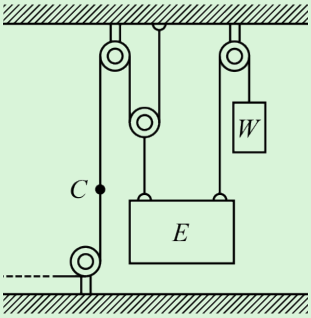
Solutions

of the Dynamics Midterm Exam 2023-1

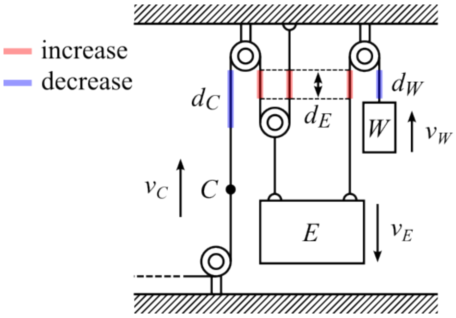
E.Hong

May 2, 2025

Question 1



Knowing that the block E is moving downward with a speed of $2\,\mathrm{m/s}$, determine (a) the velocities of C and W, (b) the relative velocity of C with respect to E and that of W with respect to E.



given : $\quad \mathbf{v}_E = 2\,\mathrm{m/s}\downarrow$

We can find some constraint equations.

$$d_C = 2d_E \quad\Rightarrow\quad v_C = 2v_E$$
$$d_W = d_E \quad\Rightarrow\quad v_W = v_E$$

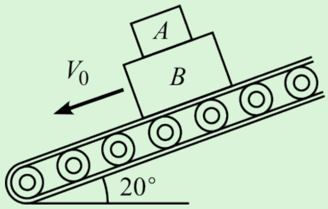
$$v_C = 2v_E = 4\,\mathrm{m/s}$$
$$v_W = v_E = 2\,\mathrm{m/s}$$

$(a) \quad \mathbf{v}_C = 4.00\,\mathrm{m/s}\uparrow\,;\ \mathbf{v}_W = 2.00\,\mathrm{m/s}\uparrow$ ◀

$$v_{C/E} = v_C - v_E = 6\,\mathrm{m/s}$$
$$v_{W/E} = v_W - v_E = 4\,\mathrm{m/s}$$

$(b) \quad \mathbf{v}_{C/E} = 6.00\,\mathrm{m/s}\uparrow\,;\ \mathbf{v}_{W/E} = 4.00\,\mathrm{m/s}\uparrow$ ◀

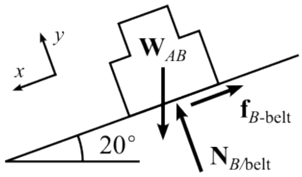
Question 2

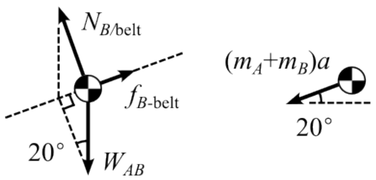


A 10-kg box A and 20-kg box B is sliding down the inclined conveyer belt in the position shown with speed V_0. Knowing that the coefficient of kinetic friction between B and the belt is 0.3, determine the minimum coefficient of static friction between A and B when A does not slip on B.

$$\text{given}: \quad \mu_k = 0.3, \quad m_A = 10\,\text{kg}, \quad m_B = 20\,\text{kg}$$

In the system of A and B, the FBD and KD is





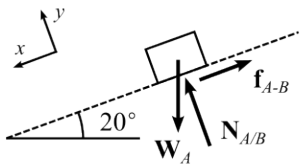
$$\sum F_y = 0$$

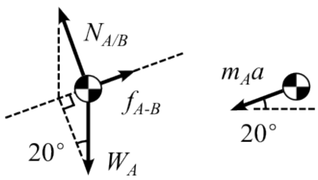
$$\begin{aligned}
&N_{B/\text{belt}} - W_{AB}\cos 20^\circ = 0\\
&N_{B/\text{belt}} = (m_A + m_B)g\cos 20^\circ\\
&f_{B-\text{belt}} = \mu_k N_{B/\text{belt}} = \mu_k(m_A + m_B)g\cos 20^\circ
\end{aligned}$$

$$\sum F_x = ma$$

$$\begin{aligned}
&W_{AB}\sin 20^\circ - f_{B-\text{belt}} = (m_A + m_B)a\\
&(m_A + m_B)g\sin 20^\circ - \mu_k(m_A + m_B)g\cos 20^\circ = (m_A + m_B)a\\
&a = g(\sin 20^\circ - \mu_k\cos 20^\circ) = 0.589702\,\text{m/s}^2
\end{aligned}$$

In the system of A, the FBD and KD is





$$\sum F_y = 0$$

$$\begin{aligned}
&N_{A/B} - W_A\cos 20^\circ = 0\\
&N_{A/B} = m_A g\cos 20^\circ
\end{aligned}$$

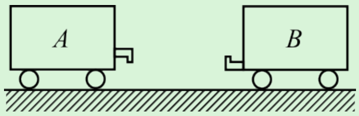
$$\sum F_x = ma$$

$$\begin{aligned}
&W_A\sin 20^\circ - f_{A-B} = m_A a\\
&m_A g\sin 20^\circ - f_{A-B} = m_A g(\sin 20^\circ - \mu_k\cos 20^\circ)\\
&f_{A-B} = \mu_k m_A g\cos 20^\circ
\end{aligned}$$

$$\begin{aligned}
&f_{A-B} \le \mu_s N_{A/B}\\
&\mu_k m_A g\cos 20^\circ \le \mu_s m_A g\cos 20^\circ\\
&\mu_k \le \mu_s\\
&\mu_{s,\min} = \mu_k = 0.3
\end{aligned}$$

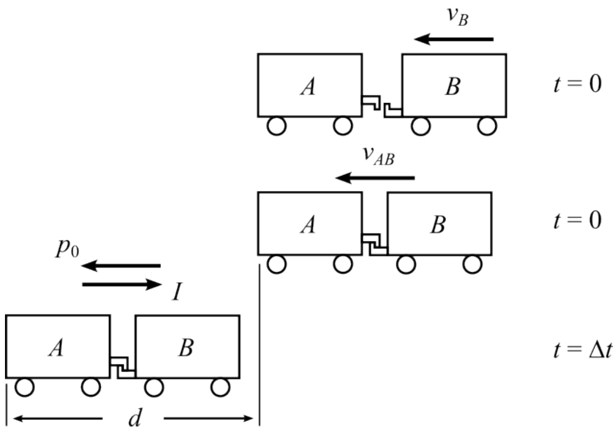
0.300 ◄

Question 3



A 40-Mg car B is moving to the left in this picutre and combined with a 20-Mg car A that is at rest. Knowing that the coefficient of friction between the wheels and ground is 0.3 and the speed of B just before combination is $4\,\mathrm{m/s}$, determine (a) the velocity of A and B just after combining and (b) the time taken for them to stop.

$$\text{given}: \quad v_B = 4\,\mathrm{m/s}, \quad \mu_k = 0.3, \quad m_A = 20\times10^3\,\mathrm{kg}, \quad m_B = 40\times10^3\,\mathrm{kg}$$



$$p_0 = m_B v_B = (m_A + m_B) v_{AB}$$

$$v_{AB} = \frac{m_B v_B}{m_A + m_B} = \frac{(40)(4)}{40+20}\,\mathrm{m/s}$$

$$= 2.67\,\mathrm{m/s}$$

$(a) \quad \mathbf{v}_{AB} = 2.67\,\mathrm{m/s} \leftarrow$ ◀

$$F = \mu_k N = \mu_k g(m_A + m_B)$$

$$I = F\Delta t = \mu_k g(m_A + m_B)\Delta t$$

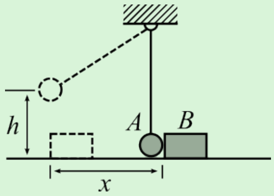
$$p_0 - I = 0$$

$$(m_A + m_B)v_{AB} - \mu_k g(m_A + m_B)\Delta t = 0$$

$$\Delta t = \frac{m_B v_B}{\mu_k g(m_A + m_B)} = \frac{(40)(4)}{(0.3)(9.81)(40+20)}\,\mathrm{s} = 0.906\,\mathrm{s}$$

$(b) \quad \Delta t = 0.906\,\mathrm{s}$ ◀

Question 4



A 1-kg block B moving to the left with a speed $2\,\mathrm{m/s}$ strikes a 0.5-kg ball A that is hanging from a cord and is initially at rest. Knowing that the coefficient of kinetic friction between B and the ground is 0.6 and the coefficient of restitution between A and B is 0.8, determine (a) the maximum h, (b) x when B stops.

v_A, v_B, v'_A and v'_B are consumed as positive number when directions of velocities of them is left.

$$\text{given}: \quad v_A = 0, \quad v_B = 2\,\mathrm{m/s}, \quad m_A = 0.5\,\mathrm{kg}, \quad m_B = 1\,\mathrm{kg}, \quad \mu_k = 0.6, \quad e = 0.8$$

$$v'_A - v'_B = e(v_B - v_A)$$

$$v'_A - v'_B = ev_B \tag{1}$$

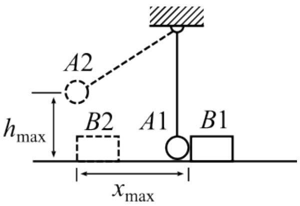
$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

$$m_B v_B = m_A v'_A + m_B v'_B \tag{2}$$

$$\begin{cases} v'_A - v'_B = ev_B & (1) \\ m_B v_B = m_A v'_A + m_B v'_B & (2) \end{cases}$$

$$v'_A = \frac{m_B(e+1)}{m_A + m_B} v_B = \frac{(1)(0.8+1)}{0.5+1}(2\,\mathrm{m/s}) = 2.4\,\mathrm{m/s}$$

$$v'_B = v'_A - ev_B = (2.4\,\mathrm{m/s}) - (0.8)(2\,\mathrm{m/s}) = -0.8\,\mathrm{m/s}$$



$$\text{we got}: \quad v'_A = 2.4\,\mathrm{m/s}, \quad v'_B = 0.8\,\mathrm{m/s}$$

$$T_{A1} + V_{g,A1} = T_{A2} + V_{g,A2}$$

$$\frac{1}{2} m_A (v'_A)^2 + 0 = 0 + m_A g h$$

$$h = \frac{(v'_A)^2}{2g} = \frac{(2.4)^2}{2(9.81)}\,\mathrm{m} = 0.294\,\mathrm{m}$$

(a) $\quad h = 0.294\,\mathrm{m}$ ◀

$$T_{B1} + U_{B1\to B2} = T_{B2}$$

$$\frac{1}{2} m_B (v'_B)^2 - \mu_k m_B g x = 0$$

$$x = \frac{(v'_B)^2}{2\mu_k g} = \frac{(-0.8)^2}{2(0.6)(9.81)} = 0.0544\,\mathrm{m}$$

(b) $\quad x = 0.0544\,\mathrm{m}$ ◀