해설

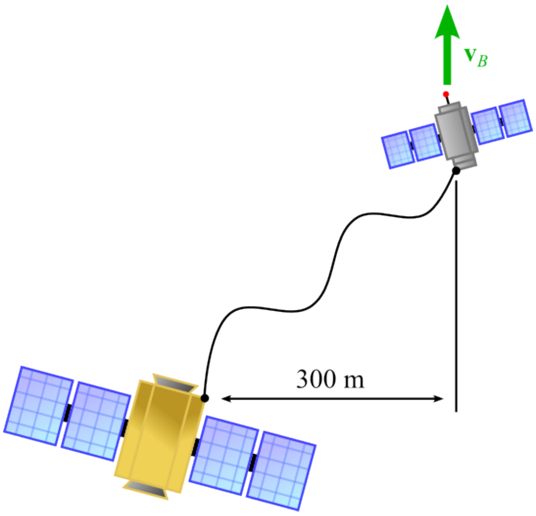
동역학(이동훈 교수님) 2025-1 기말고사

시험 실시 : 2025-06-10 18:30-20:30+α (120분+α)

기계공학과 2학년 2022**** ***

2025–06–25

Question 1 — Prob. 14.45 in textbook



The 2-kg sub-satellite B has an initial velocity $\mathbf{v}_B = 3\,\mathrm{m/s}\,\mathbf{j}$. It is connected to the 20-kg base-satellite A by a 500-m space tether. Determine the velocity of the base satellite and sub-satellite immediately after the tether becomes taut (assuming no rebound).

[SOLUTION]

$$m_A = 20\,\mathrm{kg}, \quad m_B = 2\,\mathrm{kg}$$

$$\mathbf{L} = m_B\mathbf{v}_B = m_A\mathbf{v}'_A + m_B\mathbf{v}'_B$$

$$\mathbf{v}'_A = v'_A\measuredangle \arctan\frac{4}{3} = \frac{3}{5}v'_A\mathbf{i} + \frac{4}{5}v'_A\mathbf{j}$$

$$L_x = 0 = \frac{3}{5}m_Av'_A + m_Bv'_{B,x}$$

$$L_y = m_Bv_B = \frac{4}{5}m_Av'_A + m_Bv'_{B,y}$$

$$\Rightarrow \quad v'_{B,x} = -\frac{3m_A}{5m_B}v'_A = -\frac{3(20)}{5(2)}v'_A = -6v'_A$$

$$\Rightarrow \quad v'_{B,y} = v_B - \frac{4m_A}{5m_B}v'_A = 3\,\mathrm{m/s} - \frac{4(20)}{5(2)}v'_A = 3\,\mathrm{m/s} - 8v'_A$$

$$\mathbf{v}_{B/A} = \mathbf{v}'_B - \mathbf{v}'_A = \left(v_{B,x} - \frac{3}{5}v'_A\right)\mathbf{i} + \left(v_{B,y} - \frac{4}{5}v'_A\right)\mathbf{j} = v'_{B/A}\measuredangle\left(\arctan\frac{3}{4} + 90^\circ\right)$$

$$\Rightarrow \quad -\frac{v'_{B,y} - \frac{4}{5}v'_A}{v'_{B,x} - \frac{3}{5}v'_A} = -\frac{3\,\mathrm{m/s} - 8.8v'_A}{-6.6v'_A} = \frac{3}{4}$$

$$4(3\,\mathrm{m/s} - 8.8v'_A) = 3(6.6v'_A)$$

$$v'_A = \frac{12}{4(8.8) + 3(6.6)}\,\mathrm{m/s} = \frac{12}{55}\,\mathrm{m/s} = 0.218\,\mathrm{m/s}$$

$$\theta_A = \arctan\frac{3}{4} = 36.9^\circ$$

$$v'_B = \sqrt{v'^2_{B,x} + v'^2_{B,y}} = \sqrt{(6v'_A)^2 + (3\,\mathrm{m/s} - 8v'_A)^2} = 1.813\,\mathrm{m/s}$$

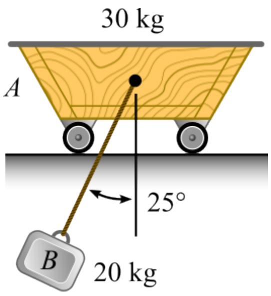
$$\theta_B = \arctan\frac{v'_{B,y}}{-v'_{B,x}} = \arctan\frac{3\,\mathrm{m/s} - 8v'_A}{6v'_A} = 43.8^\circ$$

$$180^\circ - \theta_B = 136.2^\circ$$

$$\mathbf{v}'_A = 0.218\,\mathrm{m/s}\measuredangle 36.9^\circ \quad \blacktriangleleft$$

$$\mathbf{v}'_B = 1.813\,\mathrm{m/s}\measuredangle 136.2^\circ \quad \blacktriangleleft$$

Question 2 — Prob. 14.40 in textbook



A 20-kg block B is suspended from a 2-m cord attached to a 30-kg cart A, which may roll freely on a frictionless, horizontal track. If the system is released from rest in the position shown, determine the velocities of A and B as B passes directly under A.

[SOLUTION]

Let the state 1 be the initial state and the state 2 be when B passes directly under A.

$$\cancelto{0}{T_1} + V_{g1} = T_2 + \cancelto{0}{V_{g2}}$$
$$m_B g l(1 - \cos 25°) = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$
$$m_A v_A^2 + m_B v_B^2 = 2 m_B g l(1 - \cos 25°) \tag{1}$$

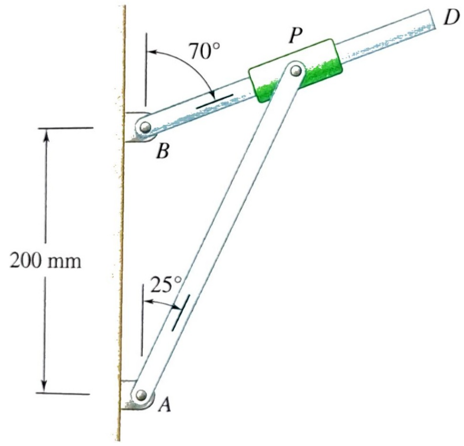
Impulses by gravity and ground are independent to horizontal component of momentum of A and B.

$$\sum p_{x1} = \sum p_{x2}$$
$$0 = -m_A v_A + m_B v_B \tag{2}$$
$$\begin{cases} m_A v_A^2 + m_B v_B^2 = 2 m_B g l(1 - \cos 25°) \\ -m_A v_A + m_B v_B = 0 \end{cases} \tag{1 \& 2}$$
$$m_A v_A = m_B v_B$$
$$m_A m_B v_A^2 + m_A^2 v_A^2 = 2 m_B^2 g l(1 - \cos 25°)$$
$$\frac{m_A}{m_B} v_A^2 + \left(\frac{m_A}{m_B}\right)^2 v_A^2 = 2 g l(1 - \cos 25°)$$
$$k(k+1) v_A^2 = 2 g l(1 - \cos 25°), \qquad k = \frac{m_A}{m_B} = 1.5$$
$$v_A = \sqrt{\frac{2 g l(1 - \cos 25°)}{k(k+1)}} = \sqrt{\frac{2(9.81)(2)(1 - \cos 25°)}{(1.5)(2.5)}}\,\text{m/s} = 0.990\,\text{m/s}$$
$$v_B = \frac{m_A}{m_B} v_A = k v_A = (1.5)(0.990)\,\text{m/s} = 1.485\,\text{m/s}$$

$$\mathbf{v}_A = 0.990\,\text{m/s} \leftarrow \quad ◀$$

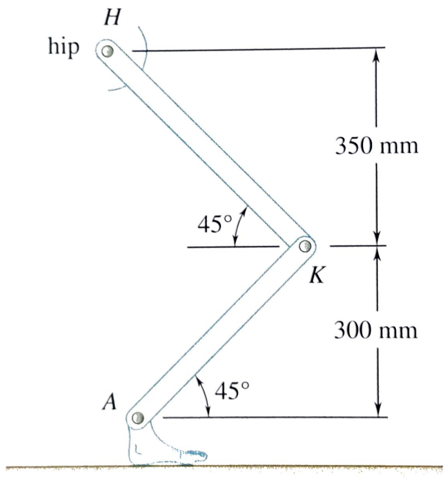
$$\mathbf{v}_B = 1.485\,\text{m/s} \rightarrow \quad ◀$$

Question 3 — variation of Prob. 15.176 in textbook



Knowing that at the instant shown the rod attached at A has an angular velocity of $5\,\mathrm{rad/s}$ counterclockwise and an angular acceleration of $2\,\mathrm{rad/s^2}$ clockwise, determine the acceleration of point D. (Length of the boom BD is $1000\,\mathrm{mm}$.)

Question 4 — Prob. 15.171 in textbook



The human leg can be crudely approximated as two rigid bars (the femur and the tibia) connected with a pin joint. At the instant shown the veolcity and acceleration of the ankle is zero. During a jump, the velocity of the ankle A is zero, the tibia AK has an angular velocity of 1.5 rad/s counterclockwise and an angular acceleration of $1\,\mathrm{rad/s^2}$ counterclockwise. Determine the relative angular velocity and angular acceleration of the femur KH with respect to AK so that the velocity and acceleration of H are both straight up at the instant shown.

*femur : 대퇴골, ankle : 발목, tibia : 정강이 뼈

[SOLUTION]

Let angluars be positive when they are conterclockwise. The origin of all fixed frame is point A.

$$\begin{aligned}\text{Given :}\quad & H = 7l, \quad h = 6l, \quad v_A = 0, \quad \mathbf{\Omega} = \boldsymbol{\omega}_{AK} = 1.5\mathbf{k}\,\mathrm{s}^{-1}, \quad \dot{\mathbf{\Omega}} = \dot{\boldsymbol{\omega}}_{AK} = 1\mathbf{k}\,\mathrm{s}^{-2}\\ & \theta_{AK} = 45^\circ, \quad \theta_{KH} = 135^\circ, \quad \dot{r}_{Hx} = 0, \quad \ddot{r}_{Hx} = 0\\ \text{Determine :}\quad & \boldsymbol{\omega} = \boldsymbol{\omega}_{KH/AK}, \quad \dot{\boldsymbol{\omega}} = \dot{\boldsymbol{\omega}}_{KH/AK}\end{aligned}$$

$$\begin{aligned}
\dot{\mathbf{r}}_H &= \dot{\mathbf{r}}'_H + \dot{\mathbf{r}}_H|_{AK} = \mathbf{\Omega} \times \mathbf{r}_{H/A} + \boldsymbol{\omega} \times \mathbf{r}_{H/K}\\
&= \Omega l(-13\mathbf{i} - \mathbf{j}) + \omega l(-7\mathbf{i} - 7\mathbf{j}) = l(-13\Omega - 7\omega)\mathbf{i} + l(-\Omega - 7\omega)\mathbf{j}\\
\dot{r}_{Hx} &= l(-13\Omega - 7\omega) = 0, \quad \omega = -\frac{13}{7}\Omega = -\frac{39}{14}\,\mathrm{rad/s} = -2.786\,\mathrm{rad/s}\\
\dot{r}_{Hy} &= l(-\Omega - 7\omega)\\
\dot{\mathbf{r}}_H|_{AK} &= \boldsymbol{\omega} \times \mathbf{r}_{H/K}\\
\ddot{\mathbf{r}}_H &= \ddot{\mathbf{r}}'_H + \ddot{\mathbf{r}}_H|_{AK}\\
&= \Big[\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{H/A}) + 2\mathbf{\Omega} \times \dot{\mathbf{r}}_H|_{AK} + \dot{\mathbf{\Omega}} \times \mathbf{r}_{H/A}\Big] + \big[\boldsymbol{\omega} \times \big(\boldsymbol{\omega} \times \mathbf{r}_{H/K}\big) + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{H/K}\big]\\
&= \Big[\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{H/A}) + 2\mathbf{\Omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{H/K}) + \dot{\mathbf{\Omega}} \times \mathbf{r}_{H/A}\Big] + \big[\boldsymbol{\omega} \times \big(\boldsymbol{\omega} \times \mathbf{r}_{H/K}\big) + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{H/K}\big]\\
&= \Big[\Omega^2 l(\mathbf{i} - 13\mathbf{j}) + 2\Omega\omega l(-7\mathbf{i} + 7\mathbf{j}) + \dot{\Omega} l(-13\mathbf{i} - \mathbf{j})\Big] + \big[\omega^2 l(7\mathbf{i} - 7\mathbf{j}) + \dot{\omega} l(-7\mathbf{i} - 7\mathbf{j})\big]\\
&= \Big(\Omega^2 l - 14\Omega\omega l - 13\dot{\Omega} l + 7\omega^2 l - 7\dot{\omega} l\Big)\,\mathbf{i} + \Big(-13\Omega^2 l + 14\Omega\omega l - \dot{\Omega} l - 7\omega^2 l - 7\dot{\omega} l\Big)\,\mathbf{j}\\
\ddot{r}_{Hx} &= \Omega^2 l - 14\Omega\omega l - 13\dot{\Omega} l + 7\omega^2 l - 7\dot{\omega} l = 0\\
\dot{\omega} &= \frac{\Omega^2 l - 14\Omega\omega l - 13\dot{\Omega} l + 7\omega^2 l}{7l} = \frac{\Omega^2 - 13\dot{\Omega}}{7} - 2\Omega\omega + \omega^2 = -2.133\,\mathrm{rad/s^2}
\end{aligned}$$

$\boldsymbol{\omega} = 2.786\,\mathrm{rad/s}$ ↻ ◀

$\dot{\boldsymbol{\omega}} = 2.133\,\mathrm{rad/s^2}$ ↻ ◀