For rank-4 tensor $T^{ijkl} \in SO(N)$, the totally symmetric tensor S^{ijkl} is the sum of all 24 T^{pqrs} where $(pqrs)$ is a permutation of $(ijkl)$.

They can be made traceless by the shift

$$\begin{aligned}\widetilde{S}^{ijkl} &= S^{ijkl} - \frac{1}{N+4}\left(\delta^{ij}S^{hhkl} + \delta^{ik}S^{hhjl} + \delta^{il}S^{hhjk} + \delta^{jk}S^{hhil} + \delta^{jl}S^{hhik} + \delta^{kl}S^{hhij}\right) \\ &+ \frac{1}{(N+2)(N+4)}\left[\left(\delta^{ij}\delta^{kl} + \delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk}\right)S^{hhmm}\right]\end{aligned}$$

The rationale behind this construction is such that if we contract, say i, j index, the first term on the RHS contributes one unit of S^{hhkl}, among the terms in the first parenthesis, the first term contributes N units of S^{hhkl} (since the outer contraction over i, j will ensure $\delta^{ij} = 1$, then summing over $i = j$ makes it N times). The 2nd, 3rd, 4th and 5th term each contributes one unit of S^{hhkl} which happens when the loop over $i = j$ coincides with either k or l. When $k \neq l$, the 6th term in the first parenthesis doesn't contribute anything (but has to appear in the expression to make it totally symmetric), neither does the entire second parenthesis at all (remember $i = j$ due to the contraction, the entire second parenthesis vanishes when $k \neq l$).

But when $k = l$, the $\delta^{kl}S^{hhij}$ term, when summed over $i = j$ gives S^{hhmm}, which is what the second parenthesis is designed to cancel. When $k = l$ and summed over $i = j$, the $\delta^{ij}\delta^{kl}$ term contributes N units of S^{hhmm}, and the other two terms each contributes one units, hence the $N + 2$ in the denominator.