Here we verify the inductive construction of C_n.

First let's see why it cannot be the case that κ is always τ_1 regardless of n.

We need C_n and κ to satisfy (36). If $\kappa = \tau_1$, we have

$$\kappa^{-1}\tau_2^T\kappa = \tau_1(-\tau_2)\tau_1 = -i\tau_3\tau_1 = \tau_2$$

Similarly

$$\kappa^{-1}\tau_1^T\kappa = \tau_1\tau_1\tau_1 = \tau_1$$

This means when $\kappa = \tau_1$, the negative sign needs to belong to the C_n term, i.e.,

$$C_n^{-1}\gamma_i^{(n)T}C_n = -\gamma_i^{(n)}$$

Then for

$$\gamma_{2k-1} = \overbrace{1\otimes\cdots\otimes 1}^{k-1}\otimes\tau_1\otimes\overbrace{\tau_3\otimes\cdots\otimes\tau_3}^{n-k}$$

we have the multiplication table for $C_n^{-1}\gamma_{2k-1}^T C_n$ below

C_n^{-1}	τ_1	$\cdots$	τ_1	τ_1	τ_1	$\cdots$	τ_1
γ_{2k-1}^T	1	$\cdots$	1	τ_1	τ_3	$\cdots$	τ_3
$C_n^{-1}\gamma_{2k-1}^T$	τ_1	$\cdots$	τ_1	1	$-i\tau_2$	$\cdots$	$-i\tau_2$
C_n	τ_1	$\cdots$	τ_1	τ_1	τ_1	$\cdots$	τ_1
$C_n^{-1}\gamma_{2k-1}^T C_n$	1	$\cdots$	1	τ_1	$-\tau_3$	$\cdots$	$-\tau_3$

which means $C_n^{-1}\gamma_{2k-1}^T C_n = (-1)^{n-k}\gamma_{2k-1}$, apparently violating our earlier requirement when $n-k$ is even.

Similarly for

$$\gamma_{2k} = \overbrace{1\otimes\cdots\otimes 1}^{k-1}\otimes\tau_2\otimes\overbrace{\tau_3\otimes\cdots\otimes\tau_3}^{n-k}$$

the multiplication table

C_n^{-1}	τ_1	$\cdots$	τ_1	τ_1	τ_1	$\cdots$	τ_1
γ_{2k}^T	1	$\cdots$	1	$-\tau_2$	τ_3	$\cdots$	τ_3
$C_n^{-1}\gamma_{2k}^T$	τ_1	$\cdots$	τ_1	$-i\tau_3$	$-i\tau_2$	$\cdots$	$-i\tau_2$
C_n	τ_1	$\cdots$	τ_1	τ_1	τ_1	$\cdots$	τ_1
$C_n^{-1}\gamma_{2k}^T C_n$	1	$\cdots$	1	τ_2	$-\tau_3$	$\cdots$	$-\tau_3$

which means $C_n^{-1}\gamma_{2k}^T C_n = (-1)^{n-k}\gamma_{2k}$, which will again violate our requirement that $C_n^{-1}\gamma_{2k}^T C_n = -\gamma_{2k}$ when $n-k$ is even.

Now we verify the inductive construction.

First we have seen that when $\kappa = \tau_1$, we require

$$C_n^{-1}\gamma_i^{(n)T}C_n = -\gamma_i^{(n)}$$

Now when $\kappa = i\tau_2$, we have

$$\kappa^{-1}\tau_2^T\kappa = (-i\tau_2)(-\tau_2)(i\tau_2) = -\tau_2$$
$$\kappa^{-1}\tau_1^T\kappa = (-i\tau_2)\tau_1(i\tau_2) = (-\tau_3)(i\tau_2) = -\tau_1$$

where we now require

$$C_n^{-1}\gamma_i^{(n)T}C_n = \gamma_i^{(n)}$$

We could have chosen either case for the base. The text chose $C_2 = i\tau_2\otimes\tau_1$, which requires

$$C_n^{-1}\gamma_i^{(n)T}C_n = (-1)^n\gamma_i^{(n)}$$

Now we should verify this is satisfied by general γ_i with $C_n = i\tau_2 \otimes \tau_1 \otimes i\tau_2 \otimes \tau_1 \otimes i\tau_2 \cdots$.

First look at the case where both n and k are even. The multiplication table for γ_{2k-1} is given by

C_n^{-1}	$-i\tau_2$	τ_1	$\cdots$	$-i\tau_2$	τ_1	$-i\tau_2$	$\cdots$	τ_1
γ_{2k-1}^T	1	1	$\cdots$	1	τ_1	τ_3	$\cdots$	τ_3
$C_n^{-1}\gamma_{2k-1}^T$	$-i\tau_2$	τ_1	$\cdots$	$-i\tau_2$	1	τ_1	$\cdots$	$-i\tau_2$
C_n	$i\tau_2$	τ_1	$\cdots$	$i\tau_2$	τ_1	$i\tau_2$	$\cdots$	τ_1
$C_n^{-1}\gamma_{2k-1}^T C_n$	1	1	$\cdots$	1	τ_1	$-\tau_3$	$\cdots$	$-\tau_3$

which produced an overall sign $(-1)^{n-k} = (-1)^n = 1$. Similarly for γ_{2k},

C_n^{-1}	$-i\tau_2$	τ_1	$\cdots$	$-i\tau_2$	τ_1	$-i\tau_2$	$\cdots$	τ_1
γ_{2k}^T	1	1	$\cdots$	1	$-\tau_2$	τ_3	$\cdots$	τ_3
$C_n^{-1}\gamma_{2k}^T$	$-i\tau_2$	τ_1	$\cdots$	$-i\tau_2$	$-i\tau_3$	τ_1	$\cdots$	$-i\tau_2$
C_n	$i\tau_2$	τ_1	$\cdots$	$i\tau_2$	τ_1	$i\tau_2$	$\cdots$	τ_1
$C_n^{-1}\gamma_{2k-1}^T C_n$	1	1	$\cdots$	1	τ_2	$-\tau_3$	$\cdots$	$-\tau_3$

which again produces an overall sign $(-1)^{n-k} = (-1)^n = 1$.

Observe that in the table above, the 2nd column is guaranteed to produce all the 1's, which don't contribute to the overall sign, thus we only need to calculate the 3rd and 4th column.

Now for k even and n odd (thus $n-k$ odd), we have the following two tables

C_n^{-1}	$\cdots$	τ_1	$-i\tau_2$	$\cdots$	τ_1	$-i\tau_2$
γ_{2k-1}^T	$\cdots$	τ_1	τ_3	$\cdots$	τ_3	τ_3
$C_n^{-1}\gamma_{2k-1}^T$	$\cdots$	1	τ_1	$\cdots$	$-i\tau_2$	τ_1
C_n	$\cdots$	τ_1	$i\tau_2$	$\cdots$	τ_1	$i\tau_2$
$C_n^{-1}\gamma_{2k-1}^T C_n$	$\cdots$	τ_1	$-\tau_3$	$\cdots$	$-\tau_3$	$-\tau_3$

C_n^{-1}	$\cdots$	τ_1	$-i\tau_2$	$\cdots$	τ_1	$-i\tau_2$
γ_{2k}^T	$\cdots$	$-\tau_2$	τ_3	$\cdots$	τ_3	τ_3
$C_n^{-1}\gamma_{2k}^T$	$\cdots$	$-i\tau_3$	τ_1	$\cdots$	$-i\tau_2$	τ_1
C_n	$\cdots$	τ_1	$i\tau_2$	$\cdots$	τ_1	$i\tau_2$
$C_n^{-1}\gamma_{2k}^T C_n$	$\cdots$	τ_2	$-\tau_3$	$\cdots$	$-\tau_3$	$-\tau_3$

which shows the overall sign for both γ_{2k-1} and γ_{2k} are $(-1)^{n-k} = (-1)^n$, as desired.

Now moving to the case where k is odd. Here the 3rd column for γ_{2k-1} becomes

$-i\tau_2$
τ_1
$-\tau_3$
$i\tau_2$
$-\tau_1$

similarly for γ_{2k}

$-i\tau_2$
$-\tau_2$
i
$i\tau_2$
$-\tau_2$

Which each contributes an extra -1 in the overall sign. Combining with the 4th column of the $n-k$ even and odd case, it's easy to verify that in all cases, the overall sign is equal to $(-1)^n$.