

해설

구조역학(박성훈 교수님) 2025-2 기말고사

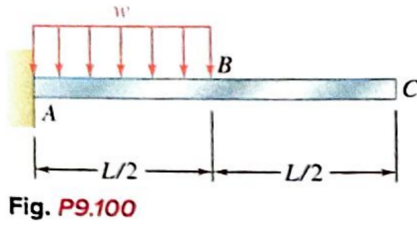
시험 실시 : 2025-10-20 16:30-19:30(180분)

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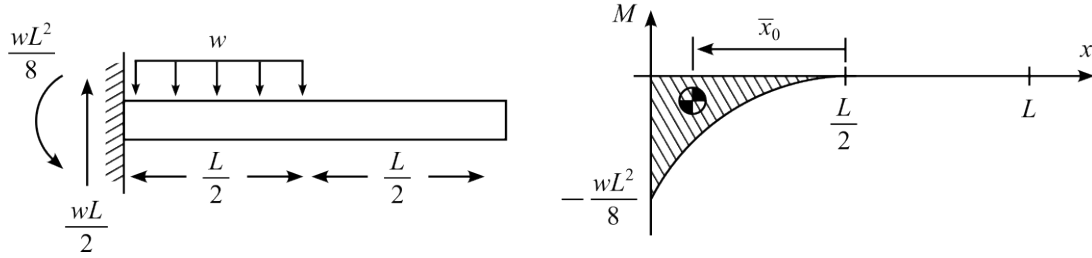
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Question 1 — Prob. 9.100



For the uniform cantilever beam and loading shown, determine the slope and deflection at (a) point B, (b) point C. Use the moment-area method to solve this problem. (부호 또는 화살표를 통해 처짐과 처짐각의 방향을 명확히 표시하시오.)



In portion AB,

$$A_m = \frac{1}{3} \cdot \frac{wL^2}{8} \cdot \frac{L}{2} = \frac{wL^3}{48}, \quad \bar{x} = \bar{x}_0 = \frac{3}{4} \cdot \frac{L}{2} = \frac{3L}{8}$$

$$\theta_{B/A} = \theta_B = -\frac{A_m}{EI} = -\frac{wL^3}{48EI}$$

$$t_{B/A} = y_B = -\frac{A_m \bar{x}}{EI} = -\frac{wL^4}{128EI}$$

$$\theta_B = -\frac{wL^3}{48EI}; \quad y_B = -\frac{wL^4}{128EI} \quad \blacktriangleleft \quad (a)$$

In portion AC,

$$\theta_{C/A} = \theta_C = -\frac{A_m}{EI} = -\frac{wL^3}{48EI}$$

$$y_C = y_B + \theta_B \times \frac{L}{2} = -\frac{7wL^4}{384EI}$$

$$\theta_C = -\frac{wL^3}{48EI}; \quad y_C = -\frac{7wL^4}{384EI} \quad \blacktriangleleft \quad (b)$$

Question 2 — Prob. 10.13

Brass
 $E = 120 \text{ GPa}$
 $\rho = 8740 \text{ kg/m}^3$

Aluminum
 $E = 70 \text{ GPa}$
 $\rho = 2710 \text{ kg/m}^3$

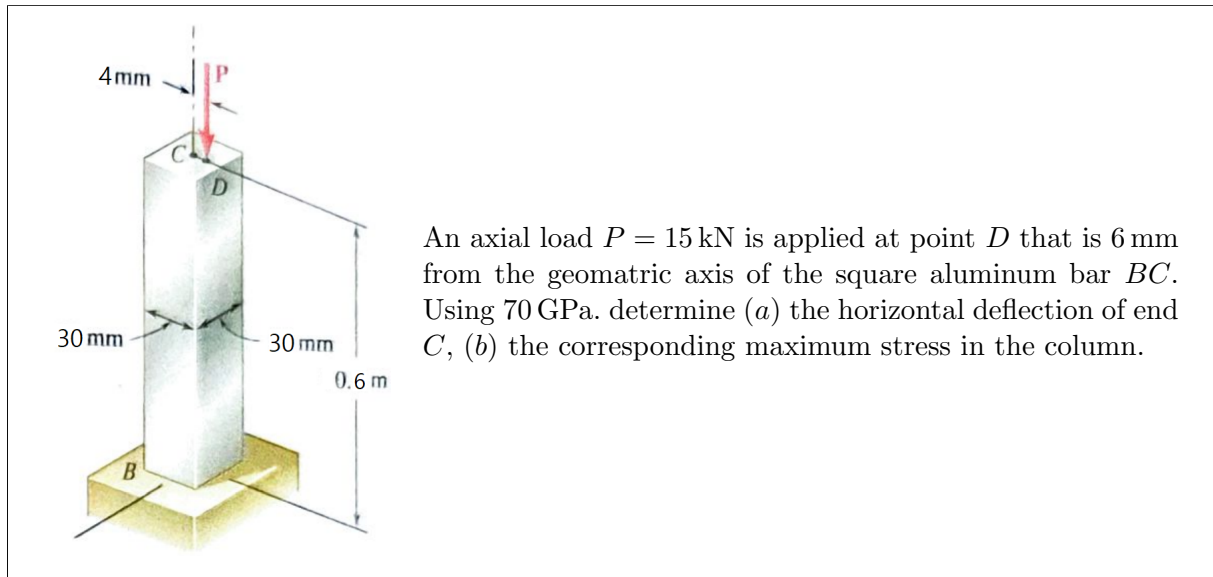
Determine (a) the critical load for the brass strut, (b) the dimension d for which the aluminum strut will have the same critical load, (c) the weight of the aluminum strut as a percent of the weight of the brass strut.

$$P_{\text{cr}} = \frac{\pi^2 E_b I_1}{L^2} = \frac{\pi^2 (120 \times 10^9) \left(\frac{1}{12} \cdot 0.02^4 \right)}{1.1^2} \text{ N} = 13050.71656 \text{ N} = 13.05 \text{ kN} \quad \blacktriangleleft \quad (a)$$

$$P_{\text{cr}} = \frac{\pi^2 E_a I_2}{L^2} = \frac{\pi^2 E_a d^4}{12 L^2} \Rightarrow d = \left(\frac{12 P_{\text{cr}} L^2}{\pi^2 E_a} \right)^{\frac{1}{4}} = 0.0228849937 \text{ m} = 22.88 \text{ mm} \quad \blacktriangleleft \quad (b)$$

$$\frac{m_a}{m_b} = \frac{\rho_a d^2 L}{\rho_b (0.02 \text{ m})^2 L} = \frac{\rho_a d^2}{\rho_b (0.02 \text{ m})^2} = \frac{(2710)(0.0228849937)^2}{(8740)(0.02)^2} = 40.60\% \quad \blacktriangleleft \quad (c)$$

Question 3 — variation of Prob. 10.35



$$I = \frac{1}{12}(0.03)^4 \text{ m}^4 = 6.75 \times 10^{-8} \text{ m}^4, \quad L_e = 2L = 1.2 \text{ m}$$

$$\text{let } H = \sec\left(\frac{L_e}{2} \sqrt{\frac{P}{EI}}\right) = \sec\left(\frac{1.2}{2} \sqrt{\frac{15 \times 10^3}{(70 \times 10^9)(6.75 \times 10^{-8})}}\right) = 2.079167354$$

$$y_{\max} = e(H - 1) = 4.32 \text{ mm} \quad \blacktriangleleft \quad (a)$$

$$\frac{ec}{r^2} = \frac{ecA}{I} = \frac{(0.004)(0.015)(0.03^2)}{6.75 \times 10^{-8}} = 0.8$$

$$\sigma_{\max} = \frac{P}{A} \left(1 + \frac{ec}{r^2} \cdot H\right) = 44.39 \text{ MPa} \quad \blacktriangleleft \quad (b)$$

Question 4 — variation of Prob. 3.36

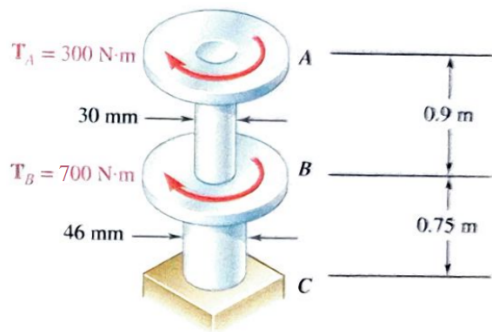


Fig. P3.36

The torques shown are exerted on pulleys *A* and *B*. Knowing that the shafts are solid and made of steel ($G = 73 \text{ GPa}$), determine the strain energy of this system.

$$T_{AB} = T_A = 300 \text{ N} \cdot \text{m}, \quad T_{BC} = T_A + T_B = 1000 \text{ N} \cdot \text{m}$$

$$J_{AB} = \frac{\pi}{2} (0.015^4) \text{ m}^4 = 7.952156404 \times 10^{-8} \text{ m}^4$$

$$J_{BC} = \frac{\pi}{2} (0.023^4) \text{ m}^4 = 4.395732149 \times 10^{-7} \text{ m}^4$$

$$U_{\text{tot}} = \frac{T_{AB}^2 L_{AB}}{2GJ_{AB}} + \frac{T_{BC}^2 L_{BC}}{2GJ_{BC}} = 18.66 \text{ J} \quad \blacktriangleleft$$

Question 5 — variation of Prob. 11.45

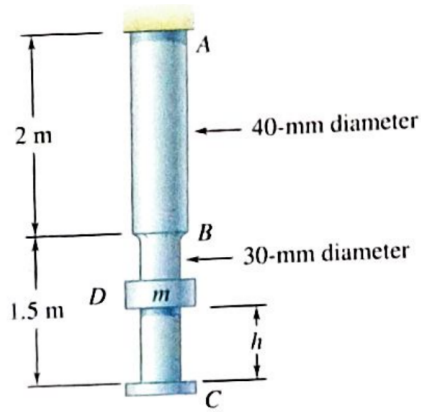


Fig. P11.126

The 50-kg collar D is released from rest in the position shown and is stopped by a plate attached at end C of the vertical rod ABC . Knowing that $E = 200$ GPa and the maximum normal stress is 250 MPa for both portions of the rod, determine the maximum height h_m .

$$P_m = \sigma_m A_{BC} = (250 \text{ MPa})(\pi \cdot 15^2 \text{ mm}^2) = 176714.5868 \text{ N}$$

$$U_m = \frac{P_m^2 L_{AB}}{2EA_{AB}} + \frac{P_m^2 L_{BC}}{2EA_{BC}} = 289.9223689 \text{ J}$$

$$U_m = \frac{1}{2} P_m \delta_m \Rightarrow \delta_m = \frac{2U_m}{P_m} = 3.28125 \text{ mm}$$

$$U_m = mg(h_m + \delta_m) \Rightarrow h_m = \frac{U_m}{mg} - \delta_m = 0.587793916 \text{ m} = 587.79 \text{ mm} \quad \blacktriangleleft$$