1. By first-order Born approximation (equation (6.72))

$$f^{(1)}(\boldsymbol{k}',\boldsymbol{k}) = -\frac{m}{2\pi\hbar^2}\int d^3x' e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{x}'}V(\boldsymbol{x}') \tag{1}$$

and the differential cross section is given by

$$\frac{d\sigma}{d\Omega_{\boldsymbol{k}'}} = \left|f^{(1)}(\boldsymbol{k}',\boldsymbol{k})\right|^2 \tag{2}$$

So the total cross section is given by the integration over the solid angle of $\boldsymbol{k}'$,

$$\begin{aligned}
\sigma_{\text{tot}} &= \int d\Omega_{\boldsymbol{k}'}\frac{d\sigma}{d\Omega_{\boldsymbol{k}'}} \\
&= \int d\Omega_{\boldsymbol{k}'}\left|f^{(1)}(\boldsymbol{k}',\boldsymbol{k})\right|^2 \\
&= \frac{m^2}{4\pi^2\hbar^4}\int d\Omega_{\boldsymbol{k}'}\int d^3x'\int d^3x'' e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{x}'}e^{-i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{x}''}V(\boldsymbol{x}')V(\boldsymbol{x}'') \\
&= \frac{m^2}{4\pi^2\hbar^4}\int d^3x'\int d^3x'' e^{i\boldsymbol{k}\cdot(\boldsymbol{x}'-\boldsymbol{x}'')}V(\boldsymbol{x}')V(\boldsymbol{x}'')\cdot\int d\Omega_{\boldsymbol{k}'}e^{-i\boldsymbol{k}'\cdot(\boldsymbol{x}'-\boldsymbol{x}'')}
\end{aligned} \tag{3}$$

where the inner-most integral is done by aligning $\hat{\boldsymbol{z}}$ with $\boldsymbol{x}'-\boldsymbol{x}''$ direction, which gives

$$\begin{aligned}
\int d\Omega_{\boldsymbol{k}'}e^{-i\boldsymbol{k}'\cdot(\boldsymbol{x}'-\boldsymbol{x}'')} &= \int_0^{2\pi}d\phi\int_0^{\pi}\sin\theta\, d\theta\, e^{-i|\boldsymbol{k}'||\boldsymbol{x}'-\boldsymbol{x}''|\cos\theta} \qquad (|\boldsymbol{k}'|=k,\ y\equiv-\cos\theta) \\
&= 2\pi\int_{-1}^{1}dy\, e^{ik|\boldsymbol{x}'-\boldsymbol{x}''|y} \\
&= 2\pi\cdot\frac{e^{ik|\boldsymbol{x}'-\boldsymbol{x}''|}-e^{-ik|\boldsymbol{x}'-\boldsymbol{x}''|}}{ik|\boldsymbol{x}'-\boldsymbol{x}''|} \\
&= 2\pi\cdot\frac{2i\sin(k|\boldsymbol{x}'-\boldsymbol{x}''|)}{ik|\boldsymbol{x}'-\boldsymbol{x}''|} \\
&= 4\pi\cdot\frac{\sin(k|\boldsymbol{x}'-\boldsymbol{x}''|)}{k|\boldsymbol{x}'-\boldsymbol{x}''|}
\end{aligned} \tag{4}$$

Now (3) becomes

$$\sigma_{\text{tot}} = \frac{m^2}{\pi\hbar^4}\int d^3x'\int d^3x'' e^{i\boldsymbol{k}\cdot(\boldsymbol{x}'-\boldsymbol{x}'')}V(\boldsymbol{x}')V(\boldsymbol{x}'')\frac{\sin(k|\boldsymbol{x}'-\boldsymbol{x}''|)}{k|\boldsymbol{x}'-\boldsymbol{x}''|} \tag{5}$$

Now for the claim of Prob 6.2 to make sense, we have to be dealing with central potentials. *However, I'm not sure how it is possible to replace the* $e^{i\boldsymbol{k}\cdot(\boldsymbol{x}'-\boldsymbol{x}'')}$ *factor in the integrand with another factor of* $\sin(k|\boldsymbol{x}'-\boldsymbol{x}''|)/(k|\boldsymbol{x}'-\boldsymbol{x}''|)$ *even taking consideration of symmetry of the central potential. Since here we are not integrating over the solid angle of* $\boldsymbol{k}$*, but over all of* $\boldsymbol{x}', \boldsymbol{x}''$*, with other factors mixed in the integrand.*

2. The first-order Born approximation is real, so only second-order contributes an imaginary part. By equation (6.86)

$$\begin{aligned}
f^{(2)}(\boldsymbol{k},\boldsymbol{k}) &= -\frac{m}{2\pi\hbar^2}\int d^3x'\int d^3x'' e^{-i\boldsymbol{k}\cdot\boldsymbol{x}'}V(\boldsymbol{x}')\left[\frac{2m}{\hbar^2}G_+(\boldsymbol{x}',\boldsymbol{x}'')\right]V(\boldsymbol{x}'')e^{i\boldsymbol{k}\cdot\boldsymbol{x}''} \\
&= -\frac{m}{2\pi\hbar^2}\int d^3x'\int d^3x'' e^{-i\boldsymbol{k}\cdot\boldsymbol{x}'}V(\boldsymbol{x}')\left[\frac{2m}{\hbar^2}\left(-\frac{1}{4\pi}\right)\frac{e^{ik|\boldsymbol{x}'-\boldsymbol{x}''|}}{|\boldsymbol{x}'-\boldsymbol{x}''|}\right]V(\boldsymbol{x}'')e^{i\boldsymbol{k}\cdot\boldsymbol{x}''} \\
&= \frac{m^2}{4\pi^2\hbar^4}\int d^3x'\int d^3x'' e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}'-\boldsymbol{x}'')}\frac{e^{ik|\boldsymbol{x}'-\boldsymbol{x}''|}}{|\boldsymbol{x}'-\boldsymbol{x}''|}V(\boldsymbol{x}')V(\boldsymbol{x}'')
\end{aligned} \tag{6}$$

If Prob 6.2(b) can be shown by Optical theorem, we would need to prove

$$\begin{aligned}\sigma_{\mathrm{tot}} &= \frac{4\pi}{k}\,\mathrm{Im}\, f^{(2)}(\boldsymbol{k},\boldsymbol{k}) \\ &= \frac{m^2}{\pi\hbar^4}\int d^3x' \int d^3x'' \frac{1}{k|\boldsymbol{x}'-\boldsymbol{x}''|}\,\mathrm{Im}\left[e^{ik|\boldsymbol{x}'-\boldsymbol{x}''|}e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}'-\boldsymbol{x}'')}\right]V(\boldsymbol{x}')V(\boldsymbol{x}'')\end{aligned} \tag{7}$$

and we should be able to replace

$$\mathrm{Im}\left[e^{ik|\boldsymbol{x}'-\boldsymbol{x}''|}e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}'-\boldsymbol{x}'')}\right]$$

by

$$\frac{\sin^2(k|\boldsymbol{x}'-\boldsymbol{x}''|)}{k|\boldsymbol{x}'-\boldsymbol{x}''|}$$

I cannot bring myself to justify this replacement even when the potential V is spherically symmetric.