Memory based learning

→ Nearest Neighbor rule
→ K-nearest neighbor classifier.

Nearest Neighbor rule

Training data ((4,5),A)
((6,5),B)

Test data
(4.5, 5.5)

Ans:

$$dist_1 = \sqrt{(4-4.5)^2 + (5-5.5)^2}$$
$$= 0.707 \quad \text{for class A}$$

$$dist_2 = \sqrt{(6-4.5)^2 + (5-5.5)^2}$$
$$= 1.58 \quad \text{for class B}$$

dist1 < dist2
Output class – A

K - nearest neighbor classifier

((3,4,5),A)
((1,2,3),B)
((4,1,5),C)
((3,2,4),A)
((2,4,1),B)
((1,4,3),C)
((4,3,2),B)
((4,2,3),C)

classify (2,3,4) using K-nearest neighbor.

$\underline{\underline{Sol^n}}$:

$$dist_1 = \sqrt{(3-2)^2 + (4-3)^2 + (5-4)^2}$$

$$= 1.73.$$

$$dist_2 = \sqrt{(1-2)^2 + (2-3)^2 + (3-4)^2}$$

$$= 1.73$$

$$dist_3 = \sqrt{(4-2)^2 + (1-3)^2 + (5-4)^2}$$

$$= 3$$

$$dist_4 = \sqrt{(3-2)^2 + (2-3)^2 + (4-4)^2}$$

$$= 1.414$$

$$dist_5 = \sqrt{(2-2)^2 + (4-3)^2 + (1-4)^2}$$

$$= 3.16$$

$$dist_6 = \sqrt{(1-2)^2 + (4-3)^2 + (3-4)^2}$$

$$= 1.73$$

$$dist_7 = (4-2)^2 + (3-3)^2 + (2-4)^2$$

$$= 2.83$$

$$dist_8 = (4-2)^2 + (2-3)^2 + (3-4)^2$$

$$= 2.45$$

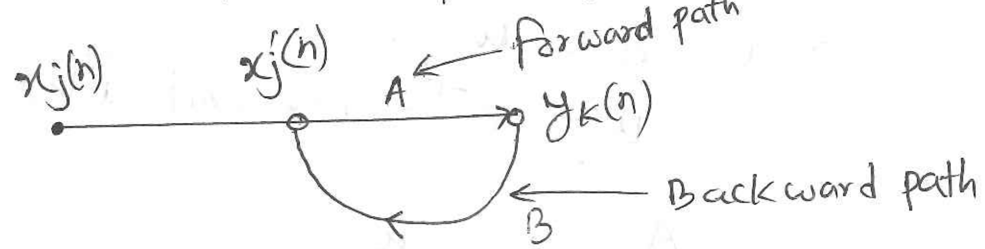
$K=4$, 4-nearest Neighbor classifier.

dist4	dist1	dist2	dist6
A	A	B	C

Taking major voting
output class = A **Ans:**

Prove that the output signal of a recurrent neural network (RNN) is the weight summation of present and past input signal.

Proof:



$x_j(n)$ = input signal

$x'_j(n)$ = Internal signal

$y_k(n)$ = output signal

$\therefore$ New input $x'_j(n) = x_j(n) + B[y_k(n)]$ - - - - (I)

output $y_k(n) = A[x'_j(n)]$ - - - - - (II)

From equation ~~(I) and~~ (II) $\Longrightarrow$

$x'_j(n) = A^{-1}[y_k(n)]$ - - (III)

From equation (I) and (III) $\Rightarrow$

$$A^{-1}[y_K(n)] = x_j(n) + B[y_K(n)]$$

$$\Rightarrow y_K(n) = A[x_j(n)] + AB[y_K(n)]$$

$$\Rightarrow y_K(n) - AB[y_K(n)] = A[x_j(n)]$$

$$\Rightarrow (1-AB)\,y_K(n) = A[x_j(n)]$$

$$\Rightarrow y_K(n) = \frac{A}{1-AB}[x_j(n)]$$

Let $A = w$

$B = z^{-1}$ = one unit delay

$$\therefore \frac{A}{1-AB} = \frac{w}{1-wz^{-1}}$$

$$= w(1-wz^{-1})^{-1}$$

$$= w[1 + w^1 z^{-1} + w^2 z^{-2} + w^3 z^{-3} + \cdots\cdots + \infty]$$

$$= w\sum_{l=0}^{\infty} w^l z^{-l}$$

$$y_K(n) = \frac{A}{1-AB}[x_j(n)]$$

$$= w\sum_{l=0}^{\infty} w^l z^{-l}[x_j(n)]$$

$$y_k(n) = w\sum_{l=0}^{\infty} w^{l} z^{-l} [x_j(n)]$$

$$= \sum_{l=0}^{\infty} w^{l+1} z^{-l} [x_j(n)]$$

$$= \sum_{l=0}^{\infty} w^{l+1} x_j(n-l)$$

Output signal $y_k(n)$ is a weighted summation of present and past samples of the input signal $x_j(n)$.

Mahalanobis distance :

class A : 2, 4
3 5
class B : 7 9
8 5

classify pattern c for data (4, 7) using Mahalanobis distance.

Soln.

Mean A : 2.5 4.5

Mean B : 7.5 7

Covariance for A :

$$\Sigma_1 = \frac{(2.5-2)^2 + (2.5-3)^2}{2}$$

$$= 0.25$$

$$\Sigma_1 = \frac{(4.5-4)^2 + (4.5-5)^2}{2}$$

$$= 0.25$$

$$\therefore \Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix}$$

$$\Rightarrow \Sigma^{-1} = \begin{bmatrix} \frac{1}{\Sigma_1} & 0 \\ 0 & \frac{1}{\Sigma_2} \end{bmatrix}$$

Mahalanobis distance : for class A

$$\text{distance} = (x-\mu)^T \Sigma^{-1} (x-\mu)$$

$$= \left[\begin{bmatrix} 4 & 7 \end{bmatrix} - \begin{bmatrix} 2.5 & 4.5 \end{bmatrix} \right]^T \begin{bmatrix} \frac{1}{\Sigma_1} & 0 \\ 0 & \frac{1}{\Sigma_2} \end{bmatrix} \left(\begin{bmatrix} 4 & 7 \end{bmatrix} - \begin{bmatrix} 2.5 & 4.5 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4-2.5 & 7-4.5 \end{bmatrix} \begin{bmatrix} \frac{1}{\Sigma_1} & 0 \\ 0 & \frac{1}{\Sigma_2} \end{bmatrix} \begin{bmatrix} 4-2.5 \\ 7-4.5 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 & 2.5 \end{bmatrix} \begin{bmatrix} \frac{1}{\Sigma_1} & 0 \\ 0 & \frac{1}{\Sigma_2} \end{bmatrix} \begin{bmatrix} 1.5 \\ 2.5 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 & 2.5 \end{bmatrix} \begin{bmatrix} \frac{1}{0.25} & 0 \\ 0 & \frac{1}{0.25} \end{bmatrix} \begin{bmatrix} 1.5 \\ 2.5 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 & 2.5 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1.5 \\ 2.5 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 \times 4 + 0 & 0 + 2.5 \times 4 \end{bmatrix} \begin{bmatrix} 1.5 \\ 2.5 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 10 \end{bmatrix} \begin{bmatrix} 1.5 \\ 2.5 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \times 1.5 + 10 \times 2.5 \end{bmatrix}$$

$$= \begin{bmatrix} 9 + 25 \end{bmatrix}$$

$$= 34$$

Covariance for B:

$$\Sigma_1 = \frac{(7.5-7)^2+(7.5-8)^2}{2}$$

$$= 0.25$$

$$\Sigma_2 = \frac{(7-9)^2+(7-5)^2}{2}$$

$$= 4$$

$$\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix}$$

$$\Rightarrow \Sigma^{-1} = \begin{bmatrix} \frac{1}{\Sigma_1} & 0 \\ 0 & \frac{1}{\Sigma_2} \end{bmatrix}$$

Mahalanobis distance for B class:

$$\text{distance} = (x-\mu)^T \Sigma^{-1} (x-\mu)$$

$$= \left([4\ 7]-[7.5\ 7]\right)^T \begin{bmatrix} \frac{1}{\Sigma_1} & 0 \\ 0 & \frac{1}{\Sigma_2} \end{bmatrix} \left([4\ 7]-[7.5\ 7]\right)$$

$$= \begin{bmatrix} 4-7.5 & 7-7 \end{bmatrix} \begin{bmatrix} \frac{1}{\Sigma_1} & 0 \\ 0 & \frac{1}{\Sigma_2} \end{bmatrix} \begin{bmatrix} 4-7.5 \\ 7-7 \end{bmatrix}$$

$$= \begin{bmatrix} -3.5 & 0 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 0.25 \end{bmatrix} \begin{bmatrix} -3.5 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3.5\times4+0 & 0+0.25\times(-3.5) \end{bmatrix} \begin{bmatrix} -3.5 \\ 0 \end{bmatrix} = \begin{bmatrix} -14 & 0 \end{bmatrix} \begin{bmatrix} -3.5 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 0 \end{bmatrix} \begin{bmatrix} -3.5 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -14 \times (-3.5) + 0 \end{bmatrix}$$

$$= 49$$

$$\text{Minimum} = \min(\text{dist}_A, \text{dist}_B)$$
$$= \min(34, 49)$$
$$= 34$$

$\therefore$ The given pattern $(4,7)$ is in class A.