HW3

2025-2 구조역학(박성훈 교수님)

Problem 9.1, 9.3, 9.10, 9.16, 9.20, 9.67, 9.72, 9.76

2025–12–27

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In the following problems, assume that the flexural rigidity EI of each beam is constant.

Problem 9.1 and 9.3

For the loading shown, determine (a) the equation of the elastic curve for the cantilever beam AB, (b) the deflection at the free end, (c) the slope at the free end.

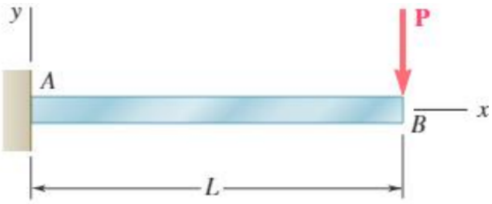


Fig. P9.1

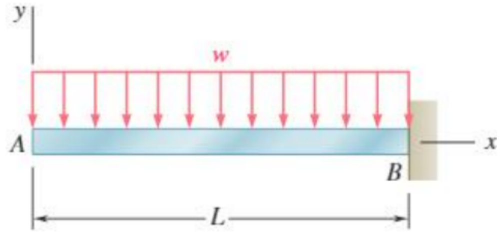


Fig. P9.3

Prob. 9.1 ▼

$$+\circlearrowleft \sum M|_A = -PL + M_A = 0$$

$$\Rightarrow \quad M_A = PL \ (\circlearrowleft)$$

$$+\uparrow \sum F_y = R_A - P = 0$$

$$\Rightarrow \quad R_A = P \ (\uparrow)$$

$$V(x) = P \ \Rightarrow \ M(x) = Px - PL$$

$$EI\theta(x) = \frac{P}{2}x^2 - PLx + c_1$$

$$\theta(0) = 0 \quad \Rightarrow \quad c_1 = 0$$

$$EIy(x) = \frac{P}{6}x^3 - \frac{PL}{2}x^2 + c_2$$

$$y(0) = 0 \quad \Rightarrow \quad c_2 = 0$$

$$\theta(x) = \frac{P}{2EI}(x^2 - 2Lx)$$

$$y(x) = \frac{P}{6EI}(x^3 - 3Lx^2) \quad ◀ \ (a)$$

$$y(L) = -\frac{PL^3}{3EI} \quad ◀ \ (b)$$

$$\theta(L) = -\frac{PL^2}{2EI} \quad ◀ \ (c)$$

Prob. 9.3 ▼

$$+\circlearrowleft \sum M|_B = wL\left(\frac{L}{2}\right) - M_B = 0$$

$$\Rightarrow \quad M_B = \frac{wL^2}{2} \ (\circlearrowright)$$

$$+\uparrow \sum F_y = R_B - wL = 0$$

$$\Rightarrow \quad R_B = wL \ (\uparrow)$$

$$V(x) = -wx \ \Rightarrow \ M(x) = -\frac{w}{2}x^2$$

$$EI\theta(x) = -\frac{w}{6}x^3 + c_1$$

$$\theta(L) = 0 \quad \Rightarrow \quad c_1 = \frac{wL^3}{6}$$

$$EIy(x) = -\frac{w}{24}x^4 + \frac{wL^3}{6}x + c_2$$

$$y(L) = 0 \quad \Rightarrow \quad c_2 = -\frac{wL^4}{8}$$

$$\theta(x) = -\frac{w}{6EI}(x^3 - L^3)$$

$$y(x) = -\frac{w}{24EI}(x^4 - 4L^3x + 3L^4) \quad ◀ \ (a)$$

$$y(0) = -\frac{wL^4}{8EI} \quad ◀ \ (b)$$

$$\theta(0) = \frac{wL^3}{6EI} \quad ◀ \ (c)$$

Problem 9.10

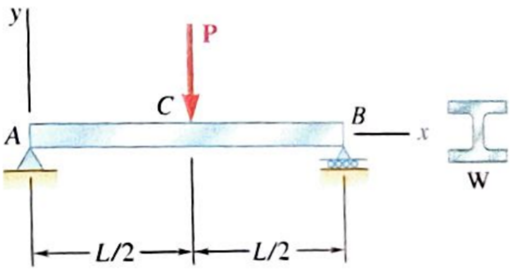


Fig. P9.10

Knowing that beam AB is a W130×23.8 rolled shape and that $P = 50\,\text{kN}$. $L = 1.25\,\text{m}$, and $E = 200\,\text{GPa}$, determine (a) the slope at A. (b) the deflection at C.

* $I = 8.91 \times 10^6\,\text{mm}^4$ $\cdots$ appendix E

$$+\circlearrowleft \sum M|_A = -\frac{PL}{2} + R_B L = 0, \quad R_B = \frac{P}{2}\ (\uparrow)$$

$$+\circlearrowleft \sum F_y = -P + R_A + R_B = 0, \quad R_A = \frac{P}{2}\ (\uparrow)$$

$$V(x) = \begin{cases} \dfrac{P}{2} & \left(0 \le x \le \dfrac{L}{2}\right) \\ \\ -\dfrac{P}{2} & \left(\dfrac{L}{2} \le x \le L\right) \end{cases} \quad \Rightarrow \quad M(x) = \begin{cases} \dfrac{P}{2}x & \left(0 \le x \le \dfrac{L}{2}\right) \\ \\ -\dfrac{P}{2}(x-L) & \left(\dfrac{L}{2} \le x \le L\right) \end{cases}$$

$$EI\theta(x) = \begin{cases} \dfrac{P}{4}x^2 + c_1 & ('') \\ \\ -\dfrac{P}{4}(x-L)^2 + c_2 & ('') \end{cases} \quad \Rightarrow \quad EIy(x) = \begin{cases} \dfrac{P}{12}x^3 + c_1 x + c_3 & ('') \\ \\ -\dfrac{P}{12}(x-L)^3 + c_2 x + c_4 & ('') \end{cases}$$

$$\theta\left(\frac{L}{2}\right) = 0 \quad \Rightarrow \quad EI\theta\left(\frac{L}{2}\right) = \frac{P}{4}\left(\frac{L}{2}\right)^2 + c_1 = -\frac{P}{4}\left(-\frac{L}{2}\right)^2 + c_2 = 0$$

$$\Rightarrow \quad c_1 = -\frac{PL^2}{16}, \quad c_2 = \frac{PL^2}{16}$$

$$y(0) = 0 \quad \Rightarrow \quad EIy(0) = c_3 = 0$$

$$y(L) = 0 \quad \Rightarrow \quad EIy(L) = c_2 L + c_4 = 0, \quad c_4 = -c_2 L = -\frac{PL^3}{16}$$

$$\theta(0) = \frac{c_1}{EI} = -\frac{PL^2}{16EI} = -2.74 \times 10^{-3} \quad \blacktriangleleft \quad (a)$$

$$y\left(\frac{L}{2}\right) = \frac{1}{EI}\left\{\frac{P}{12}\left(\frac{L}{2}\right)^3 - \frac{PL^2}{16}\left(\frac{L}{2}\right)\right\} = -\frac{PL^3}{48EI} = -1.142\,\text{mm} \quad \blacktriangleleft \quad (b)$$

Problem 9.16

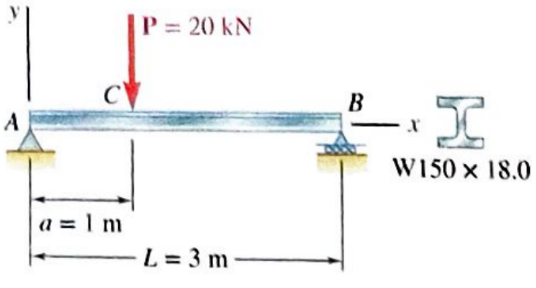


Fig. P9.16

For the beam and loading shown, determine the deflection at point C, Use $E = 200\,\text{GPa}$

from appendix E : $I = 9.20 \times 10^6\,\text{mm}^4$

$$+\circlearrowleft \sum M|_A = -P \cdot \frac{L}{3} + R_B L = 0, \quad R_B = \frac{P}{3}\uparrow$$

$$+\uparrow \sum F_y = R_A + R_B - P = 0, \quad R_A = \frac{2P}{3}\uparrow$$

$$V(x) = \begin{cases} \dfrac{2P}{3} & (0 \le x \le a) \\ -\dfrac{P}{3} & (a < x \le L) \end{cases} \quad \Rightarrow \quad M(x) = \begin{cases} \dfrac{2P}{3}x & ('') \\ -\dfrac{P}{3}x + \dfrac{PL}{3} = -\dfrac{P}{3}(x-L) & ('') \end{cases}$$

$$EI\theta(x) = \begin{cases} \dfrac{P}{3}x^2 + c_1 \\ -\dfrac{P}{6}(x-L)^2 + c_2 \end{cases} \quad \Rightarrow \quad EIy(x) = \begin{cases} \dfrac{P}{9}x^3 + c_1 x + c_3 \\ -\dfrac{P}{18}(x-L)^3 + c_2 x + c_4 \end{cases}$$

The boundary conditions are $y(0) = y(L) = 0$, $\theta_1(L/3) = \theta_2(L/3)$ and $y_1(L/3) = y_2(L/3)$.

$$EIy(0) = c_3 = 0$$

$$EIy(L) = c_2 L + c_4 = 0 \tag{1}$$

$$EI\theta\left(\frac{L}{3}\right) = \frac{PL^2}{27} + c_1 = -\frac{2PL^2}{27} + c_2 \tag{2}$$

$$EIy\left(\frac{L}{3}\right) = \frac{PL^3}{243} + c_1 L + \cancel{c_3}^{\,0} = \frac{4PL^3}{243} + c_2\frac{L}{3} + c_4 \tag{3}$$

We can make a series of equations with equations (1), (2) and (3).

$$\begin{array}{llll} & Lc_2 & +c_4 & = 0 \quad (1) \\ c_1 & -c_2 & & = -\dfrac{PL^2}{9} \quad (2) \\ \dfrac{L}{3}c_1 & -\dfrac{L}{3}c_2 & -c_4 & = \dfrac{PL^3}{81} \quad (3) \end{array} \quad \Rightarrow \quad \left(\begin{array}{ccc|c} 0 & L & 1 & 0 \\ 1 & -1 & 0 & -\frac{PL^2}{9} \\ \frac{L}{3} & -\frac{L}{3} & -1 & \frac{PL^3}{81} \end{array}\right)$$

Use a scientific calculator. $\boxed{\texttt{MODE}} \rightarrow \boxed{5}$ (Equations)

$$c_1 = -\frac{5PL^2}{81}, \quad c_2 = \frac{4PL^2}{81}, \quad c_4 = -\frac{4PL^3}{81}$$

$$\theta(x) = \begin{cases} \dfrac{P}{81EI}\left(27x^2 - 5L^2\right) \\ -\dfrac{P}{162EI}\left(27x^2 - 54Lx + 19L^2\right) \end{cases}, \quad y(x) = \begin{cases} \dfrac{P}{81EI}\left(9x^3 - 5L^2 x\right) \\ -\dfrac{P}{162EI}\left(9x^3 - 27Lx^2 + 19L^2 x - L^3\right) \end{cases}$$

$$y\left(\frac{L}{3}\right) = -\frac{4PL^3}{243EI} = -4.83\,\text{mm} \quad \blacktriangleleft$$

Problem 9.20

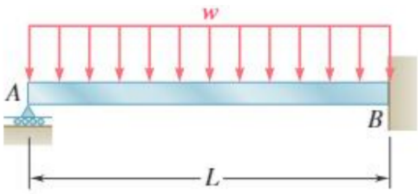


Fig. P9.20

For the beam and loading shown, determine the reaction at the roller support.

$$+\uparrow \sum F_y = R_A - wL + R_B = 0 \tag{1}$$

$$+\circlearrowleft \sum M|_B = M_B + wL\cdot\frac{L}{2} - R_A L = 0 \tag{2}$$

$$M(0)=0, \quad M(x) = R_A x - wx\cdot\frac{x}{2} = -\frac{w}{2}x^2 + R_A x$$

$$EI\theta(x) = -\frac{w}{6}x^3 + \frac{R_A}{2}x^2 + c_1$$

$$\theta(L)=0 \quad \Rightarrow \quad EI\theta(L) = -\frac{wL^3}{6} + \frac{R_A L^2}{2} + c_1 = 0 \tag{3}$$

$$EIy(x) = -\frac{w}{24}x^4 + \frac{R_A}{6}x^3 + c_1 x + c_2$$

$$y(0)=0 \quad \Rightarrow \quad EIy(0) = c_2 = 0$$

$$y(L)=0 \quad \Rightarrow \quad EIy(L) = -\frac{wL^4}{24} + \frac{R_A L^3}{6} + c_1 L = 0 \tag{4}$$

$$\begin{array}{lllll} -LR_A & +M_B & & = -\dfrac{wL^2}{2} & (2) \\ \dfrac{L^2}{2}R_A & & +c_1 & = \dfrac{wL^3}{6} & (3) \\ \dfrac{L^3}{6}R_A & & +Lc_1 & = \dfrac{wL^4}{24} & (4) \end{array} \quad \Rightarrow \quad \left(\begin{array}{ccc|c} -L & 1 & 0 & -\frac{wL^2}{2} \\ \frac{L^2}{2} & 0 & 1 & \frac{wL^3}{6} \\ \frac{L^3}{6} & 0 & L & \frac{wL^4}{24} \end{array}\right)$$

$$R_A = \frac{3wL}{8}\ (\uparrow) \quad \blacktriangleleft \quad , \quad M_B = -\frac{wL^2}{8}, \quad c_1 = -\frac{wL^3}{48}$$

From the eq.(1), $R_B = wL - R_A = \dfrac{5wL}{8}$

Use the method of superposition to solve the following problems and assume that the flexural rigidity EI of each beam is constant.

Problem 9.67

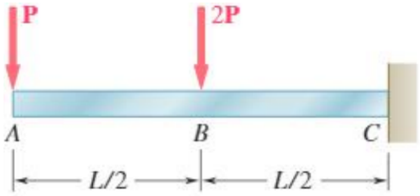


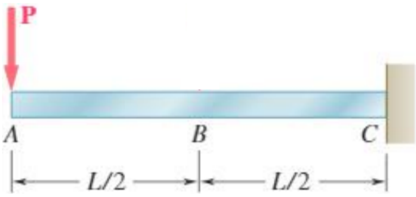
Fig. P9.67

For the cantilever beam and loading shown, determine the slope and deflection at the free end.

From the solution of prob.9.1, the slope and deflection of the end point of a cantilever beam are

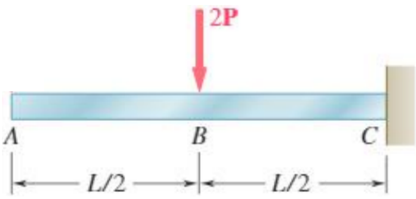
$$\theta_{\text{end}} = \pm\frac{PL^2}{2EI}, \quad y_{\text{end}} = \pm\frac{PL^3}{3EI}$$

Case 1,



$$\theta_{A1} = \frac{PL^2}{2EI}, \quad y_{A1} = -\frac{PL^3}{3EI}$$

Case 2,



$$\theta_{B2} = \frac{(2P)(\frac{L}{2})^2}{2EI} = \frac{PL^2}{4EI}$$

$$y_{B2} = -\frac{(2P)(\frac{L}{2})^3}{3EI} = -\frac{PL^3}{12EI}$$

$$\theta_{A2} = \theta_{B2} = \frac{PL^2}{4EI}$$

$$y_{A2} = y_{B2} + \theta_{B2} \times \left(-\frac{L}{2}\right) = -\frac{5PL^3}{24EI}$$

$$\theta_A = \theta_{A1} + \theta_{A2} = \frac{3PL^2}{4EI} \quad \blacktriangleleft$$

$$y_A = y_{A1} + y_{A2} = -\frac{13PL^3}{24EI} \quad \blacktriangleleft$$

Problem 9.72

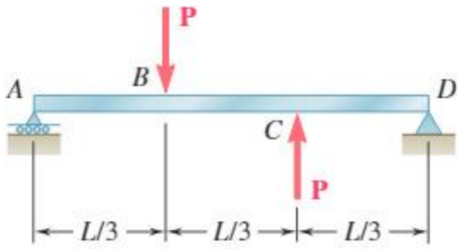


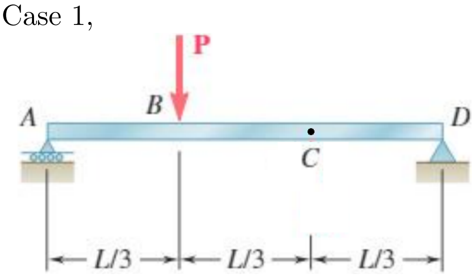
Fig. P9.72

For the beam and loading loadin shown, determine (a) the deflection at point C, (b) the slope at end A.

From the solution of prob.9.16,

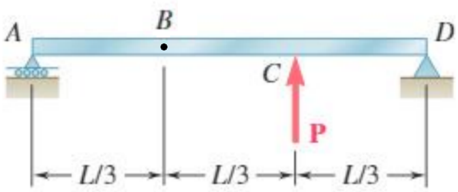
$$\theta(0) = -\frac{5PL^2}{81EI}, \quad \theta(L) = \frac{4PL^2}{81EI}$$
$$y\left(\frac{L}{3}\right) = -\frac{4PL^3}{243EI}, \quad y\left(\frac{2L}{3}\right) = -\frac{7PL^3}{486EI}$$

Case 1,



$$\theta_{A1} = -\frac{5PL^2}{81EI}, \quad y_{C1} = -\frac{7PL^3}{486EI}$$

Case 2,



$$\theta_{A2} = \frac{4PL^2}{81EI}, \quad y_{C2} = \frac{4PL^3}{243EI}$$

$$y_C = y_{C1} + y_{C2} = \frac{PL^3}{486EI} \quad ◀ \quad (a)$$
$$\theta_A = \theta_{A1} + \theta_{A2} = -\frac{PL^2}{81EI} \quad ◀ \quad (b)$$

Problem 9.76

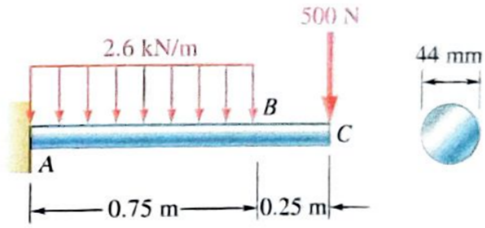


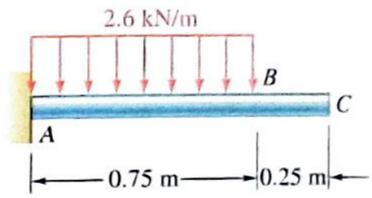
Fig. P9.75 and P9.76

For the cantilever beam and loading shown, determine the slope and deflection at point B. Use $E = 200\,\text{GPa}$.

$$I = \frac{\pi}{4}(0.022)^4\,\text{m}^4 = 1.839842 \times 10^{-7}\,\text{m}^4$$

$$\text{let} \quad w = 2.6\,\text{kN/m}, \quad P = 500\,\text{N}, \quad L = 1\,\text{m}$$

Case 1,

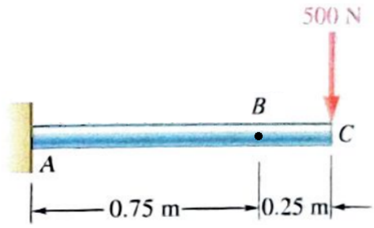


From the solution of prob.9.3,

$$\theta_{B1} = -\frac{w\left(\frac{3L}{4}\right)^3}{6EI} = -\frac{9wL^3}{128EI}$$

$$y_{B1} = -\frac{w\left(\frac{3L}{4}\right)^4}{8EI} = -\frac{81wL^4}{2048EI}$$

Case 2,



From the solution of prob.9.1,

$$\theta_{B2} = \theta\left(\frac{3L}{4}\right) = -\frac{15PL^2}{32EI}$$

$$y_{B2} = y\left(\frac{3L}{4}\right) = -\frac{27PL^3}{128EI}$$

$$\theta_B = \theta_{B1} + \theta_{B2} = -\frac{9wL^3}{128EI} - \frac{15PL^2}{32EI} = -0.01134 \quad \blacktriangleleft$$

$$y_B = y_{B1} + y_{B2} = -\frac{81wL^4}{2048EI} - \frac{27PL^3}{128EI} = -5.66\,\text{mm} \quad \blacktriangleleft$$