We are going to use the procedure on page 398 to solve this problem. Within the sphere, the radial wavefunction $A_l(r)$ is given by eq (6.141), i.e.,

$$\frac{d^2u_l}{dr^2}+\left[k^2-\frac{2m}{\hbar^2}V-\frac{l(l+1)}{r^2}\right]u_l=0 \qquad \text{where } u_l(r)=rA_l(r) \tag{1}$$

If we let

$$\kappa=\sqrt{k^2-\frac{2mV}{\hbar^2}} \tag{2}$$

then (1) is exactly the same radial equation for the l-th partial wave of the free particle with wave number κ. Therefore the physically feasible solution is $A_l(r)=c_l j_l(\kappa r)$ for $r<R$.

Then

$$\beta_l(R)\Big|_{\text{out}}=\beta_l(R)\Big|_{\text{in}}=\frac{r}{A_l}\frac{dA_l}{dr}\Big|_{r=R}=\frac{\kappa R}{j_l(\kappa R)}j_l'(\kappa R) \tag{3}$$

By (6.140)

$$\begin{aligned}\tan\delta_l&=\frac{kRj_l'(kR)-\beta_l j_l(kR)}{kRn_l'(kR)-\beta_l n_l(kR)}\\&=\frac{kj_l'(kR)-\dfrac{\kappa j_l'(\kappa R)j_l(kR)}{j_l(\kappa R)}}{kn_l'(kR)-\dfrac{\kappa j_l'(\kappa R)n_l(kR)}{j_l(\kappa R)}}\\&=\frac{kj_l'(kR)j_l(\kappa R)-\kappa j_l'(\kappa R)j_l(kR)}{kn_l'(kR)j_l(\kappa R)-\kappa j_l'(\kappa R)n_l(kR)}\end{aligned} \tag{4}$$

Our strategy is then to express all j' and n' in terms of j and n using spherical Bessel function's recurrence relation, and then use their asymptotic form at $kR(\text{or } \kappa R)\ll 1$.

In particular, with the recurrence relation

$$f_l'(x)=-f_{l+1}(x)+\frac{lf_l(x)}{x} \qquad (\text{for } f=j,n) \tag{5}$$

(4) becomes

$$\begin{aligned}\tan\delta_l&=\frac{k\left[-j_{l+1}(kR)+\dfrac{lj_l(kR)}{kR}\right]j_l(\kappa R)-\kappa\left[-j_{l+1}(\kappa R)+\dfrac{lj_l(\kappa R)}{\kappa R}\right]j_l(kR)}{k\left[-n_{l+1}(kR)+\dfrac{ln_l(kR)}{kR}\right]j_l(\kappa R)-\kappa\left[-j_{l+1}(\kappa R)+\dfrac{lj_l(\kappa R)}{\kappa R}\right]n_l(kR)}\\&=\frac{\kappa j_{l+1}(\kappa R)j_l(kR)-kj_{l+1}(kR)j_l(\kappa R)}{\kappa j_{l+1}(\kappa R)n_l(kR)-kn_{l+1}(kR)j_l(\kappa R)}\end{aligned} \tag{6}$$

Recall the "small-x" asymptotic form of spherical Bessel functions (eq 6.151)

$$j_l(x)\approx\frac{x^l}{(2l+1)!!} \qquad n_l(x)\approx-\frac{(2l-1)!!}{x^{l+1}} \tag{7}$$

(6) can be rewritten as

$$\begin{aligned}\tan\delta_l&=\frac{\kappa\dfrac{(\kappa R)^{l+1}}{(2l+3)!!}\dfrac{(kR)^l}{(2l+1)!!}-k\dfrac{(kR)^{l+1}}{(2l+3)!!}\dfrac{(\kappa R)^l}{(2l+1)!!}}{-\kappa\dfrac{(\kappa R)^{l+1}}{(2l+3)!!}\dfrac{(2l-1)!!}{(kR)^{l+1}}+k\dfrac{(2l+1)!!}{(kR)^{l+2}}\dfrac{(\kappa R)^l}{(2l+1)!!}}\\&=\frac{\dfrac{R^{2l+1}\kappa^l k^l(\kappa^2-k^2)}{(2l+3)!!(2l+1)!!}}{-\dfrac{\kappa^{l+2}}{(2l+3)(2l+1)k^{l+1}}+\dfrac{\kappa^l}{k^{l-1}}\dfrac{1}{(kR)^2}} \qquad (\text{ignore 1st term in den. since } kR\ll 1)\\&\approx\frac{R^{2l+3}k^{2l+1}(\kappa^2-k^2)}{(2l+3)!!(2l+1)!!}\end{aligned} \tag{8}$$

Since $kR \ll 1$, the phase shift δ_l is also small, so the total cross section (eq 6.127)

$$\sigma_{\text{tot}} = \frac{4\pi}{k^2} \sum_l (2l+1) \sin^2 \delta_l \tag{9}$$

is dominated by the s-wave $l = 0$, which is

$$\begin{aligned}\sigma_{\text{tot}}^{(0)} &= \frac{4\pi}{k^2} \left[\frac{kR^3}{3 \cdot 1} \left(-\frac{2mV}{\hbar^2} \right) \right]^2 \\ &= \frac{16\pi m^2 V^2 R^6}{9\hbar^4} \end{aligned}\tag{10}$$

The scattering amplitude (eq 6.126) is given by

$$f(\theta) = \frac{1}{k} \sum_l (2l+1) e^{i\delta_l} \sin \delta_l P_l(\cos\theta) \tag{11}$$

when we consider both s-wave and p-wave, we have

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= |f(\theta)|^2 \\ &= \frac{1}{k^2} \left(e^{i\delta_0} \sin \delta_0 + 3e^{i\delta_1} \sin \delta_1 \cos\theta \right) \left(e^{-i\delta_0} \sin \delta_0 + 3e^{-i\delta_1} \sin \delta_1 \cos\theta \right) \\ &\approx \frac{1}{k^2} \left[\sin^2 \delta_0 + 3 \sin \delta_0 \sin \delta_1 \cdot 2 \cos(\delta_1 - \delta_0) \cos\theta \right] \end{aligned}\tag{12}$$

which is of the form $A + B\cos\theta$. From (8), we have

$$\delta_0 \approx \frac{kR^3}{3 \cdot 1} \left(-\frac{2mV}{\hbar^2} \right) \tag{13}$$

$$\delta_1 \approx \frac{k^3 R^5}{15 \cdot 3} \left(-\frac{2mV}{\hbar^2} \right) \tag{14}$$

Finally

$$\begin{aligned}\frac{B}{A} &= \frac{6 \sin \delta_0 \sin \delta_1 \cos(\delta_0 - \delta_1)}{\sin^2 \delta_0} \qquad (\cos(\delta_0 - \delta_1) = 1 - O(\delta^2) \approx 1) \\ &\approx \frac{6\delta_1}{\delta_0} = \frac{2}{5}(kR)^2 \end{aligned}\tag{15}$$