# HW6

2025-1 고체역학(박성훈 교수님)

Problem 5.9, 5.12, 5.15, 5.19, 5.25, 5.56, 5.65, 5.70

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2025–12–23

## Problem 5.9

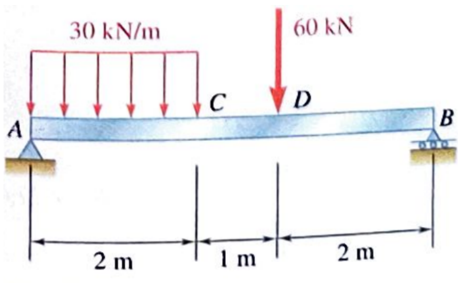



**Fig. P5.9**

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value of (*a*) the shear, (*b*) the bending moment.

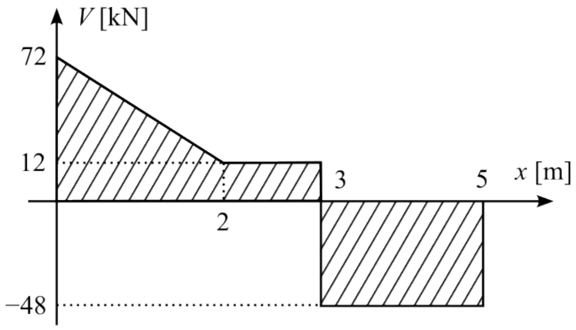
$$+\circlearrowleft \sum M|_A = -(30\,\mathrm{kN/m})(2\,\mathrm{m})(1\,\mathrm{m}) - (60\,\mathrm{kN})(3\,\mathrm{m}) + R_B(5\,\mathrm{m}) = 0 \quad \Rightarrow \quad R_B = 48\,\mathrm{kN}$$

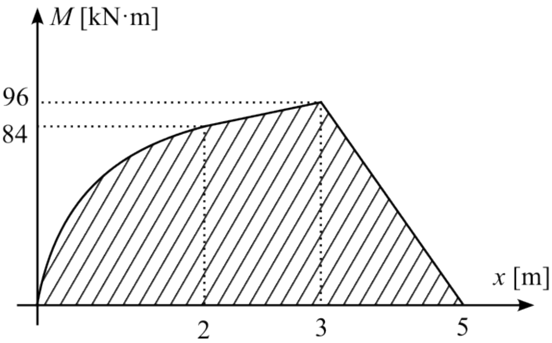
$$+\uparrow \sum F_y = R_A + R_B - (30\,\mathrm{kN/m})(2\,\mathrm{m}) - 60\,\mathrm{kN} = 0 \quad \Rightarrow \quad R_A = 72\,\mathrm{kN}$$

The units 'kN' and 'm' are omitted from the following eqations.

$$\begin{aligned}
&V(0) = R_A = 72 \\
&V(x) = R_A - wx = 72 - 30x \qquad (0 < x < 2) \\
&V(x) = 72 - (30)(2) = 12 \qquad (2 \le x < 3) \\
&V(x) = 12 - 60 = -48 \qquad (3 < x \le 5)
\end{aligned}$$

$$\begin{aligned}
&M(0) = 0 \\
&M(x) = R_A x - wx\left(\frac{x}{2}\right) = 72x - 15x^2 \qquad (0 < x \le 2) \\
&M(x) = 72x - (30)(2)(x-1) = 12x + 60 \qquad (2 < x \le 3) \\
&M(x) = 12x + 60 - 60(x-3) = -48x + 240 \quad (3 < x \le 5)
\end{aligned}$$




$|V|_{\max} = 72.0\,\mathrm{kN}$ ◀ (*a*)

$|M|_{\max} = 96.0\,\mathrm{kN\cdot m}$ ◀ (*b*)

## Problem 5.12

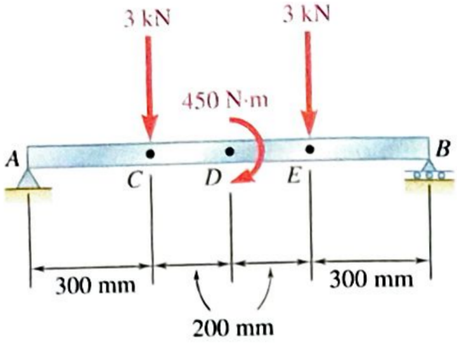



Fig. P5.12

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value of ($a$) the shear, ($b$) the bending moment.

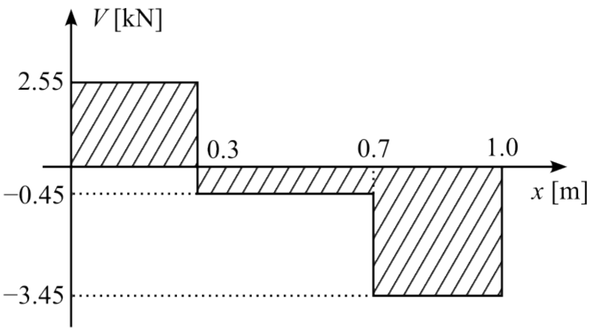
$$+\circlearrowleft \sum M|_A = -(3\,\text{kN})(0.3\,\text{m}) - 450\,\text{N}\cdot\text{m} - (3\,\text{kN})(0.7\,\text{m}) + R_B(1\,\text{m}) = 0 \;\Rightarrow\; R_B = 3.45\,\text{kN}$$

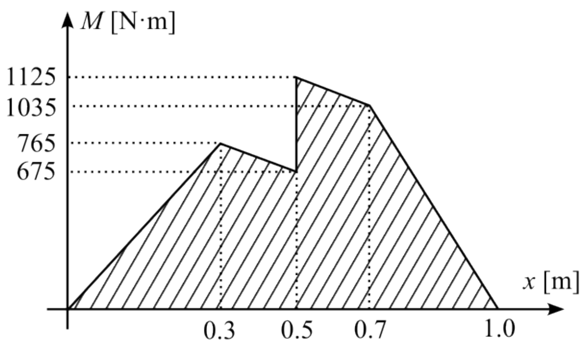
$$+\uparrow \sum F_y = R_A + R_B - 3\,\text{kN} - 3\,\text{kN} = 0 \quad\Rightarrow\quad R_A = 2.55\,\text{kN}$$

The units 'kN' and 'm' are omitted from the following eqations.

$$V(x) = R_A = 2.55 \qquad (0 \le x < 0.3)$$
$$V(x) = R_A - 3 = -0.45 \qquad (0.3 < x < 0.7)$$
$$V(x) = R_A - 3 - 3 = -3.45 \qquad (0.7 < x \le 1)$$

$$M(0) = 0$$
$$M(x) = R_A x = 2.55x \qquad (0 \le x < 0.3)$$
$$M(x) = 2.55x - 3(x - 0.3) = -0.45x + 0.9 \qquad (0.3 \le x < 0.5)$$
$$M(x) = -0.45x + 0.9 + 0.45 = -0.45x + 1.35 \qquad (0.5 < x < 0.7)$$
$$M(x) = -0.45x + 1.35 - 3(x - 0.7) = -3.45x + 3.45 \qquad (0.7 \le x \le 1)$$




$|V|_{\max} = 3.45\,\text{kN}$ ◀ ($a$)

$|M|_{\max} = 1125\,\text{N}\cdot\text{m}$ ◀ ($b$)

## Problem 5.15

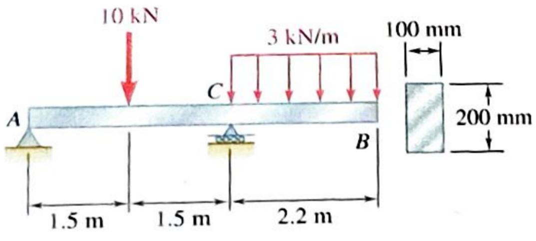



Fig. P5.15

For the beam and loading shown, determine the maximum normal stress due to bending ber on a transverse section at $C$.

$$+\circlearrowleft \sum M|_A = -(10\,\text{kN})(1.5\,\text{m}) + R_C(3\,\text{m}) - (3\,\text{kN/m})(2.2\,\text{m})(4.1\,\text{m}) = 0 \;\Rightarrow\; R_C = 14.02\,\text{kN}$$

$$+\uparrow \sum F_y = R_A + R_C - 10\,\text{kN} - (3\,\text{kN/m})(2.2\,\text{m}) = 0 \quad\Rightarrow\quad R_A = 2.58\,\text{kN}$$

$$M_C = (2.58\,\text{kN})(3\,\text{m}) - (10\,\text{kN})(1.5\,\text{m}) = -7.26\,\text{kN}\cdot\text{m}$$

$$S = \frac{1}{6}(0.1)(0.2)^2\,\text{m}^3 = \frac{1}{1500}\,\text{m}^3, \quad \sigma_m = \frac{|M_C|}{S} = 10.89\,\text{MPa} \quad \blacktriangleleft$$

## Problem 5.25

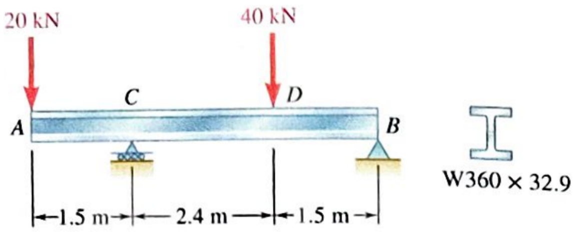
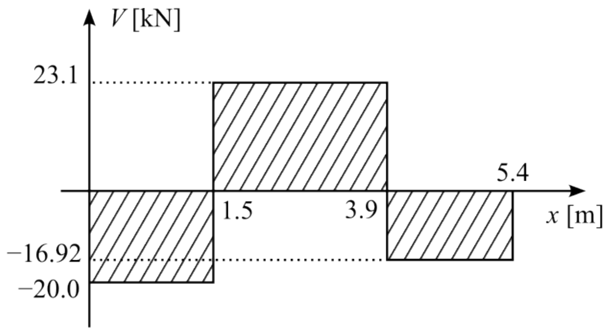


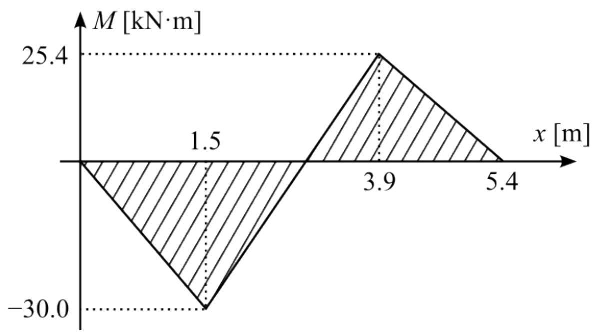

Fig. P5.25

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

$$+\circlearrowleft \sum M|_C = (20\,\text{kN})(1.5\,\text{m}) - (40\,\text{kN})(2.4\,\text{m}) + R_B(3.9\,\text{m}) = 0 \quad\Rightarrow\quad R_B = \frac{220}{13}\,\text{kN}$$

$$+\uparrow \sum F_y = R_B + R_C - 20\,\text{kN} - 40\,\text{kN} = 0 \quad\Rightarrow\quad R_C = \frac{560}{13}\,\text{kN}$$




$$M_C = (-20\,\text{kN})(1.5\,\text{m}) = -30\,\text{kN}\cdot\text{m}, \quad M_D = \left(\frac{220}{13}\,\text{kN}\right)(1.5\,\text{m}) = 25.3846\,\text{kN}\cdot\text{m}$$

$$|M|_{\max} = 30\,\text{kN}\cdot\text{m}$$

$$S = 475\times10^3\,\text{mm}^3 = 475\times10^{-6}\,\text{m}^3$$

$$\sigma_m = \frac{|M|_{\max}}{S} = \frac{30000}{475\times10^{-6}}\,\text{Pa} = 63.2\,\text{MPa} \quad \blacktriangleleft$$

## Problem 5.56

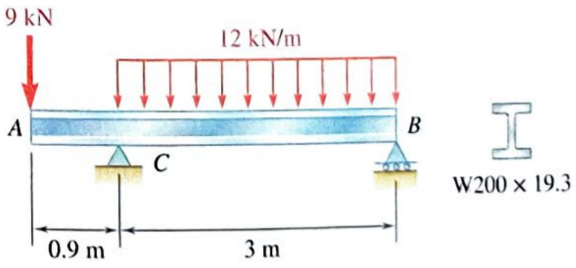
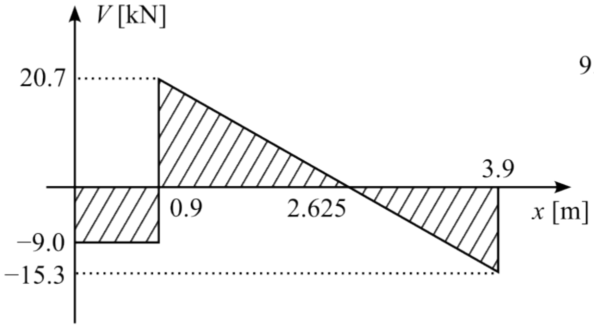


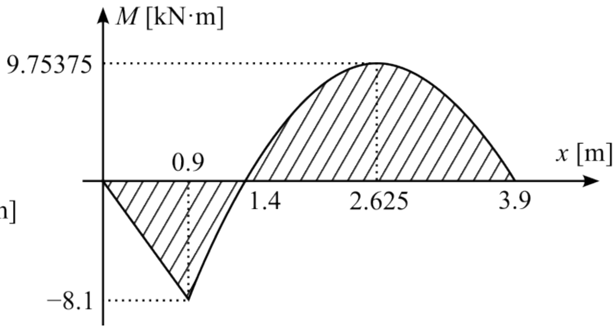

Fig. P5.56

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

$$+\circlearrowleft \sum M|_C = (9\,\text{kN})(0.9\,\text{m}) - (12\,\text{kN/m})(3\,\text{m})(1.5\,\text{m}) + R_B(3\,\text{m}) = 0 \quad \Rightarrow \quad R_B = 15.3\,\text{kN}$$

$$+\uparrow \sum F_y = R_B + R_C - 9\,\text{kN} - (12\,\text{kN/m})(3\,\text{m}) = 0 \quad \Rightarrow \quad R_C = 29.7\,\text{kN}$$




$$M_C = -(9\,\text{kN})(0.9\,\text{m}) = -8.1\,\text{kN}\cdot\text{m}$$

$$M(x = 0.265\,\text{m}) = (15.3\,\text{kN})(3.9\,\text{m} - 2.625\,\text{m}) - \frac{1}{2}(12\,\text{kN/m})(3\,\text{m} - 2.625\,\text{m})^2 = 9.75375\,\text{kN}\cdot\text{m}$$

$$|M|_{\max} = 9.75375\,\text{kN}\cdot\text{m}$$

$$S = 162 \times 10^3\,\text{mm}^3 = 162 \times 10^{-6}\,\text{m}^3$$

$$\sigma_m = \frac{|M|_{\max}}{S} = \frac{9753.75}{162 \times 10^{-6}}\,\text{Pa} = 60.2\,\text{MPa} \quad \blacktriangleleft$$

## Problem 5.65

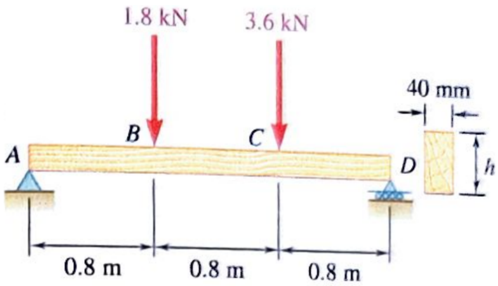
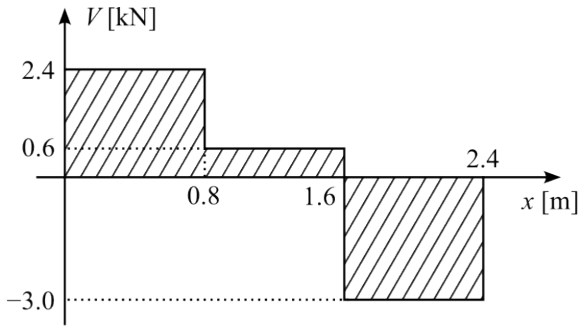


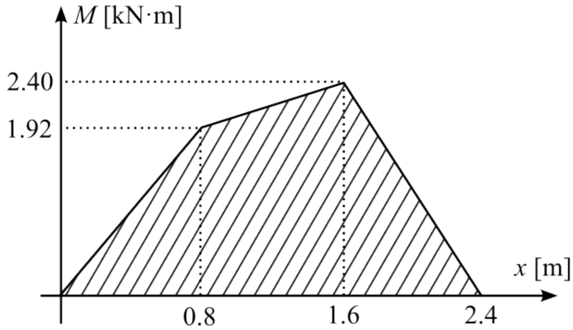

**Fig. P5.65**

For the beam and loading shown, design the the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

$$+\circlearrowleft \sum M|_A = -(1.8\,\text{kN})(0.8\,\text{m}) - (3.6\,\text{kN})(1.6\,\text{m}) + R_D(2.4\,\text{m}) = 0 \quad \Rightarrow \quad R_D = 3\,\text{kN}$$

$$+\uparrow \sum F_y = R_A + R_D - 1.8\,\text{kN} - 3.6\,\text{kN} = 0 \quad \Rightarrow \quad R_A = 2.4\,\text{kN}$$




$$|M|_{\max} = M_C = (3\,\text{kN})(0.8\,\text{m}) = 2.4\,\text{kN}\cdot\text{m}$$

$$S = \frac{1}{6}bh^2, \quad \sigma_m = \frac{|M|_{\max}}{S} = \frac{6|M|_{\max}}{bh^2} \le \sigma_{\text{all}}$$

$$\Rightarrow \quad h \ge \sqrt{\frac{6|M|_{\max}}{b\sigma_{\text{all}}}} = \sqrt{\frac{6(2400)}{(0.04)(12\times 10^6)}}\,\text{m} = 173.2\,\text{mm} \quad \blacktriangleleft$$

## Problem 5.70

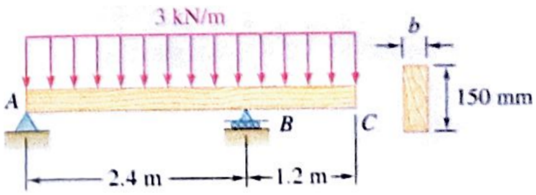
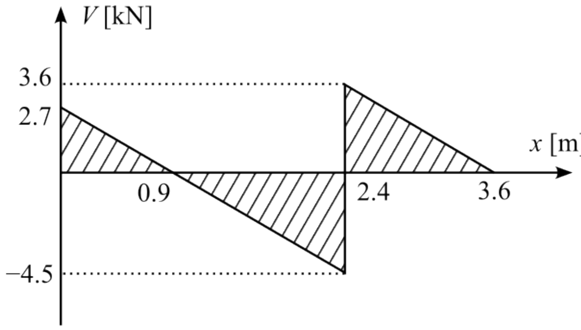


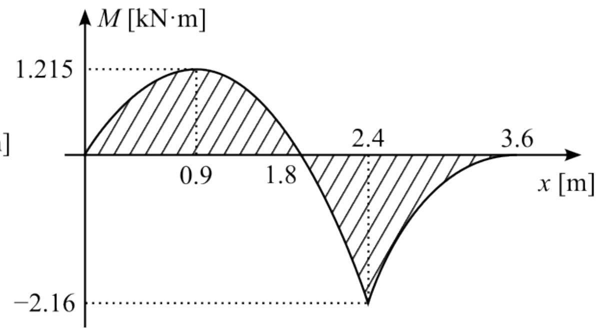

Fig. P5.70

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

$$+\circlearrowleft \sum M|_A = -(3\,\text{kN/m})(3.6\,\text{m})(1.8\,\text{m}) + R_B(2.4\,\text{m}) = 0 \quad \Rightarrow \quad R_B = 8.1\,\text{kN}$$

$$+\uparrow \sum F_y = R_A + R_B - (3\,\text{kN/m})(3.6\,\text{m}) = 0 \quad \Rightarrow \quad R_A = 2.7\,\text{kN}$$




$$M(x = 0.9\,\text{m}) = \frac{1}{2}(2.7\,\text{kN})(0.9\,\text{m}) = 1.215\,\text{kN}\cdot\text{m}, \quad M_B = -\frac{1}{2}(3.6\,\text{kN})(1.2\,\text{m}) = -2.16\,\text{kN}\cdot\text{m}$$

$$|M|_{\text{max}} = 2.16\,\text{kN}$$

$$S = \frac{1}{6}bh^2, \quad \sigma_m = \frac{|M|_{\text{max}}}{S} = \frac{6|M|_{\text{max}}}{bh^2} \le \sigma_{\text{all}}$$

$$\Rightarrow \quad b \ge \frac{6|M|_{\text{max}}}{h^2\sigma_{\text{all}}} = \frac{6(2160)}{(0.15)^2(12\times 10^6)}\,\text{m} = 48.0\,\text{mm} \quad \blacktriangleleft$$