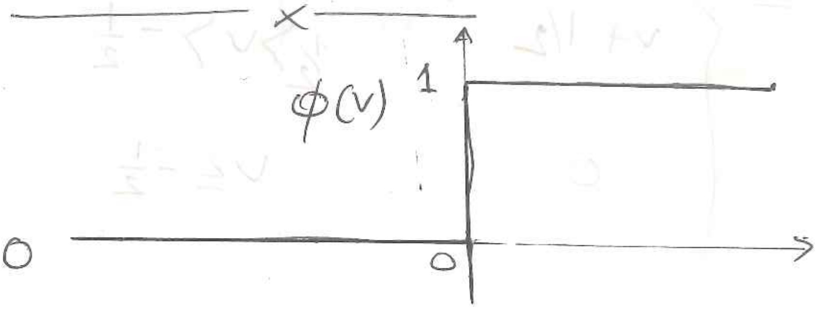
Neural Network

Types of Activation function

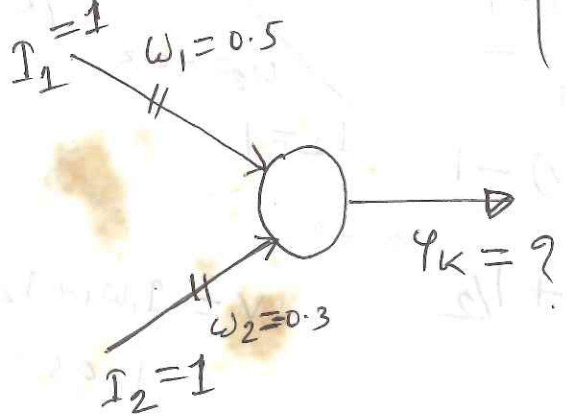
- → Threshold function
- → Piecewise-linear function
- → Sigmoid function
- → Signum function

An Activation Function decides whether a neuron should be activated or not. This means that it will decide whether the neuron's input to the network is important or not in the process of prediction using simpler mathematical operations.

Threshold function:



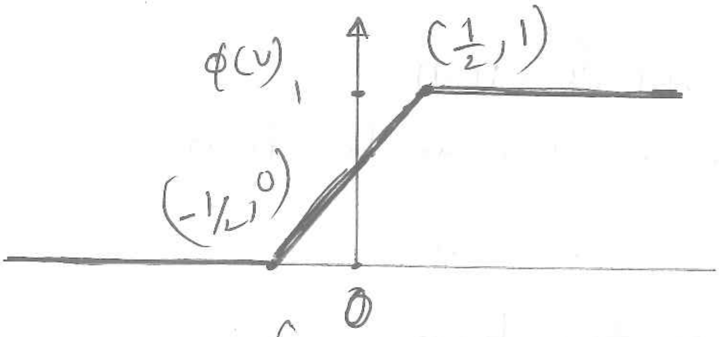
$$\phi(v) = \begin{cases} 1 & v \geq 0 \\ 0 & v < 0 \end{cases}$$



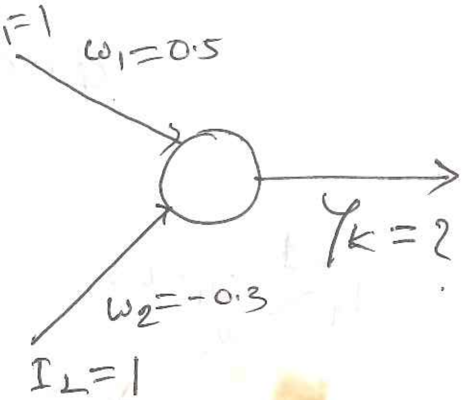
$$v = I_1 w_1 + I_2 w_2$$
$$= 1 \times 0.5 + 1 \times (-0.3)$$
$$= 0.5 - 0.3$$
$$= 0.2$$

Since, $0.2 \geq 0$, $y_K = Q(v) = 1$ Ans.

piecewise linear function.



$$\phi(v) = \begin{cases} 1 & v \geq \frac{1}{2} \\ v + 1/2 & \frac{1}{2} > v > -\frac{1}{2} \\ 0 & v \leq -\frac{1}{2} \end{cases}$$



$$\frac{x - \frac{1}{2}}{1/2 + 1/2} = \frac{y - 1}{1 - 0}$$

$$\Rightarrow \frac{v - 1/2}{1} = \frac{\phi(v) - 1}{1}$$

$$\Rightarrow v - 1/2 = \phi(v) - 1$$

$$\Rightarrow \phi(v) = v + 1/2$$

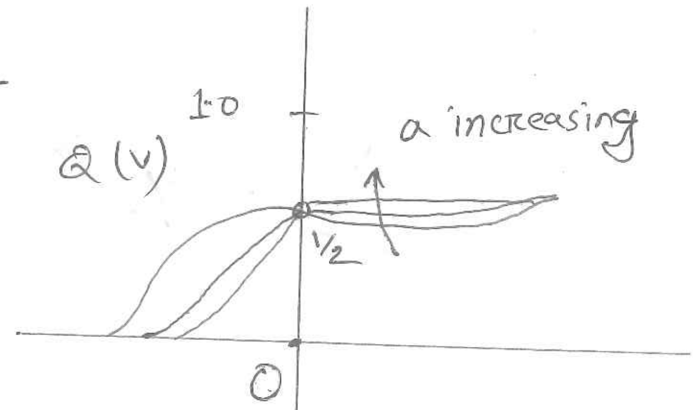
$$v = I_1 w_1 + I_2 w_2$$
$$= 1 \times 0.5 + 1 \times (-0.3)$$
$$= 0.5 - 0.3$$
$$= 0.2$$

Since $1/2 > 0.2 > -1/2$

$$\therefore Q(v) = v + 1/2$$

$$\therefore Q(0.2) = 0.2 + 1/2$$
$$= 0.2 + 0.5$$
$$= 0.7 \quad \text{Ans:}$$

Sigmoid function



where a is slope parameter.

Logistic function

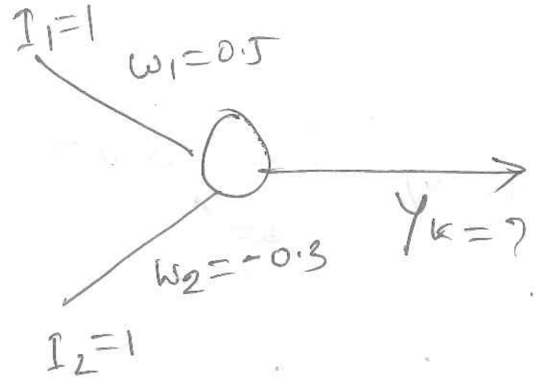
$$\phi(v) = \frac{1}{1+e^{-av}}$$

v	$Q(v)$
$-\infty$	0
∞	1
0	1/2

$$\phi(-\infty) = \frac{1}{1+e^{-1(-\infty)}} = \frac{1}{1+e^{\infty}} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0$$

$$Q(\infty) = \frac{1}{1+e^{-1(\infty)}} = \frac{1}{1+e^{-\infty}} = \frac{1}{1+\frac{1}{e^{\infty}}} = \frac{1}{1+0} = 1$$

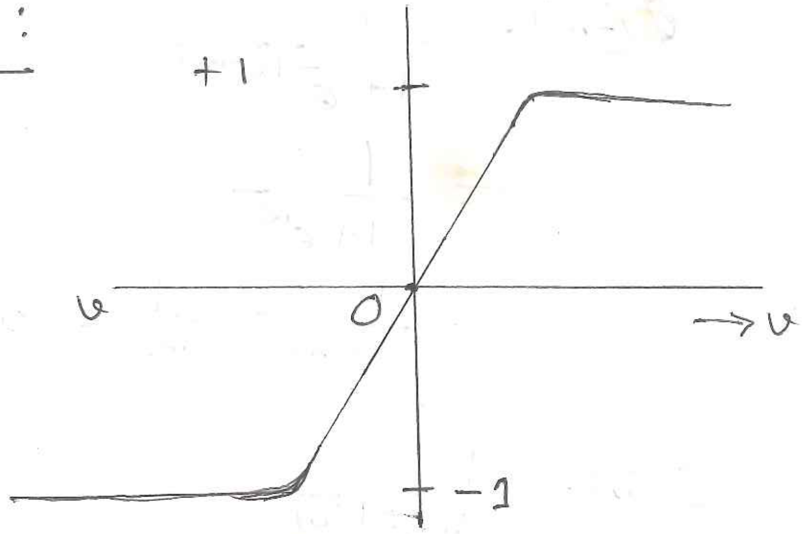
$$\phi(0) = \frac{1}{1+e^{-1(0)}} = \frac{1}{1+\frac{1}{e^{0}}} = \frac{1}{1+\frac{1}{1}} = \frac{1}{2} = 1/2$$



$$v = I_1 w_1 + I_2 w_2$$
$$= 1\times(0.5) - 1\times 0.3$$
$$= 0.5 - 0.3$$
$$= 0.2$$

$$\phi(0.2) = \frac{1}{1+e^{-1(0.2)}}$$
$$= \frac{1}{1+e^{-0.2}}$$
$$= \frac{1}{1+\frac{1}{e^{0.2}}}$$
$$= 0.5498$$

Signum function:



The hyperbolic tangent function is a mathematical function that is used to calculate the ratio of the hyperbolic sine to the hyperbolic cosine. It is represented as tanh(x).

In mathematics, hyperbolic functions are analogues of the ordinary trigonometric functions, but defined using the hyperbola rather than the circle.

$$Q(v) = \tanh(v)$$
$$\Rightarrow \tanh(0.2) = 0.1973$$

Stochastic model of a neuron

A neuron is permitted to reside in only one of two +1 or −1

$$x = \begin{cases} +1 & \text{with probability } p(v) \\ \\ -1 & \text{with probability } 1 - p(v) \end{cases}$$

Randomly determined; having a random probability distribution or pattern that may be analyzed statistically but may not be predicted precisely.

x = state of the neuron.

The decision for a neuron to fire (switch its state from off to on) is probabilistic.

$p(v)$ = Probability of firing.

$$p(v) = \frac{1}{1 + e^{-\frac{v}{T}}}$$

where v is induced local field.

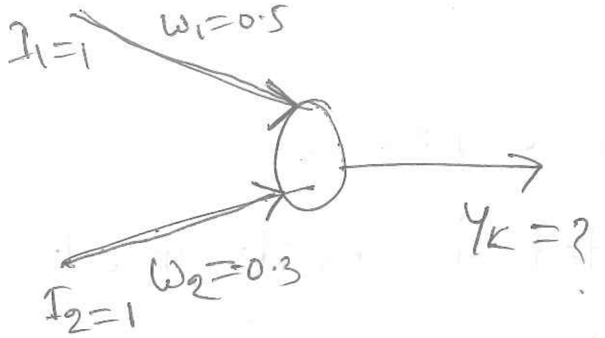
T = Pseudo temperature

T is used to control the noise level and therefore the uncertainty in firing.

when $T = 0$

$$p(v) = \frac{1}{1 + e^{-\infty}} = \frac{1}{1 + 0} = 1$$

The model is deterministic (noiseless)



$$v = I_1 w_1 + I_2 w_2$$
$$= 1 \times 0.5 + 1 \times (-0.3)$$
$$= 0.5 - 0.3$$
$$= 0.2$$

$$P(0.2) = \frac{1}{1 + e^{\frac{-0.2}{\infty}}} \quad ; (T = \infty)$$

$$= \frac{1}{1 + \frac{1}{e^0}}$$

$$= \frac{1}{1+1} = \frac{1}{2}$$

$$T = 10{,}000 \quad , \quad v = 0.2$$

$$p(0.2) = \frac{1}{1 + e^{\frac{-0.2}{10000}}}$$

$$= \frac{1}{1 + \frac{1}{e^{.00002}}}$$

$$= \frac{1}{1.99998} = 0.500005$$

Firing probability

$$1 - p(v) = 1 - 0.500005$$
$$= 0.499995$$

Not firing probability.