

From the Liénard-Wiechert formula (14.13) and (14.14)

$$\mathbf{B} = [\mathbf{n} \times \mathbf{E}]_{\text{ret}} \quad \mathbf{E} = e \left[ \frac{\mathbf{n} - \boldsymbol{\beta}}{\gamma^2 R^2 (1 - \boldsymbol{\beta} \cdot \mathbf{n})^3} \right]_{\text{ret}} + \frac{e}{c} \left\{ \frac{\mathbf{n} \times [(\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \mathbf{n})^3 R} \right\}_{\text{ret}} \quad (1)$$

the text has shown that in case of charge moving with uniform velocity, the two forms of the transverse component of the electric field (14.14) and (11.152) are equivalent.

With  $\dot{\boldsymbol{\beta}} = 0$ , longitudinal component of the electric field is

$$E_{\parallel} = e \left[ \frac{n_{\parallel} - \beta}{\gamma^2 R^2 (1 - \boldsymbol{\beta} \cdot \mathbf{n})^3} \right] = \gamma e \left\{ \frac{n_{\parallel} R - \beta R}{[\gamma R (1 - \boldsymbol{\beta} \cdot \mathbf{n})]^3} \right\} \quad (2)$$

which is understood to be evaluated at the retarded time.

Referring to Figure 14.2, we see that

$$n_{\parallel} R = R \cos \theta = \beta R + vt \quad (3)$$

giving

$$\begin{aligned} E_{\parallel} &= \frac{\gamma e v t}{[\gamma R (1 - \boldsymbol{\beta} \cdot \mathbf{n})]^3} && \text{by (14.16)} \\ &= \frac{\gamma e v t}{(b^2 + \gamma^2 v^2 t^2)^{3/2}} && (4) \end{aligned}$$

agreeing with (11.152) (except for the sign which is due to the different choice of positive direction of  $\mathbf{v}$ ).

For magnetic field, we can also recover (11.152) with the help of Figure 14.2:

$$\mathbf{B} = \mathbf{n} \times \mathbf{E} = -e \left[ \frac{\mathbf{n} \times \boldsymbol{\beta}}{\gamma^2 R^2 (1 - \boldsymbol{\beta} \cdot \mathbf{n})^3} \right] \quad \Rightarrow \quad |\mathbf{B}| = \gamma e \left\{ \frac{R \beta \sin \theta}{[\gamma R (1 - \boldsymbol{\beta} \cdot \mathbf{n})]^3} \right\} = \frac{\gamma e b \beta}{(b^2 + \gamma^2 v^2 t^2)^{3/2}} \quad (5)$$