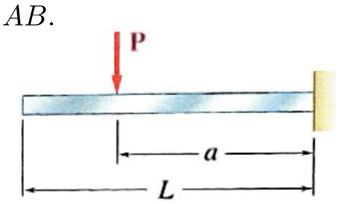


$M_1(x) = Pa = \text{const.}$
$EI\theta_1(x) = Pax + c_1, \quad \theta(L) = 0 \quad \Rightarrow \quad c_1 = -PaL$

$$EIy_1(x) = \frac{Pa}{2}x^2 - PaLx + c_2, \quad y(L) = 0 \quad \Rightarrow \quad c_2 = \frac{PaL^2}{2}$$

$$\theta_1(0) = \frac{c_1}{EI} = -\frac{PaL}{EI}, \quad y_1(0) = \frac{c_2}{EI} = \frac{PaL^2}{2EI}$$

In the case (2), x is zero at point B and the functions $\theta_2(x)$ and $y_2(x)$ are undefined at portion AB.



$V_2(x) = -P = \text{const.}$
$M_2(x) = -Px \quad (\because M(0) = 0)$
$$EI\theta_2(x) = -\frac{P}{2}x^2 + c_1^*, \quad \theta_2(a) = 0 \quad \Rightarrow \quad c_1^* = \frac{Pa^2}{2}$$

$$EIy_2(x) = -\frac{P}{6}x^3 + \frac{Pa^2}{2}x + c_2^*, \quad y_2(a) = 0 \quad \Rightarrow \quad c_2^* = -\frac{Pa^3}{3}$$

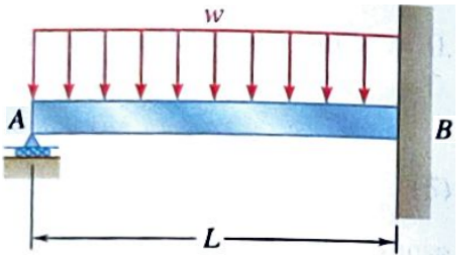
$$\theta_2(0) = \frac{c_1^*}{EI} = \frac{Pa^2}{2EI}, \quad y_2(0) = \frac{c_2^*}{EI} = -\frac{Pa^3}{3EI}$$

Superposition...

$$\theta_A = \theta_1(0) + \theta_2(0) = \frac{Pa}{2EI}(a - 2L) \quad \blacktriangleleft \quad \text{(slope)}$$

$$\begin{aligned} y_A &= y_1(0) + y_2(0) + \theta_2(0) \times (a - L) = \frac{PaL^2}{2EI} - \frac{Pa^3}{3EI} + \frac{Pa^2}{2EI}(a - L) \\ &= \frac{Pa}{6EI}\left\{3L^2 - 2a^2 + 3a(a - L)\right\} = \frac{Pa}{6EI}(a^2 - 3aL + 3L^2) \quad \blacktriangleleft \quad \text{(deflection)} \end{aligned}$$

Question 5 — variation of CA 9.5



For the loding shown, determine (a) the reaction force at $A(R_A)$, (b) equation of the elastic curve, (c) slope at A.

It's a statically indeterminate beam.

$$+\uparrow \sum F_y = R_A + R_B - wL = 0, \quad R_A + R_B = wL \quad \cdots \quad (1)$$

$$+\circlearrowleft \sum M|_A = -wL \cdot \frac{L}{2} + R_B L + M_B = 0 \quad \cdots \quad \text{We don't use this equation.}$$

$$M(x) = -wx \cdot \frac{x}{2} + R_A x = -\frac{w}{2}x^2 + R_A x$$

$$EI\theta(x) = -\frac{w}{6}x^3 + \frac{R_A}{2}x^2 + c_1, \quad \theta(L) = 0 \quad \Rightarrow \quad \frac{R_A L^2}{2} + c_1 = \frac{wL^3}{6} \quad \cdots \quad (2)$$

$$EIy(x) = -\frac{w}{24}x^4 + \frac{R_A}{6}x^3 + c_1 x + c_2, \quad y(0) = 0 \quad \Rightarrow \quad c_2 = 0$$

$$y(L) = 0 \quad \Rightarrow \quad \frac{R_A L^3}{6} + c_1 L = \frac{wL^4}{24} \quad \cdots \quad (3)$$

With equation (1), (2) and (3),

$$\begin{pmatrix} 1 & 1 & 0 \\ \frac{L^2}{2} & 0 & 1 \\ \frac{L^3}{6} & 0 & L \end{pmatrix} \begin{pmatrix} R_A \\ R_B \\ c_1 \end{pmatrix} = \begin{pmatrix} wL \\ \frac{wL^3}{6} \\ \frac{wL^4}{24} \end{pmatrix}$$

$$\Rightarrow \quad R_A = \frac{3}{8}wL \ (\uparrow) \quad \blacktriangleleft \quad (a) \quad , \quad R_B = \frac{5}{8}wL, \quad c_1 = -\frac{wL^3}{48}$$

$$EIy(x) = -\frac{w}{24}x^4 + \frac{wL}{16}x^2 - \frac{wL^3}{48}x$$

$$y(x) = -\frac{w}{48EI}(2x^4 - 3Lx^2 + L^3 x) \quad \blacktriangleleft \quad (b)$$

$$\theta(0) = \frac{c_1}{EI} = -\frac{wL^3}{48EI} \quad \blacktriangleleft \quad (c)$$