1.

$$S = \{ax^2+bx+c \mid a,b,c \in \mathbb{R},\ a=c\}$$

$$= \{px^2+qx+p \mid p,q \in \mathbb{R}\}$$

$$= \{p(x^2+1)+qx \mid p,q \in \mathbb{R}\}$$

$$= \text{span}(x^2+1,\ x)$$

$\therefore$ S is a subsp.

2.

$$S = \left\{ ax^2 + bx + c, \begin{array}{c} a, b, c \in \mathbb{R} \\ a \neq c \end{array} \right\}$$

$$\left\{ \begin{array}{l} ax^2 + bx + c \quad a \neq c \\ cx^2 + bx + a \end{array} \right. \rightarrow \in S$$

$$(a+c)x^2 + 2bx + (a+c) \notin S$$

$\therefore S$ is not a subsp.

3 (a) $at^3 + (a+b)t^2 + (a-b)t + b$

$$\Rightarrow a\underbrace{(t^3 + t^2 + t)}_{f_1(t)} + b\underbrace{(t^2 - t + 1)}_{f_2(t)}$$

$f_1(t), f_2(t) \in P_3$

& are linearly independent

If $\alpha_1 f_1(t) + \alpha_2 f_2(t) = 0$, then $\alpha_1 = \alpha_2 = 0$

$\therefore$ Basis $= \{f_1(t), f_2(t)\}$

(4) $T = \{t^3 + t^2 + t + 1,\ 2t^3 + 2t^2 + t + 1,$

$3t^3 + 3t^2 + t + 1,$

$3t^3 + 2t^2 + 2t + 1\}$

$$\begin{pmatrix} 1 & t & t^2 & t^3 \end{pmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 2 & 3 & 2 \\ 1 & 2 & 3 & 3 \end{bmatrix}$$

Look at RREF of this matrix & the corresponding leading columns

RREF

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

c_1, c_2, c_4 are leading columns & are lin. $\underbrace{\text{independant.}}_{\to \text{Prove}}$

$$\therefore \text{Basis} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \\ 3 \end{pmatrix}$$

(5) (a) $A = A^F$

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} d & e & f \\ a & b & c \end{bmatrix}$$

$$a = d, \quad b = e, \quad c = f$$

$$\therefore \begin{bmatrix} a & b & c \\ a & b & c \end{bmatrix}$$

$$\Rightarrow a \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} + b \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} + c \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) $A = A^R$

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} c & b & a \\ f & e & d \end{bmatrix}$$

$a = c \ , \ d = f$

$$A = \begin{bmatrix} a & b & a \\ d & e & d \end{bmatrix}$$

$$a = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + b \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + e \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$+ \ d \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

6.

$$V = \left\{ (v, w, x, y, z) \;\middle|\; \begin{array}{l} v + 2w + x = 0 \\ y + z = 0 \end{array} \right\}$$

$$\begin{pmatrix} v \\ w \\ x \\ y \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} -2w - x \\ w \\ x \\ -z \\ z \end{pmatrix}$$

$\therefore v, y$

$\downarrow$

basic variable

w, x, z

$\downarrow$

free variable.

$$\Rightarrow \underbrace{w\begin{pmatrix}-2\\1\\0\\0\\0\end{pmatrix} + x\begin{pmatrix}-1\\0\\1\\0\\0\end{pmatrix} + z\begin{pmatrix}0\\0\\0\\-1\\1\end{pmatrix}}_{\text{linearly independent.}}$$

so it forms basis of U.

7.)

$$V = \left\{ (v, w, x, y, z) \;\middle|\; \begin{matrix} x + 2y + z = 0 \\ v - w - y = 0 \\ w - x = 0 \end{matrix} \right\}$$

$$\underbrace{\begin{bmatrix} 0 & 0 & 1 & 2 & 1 \\ 1 & -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ w \\ x \\ y \\ z \end{bmatrix}}$$

$\to$ $R_1 + R_3$, $R_3 \to R_2$,

$R_2 + R_3$, $R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 & 1 \end{bmatrix} \Rightarrow \begin{aligned} v + y + z &= 0 \\ w + 2y + z &= 0 \\ x + 2y + z &= 0 \end{aligned}$$

$$v = -y - z$$
$$w = -2y - z$$
$$x = -2y - z$$

$y, z \rightarrow$ free

$$\begin{pmatrix} v \\ w \\ x \\ y \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} -y-z \\ -2y-z \\ -2y-z \\ y \\ z \end{pmatrix}$$

$$\Rightarrow y \begin{pmatrix} -1 \\ -2 \\ -2 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} -1 \\ -1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

(c) $W_1 = \{A = A^F\}$

$W_2 = \{A = A^R\}$

$$W_1 \cap W_2 = \left\{ A \in M_{2\times 3} \mid A = A^F = A^R \right\}$$

$$\left\{ \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} d & e & f \\ a & b & c \end{bmatrix} = \begin{bmatrix} c & b & a \\ f & e & d \end{bmatrix} \right\}$$

$a = d = c$

$b = e = b$

$c = f = a$

$d = a = f$

$e = b = e$

$f = c = d$

$$\boxed{\begin{array}{l} a = c = d = f, \\ b = e \end{array}}$$

$$A = \begin{bmatrix} a & b & a \\ a & b & a \end{bmatrix}$$

$$\Rightarrow A = a\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} + b\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Q. $\begin{bmatrix} A \\ A \end{bmatrix}$ $\begin{bmatrix} A & A \\ A & A \end{bmatrix}$ where $A_{m\times n}$

$R = RREF(A)$

$$\begin{bmatrix} A \\ A \end{bmatrix} \longrightarrow \begin{bmatrix} R \\ 0 \end{bmatrix} \qquad \begin{bmatrix} A & A \\ A & A \end{bmatrix} \longrightarrow \begin{bmatrix} R & R \\ 0 & 0 \end{bmatrix}$$

Row sp

Clm sp

Null sp