《量子信息基础》2021.4.26 随堂作业：

1. (Text book* Problem 5.23, 注意是教材的第二版)
   Support you had three (non-interacting) particles, in thermal equilibrium, in a one-dimensional harmonic potential, with a total energy $E = (9/2)\hbar\omega$
   (a) If they are distinguishable particles (but all with the same mass), what are the possible occupation-number configurations, and how many distinct (three particle) states are there for each one? What is the most probable configuration? If you picked a particle at random and measured its energy, what values might you get, and what is the probability of each one? What is the most probable energy?
   (b) Do the same for the case of identical fermions (ignoring spin).
   (c) Do the same for the case of identical bosons (ignoring spin).

The total energy of the three particles is

$$E = \left(n_1 + n_2 + n_3 + \frac{3}{2}\right)\hbar\omega = \frac{9}{2}\hbar\omega$$

$$n_1 + n_2 + n_3 = 3$$

有三个粒子，就是 3/2 的零点能

The possible combinations of $(n_1,\ n_2,\ n_3)$ are

$$(1,1,1)$$

$$(0,0,3),(0,3,0),(3,0,0)$$

$$(0,1,2),(0,2,1),(1,2,0),(2,1,0),(1,0,2),(2,0,1)$$

总结问题：

1.有多少种组态

2.每种组态有多少中不同状态（分粒子）

3.每种组态的概律

4.最概然组态

5.随机抽一个粒子，求他是某一能量对应的概率

(a) If particles are distinguishable

Configuration 1 is $(0,3,0,0,0,\dots)$, 1 distinct state, Q=1/10;

Configuration 2 is $(2,0,0,1,0,\dots)$, 3 distinct states, Q=3/10;

Configuration 3 is $(1,1,1,0,0,\dots)$, 6 distinct states, Q=6/10;

Configuration 3 is the most probable configuration.

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For $E_0 = \frac{1}{2}\hbar\omega$, $P_0 = \frac{3}{10}\times\frac{2}{3} + \frac{6}{10}\times\frac{1}{3} = \frac{4}{10}$;

For $E_1 = \frac{3}{2}\hbar\omega$, $P_1 = \frac{1}{10}\times 1 + \frac{6}{10}\times\frac{1}{3} = \frac{3}{10}$;

For $E_2 = \frac{5}{2}\hbar\omega$, $P_2 = \frac{6}{10}\times\frac{1}{3} = \frac{2}{10}$;

For $E_3 = \frac{7}{2}\hbar\omega$, $P_3 = \frac{3}{10}\times\frac{1}{3} = \frac{1}{10}$.

E_0 is the most probable energy, with probability of 4/10.

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(b) If particles are fermions

Configuration 1 and 2 are forbidden. Only one configuration left which is

$$(1, 1, 1, 0, 0, \dots)$$

This is the most probable configuration.

费米子不能处于同一能级

且不可区分，所以这里就这一种情况

For $E_0 = \frac{1}{2}\hbar\omega$, $P_0 = \frac{1}{3}$;

For $E_1 = \frac{3}{2}\hbar\omega$, $P_1 = \frac{1}{3}$;

For $E_2 = \frac{5}{2}\hbar\omega$, $P_2 = \frac{1}{3}$.

All three energies are the most probable energy.

推导和答案正确给 20 分

(c) If particles are bosons

Configuration 1 is $(0, 3, 0, 0, 0, \dots)$, 1 distinct state, Q=1/3;

Configuration 2 is $(2, 0, 0, 1, 0, \dots)$, 1 distinct states, Q=1/3;

Configuration 3 is $(1, 1, 1, 0, 0, \dots)$, 1 distinct states, Q=1/3;

All three configurations are the most probable configuration.

For $E_0 = \frac{1}{2}\hbar\omega$, $P_0 = \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{3} = \frac{1}{3}$;

For $E_1 = \frac{3}{2}\hbar\omega$, $P_1 = \frac{1}{3} \times 1 + \frac{1}{3} \times \frac{1}{3} = \frac{4}{9}$;

For $E_2 = \frac{5}{2}\hbar\omega$, $P_2 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$;

For $E_3 = \frac{7}{2}\hbar\omega$, $P_3 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

E_1 is the most probable energy, with probability of 4/9.

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2. Put 10 distinguishable particles into 4 different quantum states to let the final configuration to be $(4, 3, 2, 1)$ as the macrostate. Calculate the number of microstates in this configuration. 可区分例子，排列组合

$$P = C_{10}^4 C_6^3 C_3^2 C_1^1 = 12600$$

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* David J. Griffiths, Introduction to Quantum Mechanics (2nd Edition), Cambridge University Press (2017).