For non-relativistic motion, we can use (14.20), (14.21) for the approximate angular distribution of radiated power

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c}\left|\mathbf{n}\times\left(\mathbf{n}\times\dot{\boldsymbol{\beta}}\right)\right|^2 = \frac{e^2}{4\pi c^3}|\dot{\mathbf{v}}|^2\sin^2\Theta \qquad \text{where } \Theta \text{ is the angle between } \dot{\mathbf{v}} \text{ and } \mathbf{n} \tag{1}$$

Note that the RHS is evaluated at the retarded time, and the angular distribution is in the sense as described by the second paragraph of section 14.3, i.e., *the energy per unit area per unit time detected at an observation point at time t of radiation emitted by the charge at time $t' = t - R(t')/c$.* I.e., the angular distribution from the perspective of the moving charge, not of the observer at rest.

With this in mind, we can write

$$\mathbf{n} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}} \tag{2}$$

Thus

- For harmonic motion along z-axis, $\dot{\mathbf{v}} = -\omega_0^2 a\cos(\omega_0 t)\,\hat{\mathbf{z}}$, $\Theta = \theta$, giving

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3}\omega_0^4 a^2\cos^2(\omega_0 t)\sin^2\theta \qquad \Longrightarrow \qquad \left\langle\frac{dP}{d\Omega}\right\rangle = \frac{e^2}{8\pi c^3}\omega_0^4 a^2\sin^2\theta \tag{3}$$

- For circular motion around z-axis, we have

$$\dot{\mathbf{v}}(t) = -\omega_0^2 a\left[\cos(\omega_0 t)\,\hat{\mathbf{x}} + \sin(\omega_0 t)\,\hat{\mathbf{y}}\right] \tag{4}$$

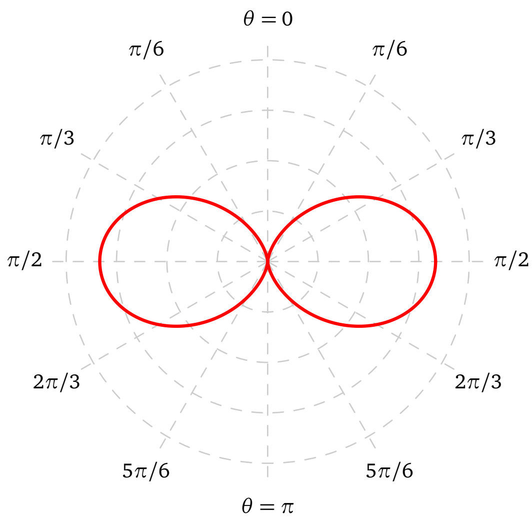
therefore

$$\cos\Theta = -\sin\theta\cos(\phi - \omega_0 t) \qquad \Longrightarrow \qquad \sin^2\Theta = 1 - \sin^2\theta\cos^2(\phi - \omega_0 t) \tag{5}$$

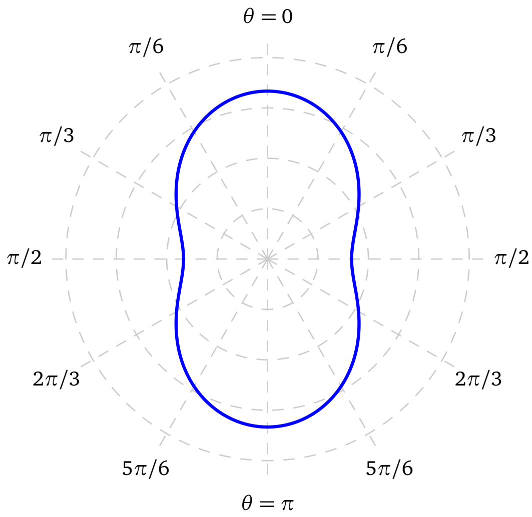
giving

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3}\omega_0^4 a^2\left[1 - \sin^2\theta\cos^2(\phi - \omega_0 t)\right] \qquad \Longrightarrow \qquad \left\langle\frac{dP}{d\Omega}\right\rangle = \frac{e^2}{4\pi c^3}\omega_0^4 a^2\left(1 - \frac{\sin^2\theta}{2}\right) \tag{6}$$

The plots are shown below.



harmonic motion: $\sin^2\theta$



circular motion: $1 - \sin^2\theta/2$