For the Runge-Lenz operator

$$\boldsymbol{M} = \frac{1}{2m}(\boldsymbol{p}\times\boldsymbol{L} - \boldsymbol{L}\times\boldsymbol{p}) - \frac{Ze^2}{r}\boldsymbol{x} \tag{1}$$

it's trivial to prove its hermicity, given the hermicity of all of $\boldsymbol{p}, \boldsymbol{L}$ and $\boldsymbol{x}$.

To prove $[\boldsymbol{M}, H] = 0$, first recall (excercise 1.31)

$$[p_k, F(\boldsymbol{x})] = -i\hbar\frac{\partial F}{\partial x_k} \tag{2}$$

then for a spherically symmetric potential $V(\boldsymbol{x}) = V(r)$, we have

$$\begin{aligned}
[L_k, H] &= \epsilon_{ijk}\left[x_i p_j, \frac{p^2}{2m} + V(r)\right] \\
&= \epsilon_{ijk}\left\{\left[x_i p_j, \frac{p^2}{2m}\right] + \left[x_i p_j, V(r)\right]\right\} \\
&= \epsilon_{ijk}\left\{\left[x_i, \frac{p_i^2}{2m}\right]p_j + x_i\left[p_j, V(r)\right]\right\} \\
&= \epsilon_{ijk}\left[\frac{2i\hbar}{2m}p_i p_j + x_i(-i\hbar)V'(r)\frac{x_j}{r}\right] = 0
\end{aligned} \tag{3}$$

The last step was due to the sum over i, j and $\epsilon_{ijk} = -\epsilon_{jik}$.

Now for $k = 1, 2, 3$,

$$[M_k, H] = \overbrace{\frac{1}{2m}\epsilon_{ijk}\left[p_i L_j - L_i p_j, H\right]}^{A} - \overbrace{\left[Ze^2\frac{x_k}{r}, H\right]}^{B} \tag{4}$$

where

$$B = Ze^2\sum_i\left[\frac{x_k}{r}, \frac{p_i^2}{2m}\right] = \frac{Ze^2}{2m}\sum_i\left\{p_i\left[\frac{x_k}{r}, p_i\right] + \left[\frac{x_k}{r}, p_i\right]p_i\right\}$$

But

$$\left[\frac{x_k}{r}, p_i\right] = i\hbar\frac{\partial}{\partial x_i}\frac{x_k}{r} = i\hbar\left(\frac{\delta_{ik}}{r} - \frac{x_i x_k}{r^3}\right) \tag{5}$$

Therefore

$$\begin{aligned}
B &= \frac{i\hbar Ze^2}{2m}\left[\left(p_k\frac{1}{r} - \sum_i p_i\frac{x_i x_k}{r^3}\right) + \left(\frac{1}{r}p_k - \sum_i\frac{x_i x_k}{r^3}p_i\right)\right] \\
&= \frac{i\hbar Ze^2}{2m}\left[\underbrace{\sum_{i\neq k}\left(p_k\frac{x_i x_i}{r^3} + \frac{x_i x_i}{r^3}p_k\right)}_{X} - \underbrace{\sum_{i\neq k}\left(p_i\frac{x_i x_k}{r^3} + \frac{x_i x_k}{r^3}p_i\right)}_{Y}\right]
\end{aligned} \tag{6}$$

where the last step uses the identity

$$\frac{1}{r} = \frac{1}{r^3}\left(x_k x_k + \sum_{i\neq k}x_i x_i\right)$$

Now let's look at term A in (4).

Since $[L_i, H] = [L_j, H] = 0$, we have

$$\begin{aligned}
\left[p_i L_j - L_i p_j, H\right] &= [p_i, H]L_j - L_i\left[p_j, H\right] \\
&= -Ze^2\left\{\left[p_i, \frac{1}{r}\right]L_j - L_i\left[p_j, \frac{1}{r}\right]\right\} \\
&= -i\hbar Ze^2\left(\frac{x_i}{r^3}L_j - L_i\frac{x_j}{r^3}\right)
\end{aligned} \tag{7}$$

Plugging (7) into A term in (4), we have

$$A = -\frac{i\hbar Ze^2}{2m} \overbrace{\epsilon_{ijk}\left(\frac{x_i}{r^3}L_j - L_i\frac{x_j}{r^3}\right)}^{W} \tag{8}$$

Now compare (6) to (8), it remains to prove $X - Y = -W$. In fact

$$\begin{aligned}
W &= \epsilon_{ijk}\left(\frac{x_i}{r^3}L_j - L_i\frac{x_j}{r^3}\right) \\
&= \epsilon_{ijk}\left(\frac{x_i}{r^3}\epsilon_{lmj}x_l p_m - \epsilon_{lmi}x_l p_m\frac{x_j}{r^3}\right) \\
&= \epsilon_{ijk}\epsilon_{lmj}\frac{x_i}{r^3}x_l p_m - \epsilon_{ijk}\epsilon_{lmi}x_l p_m\frac{x_j}{r^3} \\
\text{(relabeling } i \longleftrightarrow j \text{ in 2nd term)} \qquad &= \epsilon_{ijk}\epsilon_{lmj}\frac{x_i}{r^3}x_l p_m - \epsilon_{jik}\epsilon_{lmj}x_l p_m\frac{x_i}{r^3} \\
(\epsilon_{ijk} = -\epsilon_{jik}) \qquad &= \epsilon_{ijk}\epsilon_{lmj}\left(\frac{x_i}{r^3}x_l p_m + x_l p_m\frac{x_i}{r^3}\right)
\end{aligned} \tag{9}$$

Inspecting (9), we see that for any given non-vanishing ϵ_{ijk}, any non-zero contribution of ϵ_{lmj} must be from either $l = i, m = k$, or $l = k, m = i$, therefore

$$\begin{aligned}
W &= \overbrace{\epsilon_{ijk}\epsilon_{ikj}\left(\frac{x_i}{r^3}x_i p_k + x_i p_k\frac{x_i}{r^3}\right)}^{\text{contrib. from } l=i,m=k} + \overbrace{\epsilon_{ijk}\epsilon_{kij}\left(\frac{x_i}{r^3}x_k p_i + x_k p_i\frac{x_i}{r^3}\right)}^{\text{contrib. from } l=k,m=i} \\
&= -\left[\overbrace{\epsilon_{ijk}^2\left(\frac{x_i}{r^3}x_i p_k + x_i p_k\frac{x_i}{r^3}\right)}^{X'} - \overbrace{\epsilon_{ijk}^2\left(\frac{x_i}{r^3}x_k p_i + x_k p_i\frac{x_i}{r^3}\right)}^{Y'}\right]
\end{aligned} \tag{10}$$

Comparing (6) and (10) we can see that $X = X'$ and $Y = Y'$ since any contributing term in (10) will satisfy $i \neq k$, hence $[x_i, p_k] = [x_k, p_i] = 0$.