2.9

(a) $x_1(t) = 2\cos(4\pi t + \frac{2\pi}{3})$

Power signal

$$P_1 = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x_1(t)|^2 dt$$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$$

~~E = lim~~

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} 4\cos^2(4\pi t + \frac{2\pi}{3}) dt$$

$$= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} 2[1+\cos(8\pi t + \frac{4\pi}{3})] dt$$

$(\cos 2\alpha = 2\cos^2\alpha - 1)$

$$T_0 = \frac{2\pi}{8\pi} = \frac{1}{4}$$

$\Rightarrow P_1 = 2W$

(b) $x_2(t) = e^{-\alpha t}u(t)$

Energy signal

$$E_2 = \int_{-\infty}^{\infty} |x_2(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} e^{-2\alpha t}\, u^2(t)\, dt$$

$$= \int_{0}^{\infty} e^{-2\alpha t} dt = \frac{1}{2\alpha} W$$

(c) $x_3(t) = e^{\alpha t} u(t)$

Energy;

$$E_3 = \int_{-\infty}^{\infty} |x_3(t)|^2 dt$$

$$= \int_{-\infty}^{0} e^{2\alpha t} u^2(-t) dt$$

$$= \frac{1}{2\alpha} W$$

(d) $x_4(t) = (a^2+t^2)^{-\frac{1}{2}}$ Energy.. 解主信息

$$E_4 = \lim_{T\to\infty} \int_{-T}^{T} |x_4(t)|^2 dt$$

$$E_4 = \lim_{T\to\infty} \int_{-T}^{T} \frac{dt}{(\alpha^2+t^2)}$$

$$= \lim_{T\to\infty} \int_{-T}^{T} \frac{1}{(t^2+a^2)^{\frac{1}{4}}} dt$$

$$= \lim_{T\to\infty} \frac{1}{\alpha} \tan^{-1} \left[\frac{t}{\alpha}\right]_{-T}^{T}$$

Neither energy or Power

$$= 2 \lim_{T\to\infty} \int_{0}^{T} \frac{1}{(t^2+a^2)^{\frac{1}{4}}} dt = \infty$$

$$= \frac{1}{\alpha}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$$

$$= \frac{\pi}{\alpha}$$

$$P_4 = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{dt}{(t^2+a^2)^{\frac{1}{4}}} = 0$$

(e) $x_5(t) = e^{-\alpha|t|}$ Energy

$$E_5 = \int_{-\infty}^{\infty} |x_5(t)|^2 dt$$

$$\frac{1}{2\alpha} + \frac{1}{2\alpha}.$$

$$= \int_{-\infty}^{0} e^{\alpha t} dt + \int_{0}^{\infty} e^{-\alpha t} dt$$

$= \alpha$ $= \frac{1}{\alpha}.$

(f) $x_6(t) = e^{-\alpha t}u(t) - e^{-\alpha(t-1)}u(t-1)$

~~Energy~~ Power $\lim_{T\to\infty}\int_{-T}^{T}[e^{-\alpha t}u(t) - e^{-\alpha(t-1)}u(t+1)]^2 dt$

$E_6 = E_{61} + E_{62}$ $= \frac{1}{2\alpha} - \frac{e^{-\alpha}}{2\alpha} + \frac{1}{2\alpha}$

$= \frac{1}{2\alpha} + \frac{1}{2\alpha} = \frac{1}{\alpha} W$ $\therefore \frac{1}{\alpha}[1 - \frac{e^{-\alpha}}{2}]$

2.30

基本性质

$s(t-\tau) \Leftrightarrow S(f)e^{-j2\pi f\tau}$

$s(t)e^{-j2\pi f_0 t} \Leftrightarrow S(f+f_0)$

$s(at) \Leftrightarrow \frac{1}{|a|}S(\frac{f}{a})$

$\Pi(t) \Leftrightarrow \text{sinc}(f)$

$\text{sinc}(t) \Leftrightarrow \Pi(f)$

$\Lambda(t) \Leftrightarrow \text{sinc}^2(f)$

$\text{sinc}^2(t) \Leftrightarrow \Pi(f)$

(a) $x_1(t) = \Pi(\frac{t-1}{2})$

$\Pi(\frac{t}{2}) \leftrightarrow 2\,\text{sinc}(2f)$

$\Pi(\frac{t-1}{2}) \leftrightarrow 2\,\text{sinc}(2f)e^{-j2\pi f}$

(b) $x_2(t) = 2\,\text{sinc}[2(t-1)]$

$\text{sinc}(2t) \Leftrightarrow \frac{1}{2}\Pi(\frac{f}{2})$

$2\,\text{sinc}[2(t-1)] = \Pi(\frac{f}{2})e^{-j2\pi f}$

(c) $x_3(t) = \Lambda(\frac{t-2}{8})$

$\Lambda(t) \leftrightarrow \text{sinc}^2(f)$

$\Lambda(\frac{t}{8}) \leftrightarrow 8\,\text{sinc}^2(8f)$

$\Lambda(\frac{t-2}{8}) \leftrightarrow 8\,\text{sinc}^2(8f)e^{-j4\pi f}$

(d) $x_4(t) = \mathrm{sinc}^2\left(\frac{t-3}{4}\right)$

$$\mathrm{sinc}^2\left(\frac{t}{4}\right) \longleftrightarrow 4\Lambda(4f)$$

$$\mathrm{sinc}^2\left(\frac{t-3}{4}\right) \longleftrightarrow 4\Lambda(4f)e^{-j6\pi f}$$

(e) $x_5(t) = 5\,\mathrm{sinc}[2(t-1)] + 5\,\mathrm{sinc}[2(t+1)]$

$$\mathrm{sinc}[2(t-1)] \longleftrightarrow \frac{1}{2}\Pi\left(\frac{f}{2}\right)e^{-j2\pi f}$$

$$\mathrm{sinc}[2(t+1)] \longleftrightarrow \frac{1}{2}\Pi\left(\frac{f}{2}\right)e^{j2\pi f}$$

$$5\,\mathrm{sinc}[2(t-1)] + 5\,\mathrm{sinc}[2(t+1)] \longleftrightarrow \frac{5}{2}\Pi\left(\frac{f}{2}\right)\left(e^{-j2\pi f} + e^{j2\pi f}\right)$$

$$= 5\Pi\left(\frac{f}{2}\right)\cos(2\pi f)$$

(f) $x_6(t) = 2\Lambda\left(\frac{t-2}{8}\right) + 2\Lambda\left(\frac{t+2}{8}\right)$

$$\Lambda\left(\frac{t-2}{8}\right) \longleftrightarrow 8\,\mathrm{sinc}^2(8f)e^{-j4\pi f}$$

$$\Lambda\left(\frac{t+2}{8}\right) \longleftrightarrow 8\,\mathrm{sinc}^2(8f)e^{j4\pi f}$$

$\left(\cos 4\pi f + j\sin 4\pi f + \cos 4\pi f - j\sin 4\pi f\right)$

$$2\Lambda\left(\frac{t-2}{8}\right) + 2\Lambda\left(\frac{t+2}{8}\right) \longleftrightarrow 3\cancel{16}\,\mathrm{sinc}^2(8f)\cos(4\pi f)$$

$$\longleftrightarrow 32\,\mathrm{sinc}^2(8f)\cos(4\pi f)$$

2.41

(a) $x_1(t) = 2\cos(20\pi t + \frac{\pi}{3})$

$$G_1(f) = \delta(f+10) + \delta(f-10) \qquad P_1 = \frac{2^2}{2} = 2W$$

(b) $x_2(t) = 3\sin(30\pi t)$

$$G_2(f) = \frac{9}{4}\delta(f+15) + \frac{9}{4}\delta(f-15) \qquad P_2 = \frac{3^2}{2} = \frac{9}{2}W$$

(c) $x_3(t) = 5\sin(10\pi t - \frac{\pi}{6})$

$$G_3(f) = \frac{25}{4}\delta(f+5) + \frac{25}{4}\delta(f-5) \qquad P_3 = \frac{5^2}{2} = \frac{25}{2}W$$

(d) $x_4(t) = 3\sin(30\pi t) + 5\sin(10\pi t - \frac{\pi}{6})$

$$G_4(f) = G_2(f) + G_3(f) \qquad P_4 = P_2 + P_3 = 17W$$

2.44

(a) $x_1(t) = 2\cos(10\pi t + \frac{\pi}{3})$

$$G_1(f) = \delta(f+5) + \delta(f-5)$$

$$R_{x_1}(\tau) = F^{-1}[G_1(f)] = 2\cos(10\pi\tau)$$

(b) $x_2(t) = 2\sin(10\pi t + \frac{\pi}{3})$

$$G_2(f) = \delta(f+5) + \delta(f-5)$$

$$R_{x_2}(\tau) = F^{-1}[G_2(f)] = 2\cos(10\pi\tau)$$

(c) $x_3(t) = Re[3\exp(j10\pi t) + 4j\exp(j10\pi t)]$

$$= 3\cos(10\pi t) - 4\sin(10\pi t)$$

$$G_3(f) = \frac{25}{4}\delta(f+5) + \frac{25}{4}\delta(f-5)$$

$$R_{x_3}(\tau) = \frac{25}{2}\cos(10\pi\tau)$$

(d) $x_4(t) = x_1(t) + x_2(t)$

$$G_4(f) = G_1(f) + G_2(f)$$

$$= 2\delta(f+5) + 2\delta(f-5)$$

$$R_{x_4}(\tau) = F^{-1}[G_4(f)] = 4\cos(10\pi\tau)$$

2.53

$$X(f) = \frac{1}{3+j2\pi f}$$

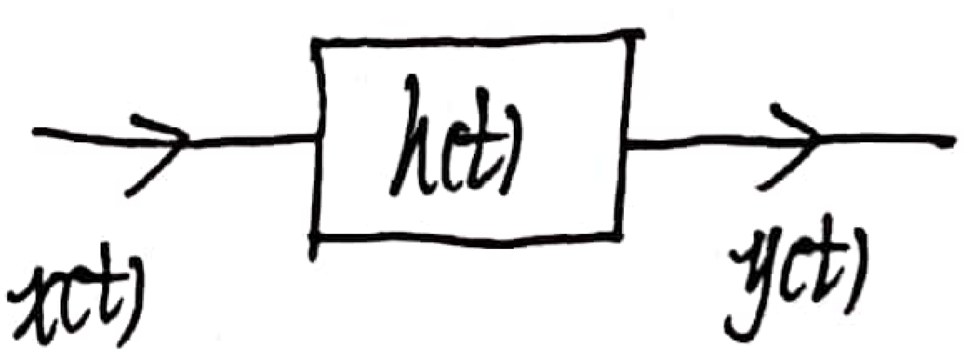
$$Y(f) = H(f)X(f)$$

$$|Y(f)|^2 = |H(f)X(f)|^2$$

$$= |H(f)|^2 |X(f)|^2$$

$$= \frac{25}{[16+(2\pi f)^2][9+(2\pi f)^2]}$$

$$|X(f)|^2 = \frac{1}{9+(2\pi f)^2}$$



$y(t) = A\,x(t-T_0)$

2.57

(a) $f_1 = \frac{48\pi}{2\pi} = 24\ \text{Hz}$

$|H(f_1)| = 4$

$\angle H(f_1) = -\frac{\frac{\pi}{2}}{75} \cdot 24\ \text{rad}$

$f_2 = \frac{126\pi}{2\pi} = 63\ \text{Hz}$

$|H(f_2)| = 2$

$\angle H(f_2) = -\frac{\frac{\pi}{2}}{75} \cdot 63\ \text{rad}$

$$y_a(t) = |H(f_1)| \cdot \cos(48\pi t + \angle H(f_1))$$

$$+ |H(f_2)| \cdot 5\cos(126\pi t + \angle H(f_2))$$

$$= 4\cos[48\pi(t - \tfrac{1}{300})] +$$

$$10\cos[126\pi(t - \tfrac{1}{300})]$$

$\frac{48\pi}{300} = \frac{4\pi}{25}$

有幅度失真，无相位失真

(b) $\cos(126\pi t)+0.5\cos(170\pi t)$

$f_1=\frac{126\pi}{2\pi}=63\text{Hz}$

$|H(f_1)|=2 \quad \angle H(f_1)=-\frac{\frac{\pi}{2}}{75}\cdot 63 \text{ rad}$

$f_2=\frac{170\pi}{2\pi}=85\text{Hz}$

$|H(f_2)|=2 \quad \angle H(f_2)=0 \text{ rad}$

$y_b(t)=|H(f_1)|\cos(126\pi t+\angle H(f_1))+|H(f_2)|\cdot 0.5\cos(170\pi t+\angle H(f_2))$

$=2\cos[126\pi(t-\frac{1}{300})]+\cos(170\pi t)$

无幅度失真，有相位失真

(c) $\cos(126\pi t)+3\cos(144\pi t)$

$f_1=\frac{126\pi}{2\pi}=63\text{Hz} \quad |H(f_1)|=2 \quad \angle H(f_1)=-\frac{\frac{\pi}{2}}{75}\cdot 63 \text{ rad}$

$f_2=\frac{144\pi}{2\pi}=72\text{Hz} \quad |H(f_2)|=2 \quad \angle H(f_2)=-\frac{\frac{\pi}{2}}{75}\cdot 72 \text{ rad}$

$y_c(t)=|H(f_1)|\cos(126\pi t+\angle H(f_1))+|H(f_2)|\cdot 3\cos(144\pi t+\angle H(f_2))$

$=2\cos[126\pi(t-\frac{1}{300})]+6\cos[144\pi(t-\frac{1}{300})]$

无幅度失真，无相位失真

(d) $\cos(10\pi t)+4\cos(50\pi t)$

$f_1=\frac{10\pi}{2\pi}=5\text{Hz} \quad f_2=\frac{50\pi}{2\pi}=25\text{Hz}$

$|H(f_1)|=4 \quad \angle H(f_1)=-\frac{\frac{\pi}{2}}{75}\cdot 5 \text{ rad}$

$|H(f_2)|=4 \quad \angle H(f_2)=-\frac{\frac{\pi}{2}}{75}\cdot 25 \text{ rad}$

$$y_d(t) = |H(f_1)|\cos(10\pi t + \angle H(f_1)) + |H(f_2)|4\cos(50\pi t + \angle H(f_2))$$

$$= 4\cos\left[10\pi\left(t - \frac{1}{300}\right)\right] + 16\cos\left[50\pi\left(t - \frac{1}{300}\right)\right]$$

无幅度失真，无相位失真

6.7

(a)

	B_1	B_2	B_3	$P(A_i)$
A_1	0.05	0.05	0.45	0.55
A_2	0.05	0.15	0.10	0.30
A_3	0.05	0.05	0.05	0.15
$P(B_j)$	0.15	0.25	0.60	1.0

(b) $$P(A_3|B_3) = \frac{P(A_3, B_3)}{P(B_3)} = \frac{0.05}{0.60} = \frac{1}{12}$$

$$P(B_2|A_1) = \frac{P(B_2, A_1)}{P(A_1)} = \frac{0.05}{0.55} = \frac{1}{11}$$

$$P(B_3|A_2) = \frac{P(B_3, A_2)}{P(A_2)} = \frac{0.10}{0.30} = \frac{1}{3}$$

6.14

(a) $$\int_0^{\infty}\int_0^{\infty} Axy\,e^{-(x+y)}\,dx\,dy = 1$$

$\Rightarrow A = 1$

(b) $$f_X(x) = \int_0^{\infty} Axy\,e^{-(x+y)}\,dy = xe^{-x}$$

$$f_Y(y) = \int_0^{\infty} Axy\,e^{-(x+y)}\,dx = ye^{-y}$$

(c) $$f_{XY}(x,y) = f_X(x)\,f_Y(y)$$

$\Rightarrow$ X, Y statistically independent

6.21

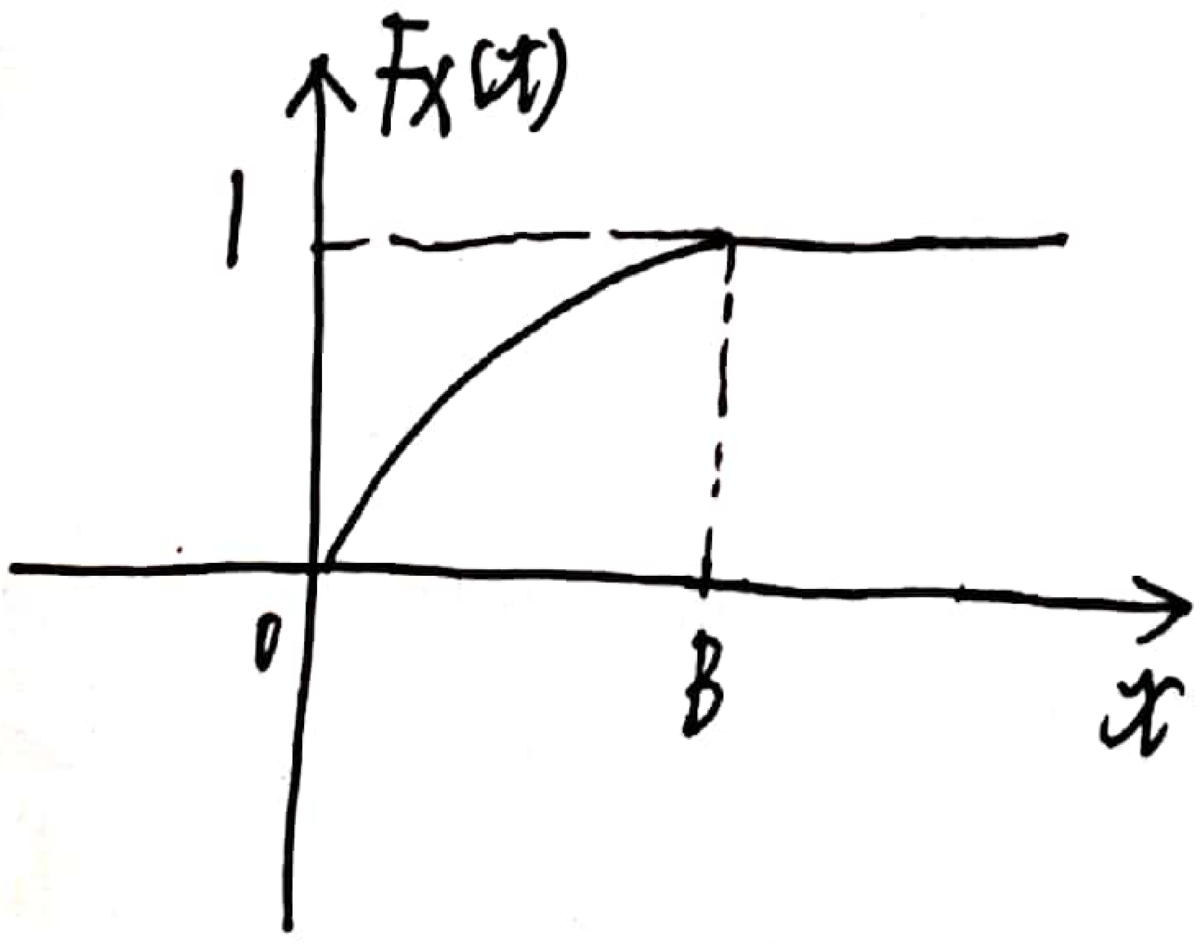
(a) $$f_X(x) = Ae^{-bx}\left[u(x) - u(x-B)\right]$$

$$\int_0^{B} f_X(x)\,dx = 1$$

$\Rightarrow b = A(1 - e^{-bB})$

(b)

$$F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx = \begin{cases} 0 & x<0 \\ \frac{A}{b}(1-e^{-bx}), & 0 \le x < B \\ 1 & x > B \end{cases}$$



(c)

$$E(X)=\int_{-\infty}^{\infty} x f_X(x)\,dx$$

$$=\int_0^B Axe^{-bx}\,dx \quad \left(\int u\,dv = uv-\int v\,du\right)$$

$$=\frac{A}{b^2}(1-e^{-bB}-ABbe^{-bB}) \quad (A=b(1-e^{-bB})^{-1})$$

$$=\frac{1}{b}[1-bB\frac{e^{-bB}}{1-e^{-bB}})$$

(d) $$E(X^2)=\int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

$$=-\frac{A}{b}B^2e^{-bB}-\frac{2A}{b}Be^{-bB}-\frac{2A}{b^3}e^{-bB}+\frac{2A}{b^3}$$

$$=\frac{2A}{b}\left[\frac{1}{b^2}-\frac{e^{-bB}}{b^2}(1+bB)\right]-\frac{A}{b}B^2e^{-bB}$$

(e) $$\sigma_X^2=[E(X)]^2-E(X^2) \quad (\text{def})$$

$$=\frac{1}{b^2}(1-bB\frac{e^{-bB}}{1-e^{-bB}})^2-\frac{2A}{b}\left[\frac{1}{b^2}-\frac{e^{-bB}}{b^2}(1+bB)\right]+\frac{A}{b}B^2e^{-bB}$$

6.25

$$f_{X|Y}(x|y) \doteq \frac{f_{XY}(x,y)}{f_Y(y)}$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}\sqrt{1-\rho^2}} \exp\left[-\frac{x^2-2\rho xy+y^2}{2\sigma^2(1-\rho^2)}+\frac{y^2}{2\sigma^2}\right]$$

$$= \frac{1}{\sqrt{2\pi\sigma^2(1-\rho^2)}} \exp\left[-\frac{(x-\rho y)^2}{2\sigma^2(1-\rho^2)}\right]$$