Machine Learning Assignment 52

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52-1

(A) Variance of the uniform distribution of X on the interval [a,b]

$$Var[X] = E[X^2] - E[X]^2$$
$$\int_a^b (x - E[X])^2 \cdot p(x))\,dx = \int_a^b x^2 \cdot p(x)\,dx - \left[\int_a^b x \cdot p(x)\,dx\right]^2$$
$$\int_a^b \left(x - \left(\frac{b+a}{2}\right)\right)^2 \cdot k\,dx = \int_a^b x^2 \cdot \frac{1}{b-a}\,dx - \left[\int_a^b x \cdot \frac{1}{b-a}\,dx\right]^2$$
$$\frac{(b-a)^2}{12} = \frac{(b-a)^2}{3} - \left(\frac{b-a}{2}\right)^2$$
$$\frac{(b-a)^2}{12} = \frac{(b-a)^2}{12}$$

(B) Variance of the probability distribution of $\lambda e^{-\lambda}$ on the interval $[0, \infty)$

$$Var[X] = E[X^2] - E[X]^2$$
$$\int_0^\infty (x - \frac{1}{\lambda})^2 \cdot \lambda e^{-\lambda} = \int_0^\infty x^2 \cdot \lambda e^{-\lambda}\,dx - \left[\int_0^\infty x \cdot \lambda e^{-\lambda}\,dx\right]^2$$
$$\frac{-1}{-\lambda^2} - (4 \cdot \infty + 1) \cdot \frac{e^{-\infty}}{-\lambda^2} = \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$$
$$\frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

(C) Variance of the Poisson distribution of $\dfrac{\lambda^n e^{-\lambda}}{n!}$ on the interval $[0, \infty)$

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\begin{aligned}
Var[N] &= E[N^2] - E[N]^2 \\
\sum_{n=0}^{\infty}(n-\lambda)^2 \cdot \frac{\lambda^n e^{-\lambda}}{n!} &= \left[\sum_{n=0}^{\infty} n^2 \cdot \frac{\lambda^n e^{-\lambda}}{n!}\right] - \left[\sum_{n=0}^{\infty} n \cdot \frac{\lambda^n e^{-\lambda}}{n!}\right]^2 \\
e^{-\lambda} \cdot \lambda^2 \cdot e^{\lambda} - 2(\lambda-1) \cdot e^{\lambda} + \lambda^2 * e^{\lambda} &= \left[e^{-\lambda} \cdot \sum_{n=0}^{\infty} \frac{n^2 \cdot \lambda^n}{n!}\right] - \left[\lambda \cdot e^{-\lambda} \cdot \sum_{n=1}^{\infty} \frac{\lambda^n}{n!}\right]^2 \\
e^{-\lambda} * e^{\lambda} * (\lambda^2 - 2(\lambda-1) + \lambda^2) &= \left[e^{-\lambda} \cdot \left(0 + \frac{1^2\lambda^1}{1!} + \frac{2^2\lambda^2}{2!} + \frac{3^2\lambda^3}{3!} + \ldots + \frac{n^2 \cdot \lambda^n}{n!}\right)\right] - \left[\lambda \cdot e^{-\lambda} \cdot e^{\lambda}\right]^2 \\
2\lambda^2 - \lambda * (2\lambda - 1) &= \left[\lambda e^{-\lambda} \cdot \left(1 + 2\lambda + \frac{3\lambda^2}{2!} + \ldots + \frac{n \cdot \lambda^{n-1}}{(n-1)!}\right)\right] - \left[\lambda^2 \cdot e^{-2\lambda} \cdot e^{2\lambda}\right] \\
\lambda &= \left[\lambda e^{-\lambda} \cdot \left[\left(1 + \lambda + \frac{\lambda^2}{2!} + \ldots + \frac{\lambda^n}{n!}\right) + \left(\lambda + \frac{2\lambda^2}{2!} + \frac{3\lambda^3}{3!} + \ldots + \frac{n\lambda^n}{n!}\right)\right]\right] - [\lambda^2] \\
\lambda &= \left[\lambda e^{-\lambda} \cdot \left[e^{\lambda} + \lambda e^{\lambda}\right]\right] - [\lambda^2] \\
\lambda &= \left[e^{-\lambda} \cdot \left[\lambda e^{\lambda} + \lambda^2 e^{\lambda}\right]\right] - [\lambda^2] \\
\lambda &= [\lambda + \lambda^2] - [\lambda^2] \\
\lambda &= \lambda
\end{aligned}
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