We are only interested in the s-state, so throughout, we have $l = 0$. Following methods from section 3.7.1, define $u(r) = R(r)r$, which should satisfy the boundary coundition

$$\lim_{r\to\infty} u(r) = 0 \tag{1}$$
$$\lim_{r\to 0} u(r) = 0 \tag{2}$$

Also the original radial wavefunction $R(r)$ should also satisfy

$$\lim_{r\to\infty} R(r) = 0 \tag{3}$$
$$\lim_{r\to\infty} R'(r) = 0 \tag{4}$$

By eq (3.271), the differential equation for u is (recall that $l = 0$)

$$-\frac{\hbar^2}{2m}\frac{d^2u(r)}{dr^2} + V(r)u(r) = Eu(r) \tag{5}$$

Multiply $u' = du/dr$, we have

$$\begin{aligned}
-\frac{\hbar^2}{2m}u'\frac{d^2u}{dr^2} + Vuu' = Euu' & \qquad \Longrightarrow \\
-\frac{\hbar^2}{2m}\frac{1}{2}\frac{d(u'^2)}{dr} + \frac{1}{2}V\frac{d(u^2)}{dr} = \frac{1}{2}E\frac{d(u^2)}{dr} & \qquad \Longrightarrow \\
-\frac{\hbar^2}{2m}\frac{d(u'^2)}{dr} + \frac{d(Vu^2)}{dr} - \frac{dV}{dr}u^2 = E\frac{d(u^2)}{dr} & \qquad \Longrightarrow \\
-\frac{\hbar^2}{2m}u'^2\Big|_0^\infty + \underbrace{Vu^2\Big|_0^\infty}_{=0} - \int_{r=0}^\infty \frac{dV}{dr}u^2dr = \underbrace{Eu^2\Big|_0^\infty}_{=0} & \qquad \Longrightarrow \\
\frac{\hbar^2}{2m}u'^2(0) - \frac{\hbar^2}{2m}u'^2(\infty) = \int_{r=0}^\infty \frac{dV}{dr}u^2dr & \qquad (6)
\end{aligned}$$

But

$$u' = R'r + R \tag{7}$$

therefore (6) becomes

$$\begin{aligned}
\frac{\hbar^2}{2m}R^2(0) = \int_{r=0}^\infty \frac{dV}{dr}R^2(r)r^2dr = \frac{1}{4\pi}\left\langle\frac{dV}{dr}\right\rangle & \qquad \Longrightarrow \\
R^2(0) = \frac{m}{2\pi\hbar^2}\left\langle\frac{dV}{dr}\right\rangle & \qquad (8)
\end{aligned}$$

which is trivially equal to $|\psi(0)|^2$ if we add the spherical harmonics $Y_{l=0}^0(\theta = 0, \phi = \text{undefined})$ to the LHS to get the full ψ.