Dirac bilinears are constructed by sandwiching $\{I, \gamma^\mu, \sigma^{\mu\nu}, \gamma^\mu\gamma_5, \gamma_5\}$ between $\bar{\psi} = \psi^\dagger\gamma^0$ and ψ. It is straightforward to show that they transform like they should, by using the "Weyl map" relations proved in earlier document.

$$\bar{\sigma}^\mu \Lambda_\mu{}^\nu = (S_R^{-1})^\dagger \bar{\sigma}^\nu S_R^{-1} = S_L \bar{\sigma}^\nu S_L^\dagger \tag{1}$$
$$\sigma^\mu \Lambda_\mu{}^\nu = (S_L^{-1})^\dagger \sigma^\nu S_L^{-1} = S_R \sigma^\nu S_R^\dagger \tag{2}$$
$$\Lambda^\nu{}_\mu \bar{\sigma}^\mu = S_R^{-1} \bar{\sigma}^\nu (S_R^{-1})^\dagger = S_L^\dagger \bar{\sigma}^\nu S_L \tag{3}$$
$$\Lambda^\nu{}_\mu \sigma^\mu = S_L^{-1} \sigma^\nu (S_L^{-1})^\dagger = S_R^\dagger \sigma^\nu S_R \tag{4}$$

Also recall

$$\psi = \begin{bmatrix} u \\ v \end{bmatrix}, \quad \gamma^\mu = \begin{bmatrix} 0 & \bar{\sigma}^\mu \\ \sigma^\mu & 0 \end{bmatrix}, \quad \bar{\psi} = \psi^\dagger\gamma^0 = \begin{bmatrix} u^\dagger & v^\dagger \end{bmatrix} \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} = \begin{bmatrix} v^\dagger & u^\dagger \end{bmatrix},$$
$$\gamma_5 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}, \quad \sigma^{\mu\nu} = \frac{i}{2} \begin{bmatrix} \bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu & 0 \\ 0 & \sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu \end{bmatrix}$$

1. $\bar{\psi}\psi$ transforms like scalar.

$$\bar{\psi}\psi = \begin{bmatrix} v^\dagger & u^\dagger \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = v^\dagger u + u^\dagger v \to v^\dagger S_L^\dagger S_R u + u^\dagger S_R^\dagger S_L v = v^\dagger u + u^\dagger v$$

2. $\bar{\psi}\gamma^\mu\psi$ transforms like vector.

$$\bar{\psi}\gamma^\mu\psi = \begin{bmatrix} v^\dagger & u^\dagger \end{bmatrix} \begin{bmatrix} 0 & \bar{\sigma}^\mu \\ \sigma^\mu & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u^\dagger\sigma^\mu & v^\dagger\bar{\sigma}^\mu \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = u^\dagger\sigma^\mu u + v^\dagger\bar{\sigma}^\mu v \quad \to$$
$$u^\dagger S_R^\dagger \sigma^\mu S_R u + v^\dagger S_L^\dagger \bar{\sigma}^\mu S_L v = u^\dagger \Lambda^\mu{}_\nu \sigma^\nu u + v^\dagger \Lambda^\mu{}_\nu \bar{\sigma}^\nu v = \Lambda^\mu{}_\nu \left(\bar{\psi}\gamma^\nu\psi\right)$$

3. $\bar{\psi}\sigma^{\mu\nu}\psi$ transforms like antisymmetric tensor (ignoring factor $i/2$).

$$\begin{aligned}\bar{\psi}\sigma^{\mu\nu}\psi &= \begin{bmatrix} v^\dagger & u^\dagger \end{bmatrix} \begin{bmatrix} \bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu & 0 \\ 0 & \sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \\ &= \begin{bmatrix} v^\dagger (\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu) & u^\dagger (\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \\ &= v^\dagger (\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu) u + u^\dagger (\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu) v \quad \to \\ & \quad v^\dagger S_L^\dagger (\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu) S_R u + u^\dagger S_R^\dagger (\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu) S_L v\end{aligned}$$

where we see that

$$S_L^\dagger \bar{\sigma}^\mu \sigma^\nu S_R = S_L^\dagger \bar{\sigma}^\mu S_L \cdot S_R^\dagger \sigma^\nu S_R = \Lambda^\mu{}_\rho \bar{\sigma}^\rho \cdot \Lambda^\nu{}_\tau \sigma^\tau$$

etc., are how the 2-index tensor components transform under Λ.

4. $\bar{\psi}\gamma^\mu\gamma_5\psi$ transforms like pseudovector.

$$\begin{aligned}\bar{\psi}\gamma^\mu\gamma_5\psi &= \begin{bmatrix} v^\dagger & u^\dagger \end{bmatrix} \begin{bmatrix} 0 & \bar{\sigma}^\mu \\ \sigma^\mu & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} v^\dagger & u^\dagger \end{bmatrix} \begin{bmatrix} 0 & -\bar{\sigma}^\mu \\ \sigma^\mu & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \\ &= \begin{bmatrix} u^\dagger\sigma^\mu & -v^\dagger\bar{\sigma}^\mu \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = u^\dagger\sigma^\mu u - v^\dagger\bar{\sigma}^\mu v\end{aligned}$$

Now compare with case 2 above, it's clear it transforms with Λ. But since under space reflection, $J_{+i} \longleftrightarrow J_{-i}$ hence $u \longleftrightarrow v$, space reflection will add a minus sign, hence $\bar{\psi}\gamma^\mu\gamma_5\psi$ is a pseudovector.

5. $\bar{\psi}\gamma_5\psi$ transforms like pseudoscalar.

$$\bar{\psi}\gamma_5\psi = \begin{bmatrix} v^\dagger & u^\dagger \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} v^\dagger & -u^\dagger \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = v^\dagger u - u^\dagger v$$

Compare with case 1, it clearly transforms like scalar under Λ, but changes sign under space reflection.