Chapter 4B.

Relational Algebra II

CSIS0278 / COMP3278

Introduction to

Database Management Systems

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Content

- Additional relational algebra operations
- Extended relational algebra operations
- Equivalence rules
- A simple illustration of query optimization

Section 1

Additional operators

Motivation

- The fundamental operators of the relational algebra introduced in Chapter 4A are sufficient to express any relational-algebra query.
- However, if we restrict ourselves to just the fundamental operators, certain common queries are lengthy to express.
  - Therefore, we define additional operators that do not add any power to the algebra, but simplify common queries

That is to say, for each additional operator, we can give an equivalent expression that use only the fundamental operators.

Additional operators

- Set intersection ($\cap$)
- Natural join ($\bowtie$)
- Assignment ($\leftarrow$)
- Left outer join ($⟕$), Right outer join ($⟖$)
- Division ($\div$)
- Many more
  - ***Θ*-join**, semi join, anti join, full outer join.

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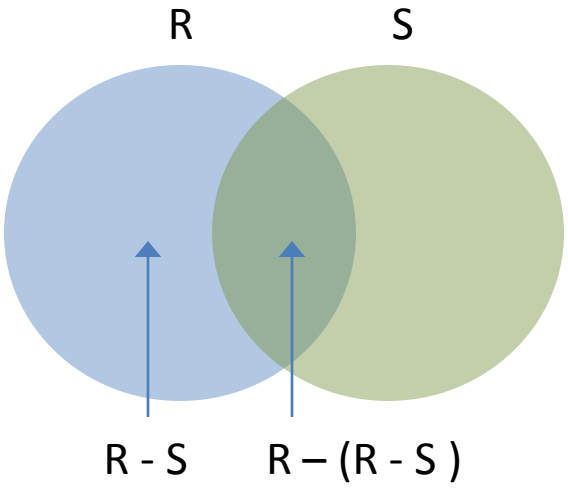
Set intersection



- $R \cap S = R - (R - S)$
  - R and S must have the **same number of attributes** and **attribute domains compatible**.
- Example
  - **Query:** Find the employee_id of employees who work in department 1 **and** department 3.

```
SELECT employee_id
FROM Works_in
WHERE department_id=1

INTERSECT

SELECT employee_id
FROM Works_in
WHERE department_id=3
```

SQL

Query processer

$$(\pi_{employee\_id}(\sigma_{department\_id=1}(\textbf{Works\_in}))) \cap (\pi_{employee\_id}(\sigma_{department\_id=3}(\textbf{Works\_in})))$$

Relational algebra with set intersection

$$(\pi_{employee\_id}(\sigma_{department\_id=1}(\textbf{Works\_in}))) - ((\pi_{employee\_id}(\sigma_{department\_id=1}(\textbf{Works\_in}))) - (\pi_{employee\_id}(\sigma_{department\_id=3}(\textbf{Works\_in}))))$$

Equivalence relational algebra with only fundamental operators

Set intersection

For your reference, there is another way to answer the same query by joining the **Works_in** table with itself.

```
SELECT DISTINCT employee_id
FROM Works_in W1, Works_in W2
WHERE
W1.employee_id = W2.employee_id AND
W1.department_id=1 AND
W2. department_id = 3
```

Query processer

Works_in

employee_id	department_id
1	1
2	1
1	3

SQL

$$\pi_{\text{employee\_id}}(\sigma_{\text{W1.department\_id=1} \wedge \text{W2.department\_id=3}}(\sigma_{\text{W1.employee\_id=W2.employee\_id}}(\rho_{\mathbf{W1}}(\mathbf{Works\_in}) \times \rho_{\mathbf{W2}}(\mathbf{Works\_in}))))$$

Set intersection

For your reference, there is another way to answer the same query by joining the **Works_in** table with itself.

```
SELECT DISTINCT employee_id
FROM Works_in W1, Works_in W2
WHERE
W1.employee_id = W2.employee_id AND
W1.department_id=1 AND
W2. department_id = 3
```

Query processer

ρ_{W1} (**Works_in**) $\times$ ρ_{W2} (**Works_in**)

W1.employee_id	W1.department_id	W2.employee_id	W2.department_id
1	1	1	1
1	1	2	1
1	1	1	3
2	1	1	1
2	1	2	1
2	1	1	3
1	3	1	1
1	3	2	1
1	3	1	3

Works_in

employee_id	department_id
1	1
2	1
1	3

SQL

$\pi_{employee\_id}(\sigma_{W1.department\_id=1 \wedge W2.department\_id=3}(\sigma_{W1.employee\_id=W2.employee\_id}(\rho_{W1}(\textbf{Works\_in}) \times \rho_{W2}(\textbf{Works\_in}))))$

Set intersection

For your reference, there is another way to answer the same query by joining the **Works_in** table with itself.

```
SELECT DISTINCT employee_id
FROM Works_in W1, Works_in W2
WHERE
W1.employee_id = W2.employee_id AND
W1.department_id=1 AND
W2. department_id = 3
```

Query processer

$\sigma_{W1.employee\_id=W2.employee\_id}(\rho_{W1}(\textbf{Works\_in}) \times \rho_{W2}(\textbf{Works\_in}))$

W1.employee_id	W1.department_id	W2.employee_id	W2.department_id
1	1	1	1
1	1	1	3
2	1	2	1
1	3	1	1
1	3	1	3

$\rho_{W1}(\textbf{Works\_in}) \times \rho_{W2}(\textbf{Works\_in})$

W1.employee_id	W1.department_id	W2.employee_id	W2.department_id
1	1	1	1
1	1	2	1
1	1	1	3
2	1	1	1
2	1	2	1
2	1	1	3
1	3	1	1
1	3	2	1
1	3	1	3

Works_in

employee_id	department_id
1	1
2	1
1	3

SQL

$\pi_{employee\_id}(\sigma_{W1.department\_id=1 \wedge W2.department\_id=3}(\sigma_{W1.employee\_id=W2.employee\_id}(\rho_{W1}(\textbf{Works\_in}) \times \rho_{W2}(\textbf{Works\_in}))))$

Set intersection

$\sigma_{W1.department\_id=1 \wedge W2.department\_id=3}(\sigma_{W1.employee\_id=W2.employee\_id}(\rho_{W1}(Works\_in) \times \rho_{W2}(Works\_in)))$

W1.employee_id	W1.department_id	W2.employee_id	W2.department_id
1	1	1	3

For your reference, there is another way to answer the same query by joining the **Works_in** table with itself.

$\sigma_{W1.employee\_id=W2.employee\_id}(\rho_{W1}(Works\_in) \times \rho_{W2}(Works\_in))$

W1.employee_id	W1.department_id	W2.employee_id	W2.department_id
1	1	1	1
1	1	1	3
2	1	2	1
1	3	1	1
1	3	1	3

$\rho_{W1}(Works\_in) \times \rho_{W2}(Works\_in)$

W1.employee_id	W1.department_id	W2.employee_id	W2.department_id
1	1	1	1
1	1	2	1
1	1	1	3
2	1	1	1
2	1	2	1
2	1	1	3
1	3	1	1
1	3	2	1
1	3	1	3

Works_in

employee_id	department_id
1	1
2	1
1	3

```
SELECT DISTINCT employee_id
FROM Works_in W1, Works_in W2
WHERE
W1.employee_id = W2.employee_id AND
W1.department_id=1 AND
W2. department_id = 3
```

Query processer

SQL

$\pi_{employee\_id}(\sigma_{W1.department\_id=1 \wedge W2.department\_id=3}(\sigma_{W1.employee\_id=W2.employee\_id}(\rho_{W1}(Works\_in) \times \rho_{W2}(Works\_in))))$

Natural join

- Usually, a query that involves a Cartesian product includes a selection operation on the result of the Cartesian product.
  - The selection operation most often requires that all **attributes that are common** to the relations that are involved in the Cartesian product **be equated**.
- $r \bowtie s = \pi_{R \cup S}(\sigma_{r.A1 = s.A1 \wedge r.A2 = s.A2 \wedge \ldots \wedge r.An = s.An} (r \times s))$

  Requires the common attributes to be equated
  - where $R \cap S = \{A1, A2, \ldots , An\}$

Natural join

- The schema of R $\bowtie$ S is R-schema $\cup$ S-schema (repeated attributes are removed)
  - For each pair of tuples t_r from R and t_s from S,
  - If t_r and t_s share the same value over each of the common attributes in R and S,
  - Tuple t will be added to the result of R $\bowtie$ S.

Natural join

R

A	B
1	1
1	2
2	3

S

A	C
1	2
2	1

- Common attributes: $R \cap S = \{A\}$
- Attributes of the resulting relation: $R \cup S = \{A,B,C\}$

$R \times S$

R.A	R.B	S.A	S.C
1	1	1	2
1	1	2	1
1	2	1	2
1	2	2	1
2	3	1	2
2	3	2	1

$\sigma_{R.A=S.A}(R \times S)$

R.A	R.B	S.A	S.C
1	1	1	2
1	2	1	2
2	3	2	1

$\pi_{A,B,C}(\sigma_{R.A=S.A}(R \times S))$

A	B	C
1	1	2
1	2	2
2	3	1

equivalent to:

$R \bowtie S$

A	B	C
1	1	2
1	2	2
2	3	1

Assignment

- It is convenient to write a relational-algebra expression by assigning parts of it to **temporary relation variables**.
- The assignment operator, denoted by "$\leftarrow$", works like assignment operator "=" in programming language.
  - temp1 $\leftarrow \pi_a(R)$
  - temp2 $\leftarrow \pi_a(S)$
  - result $\leftarrow$ temp1 – temp2

With the assignment operator, a query can be written as a sequential program.

temp1, temp2, result are called "**relation variable**".

Outer join

- The outer-join operator is an extension of the join operation to deal with missing information.

Customer

customer_id	name	address
C1	Kit	CB320
C2	Ben	CB326
C3	Jolly	CB311
C4	Yvonne	CB415

Depositor

account_id	customer_id
A1	C1
A1	C2
A2	C2
A3	C4
A4	C4

Customer ⋈ Depositor

customer_id	name	address	account_id
C1	Kit	CB320	A1
C2	Ben	CB326	A1
C2	Ben	CB326	A2
C4	Yvonne	CB415	A3
C4	Yvonne	CB415	A4

- Outer join

Natural join (e.g., joining Customer and Depositor) result in a table **without the information of customers (e.g., C3 in this case) who has no account**.

  - Natural join result, plus
  - The **tuples that do not match any tuples from the other side**.

Left outer join

$$R \mathbin{⟕} S = (R \bowtie S) \cup (R - \pi_R(R \bowtie S)) \times \{(\boldsymbol{null}, \ldots, \boldsymbol{null})\}$$

Customer

customer_id	name	address
C1	Kit	CB320
C2	Ben	CB326
C3	Jolly	CB311
C4	Yvonne	CB415

Depositor

account_id	customer_id
A1	C1
A1	C2
A2	C2
A3	C4
A4	C4

Customer ⋈ Depositor

customer_id	name	address	account_id
C1	Kit	CB320	A1
C2	Ben	CB326	A1
C2	Ben	CB326	A2
C4	Yvonne	CB415	A3
C4	Yvonne	CB415	A4

Let's illustrate why the left outer join is defined as $(R \bowtie S) \cup (R - \pi_R(R \bowtie S)) \times \{(\boldsymbol{null}, \ldots, \boldsymbol{null})\}$ through a step-by-step illustration.

Left outer join

$$R \mathbin{⟕} S = (R \bowtie S) \cup (R - \pi_R(R \bowtie S)) \times \{(null, \ldots, null)\}$$

Customer

customer_id	name	address
C1	Kit	CB320
C2	Ben	CB326
C3	Jolly	CB311
C4	Yvonne	CB415

Depositor

account_id	customer_id
A1	C1
A1	C2
A2	C2
A3	C4
A4	C4

Customer ⋈ Depositor

customer_id	name	address	account_id
C1	Kit	CB320	A1
C2	Ben	CB326	A1
C2	Ben	CB326	A2
C4	Yvonne	CB415	A3
C4	Yvonne	CB415	A4

Customer - $\pi_{\text{Customer's attributes}}$ **(Customer ⋈ Depositor)**

customer_id	name	address
C3	Jolly	CB311

Finding missed tuples in the natural join

$R - \pi_R(R \bowtie S)$ is to generate the tuples in R (i.e., Customer) that are missed in the natural join.

i.e., C3 Jolly in Customer doesn't have any matched records in Depositor, we use this part to recover Jolly's record.

Left outer join

$$R \mathbin{⟕} S = (R \bowtie S) \cup (R - \pi_R(R \bowtie S)) \times \{(null, \ldots, null)\}$$

Customer

customer_id	name	address
C1	Kit	CB320
C2	Ben	CB326
C3	Jolly	CB311
C4	Yvonne	CB415

Depositor

account_id	customer_id
A1	C1
A1	C2
A2	C2
A3	C4
A4	C4

Customer ⋈ Depositor

customer_id	name	address	account_id
C1	Kit	CB320	A1
C2	Ben	CB326	A1
C2	Ben	CB326	A2
C4	Yvonne	CB415	A3
C4	Yvonne	CB415	A4

Customer - $\pi_{\text{Customer's attributes}}$ (**Customer ⋈ Depositor**)

customer_id	name	address
C3	Jolly	CB311

Customer - $\pi_{\text{Customer's attributes}}$ (**Customer ⋈ Depositor**) $\times \{(null, \ldots, null)\}$

customer_id	name	address	account_id
C3	Jolly	CB311	*null*

Constructing the missed tuple by adding null value to extra attributes

The "$\times \{(null, \ldots, null)\}$" part simply add back the remaining column values as ***null*** because there is no matched records in S (i.e., Depositor).

Left outer join

$$R \mathbin{⟕} S = (R \bowtie S) \cup (R - \pi_R(R \bowtie S)) \times \{(\textit{null}, \ldots, \textit{null})\}$$

Customer

customer_id	name	address
C1	Kit	CB320
C2	Ben	CB326
C3	Jolly	CB311
C4	Yvonne	CB415

Depositor

account_id	customer_id
A1	C1
A1	C2
A2	C2
A3	C4
A4	C4

Customer ⋈ Depositor

customer_id	name	address	account_id
C1	Kit	CB320	A1
C2	Ben	CB326	A1
C2	Ben	CB326	A2
C4	Yvonne	CB415	A3
C4	Yvonne	CB415	A4

Customer - $\pi_{\text{Customer's attributes}}$ (**Customer ⋈ Depositor**)

customer_id	name	address
C3	Jolly	CB311

Customer - $\pi_{\text{Customer's attributes}}$ (**Customer ⋈ Depositor**) $\times$ { (***null***, … , ***null***) }

customer_id	name	address	account_id
C3	Jolly	CB311	***null***

(**Customer ⋈ Depositor**) $\cup$ (**Customer** - $\pi_{\text{Customer's attributes}}$ (**Customer ⋈ Depositor**)) $\times$ { (***null***, … , ***null***) }

customer_id	name	address	account_id
C1	Kit	CB320	A1
C2	Ben	CB326	A1
C2	Ben	CB326	A2
C3	Jolly	CB311	***null***
C4	Yvonne	CB415	A3
C4	Yvonne	CB415	A4

equivalent to: **Customer** $⟕$ **Depositor**

Right outer join

$$R \bowtie\!\!\!\!\subset S = (R \bowtie S) \cup (S - \pi_S(R \bowtie S)) \times \{(null, \ldots, null)\}$$

Customer

customer_id	name	address
C1	Kit	CB320
C2	Ben	CB326
C3	Jolly	CB311
C4	Yvonne	CB415

Depositor

account_id	customer_id
A1	C1
A1	C2
A2	C2
A3	C4
A4	C5

Customer ⋈ Depositor

customer_id	name	address	account_id
C1	Kit	CB320	A1
C2	Ben	CB326	A1
C2	Ben	CB326	A2
C4	Yvonne	CB415	A3

Let's illustrate why the right outer join is defined as
$(R \bowtie S) \cup (S - \pi_S(R \bowtie S)) \times \{(null, \ldots, null)\}$
through a step-by-step illustration.

Right outer join

$$R \overset{\subset}{\bowtie} S = (R \bowtie S) \cup (S - \pi_S(R \bowtie S)) \times \{(null, \ldots, null)\}$$

Customer

customer_id	name	address
C1	Kit	CB320
C2	Ben	CB326
C3	Jolly	CB311
C4	Yvonne	CB415

Depositor

account_id	customer_id
A1	C1
A1	C2
A2	C2
A3	C4
A4	C5

Customer ⋈ Depositor

customer_id	name	address	account_id
C1	Kit	CB320	A1
C2	Ben	CB326	A1
C2	Ben	CB326	A2
C4	Yvonne	CB415	A3

Depositor - $\pi_{\text{Depositor's attributes}}$ (**Customer ⋈ Depositor**)

customer_id	account_id
C5	A4

Finding missed tuples in the natural join

$S - \pi_S(R \bowtie S)$ is to generate the tuples in S (i.e., Depositor) that are missed in the natural join.

i.e., C5 in Depositor doesn't have any matched records in Customer, we use this part to recover C5's record.

Right outer join

$$R \overset{\subset}{\bowtie} S = (R \bowtie S) \cup (S - \pi_S(R \bowtie S)) \times \{(null, \ldots, null)\}$$

Customer

customer_id	name	address
C1	Kit	CB320
C2	Ben	CB326
C3	Jolly	CB311
C4	Yvonne	CB415

Depositor

account_id	customer_id
A1	C1
A1	C2
A2	C2
A3	C4
A4	C5

Customer ⋈ Depositor

customer_id	name	address	account_id
C1	Kit	CB320	A1
C2	Ben	CB326	A1
C2	Ben	CB326	A2
C4	Yvonne	CB415	A3

Depositor - $\pi_{\text{Depositor's attributes}}$ (**Customer ⋈ Depositor**)

customer_id	account_id
C5	A4

Depositor - $\pi_{\text{Depositor's attributes}}$ (**Customer ⋈ Depositor**) $\times \{(null, \ldots, null)\}$

customer_id	name	address	account_id
C5	*null*	*null*	A4

Constructing the missed tuple by adding *null* value to extra attributes

The "$\times \{(null, \ldots, null)\}$" part simply add back the remaining column values as ***null*** because there is no matched records in R (i.e., Customer).

Right outer join

$$R \overset{\subset}{\bowtie} S = (R \bowtie S) \cup (S - \pi_S(R \bowtie S)) \times \{(null, \ldots, null)\}$$

Customer

customer_id	name	address
C1	Kit	CB320
C2	Ben	CB326
C3	Jolly	CB311
C4	Yvonne	CB415

Depositor

account_id	customer_id
A1	C1
A1	C2
A2	C2
A3	C4
A4	C5

Customer ⋈ Depositor

customer_id	name	address	account_id
C1	Kit	CB320	A1
C2	Ben	CB326	A1
C2	Ben	CB326	A2
C4	Yvonne	CB415	A3

Depositor - $\pi_{\text{Depositor's attributes}}$ (**Customer ⋈ Depositor**)

customer_id	account_id
C5	A4

Depositor - $\pi_{\text{Depositor's attributes}}$ (**Customer ⋈ Depositor**) × { (***null***, … , ***null***) }

customer_id	name	address	account_id
C5	***null***	***null***	A4

(Customer ⋈ Depositor) ∪ (**Depositor** - $\pi_{\text{Depositor's attributes}}$ (**Customer ⋈ Depositor**)) × { (***null***, … , ***null***) }

customer_id	name	address	account_id
C1	Kit	CB320	A1
C2	Ben	CB326	A1
C2	Ben	CB326	A2
C4	Yvonne	CB415	A3
C5	***null***	***null***	A4

equivalent to: **Customer** $\overset{\subset}{\bowtie}$ **Depositor**

Division

R

A	B
1	1
2	1
2	2
3	3
4	1
4	2
4	3

S

B
1
2

R ÷ S

A
2
4

- Notation: $R \div S$
- Definition
  - Let $S \subseteq R$

The attributes in relation S is a subset of the attributes in relation R.

  - $R \div S = \{ t \mid t \in \pi_{R-S}(R) \wedge (\forall s \in S, ((t \cup s) \in R))\}$

Condition 1. A resulting tuple t has to be in the relation $\pi_{R-S}(R)$

$\pi_{R-S}(R)$

A
1
2
3
4

Condition 2. And if we combine t with each tuple $s \in S$, all the combined tuples have to be included in R.

1 For each tuple $t \in \pi_{R-S}(R)$		**2** **"$\forall s \in S, t \cup s$" part:** Generate tuples by union t with all tuples $s \in S$	**3** **"$\forall s \in S, ((t \cup s) \in R)$" part:** Then we check if both tuples generated are in R.
$t_1 \in \pi_{R-S}(R)$	A: 1	$\forall s \in S, (t_1 \cup s)$	A: 1, 1 / B: 1, 2 — In this case, not both tuples are in R, so t_1 is **NOT** in result of $R \div S$
$t_2 \in \pi_{R-S}(R)$	A: 2	$\forall s \in S, (t_2 \cup s)$	A: 2, 2 / B: 1, 2 — In this case, both tuples are in R, so t_2 is in result of $R \div S$

Division

R

A	B
1	1
2	1
2	2
3	3
4	1
4	2
4	3

S

B
1
2

R ÷ S

A
2
4

Observation

Let's focus on the result of **R ÷ S** (say, the tuple A=2), it means that

- For the tuples with values in attribute A equals to 2 in relation R,
- Those tuples's values in attribute B covers **ALL** values in attribute B of S.

- Division is used to express queries with "**all**"
  - Find the IDs of all students who have taken **all** CS courses (dpt_id = 1).

Student

student_id	name	dpt_id
1	Peter	1
2	Sharon	1
3	David	2
4	Joe	3

Takes

student_id	course_id	Grade
1	1	A
1	2	B
1	3	A+
2	3	B-
3	3	B
4	1	C
4	2	A-

Course

course_id	title	dpt_id	credit
1	Intro to DB	1	6
2	Programming I	1	6
3	Accounting	2	6

Division

- **Step 1. All part (the relation S):** What is the course ID of **all** CS courses (dpt_id = 1)?

$\pi_{course\_id}(\sigma_{dpt\_id=1}(Course))$

course_id
1
2

$\sigma_{dpt\_id=1}(Course)$

course_id	title	dpt_id	credit
1	Intro to DB	1	6
2	Programming I	1	6

Student

student_id	name	dpt_id
1	Peter	1
2	Sharon	1
3	David	2
4	Joe	3

Takes

student_id	course_id	Grade
1	1	A
1	2	B
1	3	A+
2	3	B-
3	3	B
4	1	C
4	2	A-

Course

course_id	title	dpt_id	credit
1	Intro to DB	1	6
2	Programming I	1	6
3	Accounting	2	6

Division

- **Step 2. Dividend part (the relation R):** What is the list of Takes tuples (i.e., all < student_id, course_id > pairs)?

$\pi_{student\_id,course\_id}$ (Takes)

student_id	course_id
1	1
1	2
1	3
2	3
3	3
4	1
4	2

π_{course_id} ($\sigma_{dpt\_id=1}$(Course))

course_id
1
2

Student

student_id	name	dpt_id
1	Peter	1
2	Sharon	1
3	David	2
4	Joe	3

Takes

student_id	course_id	Grade
1	1	A
1	2	B
1	3	A+
2	3	B-
3	3	B
4	1	C
4	2	A-

Course

course_id	title	dpt_id	credit
1	Intro to DB	1	6
2	Programming I	1	6
3	Accounting	2	6

Division

- **Step 3. Division:** Which student in $\pi_{\text{student\_id,course\_id}}$ (Takes) takes **all** CS courses $\pi_{\text{course\_id}}(\sigma_{\text{dpt\_id=1}}(\text{course}))$?.

$\pi_{\text{student\_id,course\_id}}$ (Takes)

student_id	course_id
1	1
1	2
1	3
2	3
3	3
4	1
4	2

$\pi_{\text{course\_id}}(\sigma_{\text{dpt\_id=1}}(\text{Course}))$

course_id
1
2

$\pi_{\text{student\_id,course\_id}}(\text{Takes}) \div \pi_{\text{course\_id}}(\sigma_{\text{dpt\_id=1}}(\text{Course}))$

student_id
1
4

Explanation: Let's focus on student_id = 1 in the result

- It means that, for the tuples in **Takes** with student_id=1,
- Those tuple's value in course_id attribute covers **all** 1 and 2 (i.e., the CS courses).
- That is to say, student with student_id 1 takes **ALL** CS courses.
- The same argument apples to student_id 4.

Division

- Division has a property: $R \times S \div S = R$

R

A	B
1	1
1	2
2	3

S

C	D
1	2
2	2

$R \times S$

A	B	C	D
1	1	1	2
1	1	2	2
1	2	1	2
1	2	2	2
2	3	1	2
2	3	2	2

$(R \times S) \div S$

A	B
1	1
1	2
2	3

= R

Self-study question

How about $(R \div S) \times S = R$?

Is this true in relational algebra?

Can you prove/ disprove it?

Section 2

Extended operators

Aggregation

- Aggregation function takes a collection of values and returns a single-valued result.
  - e.g., avg, min, max, sum, count, count-distinct
- Aggregate operation in relational algebra:
  - **Grouping** - Divides the tuples into groups.
  - **Aggregation** - Computes an aggregation function in each group to create a result tuple.

Aggregation

$$_{G_1, G_2, \ldots, G_n}\mathrm{g}_{F_1(A_1), F_2(A_2), \ldots, F_n(A_n)}(E)$$

- **E** – a relation (can be a result of relational algebra expression).
- **G_1, …, G_n** – attributes used to form groups.
  - Tuples with the same values in **G_1** to **G_n** are put into the same group.
  - **G** can be empty, which means that the whole relation is one group.
- **$F_i(A_i)$** – an aggregate function applied on an attribute.

Aggregation

Account

branch_id	account_id	balance
B1	A1	500
B2	A2	400
B2	A3	900
B1	A4	700

$$\rho_{Result(branch\_id,\ sum\_of\_balance)} ({}_{branch\_id}\, g_{sum(balance)} (Account))$$

Result

branch_id	sum_of_balance
B1	1200
B2	1300

- **Step 1.** Let's group the tuples in Account according to their branch_id.
- **Step 2.** Then aggregate the tuples in each group by summing their values in the balance attribute.
- **Step 3.** Since the resulting relation has no name after aggregation, we use renaming operator to give name to the relation and attributes.

Aggregation

- Note that grouping **can be done on multiple attributes**.
- E.g., in this case, we group tuples in Student with the same values in both dpt_id and gender attributes.
- i.e., We are finding the number of male / female students in each department.

Student

student_id	name	dpt_id	gender
1	Peter	1	M
2	Sharon	1	F
3	David	2	M
4	Joe	2	M
5	Betty	1	F

$$\rho_{Result(dpt\_id,\ gender,\ count)} ({}_{dpt\_id,\ gender}\mathcal{G}_{count()} (Student))$$

Result

dpt_id	gender	count
1	M	1
1	F	2
2	M	2

Section 3

Algebraic properties

Slides prepared by - **Dr. Chui Chun Kit**, http://www.cs.hku.hk/~ckchui/ for students in CSIS0278 / COMP3278
For other uses, please email : ckchui@cs.hku.hk

Transformation of expression

- A query can be expressed in several different ways, with different cost of evaluation.
- Two relational-algebra expressions are said to be equivalent if, on every legal database instance, the two expressions generate the same set of tuples.

In the following discussions, we treat a relation as a set of tuples, the order of the tuples is irrelevant.

Equivalence rules

- **Rule 1.** Only the final operations in a sequence of projection operations are needed.

$$\pi_{L1}(\pi_{L2}(\ldots(\pi_{Ln}(E))\ldots)) = \pi_{L1}(E)$$

$\pi_{employee\_id}(\pi_{employee\_id,salary}(Employee))$

employee_id
1
2
3

$\pi_{employee\_id,salary}(Employee)$

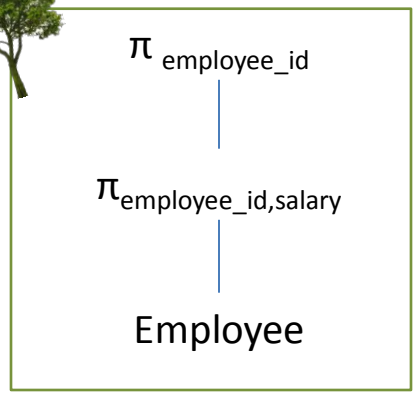
employee_id	name
1	Jones
2	Smith
3	Smith

Employee

employee_id	name	salary
1	Jones	26000
2	Smith	28000
3	Smith	24000

$\pi_{employee\_id}$ ($\pi_{employee\_id,salary}$ (Employee))

Regular expression



Expression tree

Expression tree:

- Tells which operator is executed ahead of another.
- It allow transformation of the execution order by applying the equivalence rules. (Alter the tree)

Equivalence rules

- **Rule 1.** Only the final operations in a sequence of projection operations are needed.

$$\pi_{L1}(\pi_{L2}(\ldots(\pi_{Ln}(E))\ldots)) = \pi_{L1}(E)$$

$\pi_{employee\_id}$(**Employee**)

employee_id
1
2
3

Employee

employee_id	name	salary
1	Jones	26000
2	Smith	28000
3	Smith	24000

$\pi_{employee\_id}$ ($\pi_{employee\_id,salary}$ (Employee))

Regular expression

Expression tree

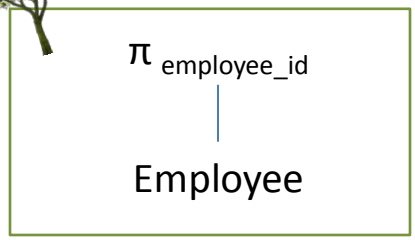
Remove it

equivalent to

$\pi_{employee\_id}$ (Employee)

Regular expression

equivalent to



Transformed expression tree

Equivalence rules

- **Rule 2.** Conjunctive selection operations can be deconstructed into a sequence of individual selections.

$$\sigma_{p1 \wedge p2}(E) = \sigma_{p1}(\sigma_{p2}(E))$$

$\sigma_{name="Smith" \wedge salary>24000}(Employee)$

$\sigma_{name="Smith"}(\sigma_{salary>24000}(Employee))$

$\sigma_{name="Smith" \wedge salary>24000}$

Employee

equivalent to

$\sigma_{name="Smith"}$

$\sigma_{salary>24000}$

Employee

$\sigma_{name="Smith"}(\sigma_{salary>24000}(\textbf{Employee}))$

employee_id	name	salary
2	Smith	28000

$\sigma_{salary>24000}(\textbf{Employee})$

employee_id	name	salary
1	Jones	26000
2	Smith	28000

Employee

employee_id	name	salary
1	Jones	26000
2	Smith	28000
3	Smith	24000

You may wonder why breaking down the conjunctive selections is useful.
We will show that it is useful **to reduce temporary result**.
We can try to push each selection predicates down the tree (to perform selection as early as possible).

Equivalence rules

- **Rule 3.** Selection operations are commutative.

$$\sigma_{p1}(\sigma_{p2}(E)) = \sigma_{p2}(\sigma_{p1}(E))$$

$\sigma_{name="Smith"}(\sigma_{salary>24000}(\text{Employee}))$

employee_id	name	salary
2	Smith	28000

$\sigma_{salary>24000}(\text{Employee})$

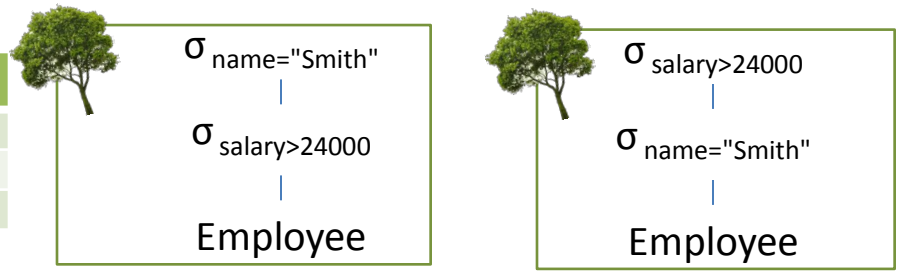
employee_id	name	salary
1	Jones	26000
2	Smith	28000
4	David	25000

Employee

employee_id	name	salary
1	Jones	26000
2	Smith	28000
3	Smith	24000
4	David	25000

$\sigma_{name="Smith"}(\sigma_{salary>24000}(\text{Employee}))$

$\sigma_{salary>24000}(\sigma_{name="Smith"}(\text{Employee}))$



$\sigma_{salary>24000}(\sigma_{name="Smith"}(\text{Employee}))$

employee_id	name	salary
2	Smith	28000

$\sigma_{name="Smith"}(\text{Employee})$

employee_id	name	salary
2	Smith	28000
3	Smith	24000

Employes

employee_id	name	salary
1	Jones	26000
2	Smith	28000
3	Smith	24000
4	David	25000

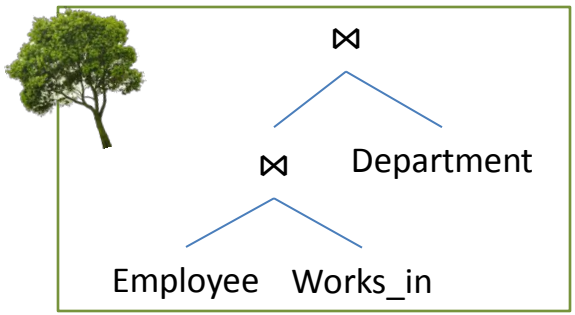
Note that the two executions have different costs. In particular, the size of their temporary relations are different.

Equivalence rules

- **Rule 4.** Natural join operations are associative.

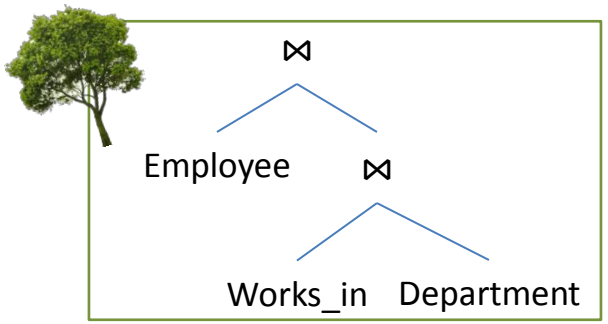
$$(E_1 \bowtie E_2) \bowtie E_3 = E_1 \bowtie (E_2 \bowtie E_3)$$

(Employee ⋈ Works_in) ⋈ Department



equivalent to

Employee ⋈ (Works_in ⋈ Department)

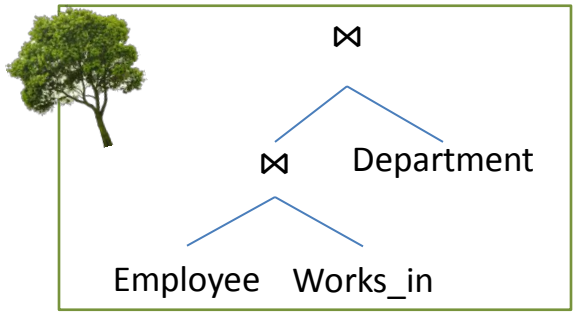


Although both expression trees return the **same resulting relation**, these two expression trees have **different costs** because the size of their temporary relations are different

Equivalence rules

Expression tree A.

(Employee ⋈ Works_in) ⋈ Department



(Employee ⋈ Works_in) ⋈ Department

employee_id	Employee. name	salary	department_id	Department. name
1	Jones	26000	1	Toys
2	Smith	28000	1	Toys
2	Smith	28000	2	Tools
3	Parker	35000	3	Food
4	Smith	24000	3	Food

Employee ⋈ Works_in

employee_id	name	salary	department_id
1	Jones	26000	1
2	Smith	28000	1
2	Smith	28000	2
3	Parker	35000	3
4	Smith	24000	3

Natural join evaluates 4*5 = 20 combinations
result temporary relation consists of 5 tuples and 4 columns.

Employee

employee_id	name	salary
1	Jones	26000
2	Smith	28000
3	Parker	35000
4	Smith	24000

Works_in

employee_id	department_id
1	1
2	1
2	2
3	3
4	3

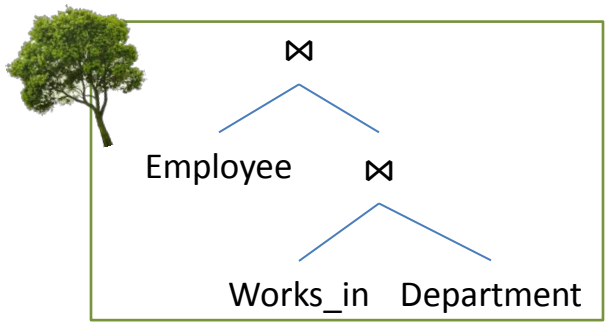
Department

department_id	name
1	Toys
2	Tools
3	Food

Equivalence rules

Expression tree B.

Employee ⋈ (Works_in ⋈ Department)



Employee ⋈ (Works_in ⋈ Department)

employee_id	Employee. name	salary	department_id	Department. name
1	Jones	26000	1	Toys
2	Smith	28000	1	Toys
2	Smith	28000	2	Tools
3	Parker	35000	3	Food
4	Smith	24000	3	Food

Works_in ⋈ Department

employee_id	department_id	name
1	1	Toys
2	1	Toys
2	2	Tools
3	3	Food
4	3	Food

Natural join evaluates 5*3 = 15 combinations
result temporary relation consists of 5 tuples and 3 columns.

Employee

employee_id	name	salary
1	Jones	26000
2	Smith	28000
3	Parker	35000
4	Smith	24000

Works_in

employee_id	department_id
1	1
2	1
2	2
3	3
4	3

Department

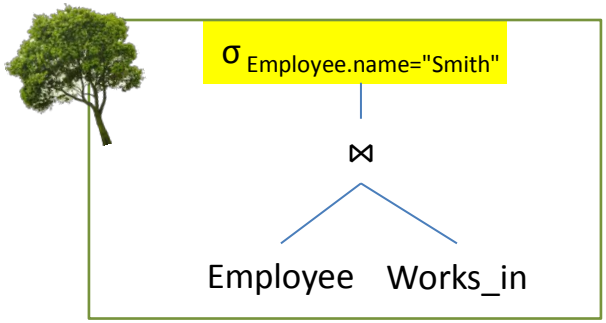
department_id	name
1	Toys
2	Tools
3	Food

Equivalence rules

- **Rule 5.** The selection operation distributes over the natural join operation under the following two conditions
  - **Rule 5a.** It distributes when all the attributes in selection condition **involve only the attributes of one of the expressions** (say, E_1) being joined.

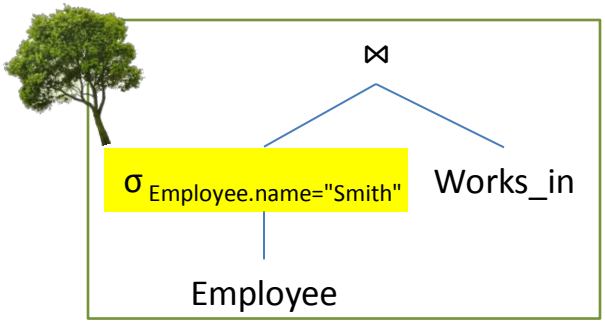
$$\sigma_p (E_1 \bowtie E_2) = (\sigma_p (E_1) \bowtie E_2)$$

$\sigma_{\text{Employee.name="Smith"}}$(Employee ⋈ Works_in)



equivalent to

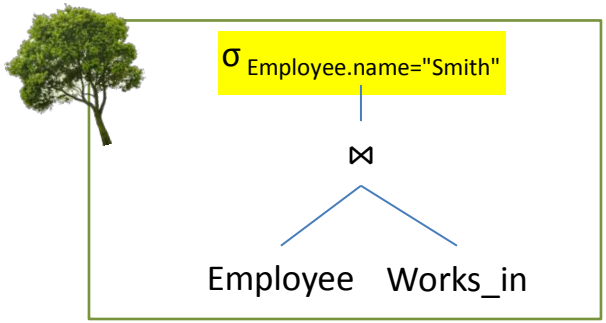
$(\sigma_{\text{Employee.name="Smith"}}$ (Employee) ⋈ Works_in)



Equivalence rules

Expression tree A.

$\sigma_{\text{Employee.name="Smith"}}$(Employee ⋈ Works_in)



$\sigma_{\text{Employee.name="Smith"}}$(**Employee ⋈ Works_in**)

employee_id	name	salary	department_id
2	Smith	28000	1
2	Smith	28000	2
4	Smith	24000	3

Employee ⋈ Works_in

employee_id	name	salary	department_id
1	Jones	26000	1
2	Smith	28000	1
2	Smith	28000	2
3	Parker	35000	3
4	Smith	24000	3

Natural join evaluates 4*5 = 20 combinations
result temporary relation consists of 5 tuples and 4 columns.

Employee

employee_id	name	salary
1	Jones	26000
2	Smith	28000
3	Parker	35000
4	Smith	24000

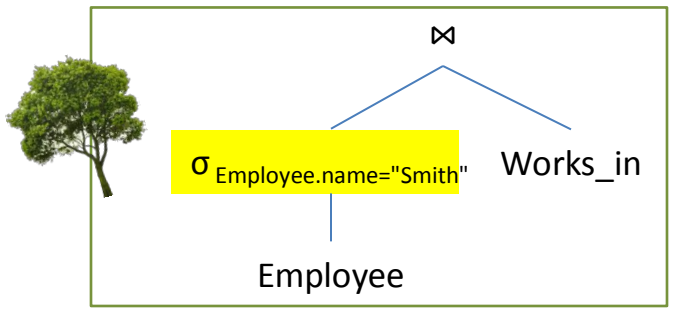
Works_in

employee_id	department_id
1	1
2	1
2	2
3	3
4	3

Equivalence rules

Expression tree B.

$\sigma_{\text{Employee.name="Smith"}}$ (Employee) ⋈ Works_in



$\sigma_{\text{Employee.name="Smith"}}$ (Employee) ⋈ Works_in

employee_id	name	salary	department_id
2	Smith	28000	1
2	Smith	28000	2
4	Smith	24000	3

Natural join evaluates 2*5 = 10 combinations
result temporary relation consists of 3 tuples and 4 columns.

$\sigma_{\text{Employee.name="Smith"}}$ (Employee)

employee_id	name	salary
2	Smith	28000
4	Smith	24000

When comparing with the equivalence expression tree 1, we can see that if we push the selection predicates down the natural join (perform selection earlier than joining), then we will probably have a **smaller temporary relation**.

Employee

employee_id	name	salary
1	Jones	26000
2	Smith	28000
3	Parker	35000
4	Smith	24000

Works_in

employee_id	department_id
1	1
2	1
2	2
3	3
4	3

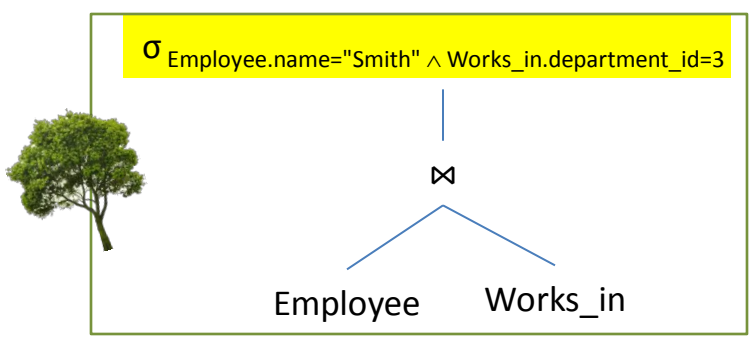
Equivalence rules

- **Rule 5b.** The selection distributes when selection condition p1 involves only the attributes of E_1 and p2 involves only the attributes of E_2.

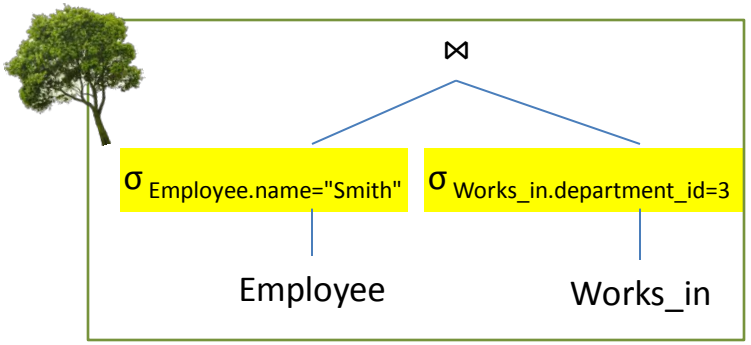
$$\sigma_{p1 \wedge p2}(E_1 \bowtie E_2) = (\sigma_{p1}(E_1) \bowtie \sigma_{p2}(E_2))$$

$\sigma_{\text{Employee.name="Smith"} \wedge \text{Works\_in.department\_id=3}}$(**Employee** ⋈ **Works_in**)

($\sigma_{\text{Employee.name="Smith"}}$ (**Employee**) ⋈ $\sigma_{\text{Works\_in.department\_id=3}}$ (**Works_in**))



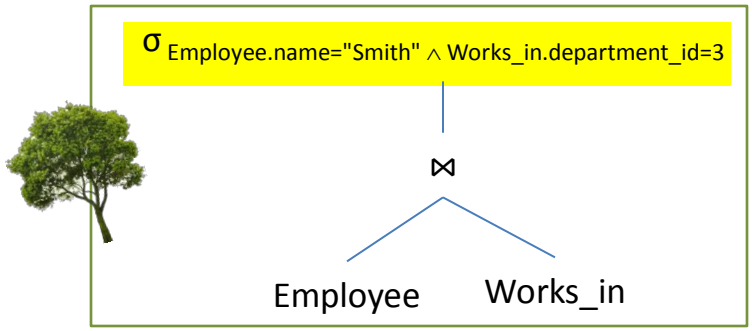
equivalent to



Equivalence rules

Expression tree A.

$\sigma_{\text{Employee.name="Smith"} \wedge \text{Works\_in.department\_id=3}}$(
Employee ⋈ Works_in
)



$\sigma_{\text{Employee.name="Smith"} \wedge \text{Works\_in.department\_id=3}}$(**Employee ⋈ Works_in**)

employee_id	name	salary	department_id
4	Smith	24000	3

Employee ⋈ Works_in

employee_id	name	salary	department_id
1	Jones	26000	1
2	Smith	28000	1
2	Smith	28000	2
3	Parker	35000	3
4	Smith	24000	3

Natural join evaluates 4*5 = 20 combinations

Employee

employee_id	name	salary
1	Jones	26000
2	Smith	28000
3	Parker	35000
4	Smith	24000

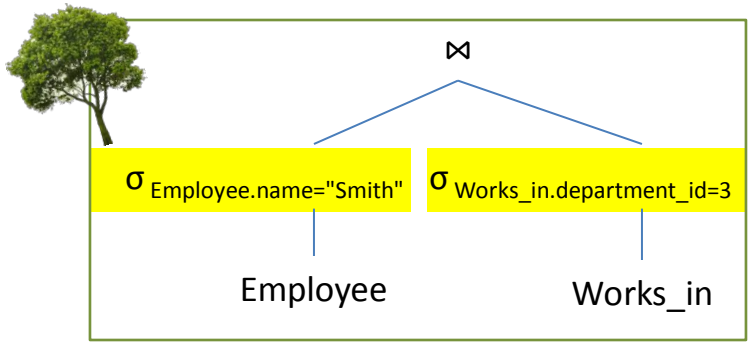
Works_in

employee_id	department_id
1	1
2	1
2	2
3	3
4	3

Equivalence rules

Expression tree B.

$$\left(\sigma_{\text{Employee.name="Smith"}} (\textbf{Employee}) \bowtie \sigma_{\text{Works\_in.department\_id=3}} (\textbf{Works\_in}) \right)$$



$$\left(\sigma_{\text{Employee.name="Smith"}} (\textbf{Employee}) \bowtie \sigma_{\text{Works\_in.department\_id=3}} (\textbf{Works\_in}) \right)$$

employee_id	name	salary	department_id
4	Smith	24000	3

Natural join evaluates 4 combinations

$\sigma_{\text{Employee.name="Smith"}}$ (**Employee**)

employee_id	name	salary
2	Smith	28000
4	Smith	24000

$\sigma_{\text{Works\_in.department\_id=3}}$ (**Works_in**)

employee_id	department_id
3	3
4	3

Employee

employee_id	name	salary
1	Jones	26000
2	Smith	28000
3	Parker	35000
4	Smith	24000

Works_in

employee_id	department_id
1	1
2	1
2	2
3	3
4	3

When comparing with the equivalence expression 1, we can see that if we push the selection predicates down the natural join (perform selection earlier than joining), the **natural join would consider fewer combinations**.

Equivalence rules

- **Rule 6.** The projection operation can distribute over the natural join operation.

$$\pi_{L1 \cup L2}(E_1 \bowtie E_2) = \pi_{L1 \cup L2}((\pi_{L1 \cup L3}(E_1)) \bowtie (\pi_{L2 \cup L3}(E_2)))$$

  - Let L1 and L2 be some attributes from E1 and E2,respectively.
  - Let L3 be attributes that are involved in join condition, but are not in $L1 \cup L2$.

Equivalence rules

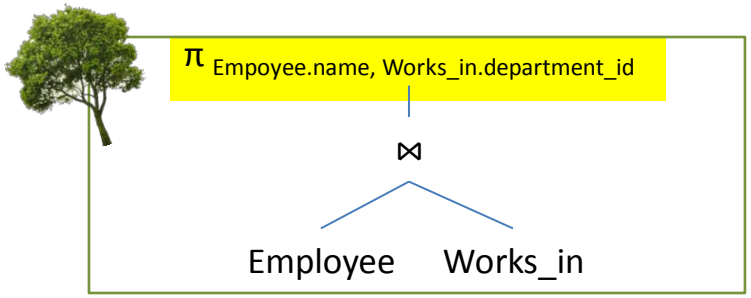
$$\pi_{L1 \cup L2}(E_1 \bowtie E_2) = \pi_{L1 \cup L2}((\pi_{L1 \cup L3}(E_1)) \bowtie (\pi_{L2 \cup L3}(E_2)))$$

$\pi_{\text{Empoyee.name, Works\_in.department\_id}}$ (**Employee ⋈ Works_in**)

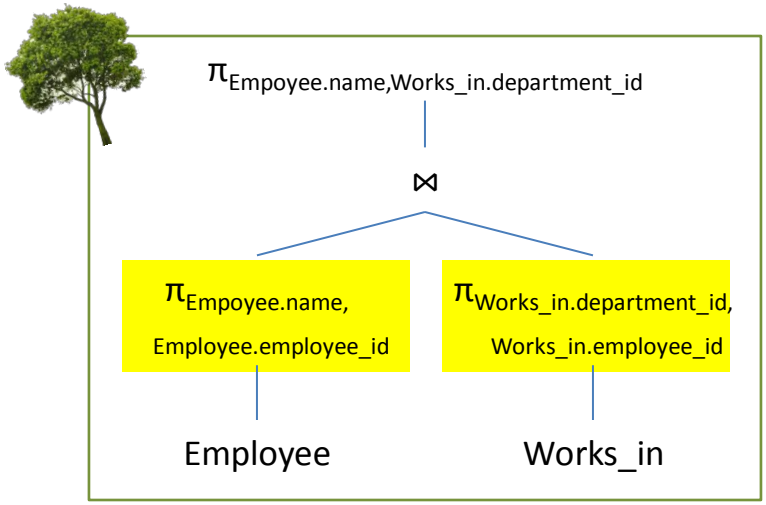
- L1 = Empoyee.name
- L2 = Works_in.department_id
- L3 = employee_id

The attribute used in natural join

$\pi_{\text{Empoyee.name, Works\_in.department\_id}}$ (
$\pi_{\text{Empoyee.name, Employee.employee\_id}}$ (Employee)
⋈
$\pi_{\text{Works\_in.department\_id, Works\_in.employee\_id}}$ (Works_in)
)



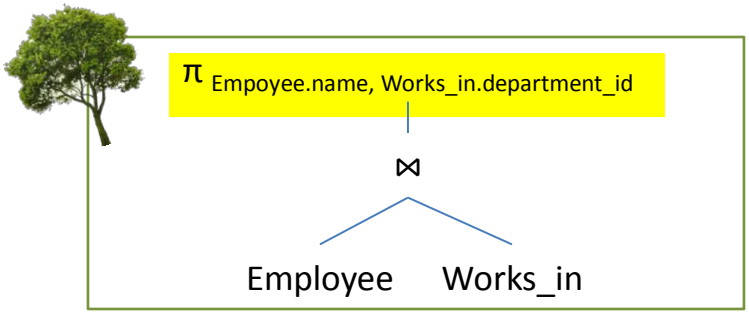
equivalent to



Equivalence rules

Expression tree A.

π Empoyee.name, Works_in.department_id
(**Employee ⋈ Works_in**)



π Empoyee.name, Works_in.department_id (**Employee ⋈ Works_in**)

name	department_id
Jones	1
Smith	1
Smith	2
Parker	3
Smith	3

Employee ⋈ Works_in

employee_id	name	salary	department_id	since
1	Jones	26000	1	2012/1/1
2	Smith	28000	1	2011/3/2
2	Smith	28000	2	2014/2/1
3	Parker	35000	3	2013/2/2
4	Smith	24000	3	2013/2/8

Natural join evaluates 4*5 = 20 combinations, result temporary relation consists of 5 columns.

Employee

employee_id	name	salary
1	Jones	26000
2	Smith	28000
3	Parker	35000
4	Smith	24000

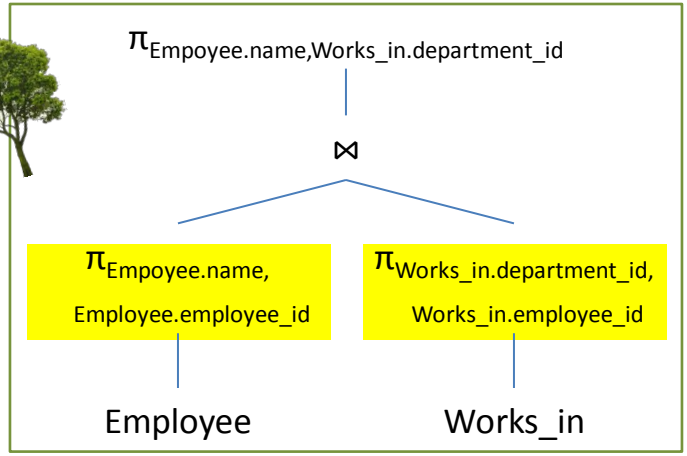
Works_in

employee_id	department_id	since
1	1	2012/1/1
2	1	2011/3/2
2	2	2014/2/1
3	3	2013/2/2
4	3	2013/2/8

Equivalence rules

Expression tree B.

π Empoyee.name, Works_in.department_id (π Empoyee.name, Employee.employee_id (Employee) $\bowtie$ π Works_in.department_id, Works_in.employee_id(Works_in))



π Empoyee.name, Works_in.department_id (π Empoyee.name, Employee.employee_id(**Employee**) $\bowtie$ π Works_in.department_id, Works_in.employee_id (**Works_in**))

name	department_id
Jones	1
Smith	1
Smith	2
Parker	3
Smith	3

π Empoyee.name, Employee.employee_id(**Employee**) $\bowtie$ π Works_in.department_id, Works_in.employee_id (**Works_in**)

employee_id	name	department_id
1	Jones	1
2	Smith	1
2	Smith	2
3	Parker	3
4	Smith	3

Natural join evaluates 4*5 = 20 combinations, result temporary relation consists of 3 columns.

π Empoyee.name, Employee.employee_id(**Employee**)

employee_id	name
1	Jones
2	Smith
3	Parker
4	Smith

π Works_in.department_id, Works_in.employee_id (**Works_in**)

employee_id	department_id
1	1
2	1
2	2
3	3
4	3

Employee

employee_id	name	salary
1	Jones	26000
2	Smith	28000
3	Parker	35000
4	Smith	24000

Works_in

employee_id	department_id	since
1	1	2012/1/1
2	1	2011/3/2
2	2	2014/2/1
3	3	2013/2/2
4	3	2013/2/8

Equivalence rules

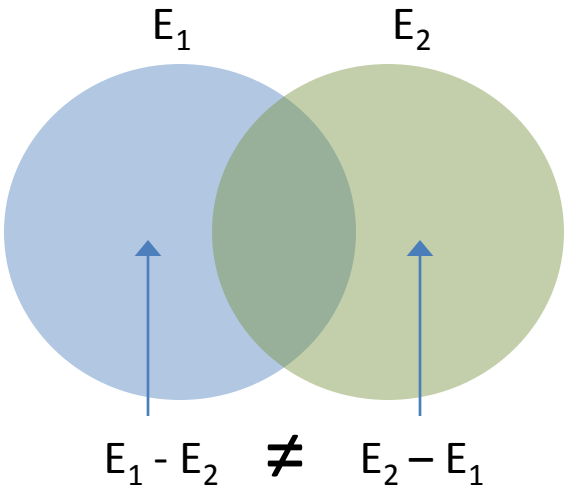
- **Rule 7.** The set operations union and intersections are commutative.

$$E_1 \cup E_2 = E_2 \cup E_1$$

$$E_1 \cap E_2 = E_2 \cap E_1$$

- The set different operation is NOT commutative

$$E_1 - E_2 \neq E_2 - E_1$$



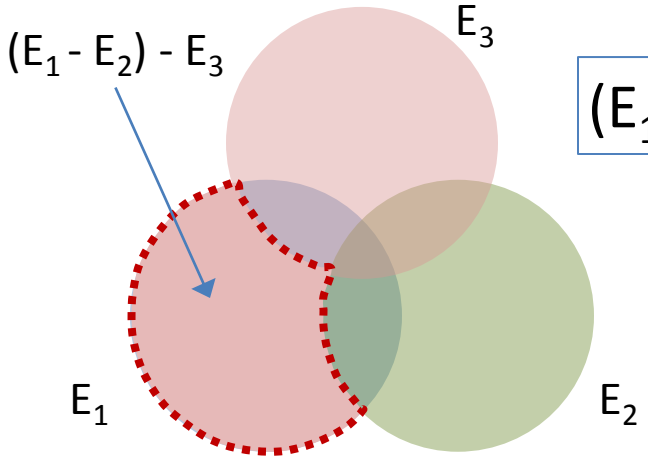
Equivalence rules

- **Rule 8.** The set operations union and intersections are associative.

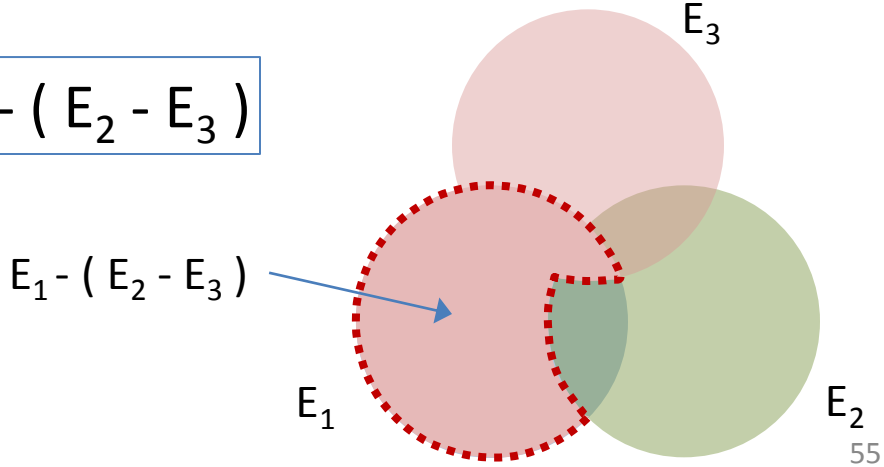
$$(E_1 \cup E_2) \cup E_3 = E_1 \cup (E_2 \cup E_3)$$

$$(E_1 \cap E_2) \cap E_3 = E_1 \cap (E_2 \cap E_3)$$

- The set different operation is NOT associative.



$$(E_1 - E_2) - E_3 \neq E_1 - (E_2 - E_3)$$



Equivalence rules

- **Rule 9.** The selection operation distributes over the union, intersection and set difference operations

$$\sigma_p(E_1 \cup E_2) = \sigma_p(E_1) \cup \sigma_p(E_2)$$

$$\sigma_p(E_1 \cap E_2) = \sigma_p(E_1) \cap \sigma_p(E_2)$$

$$\sigma_p(E_1 - E_2) = \sigma_p(E_1) - \sigma_p(E_2)$$

Audio_CD

ID	name	provider_id	stock	#tracks
CD1	One Heart	P1	55	14
CD2	Miracle	P2	4	14

DVD

ID	name	provider_id	stock	length
DVD1	Prince of Persia	P2	3	110
DVD2	Iron man 3	P3	60	90
DVD3	Legend is born: Ip Man	P3	17	90

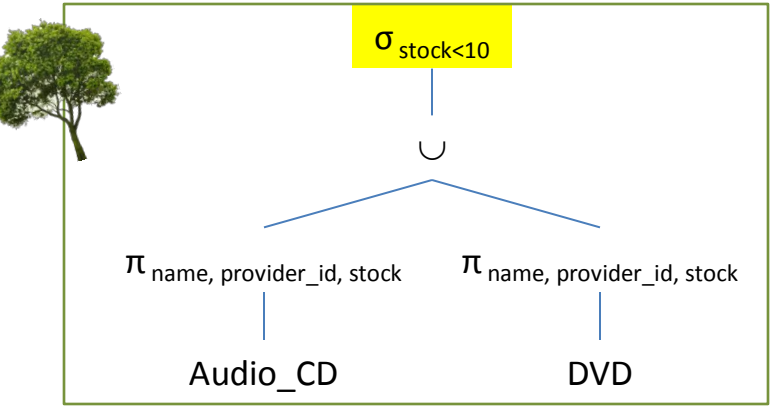
$$\sigma_{stock<10}\left(\pi_{name,\ provider\_id,\ stock}(Audio\_CD) \cup \pi_{name,\ provider\_id,\ stock}(DVD)\right)$$

equivalent to

$$\sigma_{stock<10}(\pi_{name,\ provider\_id,\ stock}(Audio\_CD)) \cup \sigma_{stock<10}(\pi_{name,\ provider\_id,\ stock}(DVD))$$

Equivalence rules

$$\sigma_{stock<10}\left(\pi_{name,\ provider\_id,\ stock}(Audio\_CD) \cup \pi_{name,\ provider\_id,\ stock}(DVD)\right)$$



$\sigma_{stock<10}(\pi_{name,\ provider\_id,\ stock}(\textbf{Audio\_CD}) \cup \pi_{name,\ provider\_id,\ stock}(\textbf{DVD}))$

name	provider_id	stock
Miracle	P2	4
Prince of Persia	P2	3

$\pi_{name,\ provider\_id,\ stock}(\textbf{Audio\_CD}) \cup \pi_{name,\ provider\_id,\ stock}(\textbf{DVD})$

name	provider_id	stock
One Heart	P1	55
Miracle	P2	4
Prince of Persia	P2	3
Iron man 3	P3	60
Legend is born: Ip Man	P3	17

Audio_CD

ID	name	provider_id	stock	#tracks
CD1	One Heart	P1	55	14
CD2	Miracle	P2	4	14

$\pi_{name,\ provider\_id,\ stock}(\textbf{Audio\_CD})$

name	provider_id	stock
One Heart	P1	55
Miracle	P2	4

DVD

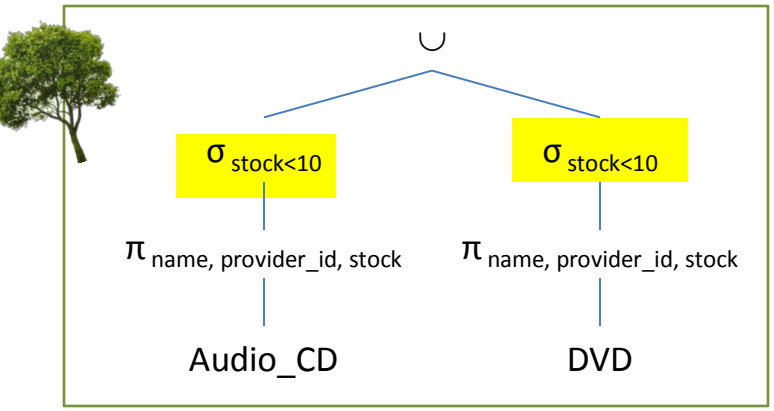
ID	name	provider_id	stock	length
DVD1	Prince of Persia	P2	3	110
DVD2	Iron man 3	P3	60	90
DVD3	Legend is born: Ip Man	P3	17	90

$\pi_{name,\ provider\_id,\ stock}(\textbf{DVD})$

name	provider_id	stock
Prince of Persia	P2	3
Iron man 3	P3	60
Legend is born: Ip Man	P3	17

Equivalence rules

$\sigma_{stock<10}(\pi_{name, provider\_id, stock}(Audio\_CD)) \cup \sigma_{stock<10}(\pi_{name, provider\_id, stock}(DVD))$



Audio_CD

ID	name	provider_id	stock	#tracks
CD1	One Heart	P1	55	14
CD2	Miracle	P2	4	14

DVD

ID	name	provider_id	stock	length
DVD1	Prince of Persia	P2	3	110
DVD2	Iron man 3	P3	60	90
DVD3	Legend is born: Ip Man	P3	17	90

$\pi_{name, provider\_id, stock}(Audio\_CD)$

name	provider_id	stock
One Heart	P1	55
Miracle	P2	4

$\pi_{name, provider\_id, stock}(DVD)$

name	provider_id	stock
Prince of Persia	P2	3
Iron man 3	P3	60
Legend is born: Ip Man	P3	17

$\sigma_{stock<10}(\pi_{name, provider\_id, stock}(Audio\_CD))$

name	provider_id	stock
Miracle	P2	4

$\sigma_{stock<10}(\pi_{name, provider\_id, stock}(DVD))$

name	provider_id	stock
Prince of Persia	P2	3

$\sigma_{stock<10}(\pi_{name, provider\_id, stock}(Audio\_CD)) \cup \sigma_{stock<10}(\pi_{name, provider\_id, stock}(DVD))$

name	provider_id	stock
Miracle	P2	4
Prince of Persia	P2	3

Equivalence rules

- **Rule 10.** The projection operation distributes over the union operation

$$\pi_L (E_1 \cup E_2) = \pi_L (E_1) \cup \pi_L (E_2)$$

- Projection does not distribute over intersection and set difference.

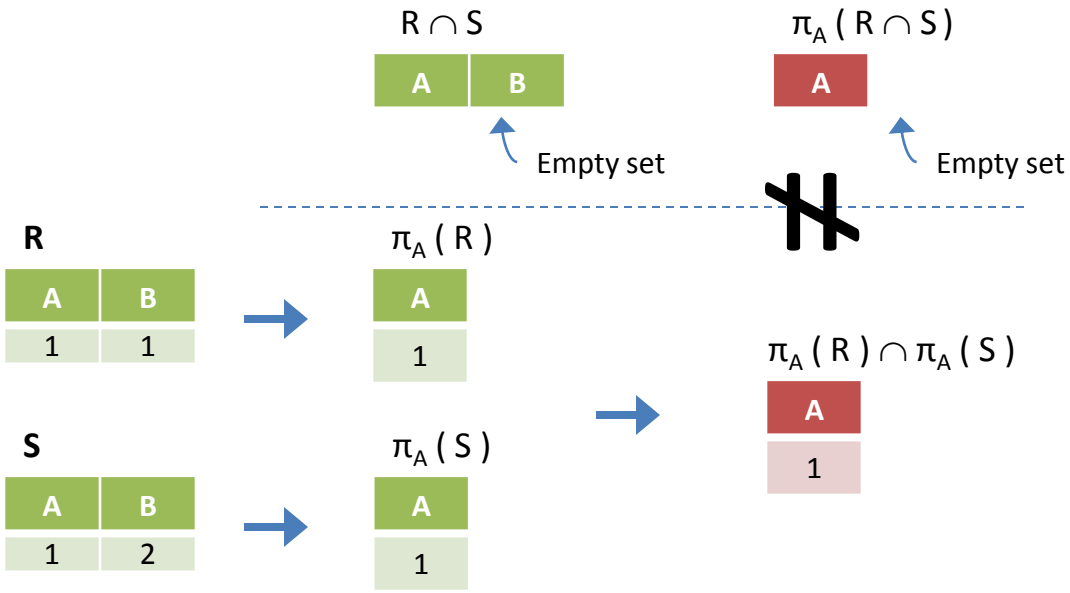
$$\pi_L (E_1 \cap E_2) \neq \pi_L (E_1) \cap \pi_L (E_2)$$

$$\pi_L (E_1 - E_2) \neq \pi_L (E_1) - \pi_L (E_2)$$

Equivalence rules

Why projection does not distribute over intersection and set difference?

To show that projection does not distribute over intersection, you only need to provide a counter example.



This counter example shows that

$\pi_A (R \cap S) \neq \pi_A (R) \cap \pi_A (S)$

Now, can you try to construct a counter example to show that projection does not distribute over **set difference**?

Equivalence rules

- Note that the equivalence rules listed are just a partial list of equivalences.
  - Please refer to the textbook for more equivalences involving extended relational operators, such as the outer join and aggregation.

Section 4

Example of query optimization

Transformation

- Find the names of all instructors in the CS department (dpt_id = 1) who have taught a course in 2nd semester, together with the course title of all the courses that the instructors teach.

```
SELECT I.name, C.title
FROM Instructor I, Teaches T, Course C
WHERE I.dpt_id = 1  AND
        T.sem=2 AND
        I.instructor_id = T.instructor_id AND
        T.course_id = C.course_id ;
```

SQL

$$\pi_{I.name,C.title}\left(\sigma_{I.dpt\_id=1 \wedge T.sem=2}\left(\rho_I(\text{Instructor}) \bowtie (\rho_T(\text{Teaches}) \bowtie \rho_C(\text{Course}))\right)\right)$$

Relational algebra

Instructor

instructor_id	name	dpt_id
1	Kit	1
2	Ben	1
3	Michael	2
4	William	3

Teaches

instructor_id	course_id	sem
1	1	1
1	2	2
2	4	1
3	3	2

Course

course_id	title	credit
1	Intro to DB	6
2	Programming I	6
3	Accounting	6
4	Algorithms	6

Transformation

$\pi_{I.name,C.title}$ (
  $\sigma_{I.dpt\_id=1 \wedge T.sem=2}$(
    ρ_I(**Instructor**) $\bowtie$ (ρ_T(**Teaches**) $\bowtie$ ρ_C(**Course**))
)
)

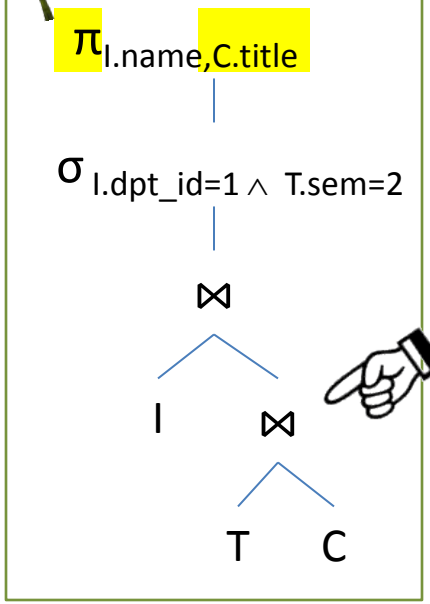
equivalent to

$\pi_{I.name,C.title}$ (
  $\sigma_{I.dpt\_id=1 \wedge T.sem=2}$(
    ρ_I(**Instructor**) $\bowtie$
    (ρ_T(**Teaches**) $\bowtie$ $\pi_{C.course\_id,\ C.title}$ (ρ_C(**Course**)))
)
)

Rule 6

$\pi_{L1 \cup L2}(E_1 \bowtie E_2) = \pi_{L1 \cup L2}((\pi_{L1 \cup L3}(E_1)) \bowtie (\pi_{L2 \cup L3}(E_2)))$

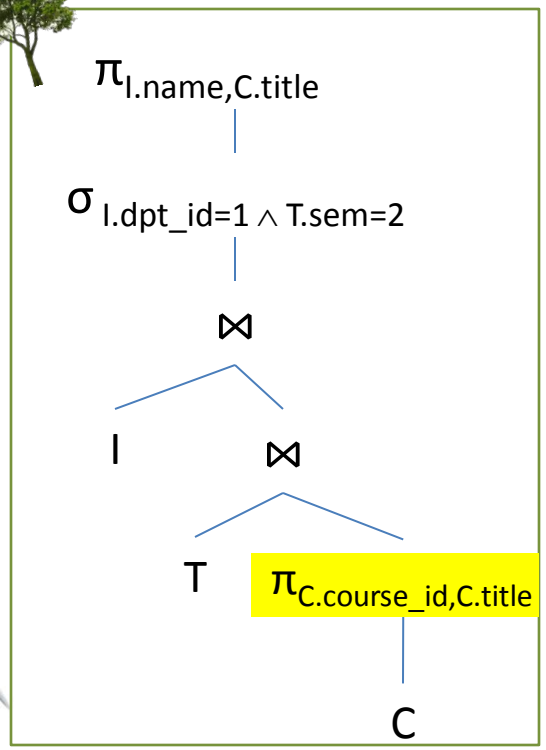
BEFORE



Let's try to push the projection $\pi_{C.title}$ downward and apply it ahead of the natural joins.

Since this natural join requires C.course_id = T.course_id, therefore, **we have to add the joining attribute C.course_id to make the projection $\pi_{C.course\_id,C.title}$**

AFTER



Transformation

$\pi_{\text{I.name,C.title}}$ (
$\sigma_{\text{I.dpt\_id =1} \wedge \text{T.sem=2}}$(
(ρ_{I}(**Instructor**) $\bowtie$ ρ_{T}(**Teaches**))
$\bowtie$ $\pi_{\text{C.course\_id, C.title}}$ (ρ_{C}(**Course**))
)
)

equivalent to

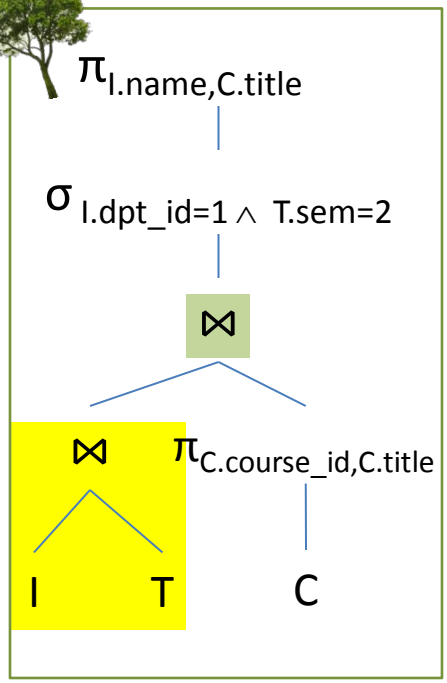
$\pi_{\text{I.name,C.title}}$ (
$\sigma_{\text{I.dpt\_id =1} \wedge \text{T.sem=2}}$(
ρ_{I}(**Instructor**) $\bowtie$
(ρ_{T}(**Teaches**) $\bowtie$ $\pi_{\text{C.course\_id, C.title}}$ (ρ_{C}(**Course**)))
)
)

Rule 4

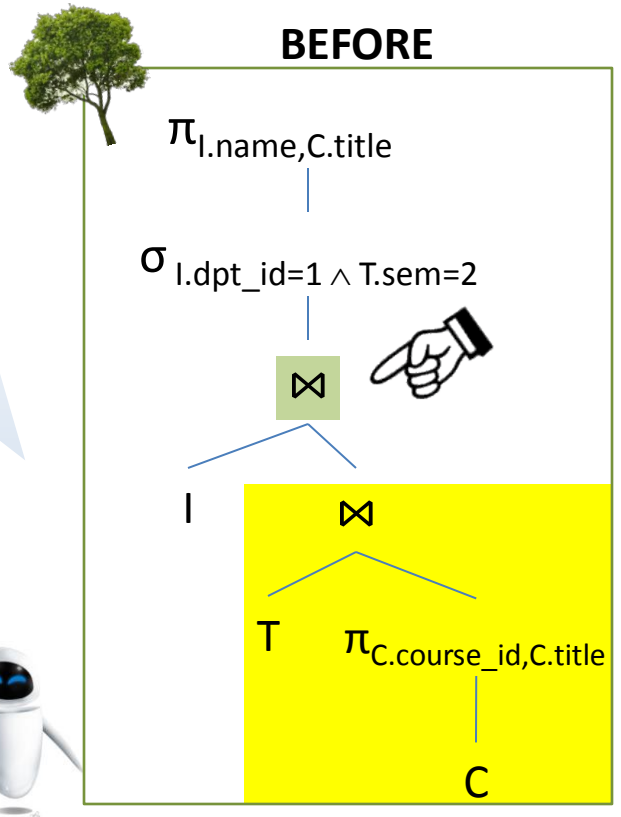
$$(E_1 \bowtie E_2) \bowtie E_3 = E_1 \bowtie (E_2 \bowtie E_3)$$

- Now we would like to push the selection down to reduce the size of the temporary result of the natural join.
- As the selection involves relations I and T only, we would like to rearrange the natural joins to make I and T under one natural join.
- Since natural joins are associative, we can make such rearrangement.

AFTER



BEFORE



Transformation

$\pi_{\text{I.name,C.title}}$ (
$\sigma_{\text{I.dpt\_id =1} \wedge \text{T.sem=2}}$(
(ρ_{I}(**Instructor**) $\bowtie$ ρ_{T}(**Teaches**))
$\bowtie$ $\pi_{\text{C.course\_id, C.title}}$ (ρ_{C}(**Course**))
)
)

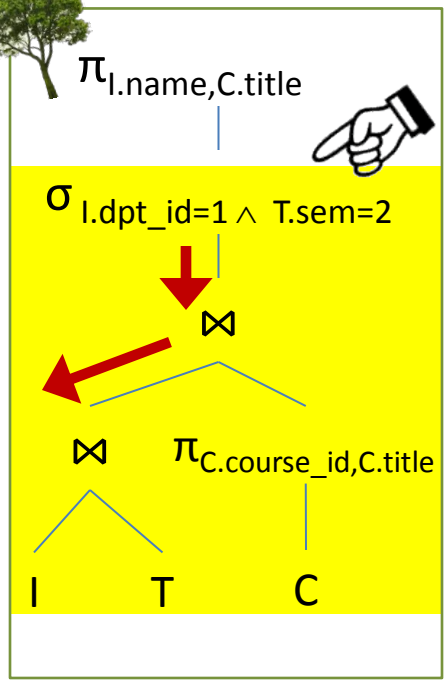
equivalent to

$\pi_{\text{I.name,C.title}}$ (
($\sigma_{\text{I.dpt\_id =1} \wedge \text{T.sem=2}}$(
ρ_{I}(**Instructor**) $\bowtie$ ρ_{T}(**Teaches**)
)
)
$\bowtie$ $\pi_{\text{C.course\_id, C.title}}$ (ρ_{C}(**Course**))
)

Rule 5a

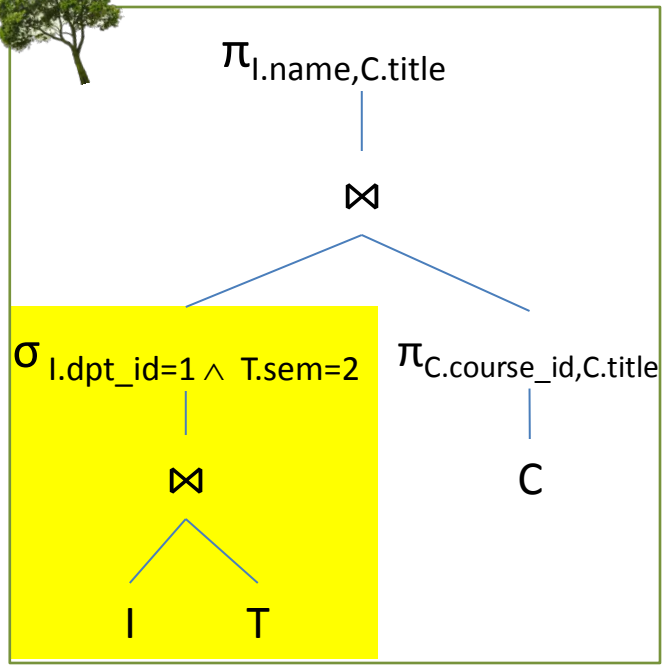
$$\sigma_{p1}(E_1 \bowtie E_2) = (\sigma_{p1}(E_1) \bowtie E_2)$$

BEFORE



- Now we can push the selection down one level.
- According to Rule 5a, we can distribute both selection predicates to the L.H.S. of the selection as the R.H.S. does not contain any attribute in the selection predicate.

AFTER



Transformation

$\pi_{I.name,C.title}$ (
($\sigma_{I.dpt\_id=1}$ (ρ_I(**Instructor**))
⋈
$\sigma_{T.sem=2}$ (ρ_T(**Teaches**))
)
⋈ $\pi_{C.course\_id,\ C.title}$ (ρ_C(**Course**))
)

equivalent to

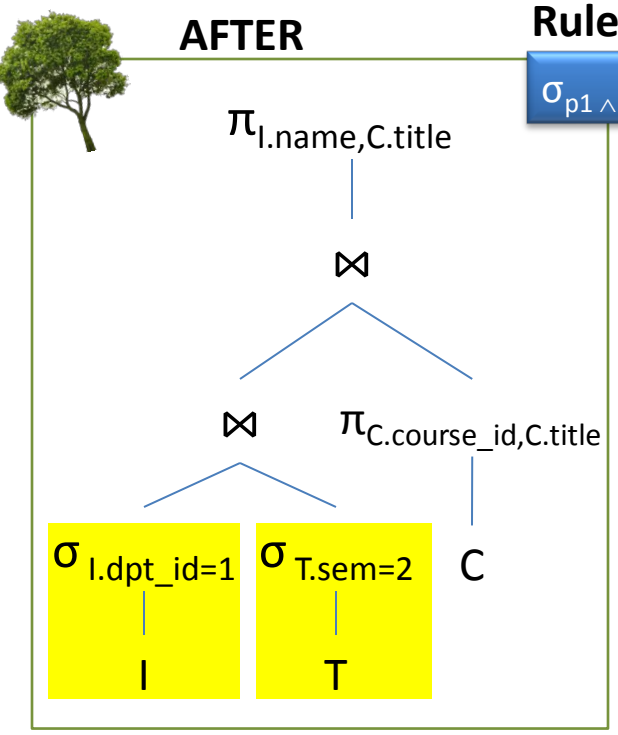
$\pi_{I.name,C.title}$ (
($\sigma_{I.dpt\_id=1\ \wedge\ T.sem=2}$(
ρ_I(**Instructor**) ⋈ ρ_T(**Teaches**)
)
)
⋈ $\pi_{C.course\_id,\ C.title}$ (ρ_C(**Course**))
)

Rule 5b

$\sigma_{p1\ \wedge\ p2}(E_1 \bowtie E_2) = (\sigma_{p1}(E_1) \bowtie \sigma_{p1}(E_2))$

- Now we can further push the selection one more level down by applying rule 5b.
- According to Rule 5b, we can distribute
  - $\sigma_{I.dpt\_id=1}$ to I
  - $\sigma_{T.sem=2}$ to T

AFTER



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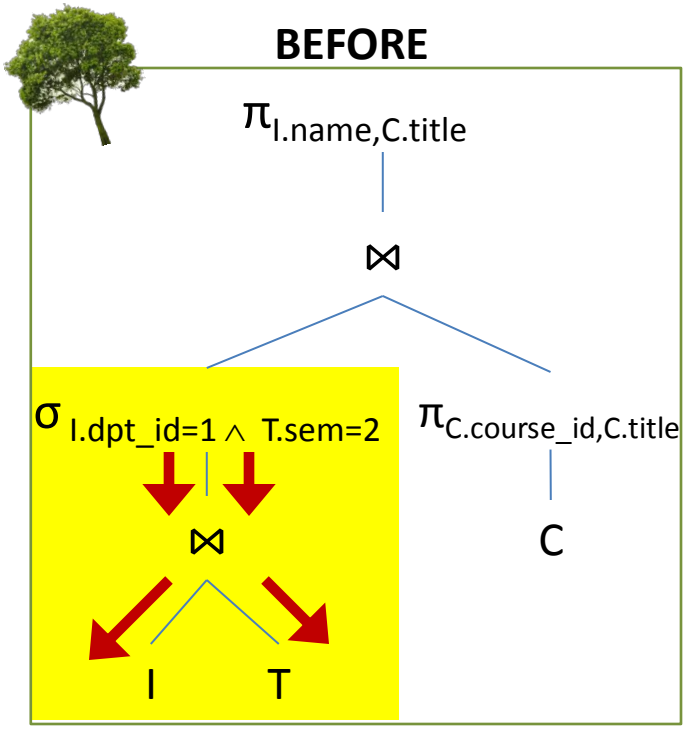
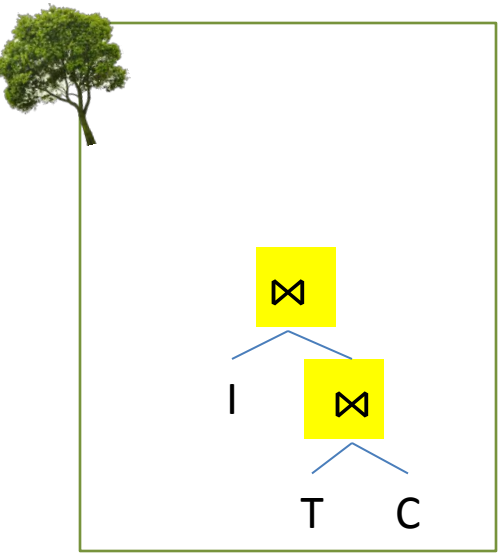


Illustration (original tree)

$\pi_{I.name,C.title}$ (
  $\sigma_{I.dpt\_id=1 \wedge T.sem=2}$(
    ρ_I(**Instructor**) ⋈
    (ρ_T(**Teaches**) ⋈ ρ_C(**Course**))
)
)



ρ_I(**Instructor**) ⋈ (ρ_T(**Teaches**) ⋈ ρ_C(**Course**))

instructor_id	name	dpt_id	course_id	sem	title	credit
1	Kit	1	1	1	Intro to DB	6
1	Kit	1	2	2	Programming I	6
2	Ben	1	4	1	Algorithms	6
3	Michael	2	3	2	Accounting	6

Natural join evaluates 4*4 = 16 combinations.

ρ_T(**Teaches**) ⋈ ρ_C(**Course**)

instructor_id	name	dpt_id	course_id	sem
1	Kit	1	1	1
1	Kit	1	2	2
2	Ben	1	4	1
3	Michael	2	3	2

Natural join evaluates 4*4 = 16 combinations.

Instructor

instructor_id	name	dpt_id
1	Kit	1
2	Ben	1
3	Michael	2
4	William	3

Teaches

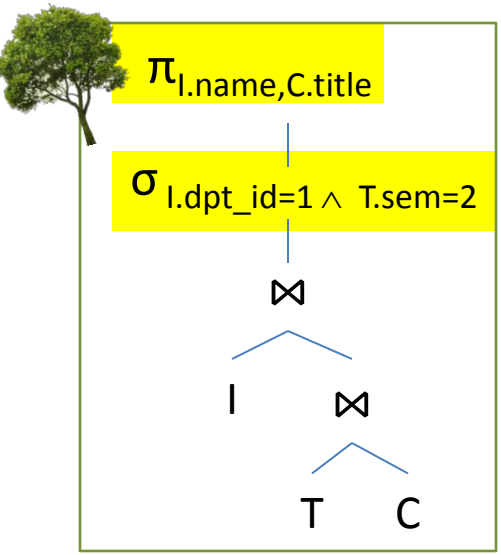
instructor_id	course_id	sem
1	1	1
1	2	2
2	4	1
3	3	2

Course

course_id	title	credit
1	Intro to DB	6
2	Programming I	6
3	Accounting	6
4	Algorithms	6

Illustration (original tree)

$\pi_{I.name,C.title}$ (
  $\sigma_{I.dpt\_id=1 \wedge T.sem=2}$(
    ρ_I(**Instructor**) ⋈
    (ρ_T(**Teaches**) ⋈ ρ_C(**Course**))
)
)



ρ_I(**Instructor**) ⋈ (ρ_T(**Teaches**) ⋈ ρ_C(**Course**))

instructor_id	name	dpt_id	course_id	sem	title	credit
1	Kit	1	1	1	Intro to DB	6
1	Kit	1	2	2	Programming I	6
2	Ben	1	4	1	Algorithms	6
3	Michael	2	3	2	Accounting	6

$\sigma_{I.dpt\_id=1 \wedge T.sem=2}$(ρ_I(**Instructor**) ⋈ (ρ_T(**Teaches**) ⋈ ρ_C(**Course**)))

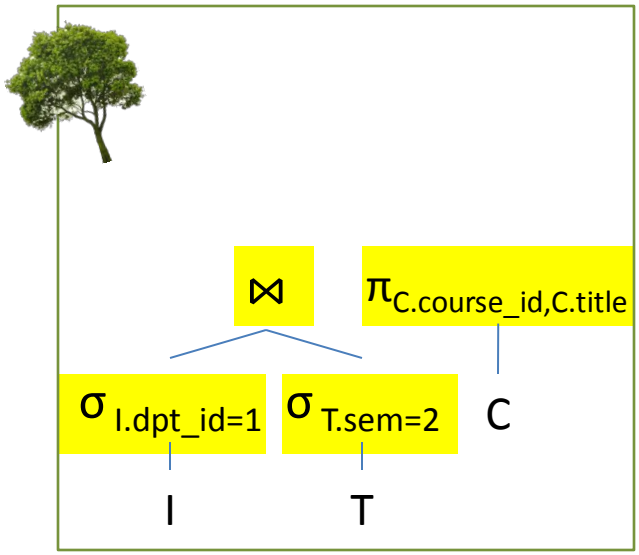
instructor_id	name	dpt_id	course_id	sem	title	credit
1	Kit	1	2	2	Programming I	6

$\pi_{I.name,C.title}$ ($\sigma_{I.dpt\_id=1 \wedge T.sem=2}$(ρ_I(**Instructor**) ⋈ (ρ_T(**Teaches**) ⋈ ρ_C(**Course**))))

name	title
Kit	Programming I

Illustration (transformed tree)

$\pi_{\text{I.name,C.title}}$ (
($\sigma_{\text{I.dpt\_id =1}}$ (ρ_{I}(**Instructor**)) ⋈ $\sigma_{\text{T.sem=2}}$ (ρ_{T}(**Teaches**)))
⋈ $\pi_{\text{C.course\_id, C.title}}$ (ρ_{C}(**Course**))
)



$\sigma_{\text{I.dpt\_id =1}}$ (ρ_{I}(**Instructor**)) ⋈ $\sigma_{\text{T.sem=2}}$ (ρ_{T}(**Teaches**))

instructor_id	name	dpt_id	course_id	sem
1	Kit	1	2	2

Natural join evaluates 2*2 = 4 combinations.

$\sigma_{\text{I.dpt\_id =1}}$ (ρ_{I}(**Instructor**))

instructor_id	name	dpt_id
1	Kit	1
2	Ben	1

$\sigma_{\text{T.sem=2}}$ (ρ_{T}(**Teaches**))

instructor_id	course_id	sem
1	2	2
3	3	2

$\pi_{\text{C.course\_id, C.title}}$ (ρ_{C}(**Course**))

course_id	title
1	Intro to DB
2	Programming I
3	Accounting
4	Algorithms

Instructor

instructor_id	name	dpt_id
1	Kit	1
2	Ben	1
3	Michael	2
4	William	3

Teaches

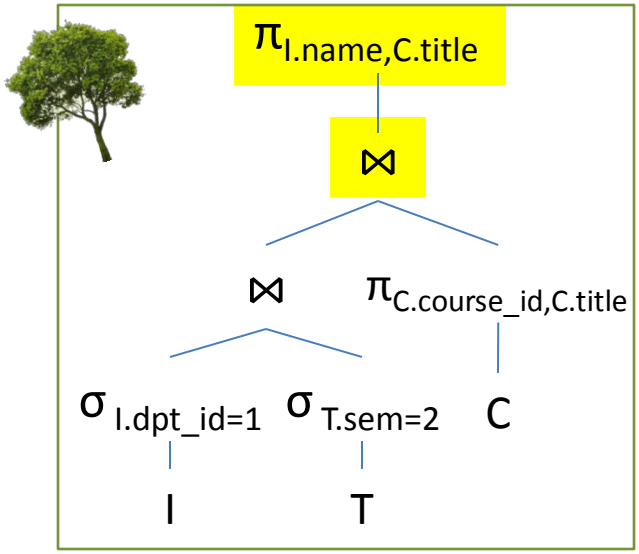
instructor_id	course_id	sem
1	1	1
1	2	2
2	4	1
3	3	2

Course

course_id	title	credit
1	Intro to DB	6
2	Programming I	6
3	Accounting	6
4	Algorithms	6

Illustration (transformed tree)

$$\pi_{I.name,C.title}\Big(\big(\sigma_{I.dpt\_id=1}(\rho_I(\textbf{Instructor}))\bowtie\sigma_{T.sem=2}(\rho_T(\textbf{Teaches}))\big)\bowtie\pi_{C.course\_id,C.title}(\rho_C(\textbf{Course}))\Big)$$



$\sigma_{I.dpt\_id=1}(\rho_I(\textbf{Instructor}))\bowtie\sigma_{T.sem=2}(\rho_T(\textbf{Teaches}))$

instructor_id	name	dpt_id	course_id	sem
1	Kit	1	2	2

Natural join evaluates 1*4 = 4 combinations.

$\pi_{C.course\_id,C.title}(\rho_C(\textbf{Course}))$

course_id	title
1	Intro to DB
2	Programming I
3	Accounting
4	Algorithms

$\sigma_{I.dpt\_id=1}(\rho_I(\textbf{Instructor}))\bowtie\sigma_{T.sem=2}(\rho_T(\textbf{Teaches}))\bowtie\pi_{C.course\_id,C.title}(\rho_C(\textbf{Course}))$

instructor_id	name	dpt_id	course_id	sem	title
1	Kit	1	2	2	Programming I

$\pi_{I.name,C.title}(\sigma_{I.dpt\_id=1}(\rho_I(\textbf{Instructor}))\bowtie\sigma_{T.sem=2}(\rho_T(\textbf{Teaches}))\bowtie\pi_{C.course\_id,C.title}(\rho_C(\textbf{Course})))$

name	title
Kit	Programming I

Summary

- **Relational algebra (RA) defines a set of algebraic operations on tables, and output tables as result.**
  - 6 fundamental operations (in Chapter 4A).
  - Additional operations does not extend the power of the fundamental operators, but they simplify the expression.
  - Extended operations add expressive power.
- **Relational algebra (RA) is the basics of query optimization.**
  - More optimization techniques are discussed in Chapter 13 of the textbook.

Chapter 4B.

END

CSIS0278 / COMP3278

Introduction to

Database Management Systems

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