1. For harmonic motion $z(t') = a\cos(\omega_0 t)$,

$$\beta\left(t'\right) = -\frac{\omega_0 a}{c}\sin\left(\omega_0 t'\right) = -\beta\sin\left(\omega_0 t'\right) \qquad \dot{v}\left(t'\right) = -\omega_0^2 a\cos\left(\omega_0 t'\right) = -\frac{\beta^2 c^2}{a}\cos\left(\omega_0 t'\right) \tag{1}$$

The power radiated per unit solid angle is given by (14.39)

$$\frac{dP(t')}{d\Omega} = \frac{e^2\dot{v}^2}{4\pi c^3}\frac{\sin^2\theta}{[1-\beta(t')\cos\theta]^5} = \frac{e^2\beta^4 c}{4\pi a^2}\frac{\sin^2\theta\cos^2(\omega_0 t')}{[1+\beta\sin(\omega_0 t')\cos\theta]^5} \tag{2}$$

2. Let $\phi = \omega_0 t'$ and $a = \beta\cos\theta$. First note that

$$\frac{\sin^2\phi}{(1+a\cos\phi)^5} = \frac{1}{a^2}\left[\frac{1}{(1+a\sin\phi)^3} - \frac{2}{(1+a\sin\phi)^4} + \frac{1}{(1+a\sin\phi)^5}\right] \tag{3}$$

Then the desired time average can be expressed as

$$\left\langle \frac{\cos^2(\omega_0 t')}{[1+\beta\cos\theta\sin(\omega_0 t)]^5}\right\rangle = \frac{1}{2\pi}\int_0^{2\pi}\frac{\cos^2\phi\, d\phi}{(1+a\sin\phi)^5} = I_5 - \left(\frac{I_3 - 2I_4 + I_5}{a^2}\right) \tag{4}$$

where

$$I_n = \frac{1}{2\pi}\int_0^{2\pi}\frac{d\phi}{(1+a\sin\phi)^n} \tag{5}$$

Let's first evaluate a more general integration

$$f(x) = \frac{1}{2\pi}\int_0^{2\pi}\frac{d\phi}{x+a\sin\phi} \qquad \text{for } |a|<|x| \tag{6}$$

Let $z = e^{i\phi}$, so that

$$\sin\phi = \frac{z-z^{-1}}{2i} \qquad d\phi = \frac{dz}{iz} \tag{7}$$

then

$$\begin{aligned} f(x) &= \frac{1}{2\pi}\oint_{|z|=1}\left[\frac{1}{x+a\left(\dfrac{z-z^{-1}}{2i}\right)}\right]\frac{dz}{iz} \\ &= \frac{1}{2\pi}\oint_{|z|=1}\frac{2dz}{az^2+2ixz-a} && \text{for } |a|<|x| \\ &= \frac{1}{2\pi}\oint_{|z|=1}\frac{2dz}{a(z-z_-)(z-z_+)} && \text{where } z_\pm = i\left(\frac{\pm\sqrt{x^2-a^2}-x}{a}\right) \end{aligned} \tag{8}$$

Only z_+ is inside the unit circle, so by residue theorem,

$$f(x) = \frac{1}{2\pi}\cdot(2\pi i)\frac{2}{a(z_+-z_-)} = \frac{1}{\sqrt{x^2-a^2}} \tag{9}$$

Taking the derivative with respect to x in (6), we get the recursion relation

$$\frac{d^{n-1}f}{dx^{n-1}} = \frac{(-1)^{n-1}(n-1)!}{2\pi}\int_0^{2\pi}\frac{d\phi}{(x+a\sin\phi)^n} \quad\Longrightarrow\quad I_n = \frac{(-1)^{n-1}}{(n-1)!}\frac{d^{n-1}}{dx^{n-1}}\left(\frac{1}{\sqrt{x^2-a^2}}\right)\Bigg|_{x=1} \tag{10}$$

This gives

$$I_1 = \frac{1}{(1-a^2)^{1/2}} \qquad I_3 = \frac{2+a^2}{2(1-a^2)^{5/2}} \qquad I_4 = \frac{2+3a^2}{2(1-a^2)^{7/2}} \qquad I_5 = \frac{8+24a^2+3a^4}{8(1-a^2)^{9/2}} \tag{11}$$

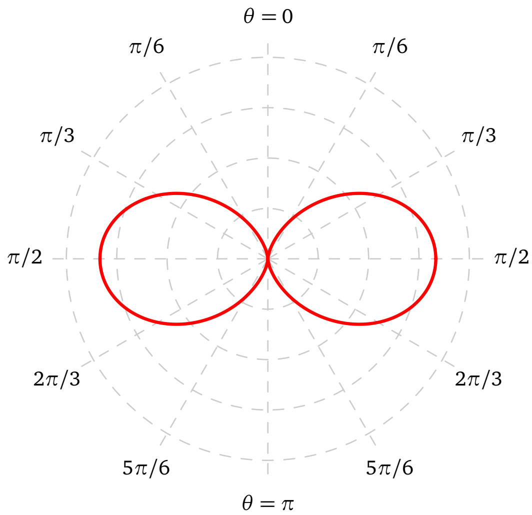
This renders the RHS of (4) to be

$$\mathrm{RHS}_{(4)} = \frac{4 - 3a^2 - a^4}{8\,(1-a^2)^{9/2}} = \frac{a^2+4}{8\,(1-a^2)^{7/2}} \tag{12}$$

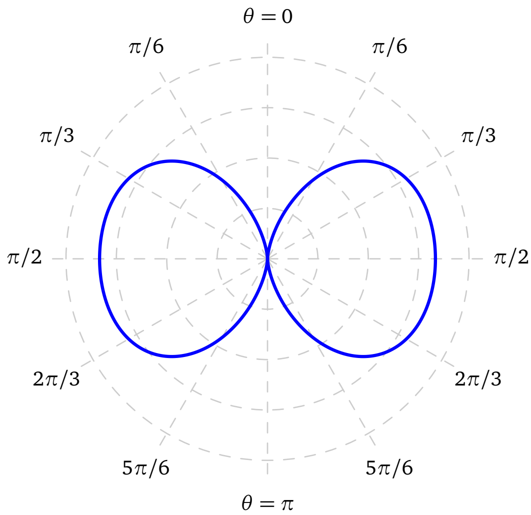
and the time-averaged power per unit solid angle is

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{e^2 c \beta^4}{32\pi a^2} \left[\frac{4 + \beta^2 \cos^2\theta}{(1-\beta^2\cos^2\theta)^{7/2}} \right] \sin^2\theta \tag{13}$$

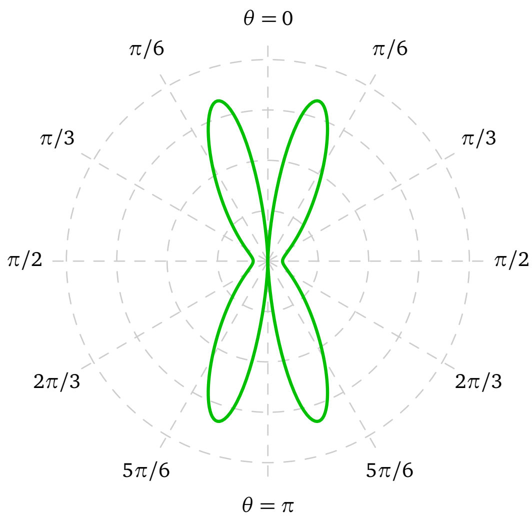
The angular distribution plots for various values of β are shown below. Each plot has its own scale.



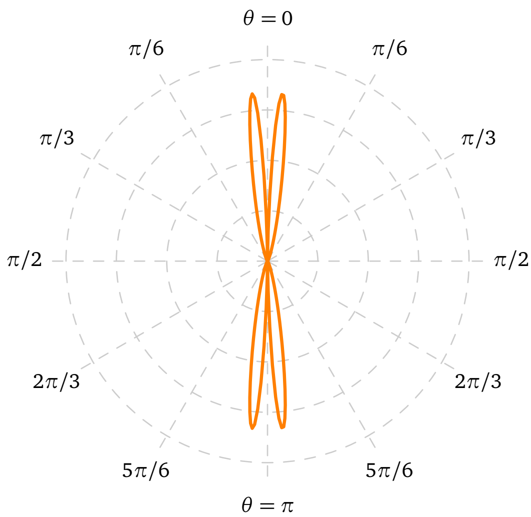
$\beta = 0.1$



$\beta = 0.5$



$\beta = 0.9$



$\beta = 0.99$