By definition (eq 7.169)

$$\hat{\mathbf{e}}_{\mathbf{k}\pm} = \mp\frac{1}{\sqrt{2}}\left(\hat{\mathbf{e}}_{\mathbf{k}}^{(1)} \pm i\hat{\mathbf{e}}_{\mathbf{k}}^{(2)}\right) \tag{1}$$

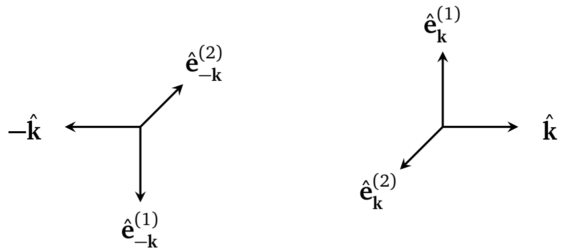
If we take $\lambda$ to be $\pm$, the above is equivalent to

$$\hat{\mathbf{e}}_{\mathbf{k}\lambda} = -\lambda\frac{1}{\sqrt{2}}\left(\hat{\mathbf{e}}_{\mathbf{k}}^{(1)} + \lambda i\hat{\mathbf{e}}_{\mathbf{k}}^{(2)}\right) \tag{2}$$

Thus

$$\begin{aligned}\hat{\mathbf{e}}_{\mathbf{k}\lambda}^* \cdot \hat{\mathbf{e}}_{\pm\mathbf{k}\lambda'} &= \frac{\lambda\lambda'}{2}\left(\hat{\mathbf{e}}_{\mathbf{k}}^{(1)} - \lambda i\hat{\mathbf{e}}_{\mathbf{k}}^{(2)}\right)\cdot\left(\hat{\mathbf{e}}_{\pm\mathbf{k}}^{(1)} + \lambda' i\hat{\mathbf{e}}_{\pm\mathbf{k}}^{(2)}\right)\\ &= \frac{\lambda\lambda'}{2}\left[\hat{\mathbf{e}}_{\mathbf{k}}^{(1)}\cdot\hat{\mathbf{e}}_{\pm\mathbf{k}}^{(1)} + \lambda\lambda'\hat{\mathbf{e}}_{\mathbf{k}}^{(2)}\cdot\hat{\mathbf{e}}_{\pm\mathbf{k}}^{(2)} - i\lambda\hat{\mathbf{e}}_{\mathbf{k}}^{(2)}\cdot\hat{\mathbf{e}}_{\pm\mathbf{k}}^{(1)} + i\lambda'\hat{\mathbf{e}}_{\mathbf{k}}^{(1)}\cdot\hat{\mathbf{e}}_{\pm\mathbf{k}}^{(2)}\right]\end{aligned} \tag{3}$$

At this point, we have to assume some convention about the orientations of $\hat{\mathbf{e}}_{\pm\mathbf{k}}^{(1)}$ and $\hat{\mathbf{e}}_{\pm\mathbf{k}}^{(2)}$ as indicated by the diagram below.




With this convention, it's clear that (3) gives

$$\hat{\mathbf{e}}_{\mathbf{k}\lambda}^* \cdot \hat{\mathbf{e}}_{\pm\mathbf{k}\lambda'} = \pm\delta_{\lambda\lambda'} \tag{4}$$

Also, the cross product is

$$\begin{aligned}\hat{\mathbf{e}}_{\mathbf{k}\lambda}^* \times \hat{\mathbf{e}}_{\pm\mathbf{k}\lambda'} &= \frac{\lambda\lambda'}{2}\left[\hat{\mathbf{e}}_{\mathbf{k}}^{(1)}\times\hat{\mathbf{e}}_{\pm\mathbf{k}}^{(1)} + \lambda\lambda'\hat{\mathbf{e}}_{\mathbf{k}}^{(2)}\times\hat{\mathbf{e}}_{\pm\mathbf{k}}^{(2)} - i\lambda\hat{\mathbf{e}}_{\mathbf{k}}^{(2)}\times\hat{\mathbf{e}}_{\pm\mathbf{k}}^{(1)} + i\lambda'\hat{\mathbf{e}}_{\mathbf{k}}^{(1)}\times\hat{\mathbf{e}}_{\pm\mathbf{k}}^{(2)}\right]\\ &= \pm i\delta_{\lambda\lambda'}\hat{\mathbf{k}}\end{aligned} \tag{5}$$

Next, let's look at the vector potential, equation (7.167), (7.168)

$$\mathbf{A}(\mathbf{x},t) = \sum_{\mathbf{k},\lambda}\hat{\mathbf{e}}_{\mathbf{k}\lambda}A_{\mathbf{k}\lambda}(\mathbf{x},t) \tag{6}$$

$$A_{\mathbf{k}\lambda}(\mathbf{x},t) = A_{\mathbf{k}\lambda}e^{-i(\omega_k t - \mathbf{k}\cdot\mathbf{x})} + A_{\mathbf{k}\lambda}^* e^{i(\omega_k t - \mathbf{k}\cdot\mathbf{x})} \tag{7}$$

I think it's confusing for the text to use boldface $\mathbf{A}_{\mathbf{k}\lambda}$, in this context, they are not vectors so they shouldn't have been in boldface.

Now we have

$$\begin{aligned}\mathbf{E} &= -\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}\\ &= \frac{i}{c}\sum_{\mathbf{k},\lambda}\omega_k\left[A_{\mathbf{k}\lambda}e^{-i(\omega_k t - \mathbf{k}\cdot\mathbf{x})} - A_{\mathbf{k}\lambda}^* e^{i(\omega_k t - \mathbf{k}\cdot\mathbf{x})}\right]\hat{\mathbf{e}}_{\mathbf{k}\lambda}\end{aligned} \tag{8}$$

$$\mathbf{E}^* = -\frac{i}{c}\sum_{\mathbf{k}',\lambda'}\omega_{k'}\left[A_{\mathbf{k}'\lambda'}^* e^{i(\omega_{k'} t - \mathbf{k}'\cdot\mathbf{x})} - A_{\mathbf{k}'\lambda'}e^{-i(\omega_{k'} t - \mathbf{k}'\cdot\mathbf{x})}\right]\hat{\mathbf{e}}_{\mathbf{k}'\lambda'}^* \tag{9}$$

which contributes to the total energy in the large box $V = L^3$ as

$$\begin{aligned}\mathscr{E}_E &= \frac{1}{8\pi}\int_V d^3x\,\mathbf{E}\cdot\mathbf{E}^*\\ &= \frac{1}{8\pi}\frac{1}{c^2}\sum_{\mathbf{k},\mathbf{k}',\lambda,\lambda'}\omega_k\omega_{k'}\left(\hat{\mathbf{e}}_{\mathbf{k}\lambda}\cdot\hat{\mathbf{e}}_{\mathbf{k}'\lambda'}^*\right)\left[I_1 + I_2 + I_3 + I_4\right]\end{aligned} \tag{10}$$

where

$$
\begin{aligned}
I_1 &= A_{\mathbf{k}\lambda}A^*_{\mathbf{k}'\lambda'}e^{-i(\omega_k-\omega_{k'})t}\int_V d^3x e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x}} \\
&= A_{\mathbf{k}\lambda}A^*_{\mathbf{k}'\lambda'}e^{-i(\omega_k-\omega_{k'})t}\cdot V\delta_{\mathbf{k},\mathbf{k}'} \qquad (11)\\
I_2 &= -A_{\mathbf{k}\lambda}A_{\mathbf{k}'\lambda'}e^{-i(\omega_k+\omega_{k'})t}\int_V d^3x e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}} \\
&= -A_{\mathbf{k}\lambda}A_{\mathbf{k}'\lambda'}e^{-i(\omega_k+\omega_{k'})t}\cdot V\delta_{\mathbf{k},-\mathbf{k}'} \qquad (12)\\
I_3 &= -A^*_{\mathbf{k}\lambda}A^*_{\mathbf{k}'\lambda'}e^{i(\omega_k+\omega_{k'})t}\int_V d^3x e^{-i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}} \\
&= -A^*_{\mathbf{k}\lambda}A^*_{\mathbf{k}'\lambda'}e^{i(\omega_k+\omega_{k'})t}\cdot V\delta_{\mathbf{k},-\mathbf{k}'} \qquad (13)\\
I_4 &= A^*_{\mathbf{k}\lambda}A_{\mathbf{k}'\lambda'}e^{i(\omega_k-\omega_{k'})t}\int_V d^3x e^{-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x}} \\
&= A^*_{\mathbf{k}\lambda}A_{\mathbf{k}'\lambda'}e^{i(\omega_k-\omega_{k'})t}\cdot V\delta_{\mathbf{k},\mathbf{k}'} \qquad (14)
\end{aligned}
$$

Plug these into (10), and invoke (4), we have

$$
\begin{aligned}
\mathscr{E}_E &= \frac{1}{8\pi}\frac{V}{c^2}\sum_{\mathbf{k},\lambda}\left[\omega_k^2\left(A_{\mathbf{k}\lambda}A^*_{\mathbf{k}\lambda}\right)+\omega_k^2(-1)\left(-A_{\mathbf{k}\lambda}A_{-\mathbf{k}\lambda}e^{-i2\omega_k t}\right)+\omega_k^2(-1)\left(-A^*_{\mathbf{k}\lambda}A^*_{-\mathbf{k}\lambda}e^{i2\omega_k t}\right)+\omega_k^2\left(A^*_{\mathbf{k}\lambda}A_{\mathbf{k}\lambda}\right)\right] \\
&= \frac{1}{8\pi}\frac{V}{c^2}\sum_{\mathbf{k},\lambda}\omega_k^2\left(A_{\mathbf{k}\lambda}A^*_{\mathbf{k}\lambda}+A^*_{\mathbf{k}\lambda}A_{\mathbf{k}\lambda}+A_{\mathbf{k}\lambda}A_{-\mathbf{k}\lambda}e^{-i2\omega_k t}+A^*_{\mathbf{k}\lambda}A^*_{-\mathbf{k}\lambda}e^{i2\omega_k t}\right) \qquad (15)
\end{aligned}
$$

For magnetic field, first recall

$$
\nabla\times(\psi\mathbf{a}) = \nabla\psi\times\mathbf{a}+\psi\nabla\cdot\mathbf{a} \qquad (16)
$$

Then we have

$$
\begin{aligned}
\mathbf{B} &= \nabla\times\mathbf{A}(\mathbf{x},t) \\
&= \nabla\times\sum_{\mathbf{k},\lambda}\hat{\mathbf{e}}_{\mathbf{k}\lambda}A_{\mathbf{k}\lambda}(\mathbf{x},t) \\
&= \sum_{\mathbf{k},\lambda}\nabla A_{\mathbf{k}\lambda}(\mathbf{x},t)\times\hat{\mathbf{e}}_{\mathbf{k}\lambda} \\
&= \sum_{\mathbf{k},\lambda}\left[i\mathbf{k}A_{\mathbf{k}\lambda}e^{-i(\omega_k t-\mathbf{k}\cdot\mathbf{x})}-i\mathbf{k}A^*_{\mathbf{k}\lambda}e^{i(\omega_k t-\mathbf{k}\cdot\mathbf{x})}\right]\times\hat{\mathbf{e}}_{\mathbf{k}\lambda} \\
&= i\sum_{\mathbf{k},\lambda}\left[A_{\mathbf{k}\lambda}e^{-i(\omega_k t-\mathbf{k}\cdot\mathbf{x})}-A^*_{\mathbf{k}\lambda}e^{i(\omega_k t-\mathbf{k}\cdot\mathbf{x})}\right](\mathbf{k}\times\hat{\mathbf{e}}_{\mathbf{k}\lambda}) \qquad (17)
\end{aligned}
$$

This is almost the same as (8) except $\omega_k\hat{\mathbf{e}}_{\mathbf{k}\lambda}/c$ was to be replaced by $\mathbf{k}\times\hat{\mathbf{e}}_{\mathbf{k}\lambda}$. So the corresponding magnetic energy in the large box will be (compare to (10))

$$
\mathscr{E}_B = \frac{1}{8\pi}\sum_{\mathbf{k},\mathbf{k}',\lambda,\lambda'}(\mathbf{k}\times\hat{\mathbf{e}}_{\mathbf{k}\lambda})\cdot\left(\mathbf{k}'\times\hat{\mathbf{e}}^*_{\mathbf{k}'\lambda'}\right)\left[I_1+I_2+I_3+I_4\right] \qquad (18)
$$

Recall the vector identity

$$
(\mathbf{a}\times\mathbf{b})\cdot(\mathbf{c}\times\mathbf{d}) = (\mathbf{a}\cdot\mathbf{c})(\mathbf{b}\cdot\mathbf{d})-(\mathbf{a}\cdot\mathbf{d})(\mathbf{b}\cdot\mathbf{c}) \qquad (19)
$$

whose second term will drop off due to the $\delta(\mathbf{k}\pm\mathbf{k}')$ in (11)-(14). Thus we have

$$
\begin{aligned}
\mathscr{E}_B &= \frac{V}{8\pi}\sum_{\mathbf{k},\lambda}\left[(k^2)\left(A_{\mathbf{k}\lambda}A^*_{\mathbf{k}\lambda}\right)+(-k^2)(-1)\left(-A_{\mathbf{k}\lambda}A_{-\mathbf{k}\lambda}e^{-i2\omega_k t}\right)+(-k^2)(-1)\left(-A^*_{\mathbf{k}\lambda}A^*_{-\mathbf{k}\lambda}e^{i2\omega_k t}\right)+(k^2)\left(A^*_{\mathbf{k}\lambda}A_{\mathbf{k}\lambda}\right)\right] \\
&= \frac{V}{8\pi}\sum_{\mathbf{k},\lambda}k^2\left(A_{\mathbf{k}\lambda}A^*_{\mathbf{k}\lambda}+A^*_{\mathbf{k}\lambda}A_{\mathbf{k}\lambda}-A_{\mathbf{k}\lambda}A_{-\mathbf{k}\lambda}e^{-i2\omega_k t}-A^*_{\mathbf{k}\lambda}A^*_{-\mathbf{k}\lambda}e^{i2\omega_k t}\right) \qquad (20)
\end{aligned}
$$

Since $k^2 = \omega_k^2/c^2$, adding (15) and (20) will yield equation (7.176)

$$
\mathscr{E} = \frac{1}{4\pi}\sum_{\mathbf{k}\lambda}\frac{\omega_k^2}{c^2}\left(A_{\mathbf{k}\lambda}A^*_{\mathbf{k}\lambda}+A^*_{\mathbf{k}\lambda}A_{\mathbf{k}\lambda}\right) \qquad (21)
$$