1. The classical angular frequency at orbiting radius ρ satisfy

$$\omega^2\rho = \frac{Ze^2}{m\rho^2} \qquad \Longrightarrow \qquad \omega^2 = \frac{Ze^2}{m\rho^3} \tag{1}$$

At principal quantum number n, the radius is

$$\rho_n = n^2 a_0 = \frac{n^2\hbar^2}{mZe^2} \qquad \Longrightarrow \qquad \omega_n = \frac{m\left(Ze^2\right)^2}{\hbar^3 n^3} \tag{2}$$

During the $n \to n-1$ transition, the frequency of the emitted photon is

$$\omega_{n\to n-1} = \frac{E_n - E_{n-1}}{\hbar} = \frac{1}{\hbar}\frac{mZ^2e^4}{2\hbar^2}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] \approx \frac{m\left(Ze^2\right)^2}{\hbar^3 n^3} = \omega_n \qquad \text{for large } n \tag{3}$$

By (14.31), the classical power radiated is

$$\begin{aligned} P &= \frac{2}{3}\frac{e^2c}{\rho^2}\beta^4\gamma^4 \qquad \text{where we take } \beta = \omega_n\rho_n/c, \gamma \approx 1 \\ &= \frac{2}{3}\frac{e^2\omega_n^4\rho_n^2}{c^3} \end{aligned} \tag{4}$$

hence by correspondence principle,

$$\frac{1}{\tau} = \frac{P}{\hbar\omega_n} = \frac{2}{3}\frac{e^2\omega_n^3\rho_n^2}{\hbar c^3} = \frac{2}{3}\frac{e^2 m\left(Ze^2\right)^4}{\hbar^6 c^3 n^5} = \frac{2}{3}\frac{e^2}{\hbar c}\left(\frac{Ze^2}{\hbar c}\right)^4\frac{mc^2}{\hbar}\frac{1}{n^5} \tag{5}$$

2. For hydrogen ($Z = 1$), the classical mean life from part (a) is

$$\tau_{\text{cl}} = \left(\frac{2}{3}\alpha^5\frac{mc^2}{\hbar}\frac{1}{n^5}\right)^{-1} \tag{6}$$

where $\alpha = e^2/\hbar c \approx 1/137$ is the fine-structure constant. Numerically, the prefactor is

$$\frac{2}{3}\alpha^5\frac{mc^2}{\hbar} \approx 1.07 \times 10^{10}\ \text{s}^{-1} \tag{7}$$

The following table compares τ_{cl} with the quantum-mechanical mean lives for three transitions with maximum l (i.e., $l = n-1 \to l' = n-2$):

Transition	n	τ_{cl} (s)	τ_{QM} (s)
$2p \to 1s$	2	2.99×10^{-9}	1.6×10^{-9}
$4f \to 3d$	4	9.56×10^{-8}	7.3×10^{-8}
$6h \to 5g$	6	7.26×10^{-7}	6.1×10^{-7}

The difference between τ_{cl} and τ_{QM} decreases as n increases, confirming the correspondence principle works better with large n.