《量子信息基础》2021.5.24 随堂作业：

1. (Text book* Problem 3.16)

   Show that two noncommuting operators（不可对易算子） cannot have a complete set of common eigenfunctions.（没有完整公共的正交基） Hint: Show that if $\hat{P}$ and $\hat{Q}$ have a complete set of common eigenfunctions, then $[\hat{P},\hat{Q}]f = 0$ for any function in Hilbert space.

Assuming $\hat{P}f_n = \lambda_n f_n$ and $\hat{Q}f_n = \mu_n f_n$, and $\{f_n\}$ are a complete set of eigenfunctions

For arbitrary wavefunction

$$f = \sum_n c_n f_n$$

$$[\hat{P},\hat{Q}]f = (\hat{P}\hat{Q} - \hat{Q}\hat{P})\sum_n c_n f_n = \hat{P}\left(\sum_n c_n \mu_n f_n\right) - \hat{Q}\left(\sum_n c_n \lambda_n f_n\right)$$
$$= \sum_n c_n \mu_n \lambda_n f_n - \sum_n c_n \lambda_n \mu_n f_n = 0$$

Therefore, $\hat{P}\hat{Q} = \hat{Q}\hat{P}$ or $f = 0$. The former contradicts to $\hat{P}$ and $\hat{Q}$ are noncommuting. The latter contradicts to f is an arbitrary wavefunction.

相当于是反证法，推出矛盾即可

推导和答案正确给 50 分

2. $\hat{D}_x(a)$ is a translation operator in one dimension. When it applies to a wavefunction
$$\hat{D}_x(a)\psi(x) = \psi(x-a)$$
   If $\hat{f}(x)$ is commutable with $\hat{D}_x(a)$, prove $\hat{f}(x) = \hat{f}(x-a)$.

Since $\hat{f}(x)$ is commutable with $\hat{D}_x(a)$,
$$[\hat{f}(x), \hat{D}(a)] = 0$$
For an arbitrary wavefunction $\psi(x)$
$$\hat{f}(x)\hat{D}_x(a)\psi(x) = \hat{f}(x)\psi(x-a) = \hat{D}_x(a)\hat{f}(x)\psi(x) = \hat{f}(x-a)\psi(x-a)$$
Since $\psi(x-a)$ is an arbitrary wavefunction
$$\hat{f}(x) = \hat{f}(x-a)$$

推导和答案正确给 50 分

还是背证明方法就可以

* David J. Griffiths, and Darrell F. Schroeter, Introduction to Quantum Mechanics (3rd Edition), Cambridge University Press (2018).