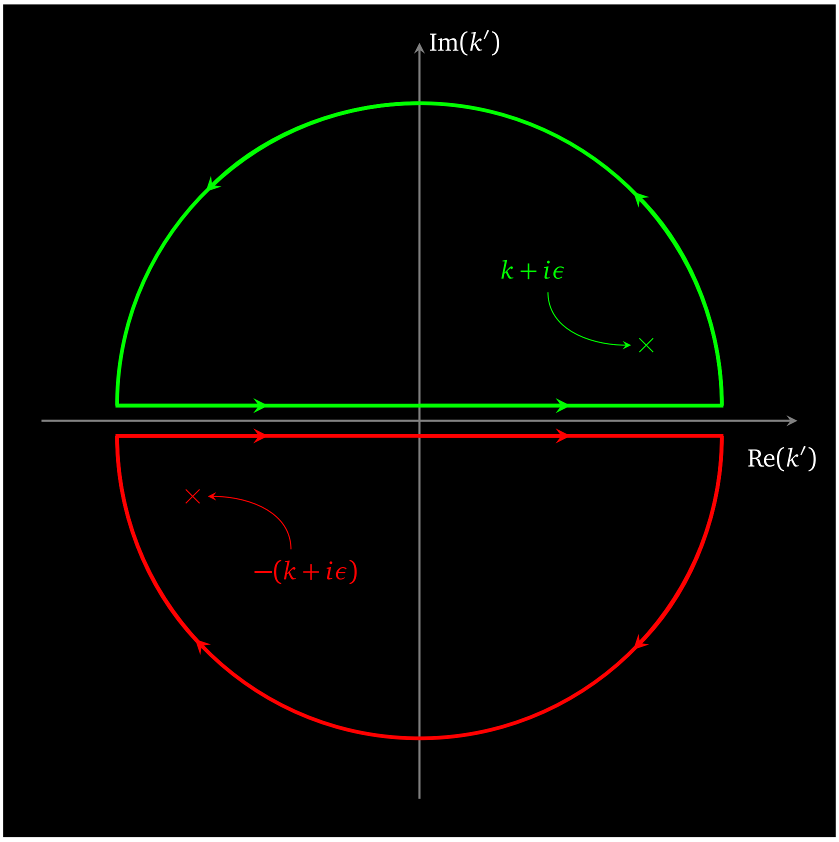
1. For one dimension, the scattering Green's function analogous to (6.37) is

$$\begin{aligned}
G(x,x') &= \frac{\hbar^2}{2m}\left\langle x\left|\frac{1}{E-H_0+i\epsilon}\right|x'\right\rangle \\
&= \frac{\hbar^2}{2m}\int_{-\infty}^{\infty} dk' \int_{-\infty}^{\infty} dk'' \langle x|k'\rangle \left\langle k'\left|\frac{1}{E-H_0+i\epsilon}\right|k''\right\rangle \langle k''|x'\rangle \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} dk' \int_{-\infty}^{\infty} dk'' e^{ik'x} e^{-ik''x'} \left(\frac{1}{k^2-k''^2+i\epsilon}\right)\langle k'|k''\rangle \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} dk' \frac{e^{ik'(x-x')}}{k^2-k'^2+i\epsilon} \\
&= -\frac{1}{2\pi}\int_{-\infty}^{\infty} dk' \frac{e^{ik'(x-x')}}{[k'+(k+i\epsilon)][k'-(k+i\epsilon)]}
\end{aligned} \tag{1}$$

The integral can be done via contour integral on the complex domain, but we have to treat the sign of $x-x'$ differently.



- $x-x' \geq 0$: We take the upper contour since the integrand at $+i\infty$ will vanish. Thus

$$\begin{aligned}
G(x,x') &= -\frac{1}{2\pi}\oint_{\text{upper}} dk' \frac{e^{ik'(x-x')}}{[k'+(k+i\epsilon)][k'-(k+i\epsilon)]} \\
&= -\frac{1}{2\pi}\cdot 2i\pi \frac{e^{ik(x-x')}}{2k} \\
&= -\frac{i}{2k}e^{ik(x-x')}
\end{aligned} \tag{2}$$

- $x - x' < 0$: We take the lower contour since the integrand at $-i\infty$ will vanish.

$$\begin{aligned} G(x,x') &= -\frac{1}{2\pi}\oint_{\text{lower}} dk' \frac{e^{-ik'|x-x'|}}{[k'+(k+i\epsilon)][k'-(k+i\epsilon)]} \\ &= -\frac{1}{2\pi}\cdot(-2i\pi)\frac{e^{-i(-k)|x-x'|}}{-2k} \\ &= -\frac{i}{2k}e^{ik|x-x'|} \end{aligned} \tag{3}$$

For either sign of $x - x'$, the result can be summarized as

$$G(x,x') = -\frac{i}{2k}e^{ik|x-x'|} \tag{4}$$

2. The Lippmann-Schwinger equation is

$$\begin{aligned} \langle x|\psi^{(+)}\rangle &= \langle x|i\rangle + \left\langle x\left|\frac{1}{E-H_0+i\epsilon}V\right|\psi^{(+)}\right\rangle \\ &= \langle x|i\rangle + \int dx' \left\langle x\left|\frac{1}{E-H_0+i\epsilon}\right|x'\right\rangle \langle x'|V|\psi^{(+)}\rangle \\ &= \langle x|i\rangle + \int dx' \frac{2m}{\hbar^2}\left(-\frac{i}{2k}e^{ik|x-x'|}\right)\left[-\frac{\gamma\hbar^2\delta(x')}{2m}\right]\langle x'|\psi^{(+)}\rangle \\ &= \frac{e^{ikx}}{\sqrt{2\pi}} + \frac{i\gamma}{2k}e^{ik|x|}\psi^{(+)}(0) \end{aligned} \tag{5}$$

(5) gives

$$\psi^{(+)}(0) = \frac{1}{\sqrt{2\pi}}\frac{1}{1-\dfrac{i\gamma}{2k}} = \frac{1}{\sqrt{2\pi}}\frac{2k}{2k-i\gamma} \tag{6}$$

Then it's clear

$$\psi^{(+)}(x) = \begin{cases} \dfrac{e^{ikx}}{\sqrt{2\pi}} + \dfrac{e^{ikx}}{\sqrt{2\pi}}\dfrac{i\gamma}{2k-i\gamma} & x \geq 0 \\ \dfrac{e^{ikx}}{\sqrt{2\pi}} + \dfrac{e^{-ikx}}{\sqrt{2\pi}}\dfrac{i\gamma}{2k-i\gamma} & x < 0 \end{cases} \tag{7}$$

By definition of $T(k)$ and $R(k)$, we can identify

$$T(k) = 1 + \frac{i\gamma}{2k-i\gamma} = \frac{2k}{2k-i\gamma} \tag{8}$$

$$R(k) = \frac{i\gamma}{2k-i\gamma} \tag{9}$$

which obviously satisfy $|T(k)|^2 + |R(k)|^2 = 1$.

3. omitted, not sure what it means.
4. Recall the bound state wave function for the attractive delta potential $V(x) = -\gamma\hbar^2\delta(x)/2m$ is given by

$$\psi(x) = Ae^{-\kappa|x|} \tag{10}$$

where the bound energy $E_B = -\hbar^2\kappa^2/2m$. If we integrate the Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x) = E_B\psi(x) \tag{11}$$

in the infinitesimal range $[-\epsilon,\epsilon]$, we get

$$-\frac{\hbar^2}{2m}\left[\psi'(\epsilon)-\psi'(-\epsilon)\right] + \int_{-\epsilon}^{\epsilon} V(x)\psi(x)dx = \int_{-\epsilon}^{\epsilon} E_B\psi(x)dx \tag{12}$$

Consider we will eventually take the limit $\epsilon \to 0$, (12) gives

$$\begin{aligned} -\frac{\hbar^2}{2m}\left[A(-\kappa)e^0 - A\kappa e^0\right] - \frac{\gamma\hbar^2}{2m}Ae^0 = 0 \qquad & \implies \\ \kappa = \frac{\gamma}{2} \qquad & \end{aligned} \tag{13}$$

which is exactly the imaginary part of the pole of $T(k), R(k)$ (ref. equation 6.211).