Here we fill the details leading to the $\mathbf{E},\mathbf{B}$ field expression (14.13), (14.14) from the field strength tensor (14.11), i.e.,

$$F^{\lambda\mu} = \frac{e}{V\cdot(x-r)}\frac{d}{d\tau}\left[\frac{(x-r)^\lambda V^\mu - (x-r)^\mu V^\lambda}{V\cdot(x-r)}\right] \tag{1}$$

where it is understood to be evaluated at the retarded time τ_0 defined by the light-cone condition

Denote

$$(x-r)^\lambda = (R, R\mathbf{n}) \equiv R n^\lambda \qquad \text{where} \qquad n^\lambda = (1,\mathbf{n}) \tag{2}$$

$$V^\lambda = (\gamma c, \gamma c\boldsymbol{\beta}) = \gamma c \beta^\lambda \qquad \text{where} \qquad \beta^\lambda = (1,\boldsymbol{\beta}) \tag{3}$$

then the numerator in the square bracket is

$$N^{\lambda\mu} \equiv (x-r)^\lambda V^\mu - (x-r)^\mu V^\lambda = \gamma c R\left(n^\lambda \beta^\mu - n^\mu \beta^\lambda\right) \tag{4}$$

Note that

$$\frac{d\left[x-r(\tau)\right]^\lambda}{d\tau} = -\frac{dr}{\frac{1}{\gamma}dt} = -V^\lambda = -\gamma c\beta^\lambda \tag{5}$$

and that the derivative of the four velocity with respect to the proper time is

$$\frac{dV^0}{d\tau} = c\frac{d\gamma}{d\tau} = c\left(\frac{d}{\frac{1}{\gamma}dt}\right)\left(\frac{1}{\sqrt{1-\boldsymbol{\beta}\cdot\boldsymbol{\beta}}}\right) = c\gamma^4 \boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}} \tag{6}$$

$$\frac{d\mathbf{V}}{d\tau} = c\left(\frac{d}{\frac{1}{\gamma}dt}\right)(\gamma\boldsymbol{\beta}) = c\gamma\frac{d}{dt}\left(\frac{\boldsymbol{\beta}}{\sqrt{1-\boldsymbol{\beta}\cdot\boldsymbol{\beta}}}\right) = c\gamma\left[\gamma\dot{\boldsymbol{\beta}} + \gamma^3\boldsymbol{\beta}\left(\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}}\right)\right] \tag{7}$$

which allows us to write more compactly as

$$\frac{dV^\lambda}{d\tau} = c\gamma^2\dot{\beta}^\lambda + c\gamma^4\beta^\lambda\left(\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}}\right) \qquad \text{where } \dot{\beta}^\lambda = \left(0,\dot{\boldsymbol{\beta}}\right) \tag{8}$$

This gives

$$\begin{aligned}
\frac{dN^{\lambda\mu}}{d\tau} = \frac{d\left[(x-r)^\lambda V^\mu - (x-r)^\mu V^\lambda\right]}{d\tau} &= \left(-\gamma c\beta^\lambda\right)\left(\gamma c\beta^\mu\right) + Rn^\lambda\frac{dV^\mu}{d\tau} - \left(-\gamma c\beta^\mu\right)\left(\gamma c\beta^\lambda\right) - Rn^\mu\frac{dV^\lambda}{d\tau} \\
&= R\left(n^\lambda\frac{dV^\mu}{d\tau} - n^\mu\frac{dV^\lambda}{dt}\right) \\
&= c\gamma^2 R\left\{n^\lambda\left[\dot{\beta}^\mu + \gamma^2\beta^\mu\left(\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}}\right)\right] - n^\mu\left[\dot{\beta}^\lambda + \gamma^2\beta^\lambda\left(\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}}\right)\right]\right\} \\
&= c\gamma^2 R\left(n^\lambda\dot{\beta}^\mu - n^\mu\dot{\beta}^\lambda\right) + c\gamma^4 R\left(\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}}\right)\left(n^\lambda\beta^\mu - n^\mu\beta^\lambda\right)
\end{aligned} \tag{9}$$

For the denominator in the square bracket, we have

$$V\cdot(x-r) = c\gamma R(1-\boldsymbol{\beta}\cdot\mathbf{n}) \tag{10}$$

$$\frac{d\left[V\cdot(x-r)\right]}{d\tau} = -c^2 + (x-r)_\lambda\frac{dV^\lambda}{d\tau} = -c^2 - c\gamma^2 R\left(\dot{\boldsymbol{\beta}}\cdot\mathbf{n}\right) + c\gamma^4 R\left(\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}}\right)(1-\boldsymbol{\beta}\cdot\mathbf{n}) \tag{11}$$

Putting all these back to (1), we have

$$\begin{aligned}
F^{\lambda\mu} &= \frac{e}{V\cdot(x-r)}\cdot\left\{\frac{\frac{dN^{\lambda\mu}}{d\tau}\left[V\cdot(x-r)\right] - \frac{d\left[V\cdot(x-r)\right]}{d\tau}N^{\lambda\mu}}{\left[V\cdot(x-r)\right]^2}\right\} \\
&= \frac{e}{\left[c\gamma R(1-\boldsymbol{\beta}\cdot\mathbf{n})\right]^3}\left\{\left[c\gamma^2 R\left(n^\lambda\dot{\beta}^\mu - n^\mu\dot{\beta}^\lambda\right) + c\gamma^4 R\left(\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}}\right)\left(n^\lambda\beta^\mu - n^\mu\beta^\lambda\right)\right]\cdot c\gamma R(1-\boldsymbol{\beta}\cdot\mathbf{n}) - \right. \\
&\qquad \left.\left[-c^2 - c\gamma^2 R\left(\dot{\boldsymbol{\beta}}\cdot\mathbf{n}\right) + c\gamma^4 R\left(\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}}\right)(1-\boldsymbol{\beta}\cdot\mathbf{n})\right]\cdot c\gamma R\left(n^\lambda\beta^\mu - n^\mu\beta^\lambda\right)\right\}
\end{aligned} \tag{12}$$

The terms involving $\left(\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}}\right)(1-\boldsymbol{\beta}\cdot\mathbf{n})$ cancel out. Separating the rest into group without acceleration and with acceleration, we have

$$F^{\lambda\mu}=e\left[\frac{n^{\lambda}\beta^{\mu}-n^{\mu}\beta^{\lambda}}{\gamma^{2}R^{2}(1-\boldsymbol{\beta}\cdot\mathbf{n})^{3}}\right]+\frac{e}{c}\left[\frac{(1-\boldsymbol{\beta}\cdot\mathbf{n})\left(n^{\lambda}\dot{\beta}^{\mu}-n^{\mu}\dot{\beta}^{\lambda}\right)+\left(\dot{\boldsymbol{\beta}}\cdot\mathbf{n}\right)\left(n^{\lambda}\beta^{\mu}-n^{\mu}\beta^{\lambda}\right)}{(1-\boldsymbol{\beta}\cdot\mathbf{n})^{3}R}\right] \tag{13}$$

For electric field, $\lambda=i,\mu=0$, this is readily seen to give

$$\begin{aligned}E^{i}=F^{i0}&=e\left[\frac{(n-\beta)^{i}}{\gamma^{2}R^{2}(1-\boldsymbol{\beta}\cdot\mathbf{n})^{3}}\right]+\frac{e}{c}\left[\frac{(1-\boldsymbol{\beta}\cdot\mathbf{n})\left(-\dot{\beta}^{i}\right)+\left(\dot{\boldsymbol{\beta}}\cdot\mathbf{n}\right)(n-\beta)^{i}}{(1-\boldsymbol{\beta}\cdot\mathbf{n})^{3}R}\right]\\&=e\left[\frac{\mathbf{n}-\boldsymbol{\beta}}{\gamma^{2}R^{2}(1-\boldsymbol{\beta}\cdot\mathbf{n})^{3}}\right]^{i}+\frac{e}{c}\left\{\frac{\mathbf{n}\times\left[(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]}{(1-\boldsymbol{\beta}\cdot\mathbf{n})^{3}R}\right\}^{i}\end{aligned} \tag{14}$$

For magnetic field, for example $\lambda=2,\mu=3$, we can see that F^{23} is the x component of the vector

$$e\left[\frac{\mathbf{n}\times\boldsymbol{\beta}}{\gamma^{2}R^{2}(1-\boldsymbol{\beta}\cdot\mathbf{n})^{3}}\right]+\frac{e}{c}\mathbf{n}\times\left\{\frac{-\mathbf{n}\times\left[(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]}{(1-\boldsymbol{\beta}\cdot\mathbf{n})^{3}R}\right\}=-\mathbf{n}\times\mathbf{E} \tag{15}$$

hence (14.13) and (14.14) are verified.