We want to show the dual tensor relation

$$\epsilon^{ijk\cdots n}R^{ip}R^{jq} = \epsilon^{pqr\cdots s}R^{kr}\cdots R^{ns} \tag{1}$$

given the determinant relation

$$\epsilon^{ijk\cdots n}R^{ip}R^{jq}R^{kr}\cdots R^{ns} = \epsilon^{pqr\cdots s} \tag{2}$$

First we should see how to interpret (1): it is a claim about any combination of $p,q,k,\cdots,n$ (because i,j on LHS and $r,\cdots,s$ on RHS are dummy indices).

For example, when $N=3$, and choose $p=2, q=1, k=2$ (due to ϵ^{pqr} on the RHS, p,q must be different, k can be free). Then we must have $r=3$, so (1) is really a claim about $R^{kr}=R^{23}$:

$$\epsilon^{ij2}R^{i2}R^{j1} = \epsilon^{213}R^{23} \qquad \text{or equivalently} \qquad R^{32}R^{11} - R^{12}R^{31} = -R^{23}$$

$$\begin{bmatrix} \utilde{R^{11}} & \utilde{R^{12}} & R^{13} \\ R^{21} & R^{22} & \underline{R^{23}} \\ \utilde{R^{31}} & \utilde{R^{32}} & R^{33} \end{bmatrix}$$

which is recognized as the cross-product relations in 3 dimensional space. Similarly for other combinations of p,q,k.

Another example for $N=4$ is by considering $p=3, q=1, k=1, n=3$, then the claim (1) becomes

$$\begin{gathered} \epsilon^{ij13}R^{i3}R^{j1} = \epsilon^{31rs}R^{1r}R^{3s} \iff \\ \epsilon^{2413}R^{23}R^{41} + \epsilon^{4213}R^{43}R^{21} = \epsilon^{3124}R^{12}R^{34} + \epsilon^{3142}R^{14}R^{32} \iff \\ -R^{23}R^{41} + R^{43}R^{21} = R^{12}R^{34} - R^{14}R^{32} \end{gathered}$$

$$\begin{bmatrix} R^{11} & \underline{R^{12}} & R^{13} & \underline{R^{14}} \\ \utilde{R^{21}} & R^{22} & \utilde{R^{23}} & R^{24} \\ R^{31} & \underline{R^{32}} & R^{33} & \underline{R^{34}} \\ \utilde{R^{41}} & R^{42} & \utilde{R^{43}} & R^{44} \end{bmatrix}$$

which is also recognized as a a relation of sub-blocks of a 4×4 orthogonal matrix.

We use $N=4$ to prove (1), the same argument applies to higher dimensions.

For $N=4$, (2) will be written as

$$\epsilon^{ijkn}R^{ip}R^{jq}R^{kr}R^{ns} = \epsilon^{pqrs} \tag{3}$$

Now define

$$u^n = \epsilon^{ijkn}R^{ip}R^{jq}R^{kr}$$

then (3) can be interpreted as an inner product relation

$$u^n R^{ns} = \epsilon^{pqrs} \tag{4}$$

between the two N-dimensional vectors u and $R^{\cdot s}$ - the s-th column of R.

Since R is orthogonal matrix, its column vectors form an orthonormal basis of the N dimensional space. (4) tells us how the vector u projects onto the s-th column $R^{\cdot s}$. If we make s range over $1\cdots N$, we know how u will decompose into this basis. In other words

$$u = \epsilon^{pqrs}R^{\cdot s}$$

with repeated index s being summed over.

Certainly for each component n, we have

$$\epsilon^{ijkn}R^{ip}R^{jq}R^{kr} = u^n = \epsilon^{pqrs}R^{ns} \tag{5}$$

Now we repeat the same argument with (5) and define $v^k = \epsilon^{ijkn}R^{ip}R^{jq}$ for a given fixed n, then the inner product relation

$$v^k R^{kr} = \epsilon^{pqrs}R^{ns}$$

implies the decomposition of v:

$$v = \epsilon^{pqrs}R^{ns}R^{\cdot r}$$

and singling out the k-th component will yield the desired equality

$$\epsilon^{ijkn}R^{ip}R^{jq} = v^k = \epsilon^{pqrs}R^{ns}R^{kr}$$