Question 1

1) $f_1(t)=\begin{cases}1 & 0\le t\le 1\\ 0 & \text{其他}\end{cases}$ $f_2(t)=\begin{cases}1 & 1\le t\le 2\\ 0 & \text{其他}\end{cases}$ $f_3(t)=\begin{cases}1 & 2\le t\le 3\\ 0 & \text{其他}\end{cases}$

2) dimension = 3

3) $s_1(t)=f_1(t)+f_2(t)$ $s_2(t)=2f_2(t)$ $s_3(t)=f_1(t)+f_2(t)+f_3(t)$

$s_4(t)=-f_3(t)$ 考察点：正交信号、信号空间

Question 2

1) $E_1=\int_0^{\frac{2\pi}{w}}|s_1(t)|^2dt=\int_0^{\frac{2\pi}{w}}3\cos^2wt\,dt=\frac{3\pi}{w}$

$E_2=\int_0^{\frac{2\pi}{w}}|s_2(t)|^2dt=\int_0^{\frac{2\pi}{w}}\left(\frac{3\sqrt{2}}{2}\sin wt+\frac{3\sqrt{2}}{2}\cos wt\right)^2dt=\frac{9\pi}{w}$

2) $f_1(t)=\frac{s_1(t)}{\sqrt{E_1}}=\sqrt{\frac{w}{\pi}}\cos wt$

$c_{12}=\int_{-\infty}^{\infty}s_2(t)f_1(t)dt=\int_0^{\frac{2\pi}{w}}\sqrt{\frac{w}{\pi}}\cos wt\left(\frac{3\sqrt{2}}{2}\sin wt+\frac{3\sqrt{2}}{2}\cos wt\right)dt$

$=\sqrt{\frac{\pi}{w}}\cdot\frac{3\sqrt{2}}{2}$

$f_2'(t)=s_2(t)-\sqrt{\frac{\pi}{w}}\cdot\frac{3\sqrt{2}}{2}\sqrt{\frac{w}{\pi}}\cos wt=\frac{3\sqrt{2}}{2}\sin wt$

$E_2'=\int_0^{\frac{2\pi}{w}}|f_2'(t)|^2dt=\frac{9\pi}{2w}$

$f_2(t)=\frac{f_2'(t)}{\sqrt{E_2'}}=\sqrt{\frac{w}{\pi}}\sin wt$

$s_1(t) = \sqrt{\frac{2\pi}{W}} f_1(t)$

$s_2(t) = \sqrt{\frac{9\pi}{2W}} f_1(t) + \sqrt{\frac{9\pi}{2W}} f_2(t)$

3) $\vec{s_1} = \left(\sqrt{\frac{2\pi}{W}}, 0\right)$ $\vec{s_2} = \left(\sqrt{\frac{9\pi}{2W}}, \sqrt{\frac{9\pi}{2W}}\right)$

$E_1 = \|\vec{s_1}\|^2 = \frac{2\pi}{W}$

$E_2 = \|\vec{s_2}\|^2 = \frac{9\pi}{2W} + \frac{9\pi}{2W} = \frac{9\pi}{W}$

4) $\vec{s_1} \cdot \vec{s_2} = \sqrt{\frac{2}{2}} \frac{\pi}{W}$

考察点：正交化过程、信号能量计算、向量的范数与内积