해설

고체역학(박성훈 교수님) 2025-1 기말고사

시험 실시 : 2025-06-16 16:30-19:00(150분)

2025–12–27

숭실대학교
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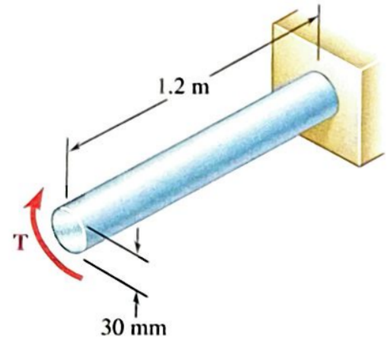
Question 1 — variation of prob. 3.95 in textbook



The solid shaft shown is made of a mild steel that is assumed to be elastoplastic with $G = 77.2\,\text{GPa}$ and $\tau_Y = 145\,\text{MPa}$. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (a) $T = 500\,\text{N}\cdot\text{m}$, (b) $T = 1200\,\text{N}\cdot\text{m}$.

$$J = \frac{\pi}{2}r^4 = \frac{\pi}{2}(15\,\text{mm})^4 = 25312.5\pi\,\text{mm}^4$$
$$T_Y = \frac{\tau_Y J}{c} = \frac{(145\,\text{MPa})(25312.5\pi\,\text{mm}^4)}{15\,\text{mm}} = \frac{(145)(25312.5\pi)}{15} \times 10^{-3}\,\text{N}\cdot\text{m} = 768.7084524\,\text{N}\cdot\text{m}$$
$$T_p = \frac{4}{3}T_Y = 1024.944603\,\text{N}\cdot\text{m}$$

(a) When $T = 500\,\text{N}\cdot\text{m}$,

$$T < T_Y \quad \Rightarrow \quad \rho_Y = 15.00\,\text{mm}$$
$$\tau_{\max} = \frac{Tc}{J} = \frac{(500\,\text{N}\cdot\text{m})(15\,\text{mm})}{25312.5\pi\,\text{mm}^4} = \frac{(500)(15)}{25312.5\pi} \times 10^3\,\text{MPa} = 94.31\,\text{MPa}$$

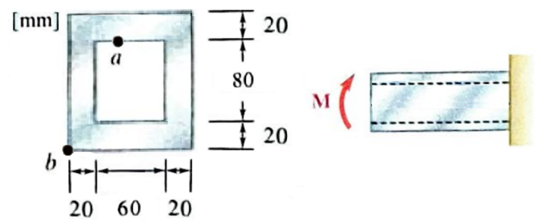
(a) $\tau_{\max} = 94.31\,\text{MPa}\,;\ \rho_Y = 15.00\,\text{mm}$ ◀

(b) When $T = 1200\,\text{N}\cdot\text{m}$,

$$T > T_p \quad \Rightarrow \quad \rho_Y = 0.00\,\text{mm}$$
$$\tau_{\max} = \tau_Y = 145.00\,\text{MPa}$$

(b) $\tau_{\max} = 145.00\,\text{MPa}\,;\ \rho_Y = 0.00\,\text{mm}$ ◀

Question 2



An aluminum beam has the cross section shown. Knowing that the magnitude of couple appling on the beam is $20\,\text{kN}\cdot\text{m}$, determine the stress at (a) point a, (b) point b.

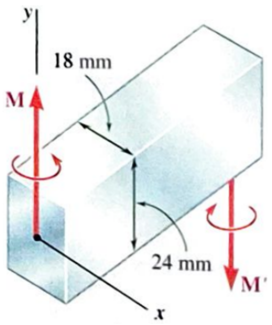
*이 문제에서 주어진 수치와 답안은 실제 실시된 시험과 차이가 있을 수 있습니다.

$$I = \frac{1}{12}(100)(120)^3 - \frac{1}{12}(60)(80)^3\,[\text{mm}^4] = 11840000\,\text{mm}^4$$
$$\sigma_a = -\frac{My_a}{I} = -\frac{(20\,\text{kN}\cdot\text{m})(40\,\text{mm})}{11840000\,\text{mm}^4} = \frac{(20)(40)}{11840000} \times 10^6\,\text{MPa} = 67.57\,\text{MPa}$$
$$\sigma_b = -\frac{My_a}{I} = -\frac{(20\,\text{kN}\cdot\text{m})(60\,\text{mm})}{11840000\,\text{mm}^4} = \frac{(20)(60)}{11840000} \times 10^6\,\text{MPa} = 101.35\,\text{MPa}$$

(a) $67.57\,\text{MPa}$ ◀

(b) $101.35\,\text{MPa}$ ◀

Question 3 — prob. 4.71 in textbook



The prismatic rod shown is made of a steel that is assumed to be elastoplastic with $E = 200\,\text{GPa}$ and $\sigma_Y = 280\,\text{MPa}$. Knowing that couples M and M' of moment 525 N-m are applied and maintained about axes parallel to the y axis, determine (a) the thickness of the elastic core, (b) the radius of curvature of the bar.

$$I_y = \frac{1}{12}(24)(18)^3\,\text{mm}^4 = 11664\,\text{mm}^4$$

$$M_Y = \frac{\sigma_Y I_y}{c} = \frac{(280\,\text{MPa})(11664\,\text{mm}^4)}{9\,\text{mm}} = \frac{(280)(11664)}{9} \times 10^3\,\text{N}\cdot\text{m} = 362.88\,\text{N}\cdot\text{m}$$

$$M = \frac{3}{2}M_Y\left(1 - \frac{x_Y^2}{3c^2}\right)$$

$$x_Y = c\sqrt{2 - \frac{3M}{M_Y}} = (9\,\text{mm})\sqrt{3 - \frac{2(525\,\text{N}\cdot\text{m})}{362.88\,\text{N}\cdot\text{m}}} = 2.936835031\,\text{mm}$$

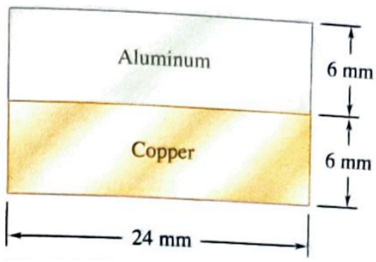
$$(\text{thickness of elastic core}) = 2x_Y = 5.87\,\text{mm}$$

(a) 5.87 mm ◄

$$\rho = -\frac{x_Y}{\varepsilon_Y} = -\frac{Ex_Y}{\sigma_Y} = -\frac{(200\,\text{GPa})(-2.936835031\,\text{mm})}{280\,\text{MPa}} = 2.10\,\text{m}$$

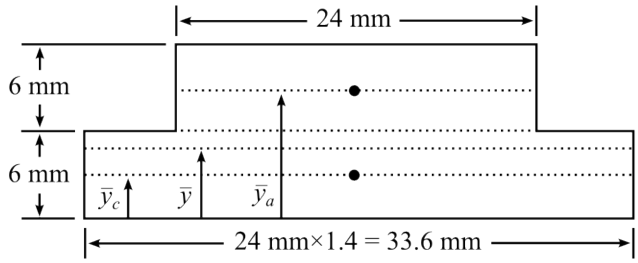
(b) 2.10 m ◄

Question 4 — prob. 4.39 in textbook



A copper strip ($E_c = 105\,\mathrm{GPa}$) and an aluminum strip ($E_a = 75\,\mathrm{GPa}$) are bonded together to form the composite beam shown. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 35\,\mathrm{N \cdot m}$, determine the maximum stress in (a) the aluminum strip, (b) the copper strip.

$$\text{let.}\quad E_c = nE_a = nE, \quad n = 1.4, \quad E = 75\,\mathrm{GPa}$$



$$\bar{y}_c = 3\,\mathrm{mm}, \quad \bar{y}_a = 9\,\mathrm{mm}$$

$$A_c = (33.6\,\mathrm{mm})(6\,\mathrm{mm}) = 201.6\,\mathrm{mm}, \quad A_a = (24\,\mathrm{mm})(6\,\mathrm{mm}) = 144\,\mathrm{mm}^2$$

$$\bar{y} = \frac{A_c\bar{y}_c + A_a\bar{y}_a}{A_c + A_a} = \frac{(201.6)(3) + (144)(9)}{201.6 + 144}\,\mathrm{mm} = 5.50\,\mathrm{mm}$$

$$I = \frac{1}{12}(33.6)(6)^3 + (201.6)(2.5)^2 + \frac{1}{12}(24)(6)^3 + (144)(3.5)^2\,[\mathrm{mm}^4] = 4060.8\,\mathrm{mm}^4$$

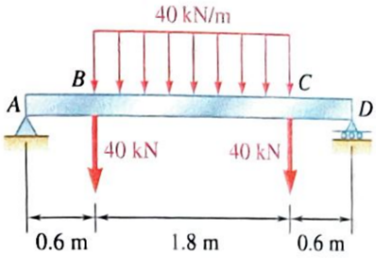
$$|\sigma_a|_{\max} = \frac{My_{\max}}{I} = \frac{(35\,\mathrm{N \cdot m})(6.5\,\mathrm{mm})}{4060.8\,\mathrm{mm}^4} = 56.02\,\mathrm{MPa}$$

(a) $56.02\,\mathrm{MPa}$ ◀

$$|\sigma_c|_{\max} = n \cdot \frac{M|y_{\min}|}{I} = (1.4)\frac{(35\,\mathrm{N \cdot m})(5.5\,\mathrm{mm})}{4060.8\,\mathrm{mm}^4} = 66.37\,\mathrm{MPa}$$

(b) $66.37\,\mathrm{MPa}$ ◀

Question 5 — varation of prob. 5.108 in textbook



Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value of (*a*) the shear, (*b*) the bending moment.

Let the centroid of the beam be point E.

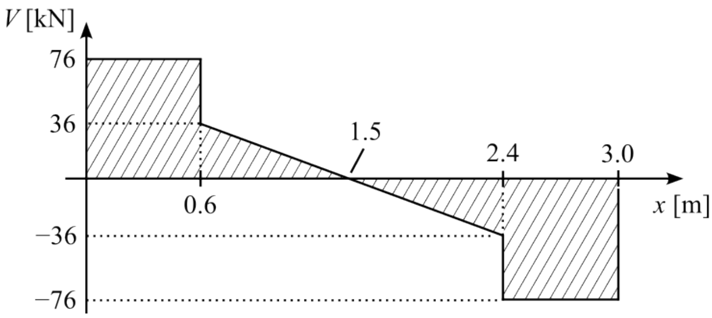
$$+\circlearrowleft \sum M|_E = -R_A(1.5\,\mathrm{m}) + R_D(1.5\,\mathrm{m}) = 0 \quad \Rightarrow \quad R_A = R_D = R$$

$$+\uparrow \sum F_y = 2R - 40\,\mathrm{kN} - (40\,\mathrm{kN/m})(1.8\,\mathrm{m}) - 40\,\mathrm{kN} = 0 \quad \Rightarrow \quad R = 76\,\mathrm{kN}$$

$0.0\,\mathrm{m}$ to $0.6\,\mathrm{m}$: $\quad V = 76\,\mathrm{kN}$

$0.6\,\mathrm{m}$ to $2.4\,\mathrm{m}$: $\quad V = 36\,\mathrm{kN} - (40\,\mathrm{kN/m})(x - 0.6\,\mathrm{m})$

$2.4\,\mathrm{m}$ to $3.0\,\mathrm{m}$: $\quad V = -76\,\mathrm{kN}$



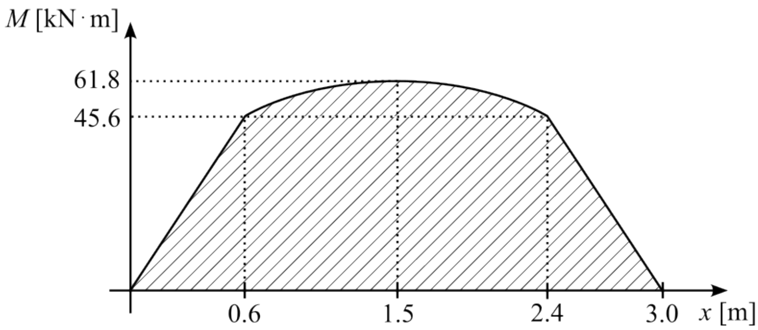
(a) $\quad 76\,\mathrm{kN}$ ◀

at $0.0\,\mathrm{m}$ and $3.0\,\mathrm{m}$: $\quad M = 0$

at $0.6\,\mathrm{m}$: $\quad M = (76\,\mathrm{kN})(0.6\,\mathrm{m}) = 45.6\,\mathrm{kN \cdot m}$

at $1.5\,\mathrm{m}$: $\quad M = (76\,\mathrm{kN})(1.5\,\mathrm{m}) - (40\,\mathrm{kN})(0.9\,\mathrm{m}) - (40\,\mathrm{kN/m})(0.9\,\mathrm{m})(0.45\,\mathrm{m}) = 61.8\,\mathrm{kN \cdot m}$

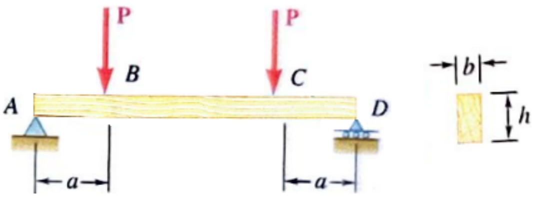
at $2.4\,\mathrm{m}$: $\quad M = 45.6\,\mathrm{kN \cdot m}$

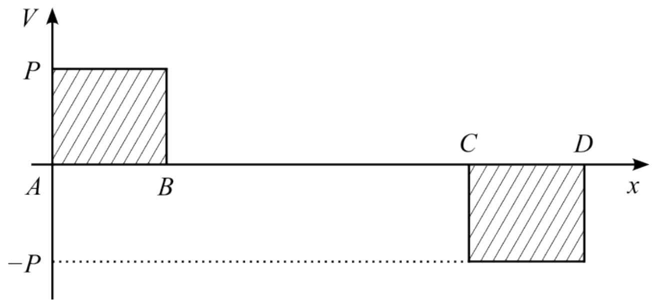


(b) $\quad 61.8\,\mathrm{kN \cdot m}$ ◀

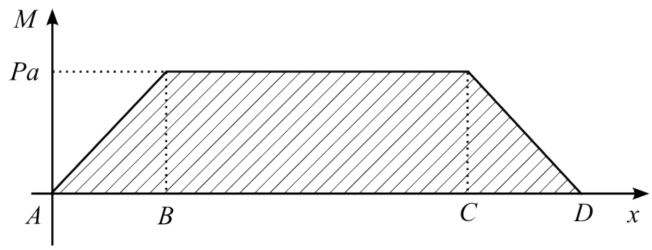
Question 6 — varation of prob. 5.5 in textbook

A timber beam AD of rectangular cross section whose width b is 100 mm and depth h is 150 mm carries loadings shown. Knowing that a is 0.5 m and the allowable normal stress and shear stress are $\sigma_{\text{all}} = 11\,\text{MPa}$ and $\tau_{\text{all}} = 1.2\,\text{MPa}$, determine the maximum permissible load P_{max}.





Because $V = \frac{dM}{dx}$ and $M = 0$ at point A and B,



$$|V|_{\text{max}} = P, \quad |M|_{\text{max}} = Pa$$

$$|\sigma|_{\text{max}} = \frac{|M|_{\text{max}}}{S} = \frac{6|M|_{\text{max}}}{bh^2} = \frac{6Pa}{bh^2} \le \sigma_{\text{all}}$$

$$P \le \frac{\sigma_{\text{all}} bh^2}{6a} = \frac{(11\,\text{MPa})(100\,\text{mm})(150\,\text{mm})^2}{6(0.5\,\text{m})} = 8.25\,\text{kN}$$

$$|\tau|_{\text{max}} = \frac{|V|_{\text{max}} Q_{\text{half}}}{It} = \frac{3|V|_{\text{max}}}{2bh} = \frac{3P}{2bh} \le \tau_{\text{all}}$$

$$P \le \frac{2\tau_{\text{all}} bh}{3} = \frac{2(1.2\,\text{MPa})(100\,\text{mm})(150\,\text{mm})}{3} = 12\,\text{kN}$$

$$P_{\text{max}} = 8.25\,\text{kN}$$

8.25 kN ◀