Let $\mathbf{r}$ be the observation point, and let $\mathbf{x}(t)$ be the position of the charge at time t. Let $t'(t)$ be the retarded time corresponding to the observation time t, of which the governing equation is

$$t' = t - \frac{R(t')}{c} \tag{1}$$

where $\mathbf{R}(t') = \mathbf{r} - \mathbf{x}(t')$ and $R = |\mathbf{R}| = [(\mathbf{r}-\mathbf{x})\cdot(\mathbf{r}-\mathbf{x})]^{1/2}$.

We use the "overdot" to denote time derivative evaluated the retarded time t'. An equivalent (informal) notation is d/dt', i.e., it will be understood that for a time dependent function f,

$$\dot{f} = \frac{df}{dt'} = \left.\frac{df}{dt}\right|_{t=t'} \tag{2}$$

It is easy to see that

$$\dot{\mathbf{R}} = -c\boldsymbol{\beta} \tag{3}$$

$$\dot{R} = \frac{1}{2}\left[\frac{-2(\mathbf{r}-\mathbf{x})\cdot c\boldsymbol{\beta}}{R}\right] = -c\boldsymbol{\beta}\cdot\mathbf{n} \tag{4}$$

where it is understood that the RHS are evaluated at the retarded time t'.

Differentiating (1) gives

$$\frac{dt}{dt'} = 1 + \frac{\dot{R}}{c} = 1 - \boldsymbol{\beta}\cdot\mathbf{n} \equiv \kappa \qquad \text{or} \qquad \frac{d}{dt} = \frac{1}{\kappa}\frac{d}{dt'} \tag{5}$$

Furthermore,

$$\dot{\mathbf{n}} = \frac{d}{dt'}\left(\frac{\mathbf{R}}{R}\right) = \frac{\dot{\mathbf{R}}}{R} - \frac{\mathbf{R}\dot{R}}{R^2} = -\frac{c\boldsymbol{\beta}}{R} + \frac{c\mathbf{n}(\boldsymbol{\beta}\cdot\mathbf{n})}{R} = \frac{c[-\boldsymbol{\beta}+\mathbf{n}(\boldsymbol{\beta}\cdot\mathbf{n})]}{R} = -\frac{c}{R}\overbrace{[\mathbf{n}\times(\boldsymbol{\beta}\times\mathbf{n})]}^{\mathbf{m}\equiv\boldsymbol{\beta}-(\boldsymbol{\beta}\cdot\mathbf{n})\mathbf{n}} \tag{6}$$

$$\dot{\kappa} = \frac{d(1-\boldsymbol{\beta}\cdot\mathbf{n})}{dt'} = -\dot{\boldsymbol{\beta}}\cdot\mathbf{n} - \boldsymbol{\beta}\cdot\dot{\mathbf{n}} = \underbrace{-\dot{\boldsymbol{\beta}}\cdot\mathbf{n}}_{\kappa_A} + \underbrace{\frac{c}{R}\boldsymbol{\beta}\cdot\mathbf{m}}_{\kappa_B} \tag{7}$$

The Heaviside-Feynman formula is

$$\mathbf{E} = e\left[\frac{\mathbf{n}}{R^2}\right]_{\rm ret} + e\left[\frac{R}{c}\right]_{\rm ret}\frac{d}{dt}\left[\frac{\mathbf{n}}{R^2}\right]_{\rm ret} + \frac{e}{c^2}\frac{d^2[\mathbf{n}]_{\rm ret}}{dt^2} \tag{8}$$

where

$$\left[\frac{R}{c}\right]_{\rm ret}\frac{d}{dt}\left[\frac{\mathbf{n}}{R^2}\right]_{\rm ret} = \frac{R}{c}\cdot\frac{1}{\kappa}\frac{d}{dt'}\left(\frac{\mathbf{n}}{R^2}\right) = \frac{R}{c\kappa}\left(\frac{\dot{\mathbf{n}}}{R^2} - \frac{2\dot{R}\mathbf{n}}{R^3}\right) = -\frac{\mathbf{m}}{\kappa R^2} + \frac{2(\boldsymbol{\beta}\cdot\mathbf{n})\mathbf{n}}{\kappa R^2} \tag{9}$$

$$\begin{aligned}
\frac{d^2[\mathbf{n}]_{\rm ret}}{dt^2} &= \frac{1}{\kappa}\frac{d}{dt'}\left(\frac{\dot{\mathbf{n}}}{\kappa}\right) = -\frac{c}{\kappa}\frac{d}{dt'}\left(\frac{\mathbf{m}}{\kappa R}\right) \\
&= -\frac{c}{\kappa}\left\{\frac{d}{dt'}\left(\frac{1}{\kappa R}\right)\mathbf{m} + \frac{\dot{\mathbf{n}}\times(\boldsymbol{\beta}\times\mathbf{n})}{\kappa R} + \frac{\mathbf{n}\times(\dot{\boldsymbol{\beta}}\times\mathbf{n})}{\kappa R} + \frac{\mathbf{n}\times(\boldsymbol{\beta}\times\dot{\mathbf{n}})}{\kappa R}\right\} \\
&= -\frac{c}{\kappa}\left\{-\frac{(\kappa_A+\kappa_B)\mathbf{m}}{\kappa^2 R} - \frac{\dot{R}\mathbf{m}}{\kappa R^2} + \frac{\dot{\mathbf{n}}\times(\boldsymbol{\beta}\times\mathbf{n})}{\kappa R} + \frac{\mathbf{n}\times(\dot{\boldsymbol{\beta}}\times\mathbf{n})}{\kappa R} + \frac{\mathbf{n}\times(\boldsymbol{\beta}\times\dot{\mathbf{n}})}{\kappa R}\right\}
\end{aligned} \tag{10}$$

Since κ_B and $\dot{\mathbf{n}}$ decay as $1/R$ already, we see that the terms in the H-F formula either decay as $1/R$ or $1/R^2$. Summing up all the $1/R$ terms gives

$$\begin{aligned}
\mathrm{HF}_{1/R} &= \frac{e}{c^2}\cdot\left(-\frac{c}{\kappa}\right)\left[-\frac{\kappa_A\mathbf{m}}{\kappa^2 R} + \frac{\mathbf{n}\times(\dot{\boldsymbol{\beta}}\times\mathbf{n})}{\kappa R}\right] \\
&= -\frac{e}{c\kappa^3 R}\left\{(\dot{\boldsymbol{\beta}}\cdot\mathbf{n})[\boldsymbol{\beta}-(\boldsymbol{\beta}\cdot\mathbf{n})\mathbf{n}] + (1-\boldsymbol{\beta}\cdot\mathbf{n})[\dot{\boldsymbol{\beta}}-(\dot{\boldsymbol{\beta}}\cdot\mathbf{n})\mathbf{n}]\right\} \\
&= -\frac{e}{c\kappa^3 R}\left\{(\dot{\boldsymbol{\beta}}\cdot\mathbf{n})\boldsymbol{\beta} + (1-\boldsymbol{\beta}\cdot\mathbf{n})\dot{\boldsymbol{\beta}} - (\dot{\boldsymbol{\beta}}\cdot\mathbf{n})\mathbf{n}\right\} \\
&= -\frac{e}{c\kappa^3 R}\left\{(\dot{\boldsymbol{\beta}}\cdot\mathbf{n})(\boldsymbol{\beta}-\mathbf{n}) - [\mathbf{n}\cdot(\boldsymbol{\beta}-\mathbf{n})]\dot{\boldsymbol{\beta}}\right\} \\
&= \frac{e}{c}\left\{\frac{\mathbf{n}\times[(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}]}{\kappa^3 R}\right\}
\end{aligned} \tag{11}$$

which matches the acceleration field in the Liénard-Wiechert formula.

Collecting all the $1/R^2$ terms, we have

$$\begin{aligned}\mathrm{HF}_{1/R^2} &= e\left(\frac{\mathbf{n}}{R^2}\right)+e\left[\frac{-\mathbf{m}+2(\boldsymbol{\beta}\cdot\mathbf{n})\mathbf{n}}{\kappa R^2}\right]-\frac{e}{c\kappa}\left\{-\frac{\kappa_B\mathbf{m}}{\kappa^2 R}-\frac{\dot{R}\mathbf{m}}{\kappa R^2}+\frac{\dot{\mathbf{n}}\times(\boldsymbol{\beta}\times\mathbf{n})}{\kappa R}+\frac{\mathbf{n}\times(\boldsymbol{\beta}\times\dot{\mathbf{n}})}{\kappa R}\right\}\\ &= \frac{e}{\kappa^3 R^2}\cdot\mathbf{X}\end{aligned} \tag{12}$$

where

$$\mathbf{X}=\overbrace{\kappa^3\mathbf{n}}^{X_1}+\overbrace{\kappa^2[-\mathbf{m}+2(\boldsymbol{\beta}\cdot\mathbf{n})\mathbf{n}]}^{X_2}+\overbrace{(\boldsymbol{\beta}\cdot\mathbf{m})\mathbf{m}}^{X_3}\overbrace{-\kappa(\boldsymbol{\beta}\cdot\mathbf{n})\mathbf{m}}^{X_4}+\overbrace{\kappa\mathbf{m}\times(\boldsymbol{\beta}\times\mathbf{n})}^{X_5}+\overbrace{\kappa\mathbf{n}\times(\boldsymbol{\beta}\times\mathbf{m})}^{X_6} \tag{13}$$

By definition of $\mathbf{m}$ (see (6)),

$$\mathbf{m}\cdot\mathbf{m}=\boldsymbol{\beta}\cdot\mathbf{m}=\beta^2-(\boldsymbol{\beta}\cdot\mathbf{n})^2 \qquad\qquad \mathbf{n}\cdot\mathbf{m}=0 \tag{14}$$

which turns (13) into

$$\begin{aligned}\mathbf{X}=\;&\overbrace{\kappa^3\mathbf{n}}^{X_1}\overbrace{-\kappa^2\boldsymbol{\beta}+3\kappa^2(\boldsymbol{\beta}\cdot\mathbf{n})\mathbf{n}}^{X_2}+\overbrace{\left[\beta^2-(\boldsymbol{\beta}\cdot\mathbf{n})^2\right]\boldsymbol{\beta}-\left[\beta^2-(\boldsymbol{\beta}\cdot\mathbf{n})^2\right](\boldsymbol{\beta}\cdot\mathbf{n})\mathbf{n}}^{X_3}\\ &\underbrace{-\kappa(\boldsymbol{\beta}\cdot\mathbf{n})\boldsymbol{\beta}+\kappa(\boldsymbol{\beta}\cdot\mathbf{n})^2\mathbf{n}}_{X_4}\underbrace{-\kappa\left[\beta^2-(\boldsymbol{\beta}\cdot\mathbf{n})^2\right]\mathbf{n}}_{X_5}\underbrace{-\kappa(\boldsymbol{\beta}\cdot\mathbf{n})\boldsymbol{\beta}+\kappa(\boldsymbol{\beta}\cdot\mathbf{n})^2\mathbf{n}}_{X_6}\end{aligned} \tag{15}$$

Collecting the coefficients of $\mathbf{n}$ and $\boldsymbol{\beta}$ respectively, we have

$$\begin{aligned}\text{coefficient of }\mathbf{n} &= \kappa^3+3\kappa^2(\boldsymbol{\beta}\cdot\mathbf{n})-\left[\beta^2-(\boldsymbol{\beta}\cdot\mathbf{n})^2\right](\boldsymbol{\beta}\cdot\mathbf{n})+\kappa(\boldsymbol{\beta}\cdot\mathbf{n})^2-\kappa\left[\beta^2-(\boldsymbol{\beta}\cdot\mathbf{n})^2\right]+\kappa(\boldsymbol{\beta}\cdot\mathbf{n})^2\\ &= \kappa^3+3\kappa^2(\boldsymbol{\beta}\cdot\mathbf{n})-\beta^2(\boldsymbol{\beta}\cdot\mathbf{n})+(\boldsymbol{\beta}\cdot\mathbf{n})^3+3\kappa(\boldsymbol{\beta}\cdot\mathbf{n})^2-\kappa\beta^2\\ &= [\kappa+(\boldsymbol{\beta}\cdot\mathbf{n})]^3-\beta^2[\kappa+(\boldsymbol{\beta}\cdot\mathbf{n})]\\ &= 1-\beta^2=\frac{1}{\gamma^2}\end{aligned} \tag{16}$$

$$\begin{aligned}\text{coefficient of }\boldsymbol{\beta} &= -\kappa^2+\left[\beta^2-(\boldsymbol{\beta}\cdot\mathbf{n})^2\right]-\kappa(\boldsymbol{\beta}\cdot\mathbf{n})-\kappa(\boldsymbol{\beta}\cdot\mathbf{n})\\ &= -[\kappa+(\boldsymbol{\beta}\cdot\mathbf{n})]^2+\beta^2\\ &= -1+\beta^2=-\frac{1}{\gamma^2}\end{aligned} \tag{17}$$

Putting these back to (12) gives

$$\mathrm{HF}_{1/R^2}=e\left(\frac{\mathbf{n}-\boldsymbol{\beta}}{\gamma^2\kappa^3R^2}\right) \tag{18}$$

which matches the velocity field in the Liénard-Wiechert formula.