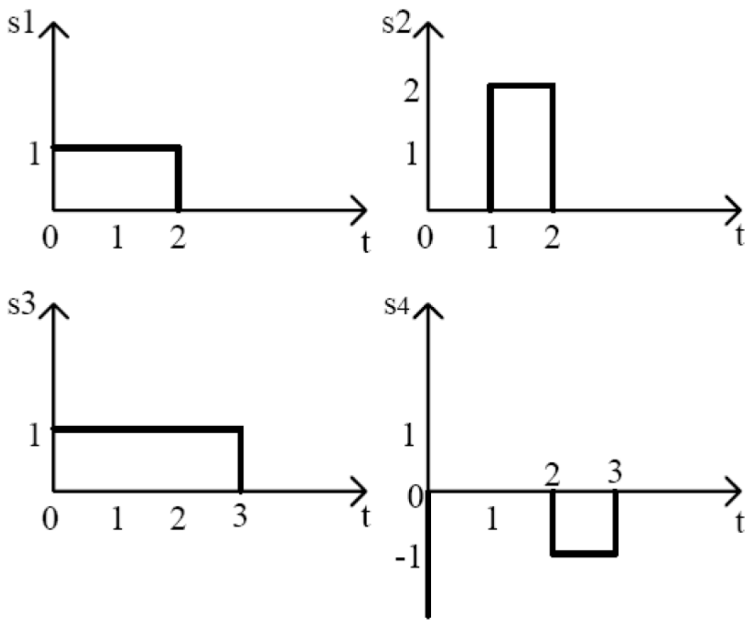
Question 1

Consider the following 4 functions



1) Write a set of orthonormal basis functions for these 4 functions.
2) What is the dimension of the space spanned by the 4 functions?
3) Use your basis in 1) to represent each of the 4 functions.
4) Use Gram-Schmidt procedure to find a set of orthonormal basis for the set spanned by the 4 functions.

Question 2

Consider the signals

$$s_1(t) = 2\cos\left(\omega t + \frac{\pi}{6}\right) + \sin(\omega t), 0 \le t \le \frac{2\pi}{\omega}$$

$$s_2(t) = 3\sin\left(\omega t + \frac{\pi}{4}\right), 0 \le t \le \frac{2\pi}{\omega}$$

1) Calculate the energy of each signal.
2) Find a set of orthonormal basis functions for these signals. Express them as a linear combination of those functions.
3) Calculate the energy again used the coordinates in the signal space.
4) Find the inner product of these two signals.

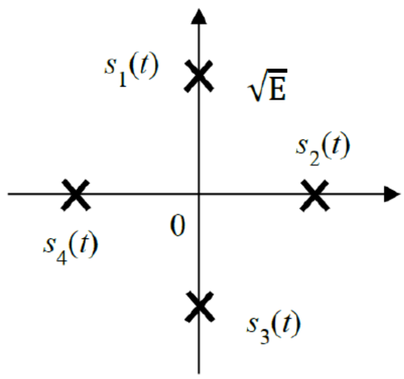
Question 3

Consider a digital communication system employing M-ary modulation over an AWGN channel with noise power-spectral density $N_0/2$. The average symbol energy is E_s

a) Assuming that 8-FSK modulation is employed, what's the dimension of 8-FSK? Using the union bound, express the BER P_b. in terms of E_s/N_0.
b) Assuming that 8-PSK modulation is employed, draw the constellation diagram and using the union bound, express the BER P_b. in terms of E_s/N_0. State all key assumptions in deriving the result.
c) Which one requires more correlators in the structure of the optimal detector (followed by a minimum distance calculator) for 8FSK and 8PSK? Point out the advantage and disadvantage for MFSK and MPSK.
d) If the system is with very limited energy, which scheme (MFSK or MPSK) will you choose?

Question 4

A digital communications system transmit 4 *equally likely* symbols (i.e., with $P(S_k) = \frac{1}{4}$ for all k) with symbol energy $\sqrt{E}$ over an AWGN channel with double-sided power spectral density $N_0/2$. The signal constellation for the concerned communication system is shown below:



a) What is the dimension of the signal space for the above signal constellation?
b) Determine the optimal decision regions Rm for $m = 1, 2, 3, 4$.
c) Determine $P(C|s_1)$: the exact probability of correct decision when S_1 is transmitted.
d) Determine the exact symbol error probability, P_{eM}, as a function of E_b and N_0 by assuming that

$(Q(\sqrt{\frac{E_b}{N_0}}))^2 \cong 0$ for high E_b/N_0

e) Sketch the block diagrams of two possible realizations of the optimum receiver structure by using correlators.

f) Give the block diagram of the optimum receiver by using the matched filter. *Your receiver must be as simple as possible.*