

The Liénard–Wiechert potentials of the collection of particles are given by

$$\Phi(\mathbf{x}, t) = \sum_j \left[\frac{q}{\kappa_j(t) R_j(t)} \right]_{\text{ret}} \quad \mathbf{A}(\mathbf{x}, t) = \sum_j \left[\frac{q \beta_j(t)}{\kappa_j(t) R_j(t)} \right]_{\text{ret}} \quad (1)$$

where

$$\kappa_j(t) = 1 - \mathbf{n}_j(t) \cdot \boldsymbol{\beta}_j(t) \quad R_j(t) = |\mathbf{x} - \mathbf{r}_j(t)| \quad \mathbf{n}_j(t) = \frac{\mathbf{x} - \mathbf{r}_j(t)}{|\mathbf{x} - \mathbf{r}_j(t)|} \quad (2)$$

and the evaluation is at the retarded time t_{ret} defined by

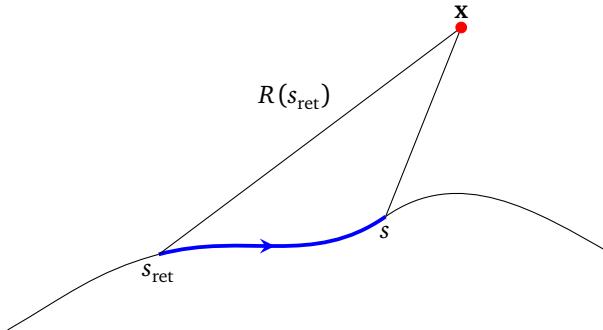
$$t - t_{\text{ret}} = \frac{R_j(t_{\text{ret}})}{c} \quad (3)$$

In the continuum limit specified by the problem, the potentials can be written in integral form with regard to the length parameter s along the path

$$\Phi(\mathbf{x}, t) = \oint_L \left[\frac{\lambda ds}{\kappa(s) R(s)} \right]_{\text{ret}} \quad \mathbf{A}(\mathbf{x}, t) = \oint_L \left[\frac{\lambda \boldsymbol{\beta}(s) ds}{\kappa(s) R(s)} \right]_{\text{ret}} = \oint \left[\frac{\lambda v ds}{c \kappa(s) R(s)} \right]_{\text{ret}} \quad (4)$$

where the retarded time evaluation is substituted by the "retarded length parameter" evaluation, defined by

$$\frac{s - s_{\text{ret}}}{v} = \frac{R(s_{\text{ret}})}{c} \quad (5)$$



We can already see that the RHS of $\Phi(\mathbf{x}, t)$ and $\mathbf{A}(\mathbf{x}, t)$ are independent of time. This is possible because of the continuum limit where the time dependence of the integrand is converted to the dependency of the length parameter s . We have static fields at any observation point \mathbf{x} .

Differentiating both sides of (5) with respect to s_{ret} gives

$$\begin{aligned} \frac{1}{v} \left(\frac{ds}{ds_{\text{ret}}} - 1 \right) &= \frac{1}{c} \frac{dR(s_{\text{ret}})}{ds_{\text{ret}}} = \frac{1}{c} \frac{d|\mathbf{x} - \mathbf{r}(s_{\text{ret}})|}{ds_{\text{ret}}} = -\frac{1}{c} \mathbf{n}(s_{\text{ret}}) \cdot \frac{d\mathbf{r}}{ds_{\text{ret}}} \\ ds &= [1 - \mathbf{n}(s_{\text{ret}}) \cdot \boldsymbol{\beta}(s_{\text{ret}})] ds_{\text{ret}} = \kappa(s_{\text{ret}}) ds_{\text{ret}} \end{aligned} \implies \quad (6)$$

This allows us to write the potentials as integrals with respect to the retarded length parameter s_{ret}

$$\Phi(x) = \oint_L \frac{\lambda ds_{\text{ret}}}{R(s_{\text{ret}})} \quad \mathbf{A}(x) = \frac{1}{c} \oint_L \frac{\lambda v ds_{\text{ret}}}{R(s_{\text{ret}})} \quad (7)$$

which are exactly the static Coulomb and Biot–Savart potentials if we treat the dummy variable s_{ret} as regular length parameter s .

Since static Coulomb and Biot-Savart fields decay as $1/R^2$, there is no radiation which requires $1/R$ decay.