《量子与统计基础》2020.4.21 随堂作业：

1. (Text book* Problem 5.26, 注意是教材的第二版)

    Use the method of Lagrange multipliers to find the rectangle of largest area, with sides parallel to the axes, that can be inscribed in the ellipse $(x/a)^2 + (y/b)^2 = 1$. What is that maximum area?

    The area of the rectangle shape is

    $$S(x,y) = 4|xy|$$

    x,y satisfy $(x/a)^2 + (y/b)^2 = 1$

    We define a new function

    $$G(x,y,\lambda) \equiv 4xy + \lambda \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 \right)$$

    $$\frac{\partial G}{\partial x} = 4y + \frac{2\lambda x}{a^2}$$

    $$\frac{\partial G}{\partial y} = 4x + \frac{2\lambda y}{b^2}$$

    $$\begin{cases} y = -\dfrac{\lambda x}{2a^2} \\ x = -\dfrac{\lambda y}{2b^2} = \dfrac{\lambda^2}{4a^2 b^2} x \end{cases}$$

    So

    $$x = 0$$

    or

    $$\lambda = \pm 2ab$$

    The later gives the maximum area, where

    $$y = \mp \frac{b}{a} x$$
    $$x = \frac{a}{\sqrt{2}}$$
    $$y = \frac{b}{\sqrt{2}}$$
    $$S_{max} = 2ab$$

    推导和答案正确给 40 分

2. Consider a system of two particles, they have six possible states $\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6$. Calculate the number of the microstates of the system, in the following conditions:
    (1) Two particles are bosons;
    (2) Two particles are fermions;

(3) Two particles are distinguishable.

(1)

$$N_b = C_6^1 + C_6^2 = 6 + 15 = 21$$

推导和答案正确给 20 分

(2)

$$N_f = C_6^2 = 15$$

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(3)

$$N_d = 6 \times 6 = 36$$

推导和答案正确给 20 分

* David J. Griffiths, Introduction to Quantum Mechanics (2nd Edition), Cambridge University Press (2017).