HW2

2025-2 구조역학(박성훈 교수님)

Problem 7.132, 7.135, 8.4, 8.13, 문제1, 문제2

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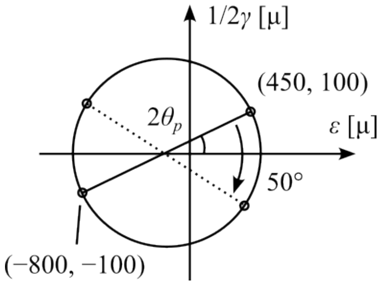
Problem 7.132

For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = -800\,\mu, \quad \varepsilon_y = +450\,\mu, \quad \gamma_{xy} = +200\,\mu, \quad \theta = 25° \circlearrowright$$



$$c = \frac{-800 + 450}{2}\mu = -175\,\mu$$

$$2\theta_p = \arctan\frac{100}{450+175} = 9.09028°$$

$$R = \sqrt{(450+175)^2 + 100^2}\,\mu = 632.94945\,\mu$$

$$\varepsilon_{x'} = c - R\cos(50° - 2\theta_p) = -653\,\mu \quad ◀$$

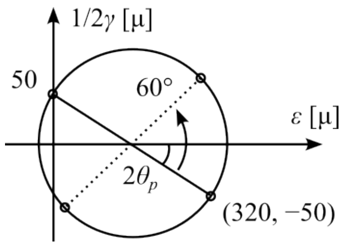
$$\varepsilon_{y'} = c + R\cos(50° - 2\theta_p) = 303\,\mu \quad ◀$$

$$\gamma_{x'y'} = -2R\sin(50° - 2\theta_p) = -829\,\mu \quad ◀$$

Problem 7.135

For the given state of plane strain, use Mohr's circle to determine the state of plane strain associated with axes x' and y' rotated through the given angle θ.

$$\varepsilon_x = 0, \quad \varepsilon_y = +320\,\mu, \quad \gamma_{xy} = -100\,\mu, \quad \theta = 30° \circlearrowleft$$



$$c = \frac{320}{2}\mu = 160\,\mu$$

$$2\theta_p = \arctan\frac{50}{320-160} = 17.35402°$$

$$R = \sqrt{(320-160)^2 + 50^2}\,\mu = 167.63055\,\mu$$

$$\varepsilon_{x'} = c - R\cos(60° - 2\theta_p) = 36.7\,\mu \quad ◀$$

$$\varepsilon_{y'} = c + R\cos(60° - 2\theta_p) = 283\,\mu \quad ◀$$

$$\gamma_{x'y'} = 2R\sin(60° - 2\theta_p) = 227\,\mu \quad ◀$$

Problem 8.4

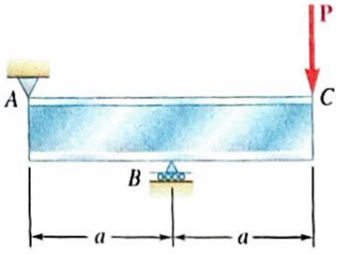
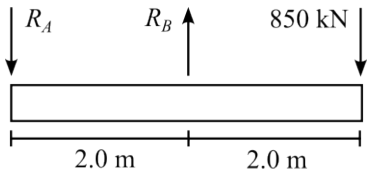


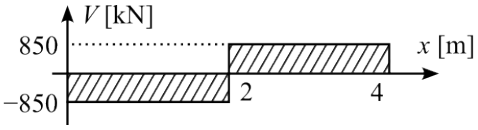
Fig. P8.3

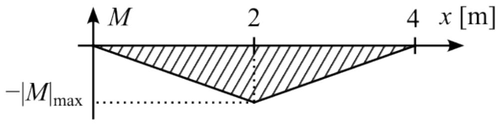
Solve Prob. 8.3, assuming that $P = 850\,\text{kN}$ and $a = 2.0\,\text{m}$.

Prob. 8.3 — An overhanging W920×449 rolled-steel beam supports a load P as shown. Knowing that $P = 700\,\text{kN}$, $a = 2.5\,\text{m}$, and $\sigma_{\text{all}} = 100\,\text{MPa}$, determine (a) the maximum value of the normal stress σ_m, in the beam, (b) the maximum value of the principal stress $\sigma_{\max}$ at the junction of the flange and web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.

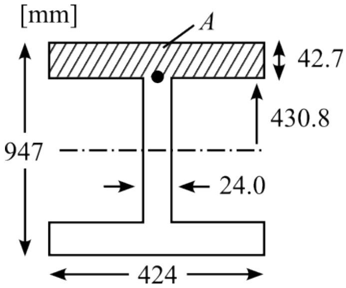


$$+\circlearrowleft \sum M|_B = R_A(2\,\text{m}) - (850\,\text{kN})(2\,\text{m}) = 0$$
$$\Rightarrow \quad R_A = 850\,\text{kN}$$
$$+\uparrow \sum F_y = -R_A + R_B - 850\,\text{kN} = 0$$
$$\Rightarrow \quad R_B = 1700\,\text{kN}$$





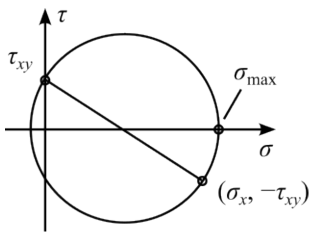
$$|M|_{\max} = (850\,\text{kN})(2\,\text{m}) = 1700\,\text{kN}\cdot\text{m}$$



$$I = 8780 \times 10^6\,\text{mm}^4$$
$$S = 18500 \times 10^3\,\text{mm}^3$$
$$y_{\text{junction}} = \left(\frac{947}{2} - 42.7\right)\text{mm} = 430.8\,\text{mm}$$
$$Q_A = A\bar{y}_A = (424)(42.7)\left(430.8 + \frac{42.7}{2}\right)\text{mm}^3$$
$$= 8186085.32\,\text{mm}^3$$

$$\sigma_m = \frac{|M|_{\max}}{S} = \frac{1700\,\text{kN}\cdot\text{m}}{18500 \times 10^3\,\text{mm}^3} = 91.9\,\text{MPa} \quad \blacktriangleleft \quad (a)$$
$$\sigma_{\text{junction}} = \frac{|M|_{\max} y_{\text{junction}}}{I} = \frac{(1700\,\text{kN})(430.8\,\text{mm})}{8780 \times 10^6\,\text{mm}^4} = 83.41230068\,\text{MPa}$$
$$|\tau_{\text{junction}}| = \frac{VQ}{It} = \frac{(850\,\text{kN})(8186085.32\,\text{mm}^3)}{(8780 \times 10^6\,\text{mm}^4)(24\,\text{mm})} = 33.02094021\,\text{MPa}$$

let. $\sigma_{\text{junction}} = \sigma_x$, $|\tau_{\text{junction}}| = \tau_{xy}$



$$c = \frac{\sigma_x}{2}$$
$$R = \sqrt{c^2 + \tau_{xy}^2} = 53.19572797\,\text{MPa}$$
$$\sigma_{\max} = c + R = 94.9\,\text{MPa} \quad \blacktriangleleft \quad (b)$$

$\sigma_{\text{all}} \quad \Rightarrow \quad \sigma_m < \sigma_{\max} < \sigma_{\text{all}} \quad \Rightarrow$ The specified shape is acceptable. $\blacktriangleleft$ (c)

Problem 8.13

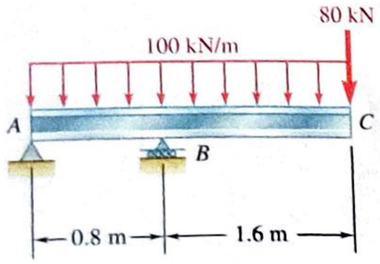
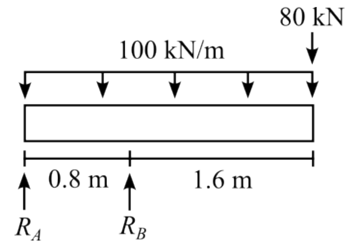


Fig. P5.77

Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \leq \sigma_{\text{all}}$. For the selected design, determine (a) the actual value of σ_m in the beam. (b) the maximum value of the principal stress $\sigma_{\max}$ at the junction of a flange and the web.

Loading of Prob. 5.77 and selected S510×98.2 shape

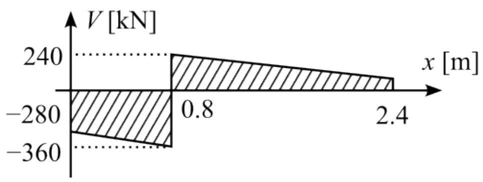


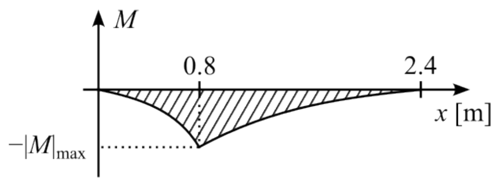
$$+\circlearrowleft \sum M|_A = R_B(0.8\,\text{m}) - (80\,\text{kN})(2.4\,\text{m}) - (100\,\text{kN/m})(2.4\,\text{m})(1.2\,\text{m}) = 0$$

$$\Rightarrow \quad R_B = 600\,\text{kN}$$

$$+\uparrow \sum F_y = R_A + R_B - (100\,\text{kN/m})(2.4\,\text{m}) - 80\,\text{kN} = 0$$

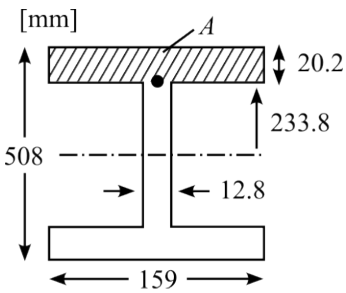
$$\Rightarrow \quad R_A = -280\,\text{kN}$$





$$|V|_m = -360\,\text{kN}$$

$$|M|_m = \frac{1}{2}(280 + 360)(0.8)\,\text{kN}\cdot\text{m} = 256\,\text{kN}\cdot\text{m}$$



$$I = 495 \times 10^6\,\text{mm}^4$$

$$S = 1950 \times 10^3\,\text{mm}^3$$

$$y_j = \left(\frac{508}{2} - 20.2\right)\text{mm} = 233.8\,\text{mm}$$

$$Q_A = A\bar{y}_A = (159)(20.2)\left(233.8 + \frac{20.2}{2}\right)\text{mm}^3$$

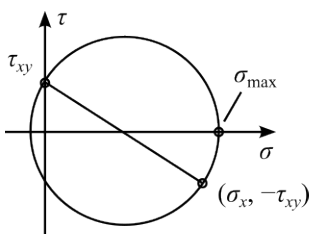
$$= 783358.02\,\text{mm}^3$$

$$\sigma_m = \frac{|M|_m}{S} = \frac{256\,\text{kN}\cdot\text{m}}{1950 \times 10^3\,\text{mm}^3} = 131.3\,\text{MPa} \quad \blacktriangleleft \quad (a)$$

$$\sigma_j = \frac{|M|_m y_j}{I} = \frac{(256\,\text{kN})(233.8\,\text{mm})}{495 \times 10^6\,\text{mm}^4} = 120.9147475\,\text{MPa}$$

$$|\tau_j| = \frac{|V|_m Q}{It} = \frac{(360\,\text{kN})(783358.02\,\text{mm}^3)}{(495 \times 10^6\,\text{mm}^4)(24\,\text{mm})} = 44.50897841\,\text{MPa}$$

let. $\sigma_j = \sigma_x, \quad |\tau_j| = \tau_{xy}$

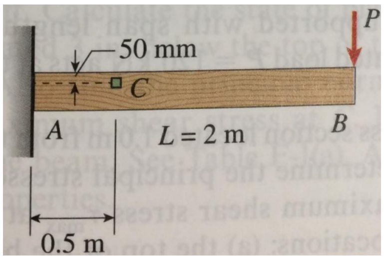


$$c = \frac{\sigma_x}{2}$$

$$R = \sqrt{c^2 + \tau_{xy}^2} = 75.07425123\,\text{MPa}$$

$$\sigma_{\max} = c + R = 135.5\,\text{MPa} \quad \blacktriangleleft \quad (b)$$

문제 1



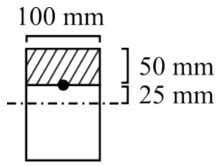
A cantilever beam with a width $b = 100\,\mathrm{mm}$ and depth $h = 150\,\mathrm{mm}$ has a length $L = 2\,\mathrm{m}$ and is subjected to a point load $P = 500\,\mathrm{N}$ at B. Calculate the state of plane stress at point C located $50\,\mathrm{mm}$ below the top of the beam and $0.5\,\mathrm{m}$ to the right of point A. Also find the principal stresses and the maximum shear stress at C. Neglect the weight of the beam.

$$R_A = 500\,\mathrm{N}\ (\uparrow), \quad M_A = (500\,\mathrm{N})(2\,\mathrm{m}) = 1000\,\mathrm{N\cdot m}\ (\circlearrowleft)$$

$$\text{at } C: \quad V = 500\,\mathrm{N}, \quad M = -1000\,\mathrm{N\cdot m} + (500\,\mathrm{N})(0.5\,\mathrm{m}) = -750\,\mathrm{kN\cdot m}$$

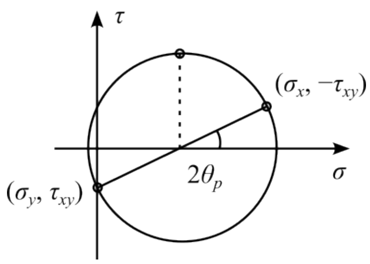
$$I = \frac{bh^3}{12} = \frac{(0.1)(0.15)^3}{12}\,\mathrm{m}^4 = 2.8125\times10^{-5}\,\mathrm{m}^4$$

$$\sigma_x = -\frac{My}{I} = -\frac{(-750\,\mathrm{N\cdot m})(0.05\,\mathrm{m})}{2.8125\times10^{-5}\,\mathrm{m}^4} = \frac{2000}{3}\,\mathrm{kPa} = 667\,\mathrm{kPa} \quad \blacktriangleleft$$



$$Q = A\bar{y} = (0.1)(0.05)(0.05)\,\mathrm{m}^3 = 2.5\times10^{-4}\,\mathrm{m}^3$$

$$\tau_{xy} = -\frac{VQ}{It} = -\frac{(500\,\mathrm{N})(2.5\times10^{-4}\,\mathrm{m}^3)}{(2.8125\times10^{-5}\,\mathrm{m}^4)(0.1\,\mathrm{m})} = -\frac{400}{9}\,\mathrm{kPa} = -44.4\,\mathrm{kPa} \quad \blacktriangleleft$$



$$c = \frac{\sigma_x}{2} = \frac{1}{2}\cdot\frac{2000}{3}\,\mathrm{kPa} = \frac{1000}{3}\,\mathrm{kPa}$$

$$\theta_p = -\frac{1}{2}\arctan\frac{-\tau_{xy}}{\sigma_x - c} = -\frac{1}{2}\arctan\frac{\left(\frac{400}{9}\right)}{\left(\frac{1000}{3}\right)} = -3.80^\circ \quad \blacktriangleleft$$

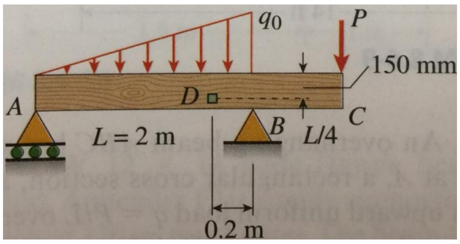
$$R = \sqrt{c^2 + \tau_{xy}^2} = 336.283\,\mathrm{kPa}$$

$$\sigma_{\max} = c + R = 670\,\mathrm{kPa} \quad \blacktriangleleft$$

$$\sigma_{\min} = c - R = -2.95\,\mathrm{kPa} \quad \blacktriangleleft$$

$$\tau_{\max} = R = 336\,\mathrm{kPa} \quad \blacktriangleleft$$

문제 2



Beam ABC with an overhang BC is subjected to a linearly varying distributed load on span AB with peak intensity $q_0 = 2500\,\mathrm{N/m}$ and a point load $P = 1250\,\mathrm{N}$ applied at C. The beam has a width $b = 100\,\mathrm{mm}$ and depth $h = 200\,\mathrm{mm}$. Find the state of plane stress at point D located 150 mm below the top of the beam and 0.2 m to the left of point B. Also find the principal stresses at D. Neglect the weight of the beam.

$$+\circlearrowleft \sum M|_A = -\frac{1}{2}(2500\,\mathrm{N/m})(2.0\,\mathrm{m})\left(\frac{4}{3}\,\mathrm{m}\right) + R_B(2.0\,\mathrm{m}) - (1250\,\mathrm{N})(2.5\,\mathrm{m}) = 0$$

$$\Rightarrow \quad R_B = \frac{19375}{6}\,\mathrm{N}$$

$$+\uparrow \sum F_y = R_A + R_B - \frac{1}{2}(2500\,\mathrm{N/m})(2.0\,\mathrm{m}) - 1250\,\mathrm{N} = 0$$

$$\Rightarrow \quad R_A = \frac{3125}{6}\,\mathrm{N}$$

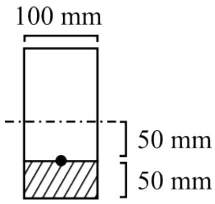
at point D,

$$V = \frac{3125}{6}\,\mathrm{N} - \frac{1}{2}\left(\frac{1.8}{2.0} \times 2500\,\mathrm{N/m}\right)(1.8\,\mathrm{m}) = -\frac{9025}{6}\,\mathrm{N}$$

$$M = -\frac{1}{2}\left(\frac{1.8}{2.0} \times 2500\,\mathrm{N/m}\right)(1.8\,\mathrm{m})\left(\frac{1}{3} \times 1.8\,\mathrm{m}\right) + \left(\frac{3125}{6}\,\mathrm{N}\right)(1.8\,\mathrm{m}) = -\frac{555}{2}\,\mathrm{N\cdot m}$$

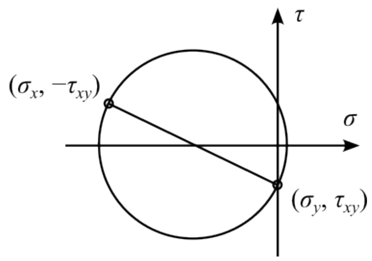
$$I = \frac{bh^3}{12} = \frac{(0.1)(0.2)^3}{12}\,\mathrm{m}^4 = \frac{2}{3} \times 10^{-4}\,\mathrm{m}^4$$

$$\sigma_x = -\frac{My}{I} = -\frac{\left(-\frac{555}{2}\,\mathrm{N\cdot m}\right)(-0.05\,\mathrm{m})}{\frac{2}{3} \times 10^{-4}\,\mathrm{m}^4} = -\frac{1665}{8}\,\mathrm{kPa} = -208\,\mathrm{kPa} \quad \blacktriangleleft$$



$$Q = A\bar{y} = (0.1)(0.05)(-0.075)\,\mathrm{m}^3 = 375 \times 10^{-6}\,\mathrm{m}^3$$

$$\tau_{xy} = -\frac{VQ}{It} = -\frac{(-\frac{9025}{6}\,\mathrm{N})(375 \times 10^{-6}\,\mathrm{m}^3)}{(\frac{2}{3} \times 10^{-4}\,\mathrm{m}^4)(0.1\,\mathrm{m})} = -\frac{5415}{64}\,\mathrm{kPa} = -84.6\,\mathrm{kPa} \quad \blacktriangleleft$$



$$c = \frac{\sigma_x}{2} = \frac{1}{2} \cdot \left(-\frac{1665}{8}\,\mathrm{kPa}\right) = -\frac{1665}{16}\,\mathrm{kPa}$$

$$R = \sqrt{c^2 + \tau_{xy}^2} = 134.1184187\,\mathrm{kPa}$$

$$\sigma_{\max} = c + R = 30.1\,\mathrm{kPa} \quad \blacktriangleleft$$

$$\sigma_{\min} = c - R = -238\,\mathrm{kPa} \quad \blacktriangleleft$$