HW6

2025-2 구조역학(박성훈 교수님)

Problem 11.7, 11.15, 11.30, 11.34, 11.42, 11.45, 11.50, 11.59

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Problem 11.7
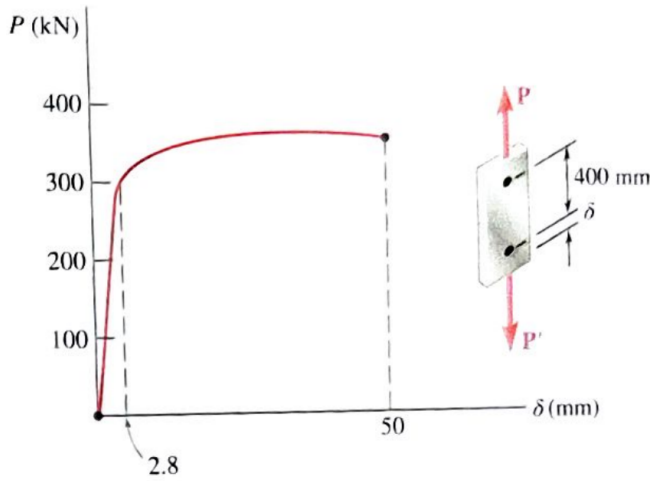

The load-deformation diagram shown has been drawn from data obtained during a tensile test of a specimen of an aluminum alloy. Knowing that the cross-sectional area of the specimen was $600\,\mathrm{mm}^2$ and that the deformation was measured using a 400-mm gage length. determine by approximate means (a) the modulus of resilience of the alloy, (b) the modulus of toughness of the alloy.

Fig. P11.7

$$\sigma_Y = \frac{P_Y}{A} = \frac{300 \times 10^3}{600 \times 10^{-6}}\,\mathrm{Pa} = 500\,\mathrm{MPa}$$

$$\varepsilon_Y = \frac{\delta_Y}{L} = \frac{2.8}{400} = 0.007$$

$$(\text{modulus of resilience}) = \frac{1}{2}\sigma_Y \varepsilon_Y = \frac{1}{2}(500 \times 10^6)(0.007)\,\mathrm{J/m^3} = 1.750\,\mathrm{MJ/m^3} \quad \blacktriangleleft \quad (a)$$

$$\varepsilon_R = \frac{\delta_R}{L} = \frac{50}{400} = 0.125$$

$$P_{\text{avg}} \approx 350\,\mathrm{kN} \quad \Rightarrow \quad \sigma_{\text{avg}} \approx \frac{350 \times 10^3}{600 \times 10^{-6}}\,\mathrm{Pa} = 583.333\,\mathrm{MPa}$$

$$(\text{modulus of resilience}) = \sigma_{\text{avg}}\varepsilon_R \approx (583.333 \times 10^6)(0.125)\,\mathrm{J/m^3} = 72.9\,\mathrm{MJ/m^3} \quad \blacktriangleleft \quad (b)$$
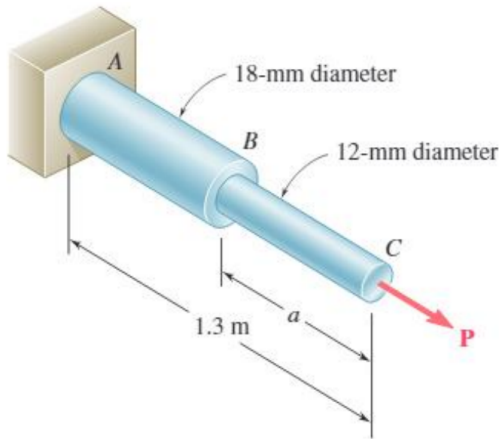
Problem 11.15

Fig. P11.15

Rod ABC is made of a steel for which the yield strength is $\sigma_Y = 450\,\text{MPa}$ and the modulus of elasticity is $E = 200\,\text{GPa}$. Knowing that a strain energy of $11.2\,\text{J}$ must be acquired by the rod as the axial load P is applied, determine the factor of safety of the rod with respect to permanent deformation when $a = 0.5\,\text{m}$.

$$A_{AB} = \pi(0.009)^2\,\text{m}^2 = 2.54469 \times 10^{-4}\,\text{m}^2, \quad A_{BC} = \pi(0.006)^2\,\text{m} = 1.130973 \times 10^{-4}\,\text{m}^2$$

$$P_Y = \sigma_Y A_{BC} = (450 \times 10^6)(1.130973 \times 10^{-4}) = 50.8938\,\text{kN}$$

$$U_Y = \frac{P_Y^2 L_{AB}}{2EA_{AB}} + \frac{P_Y^2 L_{BC}}{2EA_{BC}} = \frac{P_Y^2}{2E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$$

$$= \frac{50893.8^2}{2(200 \times 10^9)}\left(\frac{0.8}{2.54469 \times 10^{-4}} + \frac{0.5}{1.130973 \times 10^{-4}}\right)\text{J} = 48.9853\,\text{J}$$

$$(F.S.) = \frac{U_Y}{U} = \frac{48.9853\,\text{J}}{11.2\,\text{J}} = 4.37 \quad \blacktriangleleft$$
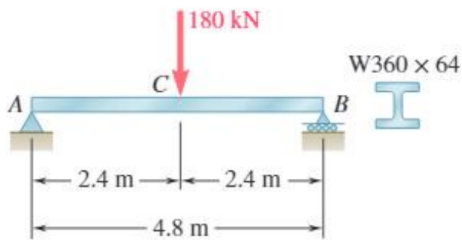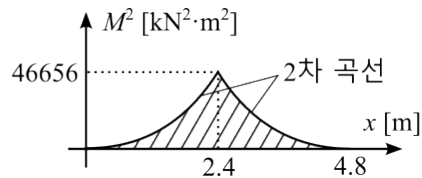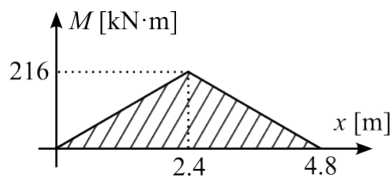
Problem 11.30

Fig. P11.30

Using $E = 200\,\text{GPa}$, determine the strain energy due to bending for the steel beam and loading shown. (Neglect the effect of shearing stresses.)
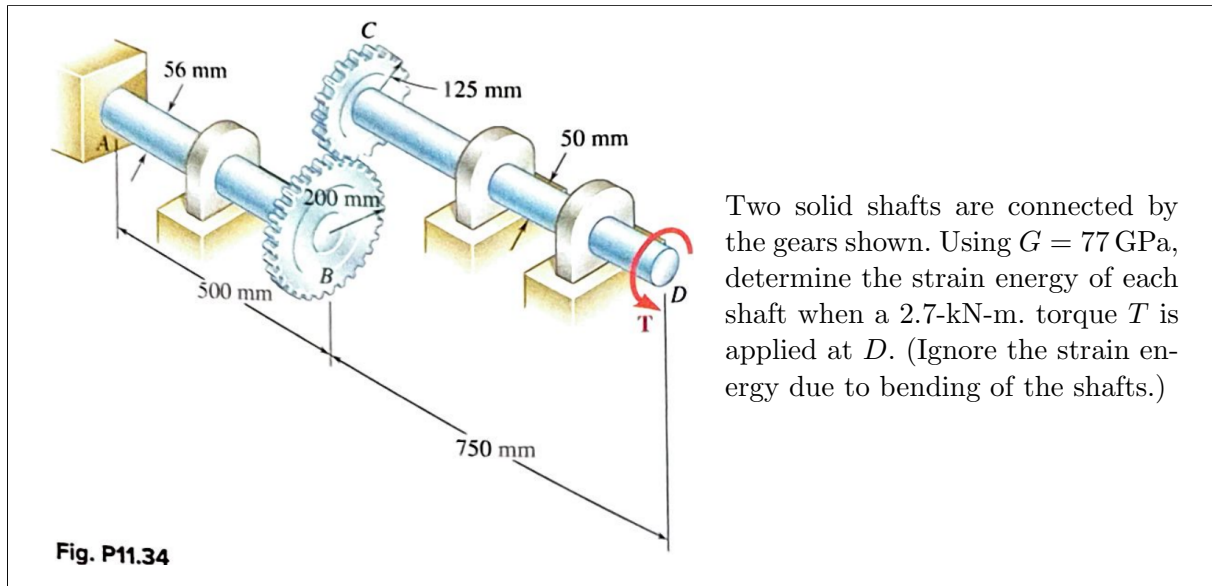

$$I = 178 \times 10^6\,\text{mm}^4 = 178 \times 10^{-6}\,\text{m}^4$$

$$\int_0^L M^2 dx = A_{m^2} = \frac{1}{3}(46656)(4.8)\,\text{kN}^2 \cdot \text{m}^3 = 74649.6\,\text{kN}^2 \cdot \text{m}^3$$

$$U = \int_0^L \frac{M^2}{2EI}dx = \frac{A_{m^2}}{2EI} = \frac{74649.6 \times 10^6}{2(200 \times 10^9)(178 \times 10^{-6})}\,\text{J} = 1048\,\text{J} \quad \blacktriangleleft$$
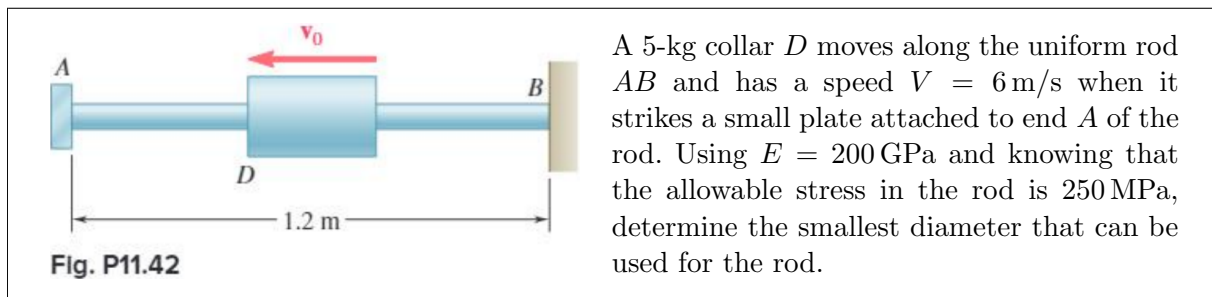
Problem 11.34

Fig. P11.34

Two solid shafts are connected by the gears shown. Using $G = 77\,\text{GPa}$, determine the strain energy of each shaft when a 2.7-kN-m. torque T is applied at D. (Ignore the strain energy due to bending of the shafts.)

$$T_{AB} = \frac{200\,\text{mm}}{125\,\text{mm}}(2.7\,\text{kN} \cdot \text{m}) = 4.32\,\text{kN} \cdot \text{m}, \quad T_{CD} = 2.7\,\text{kN} \cdot \text{m}$$

$$J_{AB} = \frac{\pi}{2}(0.025)^4\,\text{m}^4 = 9.65499 \times 10^{-7}\,\text{m}^4$$

$$J_{CD} = \frac{\pi}{2}(0.025)^4\,\text{m}^4 = 6.13592 \times 10^{-7}\,\text{m}^4$$

$$U = \frac{T_{AB}^2 L_{AB}}{2GJ_{AB}} + \frac{T_{CD}^2 L_{CD}}{2GJ_{CD}} = 120.6\,\text{J} \quad \blacktriangleleft$$

Problem 11.42

Fig. P11.42

A 5-kg collar D moves along the uniform rod AB and has a speed $V = 6\,\text{m/s}$ when it strikes a small plate attached to end A of the rod. Using $E = 200\,\text{GPa}$ and knowing that the allowable stress in the rod is $250\,\text{MPa}$, determine the smallest diameter that can be used for the rod.

$$U = \frac{1}{2}mv_0^2 = \frac{1}{2}(5)(6^2)\,\text{J} = 90\,\text{J}$$

$$P_{\text{all}} = \sigma_{\text{all}}A, \quad A = \frac{\pi}{4}d^2$$

$$U \leq U_{\text{all}} = \frac{P_{\text{all}}^2 L}{2EA} = \frac{\sigma_{\text{all}}^2 AL}{2E} = \frac{\pi\sigma_{\text{all}}^2 d^2 L}{8E}$$

$$d \geq \sqrt{\frac{8UE}{\pi\sigma_{\text{all}}^2 L}} = \sqrt{\frac{8(90)(200 \times 10^9)}{\pi(250 \times 10^6)^2(1.2)}}\,\text{m} = 24.7\,\text{mm} \quad \blacktriangleleft$$
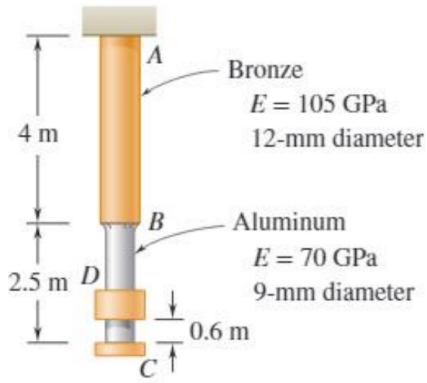
Problem 11.45

Bronze
$E = 105$ GPa
12-mm diameter

A

4 m

B — Aluminum
$E = 70$ GPa
9-mm diameter

2.5 m D

0.6 m

C

Fig. P11.45

Collar D is released from rest in the position shown and is stopped by a small plate attached at end C of the vertical rod ABC. Determine the mass of the collar for which the maximum normal stress in portion BC is 125 MPa.

$$A_{AB} = \pi(0.006)^2 \, \text{m}^2 = 1.130973 \times 10^{-4} \, \text{m}^2$$

$$A_{BC} = \pi(0.0045)^2 \, \text{m}^2 = 6.36173 \times 10^{-5} \, \text{m}^2$$

$$P_m = \sigma_m A_{BC} = 7952.16 \, \text{N} \quad (\because A_{\min} = A_{BC})$$

$$U_m = \frac{1}{2} P_m \delta_m = \frac{P_m^2 L_{AB}}{2 E_b A_{AB}} + \frac{P_m^2 L_{BC}}{2 E_a A_{BC}} \quad \Rightarrow \quad \delta_m = P_m \left(\frac{L_{AB}}{E_b A_{AB}} + \frac{L_{BC}}{E_a A_{BC}} \right) = \frac{1}{140} \, \text{m}$$

$$U \leq U_m \quad \Rightarrow \quad mg(h + \delta_m) \leq \frac{1}{2} P_m \delta_m \quad \Rightarrow \quad m \leq \frac{P_m \delta_m}{2g(h + \delta_m)} = 4.77 \, \text{kg} \quad \blacktriangleleft$$
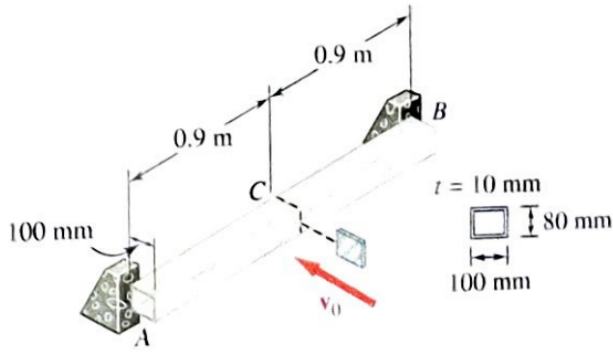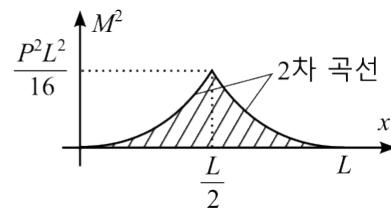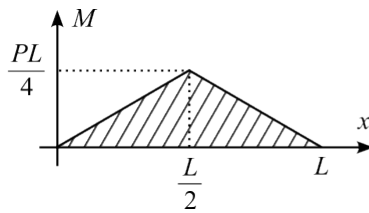
Problem 11.50

An aluminum tube having the cross section shown is struck squarely in its midsection by a 6-kg block moving horizontally with a speed of $2\,\text{m/s}$. Using $E = 70\,\text{GPa}$, determine (a) the equivalent static load, (b) the maximum stress in the beam, (c) the maximum deflection at the midpoint C of the beam.

Fig. P11.50

$$U = \frac{1}{2}mv_0^2 = \frac{1}{2}(6)(2^2)\,\text{J} = 12\,\text{J}$$

$$L = 1.8\,\text{m}, \quad M_C = \frac{P}{2} \cdot \frac{L}{2} = \frac{PL}{4}$$

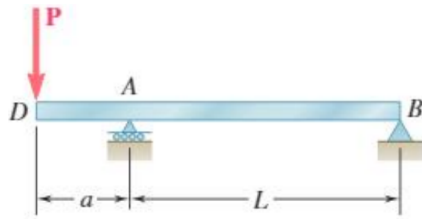$$\int_0^L M^2 dx = A_{m^2} = \frac{1}{3}\left(\frac{P^2L^2}{16}\right)L = \frac{P^2L^3}{48}$$

$$I = \frac{1}{12}\left\{(0.08)(0.1)^3 - (0.06)(0.08)^3\right\}\text{m}^4 = 4.10667 \times 10^{-6}\,\text{m}^4$$

$$U = \int_0^L \frac{M^2}{2EI}dx = \frac{A_{m^2}}{2EI} = \frac{P^2L^3}{96EI} \quad \Rightarrow \quad P = \sqrt{\frac{96UEI}{L^3}} = 7535.49\,\text{N} = 7.54\,\text{kN} \quad \blacktriangleleft \quad (a)$$

$$\sigma_m = \frac{|M|_m c}{I} = \frac{PLc}{4I} = 41.3\,\text{MPa} \quad \blacktriangleleft \quad (b)$$

$$U = \frac{1}{2}Py_C \quad \Rightarrow \quad y_C = \frac{2U}{P} = 3.18\,\text{mm} \quad \blacktriangleleft \quad (c)$$
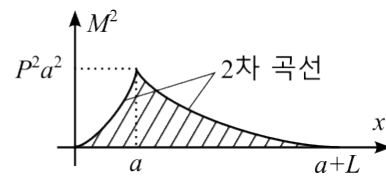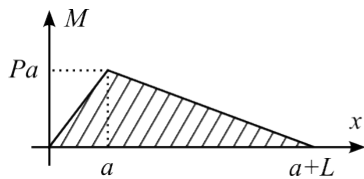
Problem 11.59

Fig. P11.59

Using the method of work and energy, determine the deflection at point D caused by the load $\mathbf{P}$.

$$+ \circlearrowleft \sum M|_A = Pa - R_B L = 0, \quad R_B = \frac{Pa}{L}$$

$$+ \uparrow \sum F_y = -P - R_B + R_A = 0, \quad R_A = P\left(1 + \frac{a}{L}\right)$$

$$\int_0^L M^2 dx = A_{m^2} = \frac{1}{3}P^2 a^2 (a + L)$$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{A_{m^2}}{2EI} = \frac{P^2 a^2 (a + L)}{6EI} = \frac{1}{2}P|y_D| \quad \Rightarrow \quad y_D = -\frac{Pa^2(a+L)}{3EI} \quad \blacktriangleleft$$