From the construction (8.15)

$$\phi(\mathbf{x},t) = \frac{1}{2}\left[\Psi(\mathbf{x},t) + \frac{i}{m}D_t\Psi(\mathbf{x},t)\right] \tag{1}$$

$$\chi(\mathbf{x},t) = \frac{1}{2}\left[\Psi(\mathbf{x},t) - \frac{i}{m}D_t\Psi(\mathbf{x},t)\right] \tag{2}$$

we have

$$\Psi = \phi + \chi \tag{3}$$

$$D_t\Psi = \frac{m}{i}(\phi - \chi) \tag{4}$$

To prove (8.16)

$$iD_t\phi = -\frac{1}{2m}\mathbf{D}^2(\phi+\chi) + m\phi \tag{5}$$

$$iD_t\chi = +\frac{1}{2m}\mathbf{D}^2(\phi+\chi) - m\chi \tag{6}$$

It's equivalent to prove

$$2imD_t\phi = -\mathbf{D}^2\Psi + 2m^2\phi \tag{7}$$

$$2imD_t\chi = \mathbf{D}^2\Psi - 2m^2\chi \tag{8}$$

which are equivalent to

$$imD_t\left(\Psi + \frac{i}{m}D_t\Psi\right) = -\mathbf{D}^2\Psi + m^2\left(\Psi + \frac{i}{m}D_t\Psi\right) \tag{9}$$

$$imD_t\left(\Psi - \frac{i}{m}D_t\Psi\right) = \mathbf{D}^2\Psi - m^2\left(\Psi - \frac{i}{m}D_t\Psi\right) \tag{10}$$

which are both equivalent to

$$\left(D_t^2 - \mathbf{D}^2 + m^2\right)\Psi = 0 \tag{11}$$

which is true by (8.14) given $D_\mu D^\mu = D_t^2 - \mathbf{D}^2$.