The determinant relations we'll be using are

$$\epsilon_{i_1 i_2 \cdots i_N} U^{i_1}_{j_1} U^{i_2}_{j_2} \cdots U^{i_N}_{j_N} = \epsilon_{j_1 j_2 \cdots j_N} \quad \Longrightarrow \quad \epsilon_{i_1 i_2 \cdots i_{N-1} p_N} U^{i_1}_{j_1} U^{i_2}_{j_2} \cdots U^{i_{N-1}}_{j_{N-1}} = \epsilon_{j_1 j_2 \cdots j_N} \left(U^\dagger\right)^{j_N}_{p_N} \tag{1}$$

$$\epsilon^{j_1 j_2 \cdots j_N} U^{i_1}_{j_1} U^{i_2}_{j_2} \cdots U^{i_N}_{j_N} = \epsilon^{i_1 i_2 \cdots i_N} \quad \Longrightarrow \quad \epsilon^{j_1 j_2 \cdots j_{N-1} q_N} U^{i_1}_{j_1} U^{i_2}_{j_2} \cdots U^{i_{N-1}}_{j_{N-1}} = \epsilon^{i_1 i_2 \cdots i_N} \left(U^\dagger\right)^{q_N}_{i_N} \tag{2}$$

Now a few more concrete examples how indices can be moved.
First, in $SU(4)$

$$\phi^{ilm} \equiv \epsilon^{jklm} \phi^i_{jk} \tag{3}$$

I.e., we can canel the two lower indices in one ϵ multiplication.

In fact, under U, the RHS transforms like

$$\epsilon^{jklm} \phi^i_{jk} \longrightarrow \epsilon^{jklm} U^i_r \left(U^\dagger\right)^p_j \left(U^\dagger\right)^q_k \phi^r_{pq} \tag{4}$$

But by (2),

$$\epsilon^{pqst} U^j_p U^k_q U^l_s U^m_t = \epsilon^{jklm} \quad \Longrightarrow \quad \epsilon^{pqst} U^l_s U^m_t = \epsilon^{jklm} \left(U^\dagger\right)^p_j \left(U^\dagger\right)^q_k$$

So (4) becomes

$$\epsilon^{jklm} \phi^i_{jk} \longrightarrow \epsilon^{jklm} U^i_r \left(U^\dagger\right)^p_j \left(U^\dagger\right)^q_k \phi^r_{pq} \longrightarrow U^i_r U^l_s U^m_t \left(\epsilon^{pqst} \phi^r_{pq}\right) = U^i_r U^l_s U^m_t \left(\phi^{rst}\right)$$

exactly how rank-3 tensor ϕ^{ilm} would transform under U.

Secondly, indices can be moved upward one by one.

$$\phi^{ilmn}_k \equiv \epsilon^{jlmn} \phi^i_{jk} \tag{5}$$

Again, under U, the RHS transforms like

$$\epsilon^{jlmn} \phi^i_{jk} \longrightarrow \epsilon^{jlmn} U^i_r \left(U^\dagger\right)^p_j \left(U^\dagger\right)^q_k \phi^r_{pq} \tag{6}$$

But by (2),

$$\epsilon^{pstu} U^j_p U^l_s U^m_t U^n_u = \epsilon^{jlmn} \quad \Longrightarrow \quad \epsilon^{pstu} U^l_s U^m_t U^n_u = \epsilon^{jlmn} \left(U^\dagger\right)^p_j$$

So (6) becomes

$$\epsilon^{jlmn} U^i_r \left(U^\dagger\right)^p_j \left(U^\dagger\right)^q_k \phi^r_{pq} = U^i_r U^l_s U^m_t U^n_u \left(U^\dagger\right)^q_k \left(\epsilon^{pstu} \phi^r_{pq}\right) = U^i_r U^l_s U^m_t U^n_u \left(U^\dagger\right)^q_k \left(\phi^{rstu}_q\right)$$

exactly how ϕ^{ilmn}_k would transform.