In the E-M field $(\phi, \mathbf{A})$, we should make the following change to the derivation of K-G equation, which originally treated free particle:

$$H = \sqrt{\mathbf{p}^2 + m^2} \qquad \longrightarrow \qquad H = \sqrt{\mathbf{\Pi}^2 + m^2} + e\phi = \sqrt{(\mathbf{p} - e\mathbf{A})^2 + m^2} + e\phi \tag{1}$$

This modification is justified by combining the two arguments: A) E-M modification of Hamiltonian as stated in eq (2.340)-(2.346) and B) the relativistic modification in eq (8.2).

Then the K-G equation in (8.5) should be modified accordingly as

$$\left(i\frac{\partial}{\partial t} - e\phi\right)^2 |\psi(t)\rangle = \left[(\mathbf{p} - e\mathbf{A})^2 + m^2\right] |\psi(t)\rangle \tag{2}$$

which in coordinate basis yields

$$\left(i\frac{\partial}{\partial t} - e\phi\right)^2 \Psi(\mathbf{x}, t) = \left[(-i\mathbf{\nabla} - e\mathbf{A})^2 + m^2\right] \Psi(\mathbf{x}, t) \tag{3}$$

Now compare (3) with the desired form

$$\left(D_\mu D^\mu + m^2\right) \Psi(\mathbf{x}, t) = 0 \tag{4}$$

It remains to show

$$D_\mu D^\mu = (-i\mathbf{\nabla} - e\mathbf{A})^2 - \left(i\frac{\partial}{\partial t} - e\phi\right)^2 \tag{5}$$

Indeed, this can be easily seen by expanding inner product between

$$D_\mu = \partial_\mu + ieA_\mu = \left(\frac{\partial}{\partial t} + ie\phi, \mathbf{\nabla} - ie\mathbf{A}\right) \qquad \text{and} \tag{6}$$

$$D^\mu = \partial^\mu + ieA^\mu = \left(\frac{\partial}{\partial t} + ie\phi, -\mathbf{\nabla} + ie\mathbf{A}\right) \tag{7}$$