

# 信号与系统第七章试题答案

$$1. \textcircled{1} H(z) = \frac{\frac{1}{3}}{1-0.5z^{-1}} + \frac{\frac{2}{3}}{1-2z^{-1}}$$

$$\sum_{n=-\infty}^{+\infty} |h[n]| < +\infty \Rightarrow \text{稳定} \Rightarrow 0.5 < |z| < 2 \text{ 包含单位圆}$$

$$\Rightarrow h[n] = \frac{1}{3} (0.5)^n u[n] - \frac{2}{3} (2)^n u[-n-1]$$

$$\textcircled{2} x[n] = 3 - u[n]$$

根据定理：某LTI系统冲激响应 $h[n]$ ，其 $z$ 变换为 $H(z)$ ，  
则 $x[n] = a^n$ 通过该系统的输出为 $H(a) a^n$ ，其中 $a$ 在  
 $H(z)$ 收敛域内。

$$3 = 3 \cdot 1^n \xrightarrow{\text{LTI}} H(1) 1^n = 3 \cdot 0 = 0$$

$$u[n] \xrightarrow{\text{LTI}} -\frac{1}{3} (0.5)^n u[n] + \frac{4}{3} (2)^n u[n]$$

$$x[n] \xrightarrow{\text{LTI}} \frac{1}{3} (0.5)^n u[n] - \frac{4}{3} (2)^n u[n]$$

2.  $\textcircled{1}$  根据上面定理，易知收敛域为 $0.5 < |z| < 2$ ，稳定非因果。

$$\textcircled{2} a^n \xrightarrow{\text{LTI}} H(a) a^n \Rightarrow H(-1) = 1$$

$$\text{则 } H(z) = \frac{-1.5z^2}{(z+2)(z-0.5)}$$

$$\frac{H(z)}{z} = \frac{-1.5z}{(z+2)(z-0.5)} = \frac{-\frac{6}{5}}{z+2} + \frac{-\frac{3}{10}}{z-0.5}$$

$$\text{则 } H(z) = \frac{-\frac{6}{5}z}{z+2} + \frac{-\frac{3}{10}z}{z-0.5} = \frac{-\frac{6}{5}}{1-2z^{-1}} + \frac{-\frac{3}{10}}{1-0.5z^{-1}}$$

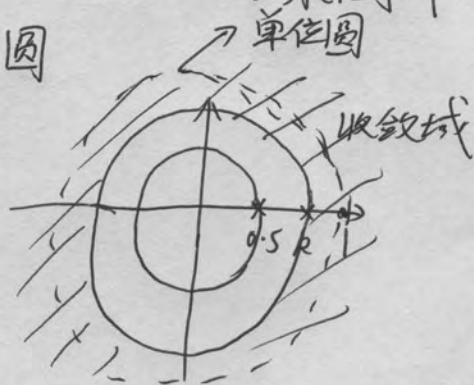
$0.5 < |z| < 2$

$$h[n] = \frac{6}{5} 2^n u[-n-1] - \frac{3}{10} (0.5)^n u[n]$$

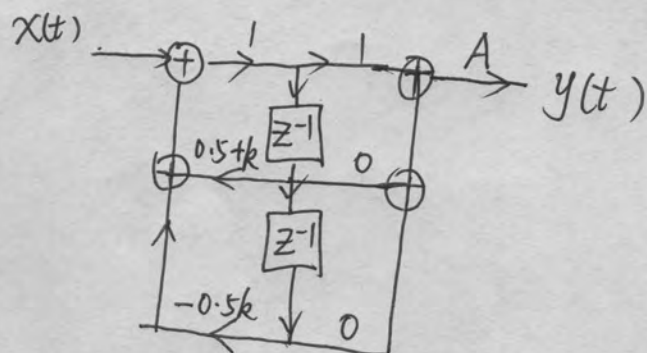
(注意收敛域为 $0.5 < |z| < 2$ )

3. ①因果系统  $\Rightarrow h[n]$  右边序列  $\Rightarrow H(z)$  收敛域为某圆外.  
 稳定  $\Rightarrow H(z)$  收敛域包含单位圆

因果 + 稳定  $\Rightarrow 0.5 < |k| < 1$



$$\textcircled{2} \quad H(z) = \frac{A}{1 - (0.5+k)z^{-1} + 0.5kz^{-2}}$$



$$\textcircled{3} \quad x[n] = (-1)^n$$

$$(-1)^n \xrightarrow{\text{LTI}} H(-1)(-1)^n = y[n]$$

所以,  $H(-1) \cdot (-1) = y[1] = -2 \Rightarrow H(-1) = 2$ , 即

$$\frac{A}{[1 - 0.5 \times (-1)][1 - \frac{1}{3}(-1)]} = 2 \Rightarrow A = 2$$

$$4. \quad \frac{X(z)}{z} = \frac{z}{z^2 - 2.5z + 1} = \frac{z}{(z-2)(z-0.5)} = \frac{\frac{4}{3}}{z-2} - \frac{\frac{1}{3}}{z-0.5}$$

$$X(z) = \frac{\frac{4}{3}z}{z-2} - \frac{\frac{1}{3}z}{z-0.5} = \frac{\frac{4}{3}}{1-2z^{-1}} - \frac{\frac{1}{3}}{1-0.5z^{-1}}$$

因为  $\sum_{n=-\infty}^{+\infty} |x[n]| < +\infty \Rightarrow \text{稳定} \Rightarrow 0.5 < |z| < 2$

$$h[n] = -\frac{4}{3} 2^n u[-n-1] - \frac{1}{3} (0.5)^n u[n]$$

5. ⑤不能。有限长  $\Rightarrow$  收敛域整个平面  $\Rightarrow$  无极点

⑥不能。绝对可和  $\Rightarrow$  收敛域包括单位圆

而左边序列  $\Rightarrow$  收敛域为  $\frac{1}{2}$  极点里边  $\Rightarrow$  不包括单位圆

⑦可以。

⑧可以。

$$6. \textcircled{1} H(z) = \frac{1-z^{-1}}{1-\frac{1}{2}z^{-1}} \cdot \frac{1-\frac{1}{2}z^{-1}}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}} = \frac{1-z^{-1}}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$$

$$\textcircled{2} \text{因果} \Rightarrow |z| > \frac{1}{2} \Rightarrow \text{收敛域包含单位圆} \Rightarrow \text{稳定}$$
$$= \frac{1-z^{-1}}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$$

$$y[n] + \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] - x[n-1]$$

$$\textcircled{3} x[n] = 1 + e^{j\frac{\pi}{2}n} + e^{j\pi n}$$

$$1 \xrightarrow{LTI} H(1) \cdot 1^n = 0$$

$$e^{j\frac{\pi}{2}n} \xrightarrow{LTI} H(e^{j\frac{\pi}{2}}) e^{j\frac{\pi}{2}n} = \frac{8j}{9-2j} e^{j\frac{\pi}{2}n}$$

$$e^{j\pi n} = (-1)^n \xrightarrow{LTI} H(-1) (-1)^n = \frac{16}{5} (-1)^n$$

所以

$$x[n] \xrightarrow{LTI} \frac{8j}{9-2j} e^{j\frac{\pi}{2}n} + \frac{16}{5} (-1)^n$$

$$7. \textcircled{1} y[n] = \frac{5}{8}y[n-1] + \frac{1}{8}y[n-2] = x[n] - \frac{1}{4}x[n-1]$$

$$\textcircled{2} H(z) = \frac{1-\frac{1}{4}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})} \quad \text{因果} \Rightarrow |z| > \frac{1}{2} \Rightarrow$$

收敛域包含单位圆  $\Rightarrow$  稳定

$$\textcircled{3} \widetilde{Y}(z) = \frac{5}{8}\{z^{-1}\widetilde{Y}(z) + y[-1]\} + \frac{1}{8}\{z^{-2}\widetilde{Y}(z) + y[-2] + y[-1]z^{-1}\}$$
$$= (1-\frac{1}{4}z^{-1})\widetilde{X}(z)$$



将  $y[-1] = -1$   $y[-2] = 0$   $\widetilde{X(z)} = \frac{1}{1 - \frac{1}{4}z^{-1}}$  代入

$$(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{3}z^{-1})\widetilde{Y(z)} = 1 - \frac{5}{6} + \frac{1}{6}z^{-1} = \frac{1}{6}(1 + z^{-1})$$

$$\widetilde{Y(z)} = \frac{1}{6} \cdot \frac{1}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{3}z^{-1})} = \frac{1}{6} \left[ \frac{3}{1 - \frac{1}{2}z^{-1}} - \frac{2}{1 - \frac{1}{3}z^{-1}} \right]$$

$$y[n] = \frac{1}{2} \left(\frac{1}{2}\right)^n u[n] - \frac{1}{3} \left(\frac{1}{3}\right)^n u[n]$$

$$= \left(\frac{1}{2}\right)^{n+1} u[n] - \left(\frac{1}{3}\right)^{n+1} u[n]$$

8. ①  $y[n] + \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] - \frac{1}{4}x[n-1]$

②  $H(z) = \frac{1 - \frac{1}{4}z^{-1}}{(1 + \frac{1}{2}z^{-1})(1 - \frac{1}{4}z^{-1})} = \frac{1}{1 + \frac{1}{2}z^{-1}}$

因果  $\Rightarrow |z| > \frac{1}{2} \Rightarrow$  稳定, 所以

$$h[n] = \left(-\frac{1}{2}\right)^n u[n]$$

③ 因为:  $w[n] + \frac{1}{4}w[n-1] - \frac{1}{8}w[n-2] = x[n]$

所以:  $\widetilde{W(z)} + \frac{1}{4}\{z^{-1}\widetilde{W(z)} + w[-1]\} - \frac{1}{8}\{z^{-2}\widetilde{W(z)} + w[-2] + w[-1]z^{-1}\} = \widetilde{X(z)}$

将  $w[-1] = 0$   $w[-2] = 8$   $\widetilde{X(z)} = \frac{1}{1 - z^{-1}}$  代入

$$\left(1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}\right)\widetilde{W(z)} = \frac{1}{1 - z^{-1}} + 1 = \frac{2 - z^{-1}}{1 - z^{-1}}$$

$$\widetilde{W(z)} = \frac{2 - z^{-1}}{(1 - z^{-1})(1 + \frac{1}{2}z^{-1})(1 - \frac{1}{4}z^{-1})}$$

$$= \frac{\frac{8}{9}}{1 - z^{-1}} + \frac{\frac{8}{9}}{1 + \frac{1}{2}z^{-1}} + \frac{\frac{2}{9}}{1 - \frac{1}{4}z^{-1}}$$

因果  $\Rightarrow w[n] = \frac{8}{9}u[n] + \frac{8}{9}\left(-\frac{1}{2}\right)^n u[n] + \frac{2}{9}\left(\frac{1}{4}\right)^n u[n]$

$$y[n] = w[n] - \frac{1}{4}w[n-1]$$

$$= \frac{8}{9}u[n] - \frac{2}{9}u[n-1] + \frac{8}{9}\left(-\frac{1}{2}\right)^n u[n] - \frac{2}{9}\left(-\frac{1}{2}\right)^{n-1} u[n-1]$$

$$+ \frac{2}{9}\left(\frac{1}{4}\right)^n u[n] - \frac{1}{18}\left(\frac{1}{4}\right)^{n-1} u[n-1]$$

化简可得:  $y[n] = \begin{cases} 2 & n=0 \\ \frac{2}{3}u[n-1] - \frac{2}{3}\left(-\frac{1}{2}\right)^{n-1}u[n-1] & n>0 \end{cases}$

或  $y[n] = 2\delta[n] + \frac{2}{3}u[n-1] - \frac{2}{3}\left(-\frac{1}{2}\right)^{n-1}u[n-1]$

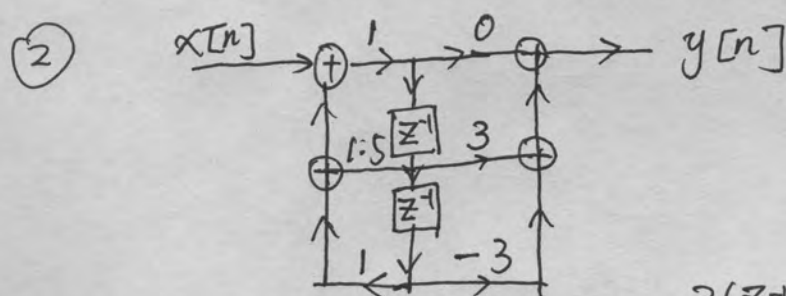
9. ①  $H(z) = \frac{A(z-1)}{(z+0.5)(z-2)}$

因为  $a^n \xrightarrow{LTI} H(a) \cdot a^n$  ( $a$ 属于收敛域时成立)

所以  $(-1)^n \xrightarrow{LTI} H(-1)(-1)^n = -4(-1)^n$

即  $\frac{A(-1-1)}{(-1+0.5)(-1-2)} = -4 \Rightarrow A = 3$

因为  $-1$ 在  $H(z)$ 收敛域内, 所以收敛域为  $0.5 < |z| < 2$ , 非因果, 稳定。



③  $Y(z) = X(z)H(z) = \frac{3(z-1)}{(z+0.5)(z-2)} \cdot \frac{1}{1-z^{-1}}$

$$= \frac{3(z-1)z^{-1}}{(1+0.5z^{-1})(1-2z^{-1})(1-z^{-1})}$$

$$= \frac{-\frac{6}{5}}{1+0.5z^{-1}} + \frac{\frac{6}{5}}{1-2z^{-1}}$$

收敛域  $0.5 < |z| < 2$

$$y[n] = -\frac{6}{5}\left(-\frac{1}{2}\right)^n u[n] - \frac{6}{5}(2)^n u[-n-1]$$

10. ①  $H(z) = \frac{1-z^{-1}}{1+\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}} = \frac{1-z^{-1}}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}$

因果  $\Rightarrow$  收敛域  $|z| > \frac{1}{2}$

②  $h[n] = \frac{9}{5}\left(-\frac{1}{2}\right)^n u[n] - \frac{4}{5}\left(\frac{1}{3}\right)^n u[n]$

$$17. \textcircled{1} H(z) = \frac{Az^n}{(z - \frac{1}{3})(z - \frac{1}{2})}$$

因果序列初值定理

$$h[0] = \lim_{z \rightarrow +\infty} \frac{Az^n}{(z - \frac{1}{3})(z - \frac{1}{2})} = 2 \Rightarrow \begin{cases} n=2 \\ A=2 \end{cases}$$

$$\text{则 } H(z) = \frac{2z^2}{(z - \frac{1}{3})(z - \frac{1}{2})} = \frac{2}{(1 - \frac{1}{3}z^{-1})(1 - \frac{1}{2}z^{-1})}$$

$$= \frac{-4}{1 - \frac{1}{3}z^{-1}} + \frac{6}{1 - \frac{1}{2}z^{-1}} \quad \text{因果} \Rightarrow \text{收敛域 } |z| > \frac{1}{2}$$

$$\text{则 } h[n] = [-4(\frac{1}{3})^n + 6(\frac{1}{2})^n]u[n]$$

$$\textcircled{2} \text{ 设 } x_1[n] = u[n] - \frac{1}{2}u[n-1], \text{ 则}$$

$$Y_1(z) = H(z)X_1(z) = \frac{2(1 - \frac{1}{2}z^{-1})}{(1 - \frac{1}{3}z^{-1})(1 - \frac{1}{2}z^{-1})(1 - z^{-1})}$$

$$= \frac{1}{1 - \frac{1}{3}z^{-1}} \cdot \frac{1}{1 - z^{-1}}$$

$$= \frac{2}{(1 - \frac{1}{3}z^{-1})(1 - z^{-1})} = \frac{-1}{1 - \frac{1}{3}z^{-1}} + \frac{3}{1 - z^{-1}}$$

$$y_1[n] = -(\frac{1}{3})^n u[n] + 3u[n]$$

$$\text{设 } x_2[n] = \cos \pi n = (-1)^n$$

$$(-1)^n \xrightarrow{\text{LTI}} H(-1)(-1)^n = (-1)^n$$

$$y[n] = y_1[n] + y_2[n] = -(\frac{1}{3})^n u[n] + 3u[n] + (-1)^n$$

$$12. \textcircled{1} y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = -\frac{2}{3}x[n] - \frac{2}{3}kx[n-1]$$

$$\textcircled{2} H(z) = \frac{-\frac{2}{3}(1 + kz^{-1})}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{3}z^{-1})}$$

因果  $\Rightarrow$  收敛域  $|z| > \frac{1}{2} \Rightarrow$  稳定。

$$\textcircled{3} \quad \widetilde{Y}(z) = \frac{5}{6} (z^{-1} \widetilde{Y}(z) + y[-1]) + \frac{1}{6} (z^{-2} \widetilde{Y}(z) + y[-2] + y[-1]z^{-1})$$

$$= -\frac{2}{3} (1 + kz^{-1}) \widetilde{X}(z)$$

将  $y[-1]=2$   $y[-2]=0$   $\widetilde{X}(z) = \frac{1}{1+z^{-1}}$  代入

$$\widetilde{Y}(z) = \frac{-\frac{2}{3}(1+kz^{-1})}{(1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2})(1+z^{-1})} + \frac{\frac{5}{3}-\frac{1}{3}z^{-1}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}} \quad \textcircled{1}$$

假设  $\widetilde{Y}(z) = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1-\frac{1}{3}z^{-1}} + \frac{C}{1+z^{-1}}$

则 C 必须为 0, 否则  $y[n]$  将出现  $C(-1)^n u[n]$  项, 不满足  $\lim_{n \rightarrow \infty} y[n] = 0$  条件。所以  $k=1$ , 使

① 中第一项分子分母的  $(1+z^{-1})$  相消。当  $k=1$  时,

$$\widetilde{Y}(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \quad |z| > \frac{1}{2}, \text{ 所以}$$

$$y[n] = \left(\frac{1}{2}\right)^n u[n]$$

13. ①  $H(z) = \frac{1}{6} \left[ \frac{1}{1-0.25z^{-1}} + \frac{1}{1-0.5z^{-1}} \right] = \frac{\frac{1}{3} - \frac{1}{8}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$

$h[n]$  因果  $\Rightarrow$  收敛域  $|z| > 0.5 \Rightarrow$  稳定

②  $y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = \frac{1}{3}x[n] - \frac{1}{8}x[n-1]$

③  $Y(z) = H(z)X(z) = \frac{\frac{1}{3} - \frac{1}{8}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z^{-1})}$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2}z^{-1}} + \frac{-\frac{1}{2}}{1-\frac{1}{4}z^{-1}} + \frac{\frac{1}{3}}{1-\frac{1}{3}z^{-1}}$$

$$y[n] = \frac{1}{2} \left(\frac{1}{2}\right)^n u[n] - \frac{1}{2} \left(\frac{1}{4}\right)^n u[n] + \frac{1}{3} \left(\frac{1}{3}\right)^n u[n]$$

14. ①  $H(j\omega) = \frac{2}{(j\omega)^2 + 4(j\omega) + 3}$

$$H(s) = \frac{2}{s^2 + 4s + 3} = \frac{2}{(s+1)(s+3)}$$

$$h(t) = e^{-t}u(t) - e^{-3t}u(t)$$

因果  $\Rightarrow \operatorname{Re}\{s\} > -1$

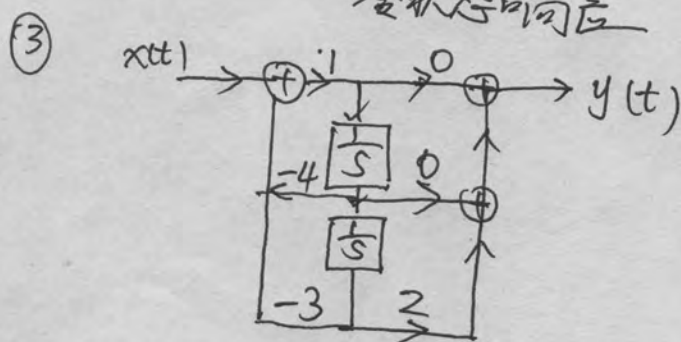


②  $s^2 \widetilde{Y}(s) - s y(0) - y'(0) + 4[s \widetilde{Y}(s) - y(0)] + 3 \widetilde{Y}(s) = 2X(s)$   
 将  $y(0)=1$   $y'(0)=-1$ ,  $X(s)=\frac{1}{s}$  代入

$$(s^2+4s+3)\widetilde{Y}(s) = \underbrace{\frac{2}{s}}_{\text{零状态响应}} + \underbrace{s+3}_{\text{零输入响应}}$$

$$\begin{aligned} \widetilde{Y}(s) &= \frac{2}{s(s+1)(s+3)} + \frac{1}{s+1} \\ &= \frac{\frac{2}{3}}{s} + \frac{-1}{s+1} + \frac{\frac{1}{3}}{s+3} + \frac{1}{s+1} \end{aligned}$$

$$y(t) = \underbrace{\frac{2}{3}u(t) - e^{-t}u(t) + \frac{1}{3}e^{-3t}u(t)}_{\text{零状态响应}} + \underbrace{e^{-t}u(t)}_{\text{零输入响应}}$$



15. ① 设阶跃响应  $S[n]$  的 ~~z~~ 变换为  $S(z)$ , 则

$$S(z) = \frac{H(z)}{1-z^{-1}} \quad |z| > 1$$

由  $h[n]$  是因果序列易推出  $S[n]$  也是因果序列, 利用因果序列终值定理, 有

$$S[+\infty] = \lim_{z \rightarrow 1} (z-1) \frac{H(z)}{1-z^{-1}} = \lim_{z \rightarrow 1} z H(z)$$

$$\begin{aligned} \text{② } H(z) &= \frac{3z^2}{(z-\frac{1}{3})(z-\frac{1}{2})} = \frac{3}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{2}z^{-1})} \\ &= \frac{-6}{1-\frac{1}{3}z^{-1}} + \frac{9}{1-\frac{1}{2}z^{-1}} \quad |z| > \frac{1}{2} \text{ (因果序列)} \end{aligned}$$



$$h[n] = -6\left(\frac{1}{3}\right)^n u[n] + 9\left(\frac{1}{2}\right)^n u[n]$$

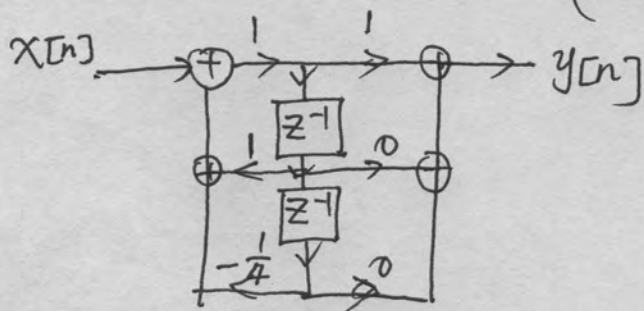
$$(3) \quad x[n] = 1^n + (-1)^n$$

$$1 \xrightarrow{\text{LTI}} H(1) = 9$$

$$(-1)^n \xrightarrow{\text{LTI}} H(-1)(-1)^n = \frac{3}{2}(-1)^n$$

$$x[n] \xrightarrow{\text{LTI}} 9 + \frac{3}{2}(-1)^n$$

$$16. \quad (1) \quad k = \frac{1}{2} \text{ 代入 } H(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)^2} = \frac{1}{1 - z^{-1} + \frac{1}{4}z^{-2}}$$



$$H(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)^2}$$

$$\text{因为 } n\left(\frac{1}{2}\right)^n u[n] \xrightarrow{z} \frac{\frac{1}{2}z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)^2}$$

$$\text{则 } -n\left(\frac{1}{2}\right)^n u[n] \xrightarrow{z} \frac{-\frac{1}{2}z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)^2} = \frac{1}{1 - \frac{1}{2}z^{-1}} - \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)^2} \quad (1)$$

$$\text{因为: } \left(\frac{1}{2}\right)^n u[n] \xrightarrow{z} \frac{1}{1 - \frac{1}{2}z^{-1}} \quad (2)$$

(2) - (1) 得:

$$\left(\frac{1}{2}\right)^n u[n] + n\left(\frac{1}{2}\right)^n u[n] \xrightarrow{z} \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)^2}$$

$$(2) \quad (-1)^n \xrightarrow{\text{LTI}} H(-1)(-1)^n \Rightarrow H(-1) = -2 \Rightarrow k = -\frac{4}{3}$$

收敛域应包含 -1, 所以  $0.5 < |z| < \frac{4}{3}$ , 非因果。