

We shall elaborate the sentence "In addition, the $0 \rightarrow 0$ transition is forbidden", under equation (3.482). The meaning of this is specifically that

$$\langle j_1 = l, j_2 = 1; m_1 = 0, m_2 = 0 | j = l, m = 0 \rangle = 0 \quad (1)$$

I.e., when we add two angular momenta $|j_1 = l, m_1 = 0\rangle$, and $|j_2 = 1, m_2 = 0\rangle$, one of the possible resulting states $|j = l, m = 0\rangle$ (in the composite angular momentum basis) is always orthogonal to the tensor product basis ket $|j_1 = l, m_1 = 0\rangle \otimes |j_2 = 1, m_2 = 0\rangle$.

This C-G coefficient is not obviously forbidden by the usual selection rule, since $|l - 1| \leq j = l \leq |l + 1|$, and $m_1 + m_2 = 0 + 0 = m$.

In fact, (1) can be proved by the Wigner-Eckart theorem.

Consider the vector operator \mathbf{J} , we can transform it into the spherical tensor form

$$T_{\pm 1}^{(1)} = \mp(J_x \pm iJ_y), \quad T_0^{(1)} = J_z \quad (2)$$

By Wigner-Eckart theorem, equation 3.474, for $j' = j = l, m' = m = 0$, we have

$$\left\langle j' = l, m' = 0 \left| T_q^{(1)} \right| j = l, m = 0 \right\rangle = \langle j = l, k = 1; m = 0, q | j' = l, k = 1; j' = l, m = 0 \rangle \frac{\langle j' = l || T^{(1)} || j = l \rangle}{\sqrt{2l + 1}} \quad (3)$$

But for $q = 0$, the LHS vanishes because $T_0^{(1)} = J_z$, which means the CG coefficient on the RHS must vanish (the second factor must be non-zero since it's the same for all m and q values), which proves (1).