Solutions

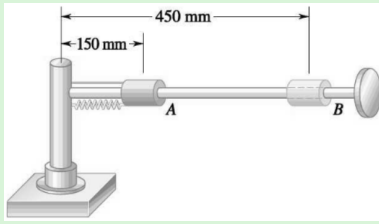## of the Dynamics Midterm Exam 2025-1

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Question 1

A 1.5 kg collar can slide on a horizontal rod, which is free to rotate about a vertical shaft. The collar is initially held at A by a cord attached to the shaft. A spring of constant 35 N/m is attached to the collar and to the shaft and is undeformed when the collar is at A. As the rod rotates at the rate is 19 rad/s, the cord is cut and the collar moves out along the rod. Neglecting friction and the mass of the rod, determine (a) the acceleration of the collar relative to the rod at A, (c) the transverse component of the velocity of the collar at B.

$$\sum F_r = 0 = m(\ddot{r} - r\dot{\theta}^2) \quad \Rightarrow \quad \ddot{r} = r\dot{\theta}^2$$
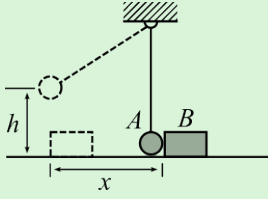
$$a_{\text{collar/rod}} = \ddot{r} = r\dot{\theta}^2 = (0.15\,\text{m})(19\,\text{s}^{-1})^2 = 54.15\,\text{m/s}^2$$

$$(a) \quad a_{\text{collar/rod}} = 54.15\,\text{m/s}^2\,\mathbf{e}_r \quad \blacktriangleleft$$

$$v_{\theta,B} = r_B\dot{\theta} = (0.45\,\text{m})(19\,\text{s}^{-1}) = 8.55\,\text{m/s}$$

$$(b) \quad v_\theta = 8.55\,\text{m/s} \quad \blacktriangleleft$$

Question 2

A 1.1-kg block B moving to the left with a speed $v_0 = 3\,\text{m/s}$ strikes a 0.2-kg ball A that is hanging from a cord and is initially at rest. Knowing that the coefficient of kinetic friction between B and the ground is 0.6 and the coefficient of restitution between A and B is 0.7, determine (a) the maximum h in [m], (b) x when B stops in [mm].

v_A, v_B, v'_A and v'_B are consumed as positive number when directions of velocities of them is left.

given : $\quad v_A = 0, \quad v_B = 3\,\text{m/s}, \quad m_A = 0.5\,\text{kg}, \quad m_B = 1\,\text{kg}, \quad \mu_k = 0.6, \quad e = 0.7$

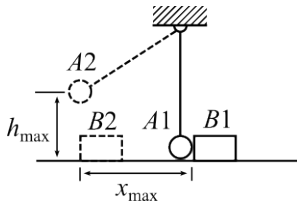$$v'_A - v'_B = e(v_B - v_A)$$
$$v'_A - v'_B = e v_B \tag{1}$$
$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$
$$m_B v_B = m_A v'_A + m_B v'_B \tag{2}$$

$$\begin{cases} v'_A - v'_B = e v_B & (1) \\ m_B v_B = m_A v'_A + m_B v'_B & (2) \end{cases}$$

$$v'_A = \frac{m_B(e+1)}{m_A + m_B} v_B = \frac{(1.1)(0.7+1)}{0.2 + 1.1}(3\,\text{m/s}) = \frac{561}{130}\,\text{m/s}$$

$$v'_B = v'_A - e v_B = \left(\frac{561}{130}\,\text{m/s}\right) - (0.7)(3\,\text{m/s}) = \frac{144}{65}\,\text{m/s}$$

we got : $\quad v'_A = \dfrac{561}{130}\,\text{m/s}, \quad v'_B = \dfrac{144}{65}\,\text{m/s}$

$$T_{A1} + V_{g,A1} = T_{A2} + V_{g,A2}$$

$$\frac{1}{2} m_A (v'_A)^2 + 0 = 0 + m_A g h$$

$$h = \frac{(v'_A)^2}{2g} = \frac{\left(\frac{561}{130}\right)^2}{2(9.81)}\,\text{m} = 0.949\,\text{m}$$
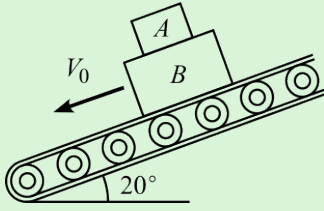
$$(a) \quad h = 0.949\,\text{m} \quad \blacktriangleleft$$

$$T_{B1} + U_{B1 \to B2} = T_{B2}$$

$$\frac{1}{2} m_B (v'_B)^2 - \mu_k m_B g x = 0$$

$$x = \frac{(v'_B)^2}{2\mu_k g} = \frac{\left(\frac{144}{65}\right)^2}{2(0.6)(9.81)} = 0.417\,\text{m} = 417\,\text{mm}$$
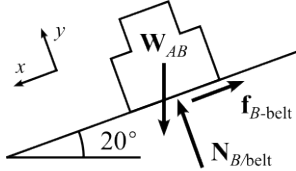
$$(b) \quad x = 417\,\text{mm} \quad \blacktriangleleft$$

3

A 10-kg box A and 20-kg box B is sliding down the inclined conveyer belt in the position shown with speed V_0. Knowing that the coefficient of kinetic friction between B and the belt is 0.3, determine the minimum coefficient of static friction between A and B when A does not slip on B.

given : $\quad \mu_k = 0.3, \quad m_A = 10\,\text{kg}, \quad m_B = 20\,\text{kg}$

In the system of A and B, the FBD and KD is
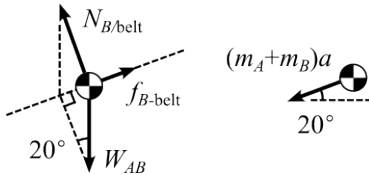

$$\sum F_y = 0$$

$N_{B/\text{belt}} - W_{AB} \cos 20° = 0$
$N_{B/\text{belt}} = (m_A + m_B)g \cos 20°$
$f_{B-\text{belt}} = \mu_k N_{B/\text{belt}} = \mu_k (m_A + m_B)g \cos 20°$
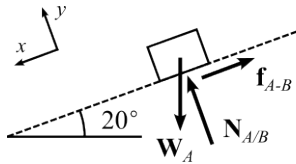
$$\sum F_x = ma$$

$W_{AB} \sin 20° - f_{B-\text{belt}} = (m_A + m_B)a$
$(m_A + m_B)g \sin 20° - \mu_k (m_A + m_B)g \cos 20° = (m_A + m_B)a$
$a = g(\sin 20° - \mu_k \cos 20°) = 0.589702\,\text{m/s}^2$
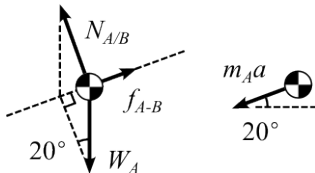
In the system of A, the FBD and KD is

$$\sum F_y = 0$$

$N_{A/B} - W_A \cos 20° = 0$
$N_{A/B} = m_A g \cos 20°$

$$\sum F_x = ma$$

$W_A \sin 20° - f_{A-B} = m_A a$
$m_A g \sin 20° - f_{A-B} = m_A g(\sin 20° - \mu_k \cos 20°)$
$f_{A-B} = \mu_k m_A g \cos 20°$

$f_{A-B} \leq \mu_s N_{A/B}$
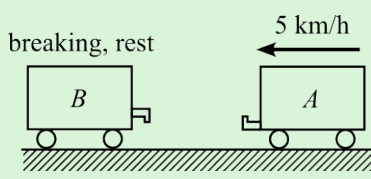$\mu_k m_A g \cos 20° \leq \mu_s m_A g \cos 20°$
$\mu_k \leq \mu_s$
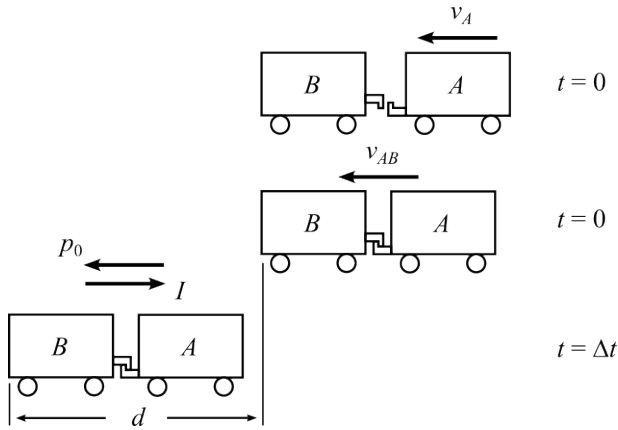$\mu_{s,\text{min}} = \mu_k = 0.3$

0.300 ◄

Question 4

5 km/h

breaking, rest

B

A

A 20-Mg train A is moving to the left in this picutre and combined with a 40-Mg train B that is at rest. And its brake is working so the coefficient of friction between B and the ground is 0.3. When the speed of B just before combination is 5 km/h, determine (a) the velocity of A and B just after combining and (b) the time taken for them to stop.

given : $\quad v_A = 5\,\text{km/h} = \dfrac{25}{18}\,\text{m/s}, \quad \mu_k = 0.3, \quad m_A = 20 \times 10^3\,\text{kg}, \quad m_B = 40 \times 10^3\,\text{kg}$

v_A

B   A $t = 0$

v_{AB}

B   A $t = 0$

p_0

I

B   A $t = \Delta t$

d

$p_0 = m_A v_A = (m_A + m_B) v_{AB}$

$v_{AB} = \dfrac{m_A v_A}{m_A + m_B} = \dfrac{(20)(5\,\text{km/h})}{40 + 20}$

$= 1.667\,\text{km/h}$

$(a) \quad \mathbf{v}_{AB} = 1.667\,\text{km/h} \leftarrow \quad \blacktriangleleft$

$F = \mu_k N = \mu_k m_B g$

$I = F\Delta t = \mu_k m_B g \Delta t$

$p_0 - I = 0$

$(m_A + m_B)v_{AB} - \mu_k m_B g \Delta t = 0$

$\Delta t = \dfrac{m_A v_A}{\mu_k m_B g} = \dfrac{(20)\left(\frac{25}{18}\,\text{m/s}\right)}{(0.3)(40)(9.81\,\text{m/s}^2)} = 0.236\,\text{s}$

$(b) \quad \Delta t = 0.236\,\text{s} \quad \blacktriangleleft$