HW4

2025-1 고체역학(박성훈 교수님)

Problem 3.67, 3.68, 3.71, 3.74, 3.92, 3.95, 3.98, 3.116

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Problem 3.67

Determine the maximum shearing stress in a solid shaft of 18-mm diameter as it transmits 3.4 kW at a frequency of (*a*) 25 Hz, (*b*) 50 Hz.

$$c = 9\,\text{mm}, \quad T = \frac{P}{2\pi f}, \quad J = \frac{\pi}{2}c^4$$
$$\tau_{\max} = \frac{Tc}{J} = \frac{P}{\pi^2 f c^3}$$

When (*a*) $f = 25\,\text{Hz}$,

$$\tau_{\max} = \frac{3.4\,\text{kW}}{\pi^2(25\,\text{Hz})(9\,\text{mm})^3} = \frac{3400}{\pi^2(25)(0.009)^3}\,\text{Pa} = 18.90\,\text{MPa} \quad \blacktriangleleft \quad (a)$$

When (*b*) $f = 50\,\text{Hz}$,

$$\tau_{\max} = \frac{3.4\,\text{kW}}{\pi^2(50\,\text{Hz})(9\,\text{mm})^3} = \frac{3400}{\pi^2(25)(0.009)^3}\,\text{Pa} = 9.45\,\text{MPa} \quad \blacktriangleleft \quad (b)$$

Problem 3.68

While a steel shaft of the cross section shown rotates at 120 rpm, a stroboscopic measurement indicates that the angle of twist is 2° in a 4-m length. Using $G = 77.2\,\text{GPa}$, determine the power being transmitted.

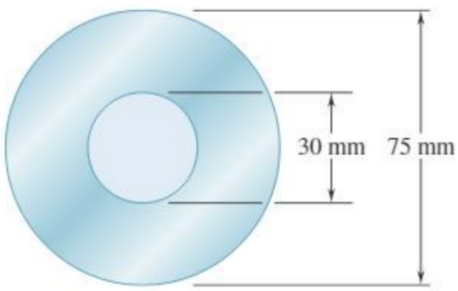


Fig. P3.68

$$f = 120\,\text{rpm} = 2\,\text{Hz}, \quad \phi = 2^\circ = \frac{\pi}{90}\,\text{(rad)}$$
$$J = \frac{\pi}{2}\left\{\left(\frac{75}{2}\,\text{mm}\right)^4 - \left(\frac{30}{2}\,\text{mm}\right)^4\right\} = 3.026789531 \times 10^{-6}\,\text{m}^4$$
$$T = \frac{G\phi J}{L}, \quad T = \frac{P}{2\pi f}$$
$$\frac{P}{2\pi f} = \frac{G\phi J}{L}$$
$$P = \frac{2\pi f G\phi J}{L} = \frac{2\pi(2\,\text{Hz})(77.2\,\text{GPa})\left(\frac{\pi}{90}\right)(3.026789531 \times 10^{-6}\,\text{m}^4)}{4\,\text{m}}$$
$$= \frac{2\pi(2)(77.2 \times 10^9)\left(\frac{\pi}{90}\right)(3.026789531 \times 10^{-6})}{4}\,\text{W} = 25.6\,\text{kW} \quad \blacktriangleleft$$

Problem 3.71

The hollow steel shaft shown ($G = 77.2\,\text{GPa}$, $\tau_{\text{all}} = 50\,\text{MPa}$) rotates at 240 rpm. Determine (a) the maximum power that can be transmitted, (b) the corresponding angle of twist of the shaft.

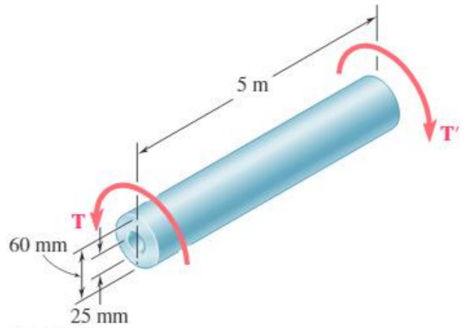


Fig. P3.71

$$f = 240\,\text{rpm} = 4\,\text{Hz}$$

$$J = \frac{\pi}{2}\left\{\left(\frac{60}{2}\,\text{mm}\right)^4 - \left(\frac{25}{2}\,\text{mm}\right)^4\right\} = 1.233995505 \times 10^{-6}\,\text{m}^4$$

$$T_{\max} = \frac{\tau_{\text{all}} J}{c}, \quad P_{\max} = 2\pi f T_{\max}$$

$$P_{\max} = \frac{2\pi f \tau_{\text{all}} J}{c} = \frac{2\pi(4\,\text{Hz})(50\,\text{MPa})(1.233995505 \times 10^{-6}\,\text{m}^4)}{30\,\text{mm}}$$

$$= \frac{2\pi(4)(50 \times 10^6)(1.233995505 \times 10^{-6})}{0.03}\,\text{W} = 51.7\,\text{kW} \quad \blacktriangleleft \quad (a)$$

$$\phi_{\max} = \frac{\gamma_{\text{all}} L}{c} = \frac{\tau_{\text{all}} L}{Gc} = \frac{(50\,\text{MPa})(5\,\text{m})}{(77.2\,\text{GPa})(30\,\text{mm})} = \frac{(50 \times 10^6)(5)}{(77.2 \times 10^9)(0.03)} = \frac{125}{1158} = 6.18° \quad \blacktriangleleft \quad (b)$$

Problem 3.74

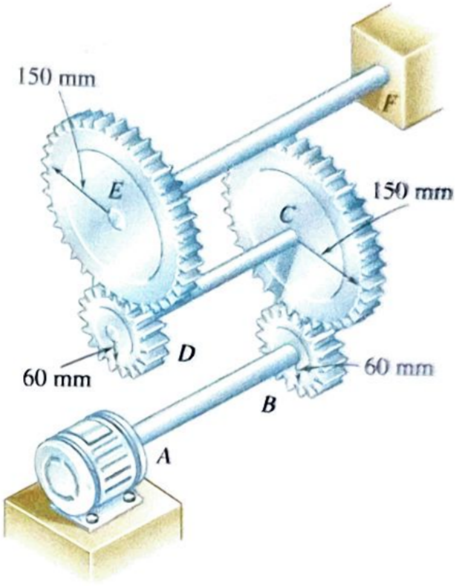


Fig. P3.74 and P3.75

Three shafts and four gears are used to form a gear train that will transmit power from the motor at A to a machine tool at F. (Bearings for the shafts are omitted in the sketch.) The diameter of each shaft is as follows: $d_{AB} = 16\,\mathrm{mm}$, $d_{CD} = 20\,\mathrm{mm}$, $d_{EF} = 28\,\mathrm{mm}$. Knowing that the frequency of the motor is 24 Hz and that the allowable shearing stress for each shaft is 75 MPa, determine the maximum power that can be transmitted.

$$T_{\max} = \frac{\tau_{\text{all}} J}{c}, \quad P_{\max} = 2\pi f T_{\max} = \frac{2\pi f \tau_{\text{all}} J}{c}$$

$$f_{AB} = 24\,\mathrm{Hz}, \quad f_{CD} = \frac{60\,\mathrm{mm}}{150\,\mathrm{mm}} f_{AB} = 9.6\,\mathrm{Hz}, \quad f_{EF} = \frac{60\,\mathrm{mm}}{150\,\mathrm{mm}} f_{CD} = 3.84\,\mathrm{Hz}$$

$$J_{AB} = \frac{\pi}{2}\left(\frac{d_{AB}}{2}\right)^4 = \frac{\pi}{2}(8\,\mathrm{mm})^4 = 2.048\pi \times 10^{-9}\,\mathrm{m}^4$$

$$P_{\max,AB} = \frac{2\pi(24\,\mathrm{Hz})(75\,\mathrm{MPa})(2.048\pi \times 10^{-9}\,\mathrm{m}^4)}{8\,\mathrm{mm}} = \frac{2\pi(24)(75 \times 10^6)(2.048\pi \times 10^{-9})}{0.008}\,\mathrm{W}$$
$$= 9.10\,\mathrm{kW}$$

$$J_{CD} = \frac{\pi}{2}\left(\frac{d_{CD}}{2}\right)^4 = \frac{\pi}{2}(10\,\mathrm{mm})^4 = 5\pi \times 10^{-9}\,\mathrm{m}^4$$

$$P_{\max,CD} = \frac{2\pi(9.6\,\mathrm{Hz})(75\,\mathrm{MPa})(5\pi \times 10^{-9}\,\mathrm{m}^4)}{10\,\mathrm{mm}} = \frac{2\pi(9.6)(75 \times 10^6)(5\pi \times 10^{-9})}{0.01}\,\mathrm{W}$$
$$= 7.11\,\mathrm{kW}$$

$$J_{EF} = \frac{\pi}{2}\left(\frac{d_{EF}}{2}\right)^4 = \frac{\pi}{2}(14\,\mathrm{mm})^4 = 1.9208\pi \times 10^{-9}\,\mathrm{m}^4$$

$$P_{\max,EF} = \frac{2\pi(3.84\,\mathrm{Hz})(75\,\mathrm{MPa})(1.9208\pi \times 10^{-9}\,\mathrm{m}^4)}{14\,\mathrm{mm}} = \frac{2\pi(3.84)(75 \times 10^6)(1.9208\pi \times 10^{-9})}{0.014}\,\mathrm{W}$$
$$= 7.80\,\mathrm{kW}$$

$$9.10\,\mathrm{kW} > 7.80\,\mathrm{kW} > 7.11\,\mathrm{kW} \quad \Rightarrow \quad P_{\max} = 7.11\,\mathrm{kW} \quad \blacktriangleleft$$

Problem 3.92

The solid circular shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145\,\text{MPa}$. Determine the magnitude T of the applied torques when the plastic zone is (a) 16 mm deep, (b) 24 mm deep.

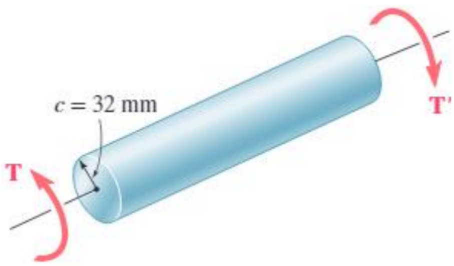


Fig. P3.92

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(32\,\text{mm})^4 = 5.24288\pi \times 10^{-7}\,\text{m}^4$$

$$T_Y = \frac{\tau_Y J}{c} = \frac{(145\,\text{MPa})(5.24288\pi \times 10^{-7}\,\text{m}^4)}{32\,\text{mm}} = \frac{(145 \times 10^6)(5.24288\pi \times 10^{-7})}{0.032}\,\text{N}\cdot\text{m}$$

$$= 7.463418835\,\text{kN}\cdot\text{m}$$

When $\rho_Y = 16\,\text{mm}$,

$$\frac{\rho_Y}{c} = \frac{16\,\text{mm}}{32\,\text{mm}} = \frac{1}{2}$$

$$T = \frac{4}{3}T_Y\left\{1 - \frac{1}{4}\left(\frac{\rho_Y}{c}\right)^3\right\} = \frac{4}{3}T_Y\left\{1 - \frac{1}{4}\left(\frac{1}{2}\right)^3\right\} = \frac{31}{24}T_Y = \frac{31}{24}(7.463418835\,\text{kN}\cdot\text{m})$$

$$= 9.64\,\text{kN}\cdot\text{m} \quad \blacktriangleleft \quad (a)$$

When $\rho_Y = 32\,\text{mm} - 24\,\text{mm} = 8\,\text{mm}$,

$$\frac{\rho_Y}{c} = \frac{8\,\text{mm}}{32\,\text{mm}} = \frac{1}{4}$$

$$T = \frac{4}{3}T_Y\left\{1 - \frac{1}{4}\left(\frac{\rho_Y}{c}\right)^3\right\} = \frac{4}{3}T_Y\left\{1 - \frac{1}{4}\left(\frac{1}{4}\right)^3\right\} = \frac{85}{64}T_Y = \frac{85}{64}(7.463418835\,\text{kN}\cdot\text{m})$$

$$= 9.91\,\text{kN}\cdot\text{m} \quad \blacktriangleleft \quad (b)$$

Problem 3.95

The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 77.2\,\text{GPa}$ and $\tau_Y = 145\,\text{MPa}$. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (*a*) $T = 600$ N-m, (*b*) $T = 1000$ N-m.

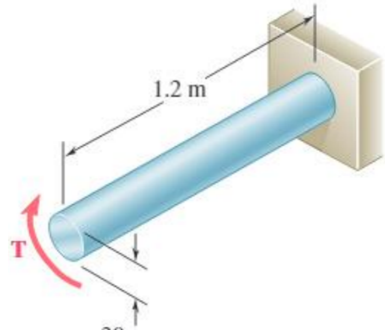


Fig. P3.95 and P3.96

$$c = \frac{30\,\text{mm}}{2} = 15\,\text{mm}$$
$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(15\,\text{mm})^4 = 2.53125\pi \times 10^{-8}\,\text{m}^4$$
$$T_Y = \frac{\tau_Y J}{c} = \frac{(145\,\text{MPa})(2.53125\pi \times 10^{-8}\,\text{m}^4)}{15\,\text{mm}} = \frac{(145 \times 10^6)(2.53125\pi \times 10^{-8})}{0.015}\,\text{N}\cdot\text{m}$$
$$= 768.7084524\,\text{N}\cdot\text{m}$$

When (*a*) $T = 600\,\text{N}\cdot\text{m} < T_Y$,

$$\tau_{\max} = \frac{Tc}{J} = \frac{(600\,\text{N}\cdot\text{m})(15\,\text{mm})}{2.53125\pi \times 10^{-8}\,\text{m}^4} = \frac{(600)(0.015)}{2.53125\pi \times 10^{-8}}\,\text{Pa} = 113.2\,\text{MPa}$$
$$\rho_Y = c = 15\,\text{mm}$$

113.2 MPa ; 15.00 mm ◄ (*a*)

When (*b*) $T = 1000\,\text{N}\cdot\text{m} > T_Y$,

$$\tau_{\max} = \tau_Y = 145\,\text{MPa}$$
$$T = \frac{4}{3}T_Y\left\{1 - \frac{1}{4}\left(\frac{\rho_Y}{c}\right)^3\right\}$$
$$\rho_Y = c\sqrt[3]{4 - \frac{3T}{T_Y}} = (15\,\text{mm})\sqrt[3]{4 - \frac{3(1000\,\text{N}\cdot\text{m})}{768.7084524\,\text{N}\cdot\text{m}}} = 6.90\,\text{mm}$$

145.0 MPa ; 6.90 mm ◄ (*b*)

Problem 3.98

The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 77.2\,\text{GPa}$ and $\tau_Y = 145\,\text{MPa}$. Determine the angle of twist caused by the application of a torque of magnitude (a) $T = 600$ N-m, (b) $T = 1000$ N-m.

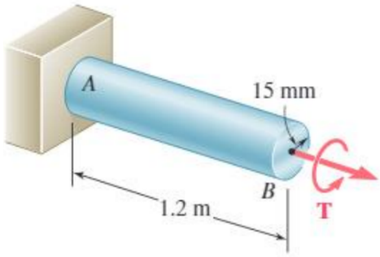


Fig. P3.98

From the solution of Prob.3.95,

$$J = 2.53125\pi \times 10^{-8}\,\text{m}^4$$
$$T_Y = 768.7084524\,\text{N}\cdot\text{m}$$

When (a) $T = 600\,\text{N}\cdot\text{m} < T_Y$,

$$\phi = \frac{TL}{GJ} = \frac{(600\,\text{N}\cdot\text{m})(1.2\,\text{m})}{(77.2\,\text{GPa})(2.53125\pi \times 10^{-8}\,\text{m}^4)} = \frac{(600)(1.2)}{(77.2 \times 10^9)(2.53125\pi \times 10^{-8})} = 6.72^\circ$$

▲ (a)

When (b) $T = 1000\,\text{N}\cdot\text{m} > T_Y$,

$$\phi_Y = \frac{\gamma_Y L}{c} = \frac{\tau_Y L}{Gc} = \frac{(145\,\text{MPa})(1.2\,\text{m})}{(77.2\,\text{GPa})(15\,\text{mm})} = \frac{(145 \times 10^6)(1.2)}{(77.2 \times 10^9)(0.015)} = \frac{29}{193}$$

$$T = \frac{4}{3}T_Y\left\{1 - \frac{1}{4}\left(\frac{\phi_Y}{\phi}\right)^3\right\}$$

$$\phi = \phi_Y\left(4 - \frac{3T}{T_Y}\right)^{-\frac{1}{3}} = \frac{29}{193}\left(4 - \frac{3(1000\,\text{N}\cdot\text{m})}{768.7084524\,\text{N}\cdot\text{m}}\right)^{-\frac{1}{3}} = 18.71^\circ \quad ◀ \quad (b)$$

Problem 3.116

The solid shaft shown is made of a steel that is assumed to be elastoplastic with $\tau_Y = 145\,\text{MPa}$ and $G = 77.2\,\text{GPa}$. The torque is increased in magnitude until the shaft has been twisted through $6°$; the torque is then removed. Determine (a) the magnitude and location of the maximum residual shearing stress, (b) the permanent angle of twist.

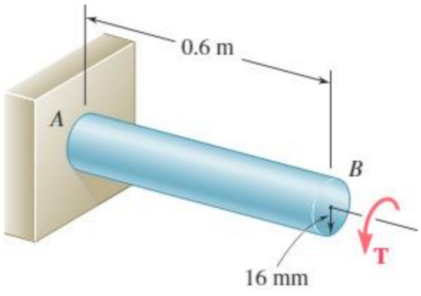


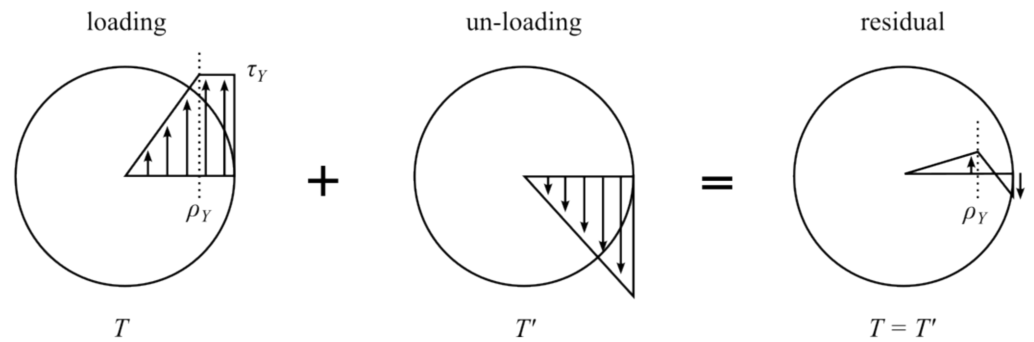
Fig. *P3.116*

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(16\,\text{mm})^4 = 3.2768\pi \times 10^{-8}\,\text{m}^4$$

$$T_Y = \frac{\tau_Y J}{c} = \frac{(145\,\text{MPa})(3.2768\pi \times 10^{-8}\,\text{m}^4)}{16\,\text{mm}} = \frac{(145 \times 10^6)(3.2768\pi \times 10^{-8})}{0.016}\,\text{N}\cdot\text{m}$$
$$= 932.9273544\,\text{N}\cdot\text{m}$$

$$\phi_Y = \frac{\tau_Y L}{Gc} = \frac{(145\,\text{MPa})(0.6\,\text{m})}{(77.2\,\text{GPa})(16\,\text{mm})} = \frac{(145 \times 10^6)(0.6)}{(77.2 \times 10^9)(0.016)} = \frac{435}{6176} = 4.035567372°$$

$$R = \frac{\rho_Y}{c} = \frac{\phi_Y}{\phi} = \frac{4.035567372°}{6°} = 0.672594562$$



$$\text{loading}: \quad T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}R^3\right) = \frac{4}{3}(932.9273544\,\text{N}\cdot\text{m})\left\{1 - \frac{1}{4}(0.672594562)^3\right\}$$
$$= 1149.282337\,\text{N}\cdot\text{m}$$

$$\text{un-loading}: \quad T' = \frac{\tau'_{\text{max}} J}{c}$$

$$T = T', \quad T = \frac{\tau'_{\text{max}} J}{c}$$

$$\tau'_{\text{max}} = \frac{Tc}{J} = \frac{(1149.282337\,\text{N}\cdot\text{m})(16\,\text{mm})}{3.2768\pi \times 10^{-8}\,\text{m}^4} = \frac{(1149.282337)(0.016)}{3.2768\pi \times 10^{-8}}\,\text{Pa}$$
$$= 178.6269189\,\text{MPa}$$

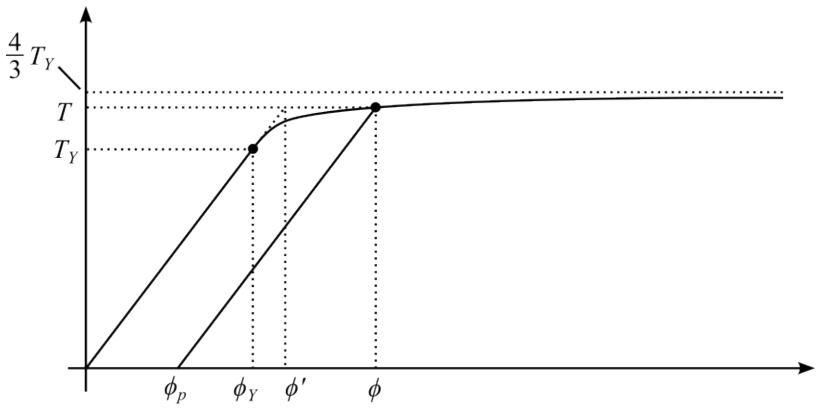
$$\tau_{\text{res}}(\rho) = |\tau(\rho) - \tau'(\rho)|$$

$$\tau_{\text{res}}(\rho_Y) = |\tau(\rho_Y) - \tau'(\rho_Y)| = |\tau_Y - R\tau'_{\text{max}}|$$
$$= |145\,\text{MPa} - (0.672594562)(178.6269189\,\text{MPa})| = 24.9\,\text{MPa}$$

$$\tau_{\text{res}}(c) = |\tau(c) - \tau'(c)| = |\tau_Y - \tau'_c| = |145\,\text{MPa} - 178.6269189\,\text{MPa}|$$
$$= 33.6\,\text{MPa}$$

$$\left.\begin{aligned} \tau_{\text{res}}(\rho_Y) &= 24.9\,\text{MPa} \\ \tau_{\text{res}}(c) &= 33.6\,\text{MPa} \end{aligned}\right\} \quad \Rightarrow \quad \tau_{\text{res,max}} = 33.6\,\text{MPa}$$

$$33.6\,\text{MPa} \quad \text{at} \quad \rho = 16.00\,\text{mm} \quad ◄ \quad (a)$$



$$\phi_p = \phi - \phi' = \phi - T \cdot \frac{\phi_Y}{T_Y} = 6^\circ - \frac{1149.282337\,\text{N}\cdot\text{m}}{932.9273544\,\text{N}\cdot\text{m}}(4.035567372^\circ) = 1.028^\circ \quad ◄ \quad (b)$$