《量子信息基础》2022.5.17 随堂作业：

1. In the EPRB experiment, state whether each of the following states is entangled（纠缠） (i.e. can it be factored into a product of states of the individual particles, in which case it is not entangled)（能被分解为两个单粒子的乘积，就是不纠缠的）
   (a) $\frac{1}{\sqrt{2}}(|0_1, 1_2\rangle - |0_1, 0_2\rangle)$
   (b) $\frac{1}{\sqrt{2}}(|0_1, 1_2\rangle - |1_1, 0_2\rangle)$
   (c) $\frac{3}{5}|0_1, 1_2\rangle + \frac{4i}{5}|1_1, 1_2\rangle$
   (d) $\frac{1}{2}(|0_1, 0_2\rangle + |0_1, 1_2\rangle + |1_1, 0_2\rangle + |1_1, 1_2\rangle)$

(a) $\frac{1}{\sqrt{2}}(|0_1, 1_2\rangle - |0_1, 0_2\rangle) = \frac{1}{\sqrt{2}}|0_1\rangle(|1_2\rangle - |0_2\rangle)$
Not entangled

推导和答案正确给 20 分

(b) $\frac{1}{\sqrt{2}}(|0_1, 1_2\rangle - |1_1, 0_2\rangle)$
Entangled

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(c) $\frac{3}{5}|0_1, 1_2\rangle + \frac{4i}{5}|1_1, 1_2\rangle = \left(\frac{3}{5}|0_1\rangle + \frac{4i}{5}|1_1\rangle\right)|1_2\rangle$
Not entangled

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(d) $\frac{1}{2}(|0_1, 0_2\rangle + |0_1, 1_2\rangle + |1_1, 0_2\rangle + |1_1, 1_2\rangle) = \frac{1}{2}\big(|0_1\rangle(|0_2\rangle + |1_2\rangle) + |1_1\rangle(|0_2\rangle + |1_2\rangle)\big) = \frac{1}{2}(|0_1\rangle + |1_1\rangle)(|0_2\rangle + |1_2\rangle)$
Not entangled

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2. Consider a Bell state $\frac{1}{\sqrt{2}}(|0_1, 0_2\rangle + |1_1, 1_2\rangle)$ of two photons.
   (a) Express it now on a basis of $|\theta\rangle$ and $|\theta + \pi/2\rangle$ by rotating the original basis by an angle θ. Hint: on such a basis, $|1\rangle = \sin\theta|\theta\rangle + \cos\theta\,|\theta + \pi/2\rangle$ and also a similar expression for $|0\rangle$.
   (b) In the new basis of such sate, show that the two photons will always come out of the same arm of each polarizer when two aligned polarizers are used to examine the pair of photons.

(a) （0 代表 x 方向的单位向量，1 代表 y 方向的单位向量）

$$|1\rangle = \sin\theta|\theta\rangle + \cos\theta\,|\theta+\pi/2\rangle$$
$$|0\rangle = \cos\theta|\theta\rangle - \sin\theta\,|\theta+\pi/2\rangle$$

$$\langle 1|0\rangle = (\langle\theta|\sin\theta + \langle\theta+\pi/2|\cos\theta)(\cos\theta|\theta\rangle - \sin\theta\,|\theta+\pi/2\rangle)$$
$$= \sin\theta\cos\theta - \cos\theta\sin\theta = 0$$

$$\begin{aligned}
&\frac{1}{\sqrt{2}}(|0_1,0_2\rangle + |1_1,1_2\rangle)\\
&= \frac{1}{\sqrt{2}}\left(\cos\theta|\theta_1\rangle - \sin\theta\left|\theta_1+\frac{\pi}{2}\right\rangle\right)\left(\cos\theta|\theta_2\rangle - \sin\theta\left|\theta_2+\frac{\pi}{2}\right\rangle\right)\\
&+ \frac{1}{\sqrt{2}}\left(\sin\theta|\theta_1\rangle + \cos\theta\left|\theta_1+\frac{\pi}{2}\right\rangle\right)\left(\sin\theta|\theta_2\rangle + \cos\theta\left|\theta_2+\frac{\pi}{2}\right\rangle\right)\\
&= \frac{1}{\sqrt{2}}\cos\theta\cos\theta|\theta_1,\theta_2\rangle \cancel{- \frac{1}{\sqrt{2}}\cos\theta\sin\theta\left|\theta_1,\theta_2+\frac{\pi}{2}\right\rangle}\\
&\cancel{- \frac{1}{\sqrt{2}}\sin\theta\cos\theta\left|\theta_1+\frac{\pi}{2},\theta_2\right\rangle} + \frac{1}{\sqrt{2}}\sin\theta\sin\theta\left|\theta_1+\frac{\pi}{2},\theta_2+\frac{\pi}{2}\right\rangle\\
&+ \frac{1}{\sqrt{2}}\sin\theta\sin\theta|\theta_1,\theta_2\rangle \cancel{+ \frac{1}{\sqrt{2}}\sin\theta\cos\theta\left|\theta_1,\theta_2+\frac{\pi}{2}\right\rangle}\\
&\cancel{+ \frac{1}{\sqrt{2}}\cos\theta\sin\theta\left|\theta_1+\frac{\pi}{2},\theta_2\right\rangle} + \frac{1}{\sqrt{2}}\cos\theta\cos\theta\left|\theta_1+\frac{\pi}{2},\theta_2+\frac{\pi}{2}\right\rangle\\
&= \frac{1}{\sqrt{2}}|\theta_1,\theta_2\rangle + \frac{1}{\sqrt{2}}\left|\theta_1+\frac{\pi}{2},\theta_2+\frac{\pi}{2}\right\rangle
\end{aligned}$$

(b)

There are only the $|\theta_1,\theta_2\rangle$ or $\left|\theta_1+\frac{\pi}{2},\theta_2+\frac{\pi}{2}\right\rangle$ states. So the photons will only come out of the same arm of the polarizers.（偏振器的同一臂）

(a)和(b)推导和答案正确给 20 分