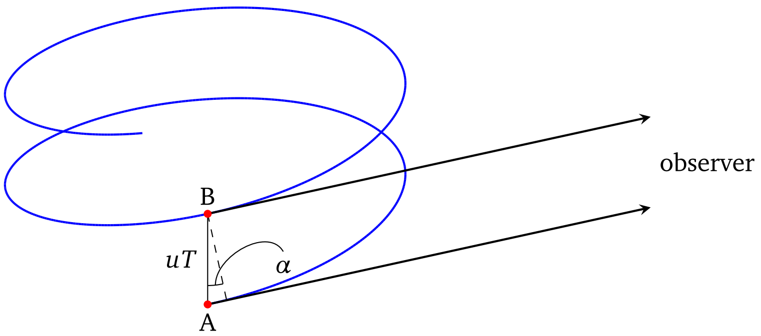
1. **Prob 14.17**



(a) Let the helix be described by

$$\mathbf{x}(t) = a\cos(\omega_B t)\hat{\mathbf{x}} + a\sin(\omega_B t)\hat{\mathbf{y}} + ut\hat{\mathbf{z}} \tag{1}$$

We approximately treat the speed of the charge as c, and the pitch angle α gives

$$\tan\alpha = \frac{u}{\omega_B a} \qquad u = c\sin\alpha \qquad \omega_B a = c\cos\alpha \tag{2}$$

Let A and B be two points on the helix where the charge's velocity points towards the observer. The fundamental frequency as seen by the observer is given by the inverse of the time difference for the photons emitted at A and B to reach the observer. When $\alpha \neq 0$, this time difference must be adjusted by the fact that B is closer than A, i.e., the Doppler effect.

Let $T = 2\pi/\omega_B$, we can see that this time difference is exactly

$$T_{\text{obs}} = T - \frac{uT\sin\alpha}{c} = T\left(1 - \sin^2\alpha\right) = T\cos^2\alpha \tag{3}$$

In other words, the observed frequency is

$$\omega_0 = \frac{\omega_B}{\cos^2\alpha} \tag{4}$$

We can calculate the radius of curvature of the helix

$$\rho = \frac{|\dot{\mathbf{x}}|^3}{|\dot{\mathbf{x}} \times \ddot{\mathbf{x}}|} = \frac{\left(a^2\omega_B^2 + u^2\right)^{3/2}}{a\omega_B^2\sqrt{a^2\omega_B^2 + u^2}} = \frac{a}{\cos^2\alpha} \tag{5}$$

Then by (14.81)

$$\omega_c = \frac{3}{2}\gamma^3\left(\frac{c}{\rho}\right) = \frac{3}{2}\gamma^3\left(\frac{c\cos^2\alpha}{a}\right) = \frac{3}{2}\gamma^3\omega_B\cos\alpha \tag{6}$$

(b) This is just simple algebra from (14.79) while noticing

$$\frac{d^2P}{d\omega d\Omega} = \frac{1}{T}\frac{d^2I}{d\omega d\Omega} = \frac{\omega_0}{2\pi}\frac{d^2I}{d\omega d\Omega} \tag{7}$$

2. **Prob 14.18**

(a) If we rewrite (14.79) in terms of ω_c, we have

$$\frac{d^2I}{d\omega d\Omega} = \frac{3e^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2\left(1+\gamma^2\theta^2\right)^2\left[K_{2/3}^2(\xi) + \left(\frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}\right)K_{1/3}^2(\xi)\right] \tag{8}$$

Since integrating the above over solid angles yields (14.91), we have

$$\frac{3e^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2\int\left(1+\gamma^2\theta^2\right)^2\left[K_{2/3}^2(\xi) + \left(\frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}\right)K_{1/3}^2(\xi)\right]d\Omega = \frac{\sqrt{3}e^2\gamma}{c}\left(\frac{\omega}{\omega_c}\right)\int_{\omega/\omega_c}^{\infty}K_{5/3}(t)\,dt \tag{9}$$

The integration of (14.17) (b) over solid angles is

$$\int \frac{d^2P}{d\omega d\Omega} d\Omega = \frac{3e^2\gamma^2}{8\pi^3 c} \frac{\omega_B}{\cos^2\alpha} \left(\frac{\omega}{\omega_c}\right)^2 \int \left(1+\gamma^2\psi^2\right)^2 \left[K_{2/3}^2(\xi) + \left(\frac{\gamma^2\psi^2}{1+\gamma^2\psi^2}\right) K_{1/3}^2(\xi)\right] d\Omega \tag{10}$$

where ψ is the angle of observation measured relative to the pitch angle α.

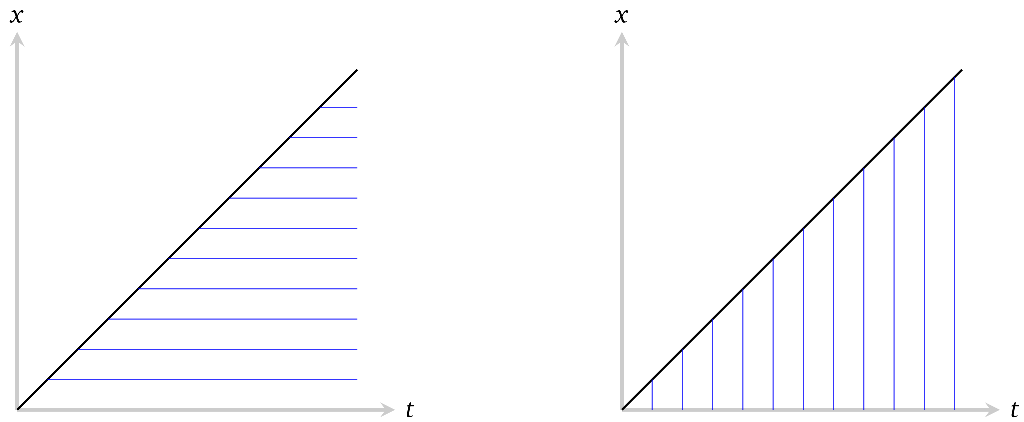
I think Jackson meant to let us use $d\Omega = \cos\alpha d\theta d\phi$ here, which cancels one $\cos\alpha$ in the denominator, and then invoking (9) should give us the claimed result

$$\frac{dP}{d\omega} = \frac{\sqrt{3}e^2\gamma\omega_B}{2\pi c\cos\alpha} \left(\frac{\omega}{\omega_c}\right) \int_{\omega/\omega_c}^{\infty} K_{5/3}(t)\,dt \tag{11}$$

However, I think this is quite inconsistent, since if we treat $d\Omega = \cos\alpha d\theta d\phi$ in (10), we cannot just invoke (9) because in (9) both the integrand and the solid angle element are in θ, the latitude angle, but in (10), the integrand is a function of ψ, while the solid angle element is now in the polar angle θ. It is not clear that we can say $d\theta = d\psi$ in (10) with any straightforward approximation.

(b) The total power is given by the integration

$$\begin{aligned} P = \int_0^\infty \frac{dP}{d\omega} d\omega &= \frac{\sqrt{3}e^2\gamma\omega_B}{2\pi c\cos\alpha} \int_0^\infty d\omega \left(\frac{\omega}{\omega_c}\right) \int_{\omega/\omega_c}^{\infty} K_{5/3}(t)\,dt \\ &= \frac{\sqrt{3}e^2\gamma\omega_B\omega_c}{2\pi c\cos\alpha} \int_0^\infty dx\,x \int_x^\infty K_{5/3}(t)\,dt \qquad \text{by (6)} \\ &= \frac{3\sqrt{3}e^2\gamma^4\omega_B^2}{4\pi c} \int_0^\infty dx\,x \int_x^\infty K_{5/3}(t)\,dt \end{aligned} \tag{12}$$



The diagram on the left schematically shows the double integral in the order given by (12), while the one on the right schematically shows the switched order, i.e.,

$$\begin{aligned} \int_0^\infty dx\,x \int_x^\infty K_{5/3}(t)\,dt &= \int_0^\infty dt K_{5/3}(t) \int_0^t x\,dx \\ &= \frac{1}{2}\int_0^\infty dt\,t^2 K_{5/3}(t) \qquad \text{by DLMF (10.43.E19)} \\ &= \frac{1}{2}\cdot 2^1 \Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{7}{3}\right) \\ &= \left(\frac{4}{3}\right)\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{1}{3}\right) \qquad \Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin(\pi z)} \\ &= \frac{8\pi}{9\sqrt{3}} \end{aligned} \tag{13}$$

turning (12) into

$$P = \frac{2e^2\gamma^4\omega_B^2}{3c} \tag{14}$$