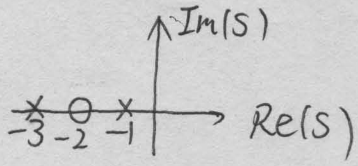
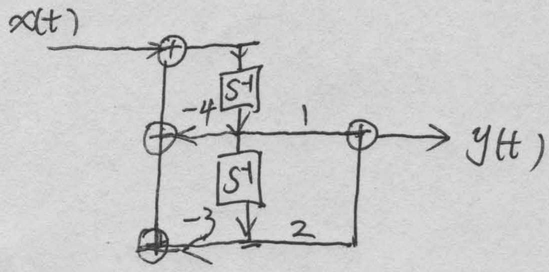
信号与系统第六章试题答案

1. ① $H(s) = \dfrac{s+2}{s^2+4s+3} = \dfrac{s+2}{(s+1)(s+3)}$



系统因果 $\Rightarrow$ 右边序列 $\Rightarrow$ 收敛域 $Re\{s\} > -1 \Rightarrow$
收敛域包含 $Im(s)$ 轴 $\Rightarrow$ 稳定。

②



③ $s^2\widetilde{Y}(s) - sy(0) - y'(0) + 4[s\widetilde{Y}(s) - y(0)] + 3\widetilde{Y}(s)$
$= (s+2)\widetilde{X}(s)$

将 $y(0^-) = y'(0^-) = 1$，$\widetilde{X}(s) = \dfrac{1}{s+2}$ 代入化简

~~H(s) = Y(s)~~ $\widetilde{Y}(s) = \dfrac{s+6}{(s+1)(s+3)} = \dfrac{\frac{5}{2}}{s+1} - \dfrac{\frac{3}{2}}{s+3}$

$y(t) = \frac{5}{2}e^{-t}u(t) - \frac{3}{2}e^{-3t}u(t)$

④ $x(t) = u(-t) + 2u(t) = 1 + u(t)$

因为 $e^{st} \xrightarrow{LTI} H(s)e^{st}$

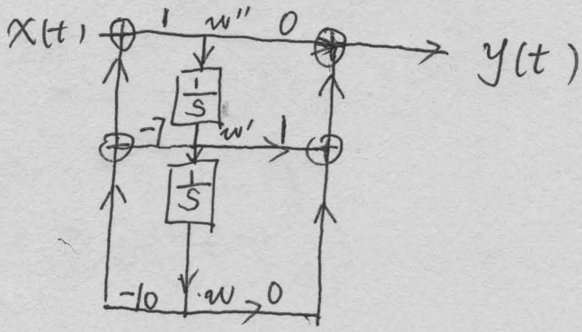
所以 $1 = e^{0t} \xrightarrow{LTI} H(0)e^{0t} = H(0) = \frac{2}{3}$

$u(t) \xrightarrow{LTI} \frac{2}{3}u(t) - \frac{1}{2}e^{-t}u(t) - \frac{1}{6}e^{-3t}u(t)$

因此，根据线性性质，

$x(t) \xrightarrow{LTI} y(t) = \frac{2}{3} + \frac{2}{3}u(t) - \frac{1}{2}e^{-t}u(t) - \frac{1}{6}e^{-3t}u(t)$

2、 ①



②
$$s^2Y(s) - s\,y(0^-) - y'(0^-) + 7\left[sY(s) - y(0^-)\right] + 10Y(s)$$
$$= sX(s)$$

将 $y(0^-) = y'(0^-) = 1$ ， $X(s) = \frac{1}{s^2}$ 代入，

$$(s^2+7s+10)Y(s) = \underbrace{s+8}_{\text{零输入项}} + \underbrace{s\cdot\frac{1}{s^2}}_{\text{零状态项}}$$

$$= \frac{s+8}{(s+2)(s+5)} + \frac{1}{s(s+2)(s+5)}$$

$$= \left(\frac{2}{s+2} - \frac{1}{s+5}\right) + \left(\frac{\frac{1}{10}}{s} - \frac{\frac{1}{6}}{s+2} + \frac{\frac{1}{15}}{s+5}\right)$$

所以：

$$y(t) = \underbrace{2e^{-2t}u(t) - e^{-5t}u(t)}_{\text{零输入响应}} + \underbrace{\frac{1}{10}u(t) - \frac{1}{6}e^{-2t}u(t) + \frac{1}{15}e^{-5t}u(t)}_{\text{零状态响应}}$$

$$= \underbrace{\frac{11}{6}e^{-2t}u(t) - \frac{14}{15}e^{-5t}u(t)}_{\text{自由响应}} + \underbrace{\frac{1}{10}u(t)}_{\text{强迫响应}}$$

3. ① $H(s)=\dfrac{A(s-3)}{(s+1)(s-2)}$

设 $s(t)\xrightarrow{L} S(s)$，则

$$S(s)=\frac{H(s)}{s}=A\cdot\frac{s-3}{s(s+1)(s-2)}=A\left[\frac{\frac{3}{2}}{s}-\frac{\frac{4}{3}}{s+1}-\frac{\frac{1}{3}}{s-2}\right]$$

因此 ① $s(t)=A\cdot\left[\frac{3}{2}u(t)-\frac{4}{3}e^{-t}u(t)-\frac{1}{3}e^{2t}u(t)\right]$ $Re(s)>2$

② $s(t)=A\left[\frac{3}{2}u(t)-\frac{4}{3}e^{-t}u(t)+\frac{1}{3}e^{2t}u(-t)\right]$ $0<Re\{s\}<2$

③ $s(t)=A\left[-\frac{3}{2}u(-t)-\frac{4}{3}e^{-t}u(t)+\frac{1}{3}e^{2t}u(-t)\right]$ $-1<Re\{s\}<0$

④ $s(t)=A\left[-\frac{3}{2}u(-t)+\frac{4}{3}e^{-t}u(-t)+\frac{1}{3}e^{2t}u(-t)\right]$ $Re\{s\}<-1$

上述4种情况中，只有情况② 满足当 $A=2$ 时，

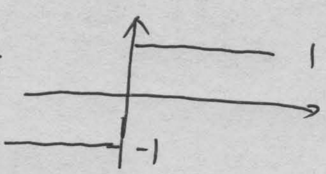
$$\lim_{t\to+\infty}s(t)=3$$

因此 $H(s)=\dfrac{2(s-3)}{(s+1)(s-2)}$ $0<Re\{s\}<2$

系统非因果，不稳定。

② $s(t)=3u(t)-\frac{8}{3}e^{-t}u(t)+\frac{2}{3}e^{2t}u(-t)$

③ $sgn(t)=$



$=-1+2u(t)$

因为 $e^{st}\xrightarrow{LTI}H(s)e^{st}$

所以 $e^{0t}=1\xrightarrow{LTI}H(0)e^{0t}=3$

$2u(t)\xrightarrow{LTI}2s(t)=6u(t)-\frac{8}{3}e^{-t}u(t)+\frac{2}{3}e^{2t}u(-t)$

$sgn(t)=2u(t)-1\xrightarrow{LTI}6u(t)-\frac{8}{3}e^{-t}u(t)+\frac{2}{3}e^{2t}u(-t)-3$

4、 ① $X(s) = 1 + \frac{4}{s-2}$

$t>0$ 时 $x(t)=0 \Rightarrow$ 左边序列 $\longrightarrow$ 收敛域 $Re\{s\}<2$

$Y(s) = \frac{2}{3}\cdot\frac{1}{s-2} + \frac{1}{3}\frac{1}{s+1}$ $\quad -1 < Re\{s\} < 2$

$= \frac{s}{(s+1)(s-2)}$

$H(s) = \frac{Y(s)}{X(s)} = \frac{s}{(s+1)(s+2)}$ $\quad -1<Re\{s\}<2$

② $H(s) = -\frac{1}{s+1} + \frac{2}{s+2}$ $\quad Re\{s\} > -1$ (极点相消
收敛域扩大

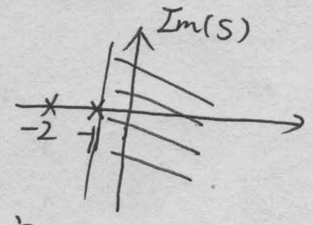
$h(t) = +2e^{-2t}u(t) - e^{-t}u(t)$

③ $e^{st} \xrightarrow{LTI} H(s)e^{st}$

因此， $e^{3t} \xrightarrow{LTI} H(3)e^{3t} = \frac{3}{20}e^{3t}$

5. ① $H(s) = \frac{s+3}{s^2+3s+2} = \frac{s+3}{(s+1)(s+2)}$

因果系统 $\Rightarrow$ 右边序列 $\Rightarrow$



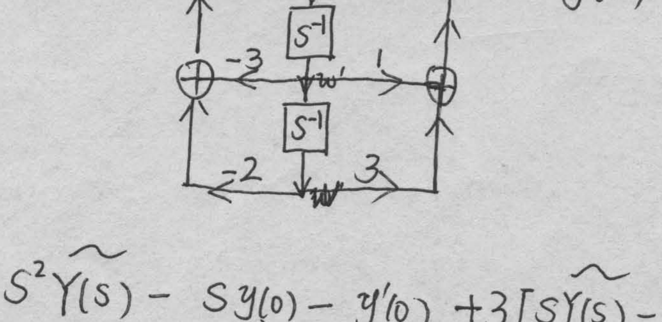
收敛域

$\Rightarrow$ 收敛域包括 $Im\{s\}$ 轴 $\Rightarrow$ 稳定

$H(s) = \frac{2}{s+1} - \frac{1}{s+2}$ $\quad Re\{s\} > -1$

$h(t) = 2e^{-t}u(t) - e^{-2t}u(t)$

②

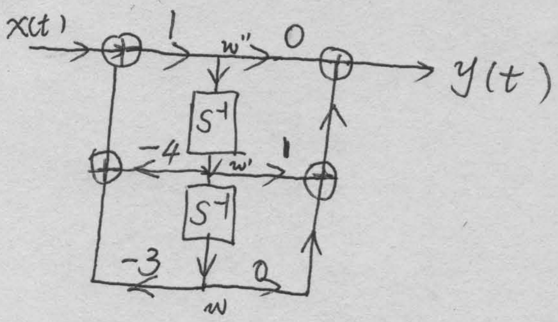


③ $s^2\widetilde{Y(s)} - sy(0) - y'(0) + 3[s\widetilde{Y(s)} - y(0)] + 2\widetilde{Y(s)} = (s+3)\widetilde{X(s)} = 1$

代入化简

$$\widetilde{Y}(s) = \frac{s+4}{(s+1)(s+2)} = \frac{3}{s+1} - \frac{2}{s+2}$$

$$y(t) = 3e^{-t}u(t) - 2e^{-2t}u(t)$$

6、① 

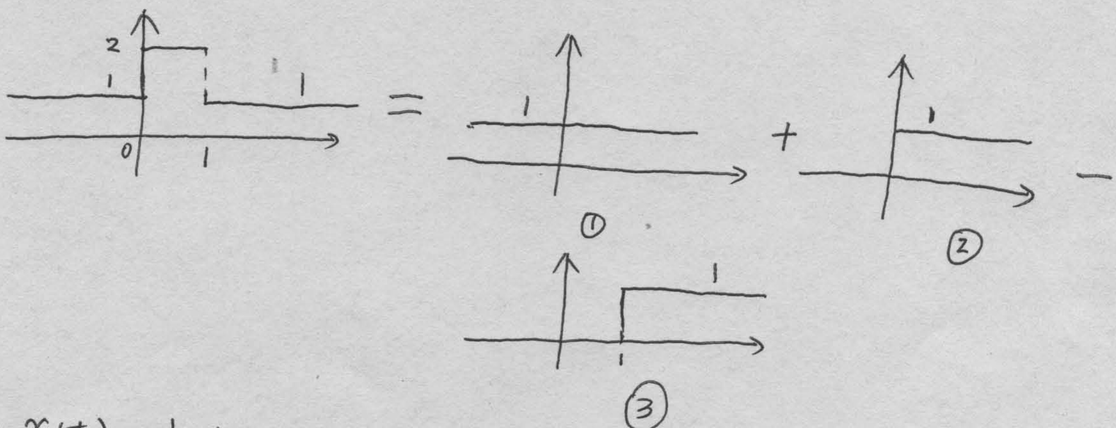
$$y''(t) + 4y'(t) + 3y(t) = x'(t)$$

② $$s^2\widetilde{Y}(s) - s\,y(0) - y'(0) + 4[s\widetilde{Y}(s) - y(0)] + 3\widetilde{Y}(s) = s\widetilde{\hat{X}}(s)$$

代入化简

$$Y(s) = \frac{s+5}{(s+1)(s+3)} = \frac{2}{s+1} - \frac{1}{s+3}$$

$$y(t) = 2e^{-t}u(t) - e^{-3t}u(t)$$

③ 

$$x(t) = 1 + u(t) - u(t-1)$$

$$1 = e^{0t} \xrightarrow{LTI} H(0)e^{0t} = 0$$

$u(t) \xrightarrow{LTI} \frac{1}{2}e^{-t}u(t) - \frac{1}{2}e^{-3t}u(t)$ 这里与第2小问不太一样，因为第二小问初始条件不为0，这是第2小问的零状态响应

$$u(t-1) \xrightarrow{LTI} \frac{1}{2}e^{-(t-1)}u(t-1) - \frac{1}{2}e^{-3(t-1)}u(t-1)$$

所以 $x(t) \xrightarrow{LTI} y(t) = \frac{1}{2}[e^{-t}u(t) - e^{-(t-1)}u(t-1) - e^{-3t}u(t) + e^{-3(t-1)}u(t-1)]$

7. ① $y''(t)+4y'(t)+3y(t)=x''(t)+2x'(t)$

② $H(s)=\dfrac{s^2+2s}{s^2+4s+3}=\dfrac{s(s+2)}{(s+1)(s+3)}$

系统因果 $\Rightarrow$ 右边序列 $\Rightarrow$ 收敛域 $Re\{s\}>-1$

$$H(s)=1-\frac{2s+3}{(s+1)(s+3)}=1-\left[\frac{\frac{1}{2}}{s+1}+\frac{\frac{3}{2}}{s+3}\right]$$

$$h(t)=\delta(t)-\frac{1}{2}e^{-t}u(t)-\frac{3}{2}e^{-3t}u(t)$$

③ $Y(s)=X(s)H(s)=\dfrac{s(s+2)}{(s+1)(s+3)}\cdot\dfrac{1}{s}$ $\quad Re\{s\}>0$

$$=\frac{s+2}{(s+1)(s+3)}=\frac{\frac{1}{2}}{s+1}+\frac{\frac{1}{2}}{s+3}$$

$$y(t)=\frac{1}{2}e^{-t}u(t)+\frac{1}{2}e^{-3t}u(t)$$

④ 首先写出 $w(t)$ 与 $x(t)$ 关系

$$w''(t)+4w'(t)+3w(t)=x'(t)$$

$$\cancel{W(s)=\frac{s}{s+}}$$

$$s^2\widetilde{W(s)}-sw(0_-)-w'(0_-)+4[s\widetilde{W(s)}-w(0_-)]+3\widetilde{W(s)}=s\widetilde{X(s)}$$

$\widetilde{X(s)}=\frac{1}{s}$, $\quad w(0_-)=1\quad w'(0_-)=0$ 代入化简得:

$$\widetilde{W(s)}=\frac{s+5}{(s+1)(s+3)}$$

因为 $\widetilde{y(t)}=2w(t)+\int_{-\infty}^{t}w(\tau)d\tau$

则 $\widetilde{Y(s)}=2\widetilde{W(s)}+\dfrac{\widetilde{W(s)}}{s}+\dfrac{\int_{-\infty}^{0}w(\tau)d\tau}{s}$ 命题不太严谨，设此项为0 $\quad P_{231}(6\text{-}71)$

$$=\frac{2(s+\frac{1}{2})}{s}\widetilde{W(s)}=\frac{2(s+\frac{1}{2})(s+5)}{s(s+1)(s+3)}$$

$$=\frac{\frac{5}{3}}{s}+\frac{2}{s+1}-\frac{\frac{5}{3}}{s+3}$$

所以 $y(t)=\left(\frac{5}{3}+2e^{-t}-\frac{5}{3}e^{-3t}\right)u(t)$

8. ① $H(s)=\frac{3s+a}{s^2+4s+3}=\frac{3s+a}{(s+1)(s+3)}$ 因果系统 $\Longrightarrow$ $Re\{s\}>-1$

$S(s)=\frac{H(s)}{s}=\frac{3s+a}{s(s+1)(s+3)}$ $Re\{s\}>0$

$\lim_{t\to+\infty} s(t)=\lim_{s\to 0} sS(s)=\left.\frac{3s+a}{(s+1)(s+3)}\right|_{s=0}=1$

所以 $a=3$

$H(s)=\frac{3(s+1)}{(s+1)(s+3)}=\frac{3}{s+3}$ $Re\{s\}>-3$

$h(t)=3e^{-3t}u(t)$

② $u(-t)=1-u(t)$

$1=e^{0t}\xrightarrow{LTI}H(0)e^{0t}=1$

$u(t)\xrightarrow{LTI}u(t)-e^{-3t}u(t)$

$u(-t)=1-u(t)\xrightarrow{LTI}y(t)=1-u(t)+e^{-3t}u(t)$

$t>0$ 时，响应 $y(t)=e^{-3t}$

9. $H(s)=\frac{s+3}{(s+1)(s+2)}=\frac{2}{s+1}-\frac{1}{s+2}$ 右边信号 $\Rightarrow$ $Re\{s\}>-1$

$h(t)=2e^{-t}u(t)-e^{-2t}u(t)$

10. ① 设 $x(t)\xrightarrow{LTI}y_{zs}(t)$ 零状态响应

则 $y_1(t)=y_{zs}(t)+y_{zi}(t)$

$y_2(t)=2y_{zs}(t)+y_{zi}(t)$

解出 $\begin{cases} y_{zs}(t)=(\cos t-e^{-t})u(t) & \text{零状态响应} \\ y_{zi}(t)=2e^{-t}u(t) & \text{零输入响应} \end{cases}$

~~② $H(s)=Y_{zi}(s)=$~~

② $X(t) = \cos t - \sin t$

$$X(s) = \frac{s-1}{s^2+1}$$

$$Y_{zs}(s) = \frac{s}{s^2+1} - \frac{1}{s+1} = \frac{s-1}{(s^2+1)(s+1)}$$

$$H(s) = \frac{Y_{zs}(s)}{X(s)} = \frac{1}{s+1}, \quad h(t) = e^{-t}u(t)$$

11. $H(s) = 1 - \frac{2}{s+2}$　因果 $\Rightarrow Re\{s\} > -2$

$$x(t) = 1 + u(t) - u(t-1)$$

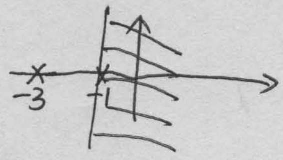
$$1 \xrightarrow{LTI} H(0) = 0$$

$$u(t) \xrightarrow{LTI} e^{-2t}u(t)$$

$$u(t-1) \xrightarrow{LTI} e^{-2(t-1)}u(t-1)$$

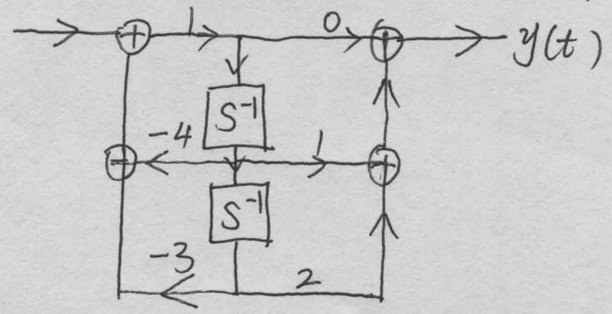
$$x(t) \xrightarrow{LTI} e^{-2t}u(t) - e^{-2(t-1)}u(t-1)$$

12. ① $H(s) = \frac{s+2}{s^2+4s+3}$,



, 稳定

②



③ $s^2\widetilde{Y(s)} - s - 1 + 4[s\widetilde{Y(s)} - 1] + 3\widetilde{Y(s)} = (s+2)\widetilde{X(s)} = 1$

$$\widetilde{Y(s)} = \frac{s+6}{s^2+4s+3} = \frac{\frac{5}{2}}{s+1} - \frac{\frac{3}{2}}{s+3}$$

$$y(t) = \frac{5}{2}e^{-t}u(t) - \frac{3}{2}e^{-3t}u(t)$$

④ $x(t) = u(-t) + 2u(t) = 1 + u(t)$

$1 \xrightarrow{LTI} H(0) = \frac{2}{3}$　$u(t) \xrightarrow{LTI} \left(\frac{2}{3} - \frac{1}{2}e^{-t} - \frac{1}{6}e^{-3t}\right)u(t)$

$$x(t) \xrightarrow{LTI} \frac{2}{3} + \left(\frac{2}{3} - \frac{1}{2}e^{-t} - \frac{1}{6}e^{-3t}\right)u(t)$$