

1. From problem 14.8, we have shown that the radiated power can be expressed in terms of the force  $\mathbf{F}$  acting on the charge

$$P = \frac{2}{3} \frac{z^2 e^2}{m^2 c^3} \gamma^2 [F^2 - (\boldsymbol{\beta} \cdot \mathbf{F})^2] \quad (1)$$

For the charge moving inside electric and magnetic field, the force is

$$\mathbf{F} = ze(\mathbf{E} + \boldsymbol{\beta} \times \mathbf{B}) \quad (2)$$

giving

$$P = \frac{2}{3} \frac{z^4 e^4}{m^2 c^3} \gamma^2 [(\mathbf{E} + \boldsymbol{\beta} \times \mathbf{B})^2 - (\boldsymbol{\beta} \cdot \mathbf{E})^2] \quad (3)$$

2. From the component form of  $F^{\mu\nu}$  and  $F_{\lambda\mu}$  (see 11.137, 11.238), it is straightforward to verify that for the 4-vector  $p^\nu = \gamma mc(1, \boldsymbol{\beta})$ , we have

$$F^{\mu\nu} p_\nu = \gamma mc(-\boldsymbol{\beta} \cdot \mathbf{E}, -\mathbf{E} - \boldsymbol{\beta} \times \mathbf{B}) \quad (4)$$

$$p^\lambda F_{\lambda\mu} = \gamma mc(\boldsymbol{\beta} \cdot \mathbf{E}, -\mathbf{E} - \boldsymbol{\beta} \times \mathbf{B}) \quad (5)$$

enabling us to write (3) as

$$P = \frac{2}{3} \frac{z^4 e^4}{m^4 c^5} (F^{\mu\nu} p_\nu p^\lambda F_{\lambda\mu}) = \frac{2}{3} \frac{z^4 r_0^2}{m^2 c} (F^{\mu\nu} p_\nu p^\lambda F_{\lambda\mu}) \quad \text{where} \quad r_0 = \frac{e^2}{mc^2} \quad (6)$$

Note: this covariant form of  $P$  is a direct substitution of the covariant equation of motion (12.3) into the Larmor formula (14.24).