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# ***Optimization of functions of several variables with Matlab***

*Statistics and Mathematical Analysis*

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**Question 1 (3p)**

Explain what criterion you apply to decide that the gradient descent algorithm has converged.

*We apply two criterias at the same time. Whenever one is reached, it exits the program. The first one defines a maximum number of iterations and the second one determines when the step size related to the learning rate (alpha) approaches zero. Since the tolerance is also near zero, we break the for loop every time that the real tolerance is less than the defined one.*

**Question 2 (2p)**

Complete the following table, indicating the coordinates of the real absolute minimum of each function.

| <b>Function</b>      | <b><i>x_real</i></b> | <b><i>y_real</i></b> |
|----------------------|----------------------|----------------------|
| Circular Paraboloide | 0                    | 0                    |
| Elliptic Paraboloide | -20                  | -30                  |
| Inverted Gaussian    | 1                    | 1                    |
| Arbitrary function   | 41                   | -39                  |

Briefly explain how you find the position of the real absolute minimum of the function.

*We define the real minimum with the matlab function min twice. Once for each column of z, a matrix of 201\*201, and another to find the final minimum of this previous collection. Finally, we need its coordinates, so we use matlab find function which basically goes through every value of matrix z and check if it matches with the one stored (M). If it matches, it saves the row as well as the column, our real\_x and real\_y.*

### Question 3 (5p)

For each of the four functions to optimize, complete the following table:

| Function               | x_initial | y_initial | alfa   | x_final | y_final | Number of iterations | minim_real-mini<br>m_final |
|------------------------|-----------|-----------|--------|---------|---------|----------------------|----------------------------|
| Circular<br>Paraboloid | 5         | 5         | 0.1    | 0.4     | 0.4     | 10                   | 0                          |
|                        | 5         | 5         | 0.01   | 0.48    | 0.48    | 109                  | 0                          |
| Elliptic<br>Paraboloid | 1         | 100       | 0.1    | -19.64  | -29.6   | 22                   | 0                          |
|                        | 100       | 1         | 0.1    | -19.52  | -29.6   | 31                   | 0                          |
| Inverted<br>Gaussian   | 50        | 50        | 0.1    | E       | E       | E                    | E                          |
|                        | 50        | 50        | $10^4$ | 1.4681  | 1.4681  | 87                   | 0                          |
| Arbitrary<br>function  | 150       | 150       | $10^4$ | 40.69   | -39.42  | 24                   | 0                          |
|                        | 50        | 190       | $10^4$ | 41.43   | -38.71  | 17                   | 0                          |

Briefly justify the differences you observe in the results obtained for each function, explaining how accurate the algorithm is in finding the absolute minimum:

#### Circular paraboloid

Since  $x_{\text{initial}}$  and  $y_{\text{initial}}$  have the same values, the difference is in the  $\alpha$  (learning rate). The number of iterations increases since the  $\alpha$  has decreased with respect to 0.1. Taking into account the coordinates of the real minimum, we conclude that it's 100% accurate.

#### Elliptic paraboloid

We observe that the more  $x_{\text{initial}}$  we have, the more iterations it took to converge to the minimum. Similarly to circular paraboloid, we can say that it's very accurate.

#### Inverted Gaussian

The inverted gaussian depends on the  $\alpha$  value. If it's decimal, it gives us "Error in Gradient\_Descendent  $x_{\text{final}} = \text{positions}(1, \text{iterator})$ ;", so we assume that the algorithm moves so slow that the iterator exceeds array out bounds. However, if  $\alpha$  is big enough ( $10^4$  for example), the accuracy is 100% again.

**Arbitrary function**

The number of iterations increases having both  $x_{\text{initial}}$  and  $y_{\text{initial}}$  the same values, and same  $\alpha$ . The difference between those coordinates in the first case is smaller (1'27) than the one in the second case (2'72).

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