Euclidean Ellipses

Drawing Mathematics with Desmos | Justin Skycak

Setup. Navigate to https://www.desmos.com/calculator. Be sure to sign in so that you can save your graph.

Demonstration - Circles. Observe the graph as you type each of the following inputs. In general, the graph of $(x - a)^2 + (y - b)^2 = r^2$ makes a circle with radius r centered at the point (a, b)

$$(x - 0)^2 + (y - 0)^2 = 1^2$$

$$(x - 2)^2 + (y - 3)^2 = 2^2$$

$$(x + 5)^2 + (y - 2)^2 = 3^2$$

Demonstration - Ellipses. Observe the graph as you type each of the following inputs. In general, the graph of $\left(\frac{x-a}{A}\right)^2 + \left(\frac{y-b}{B}\right)^2 = 1$ makes an ellipse with horizontal radius A and vertical radius B centered at the point (a, b).

$$\left(\frac{x-0}{1}\right)^2 + \left(\frac{y-0}{2}\right)^2 = 1$$

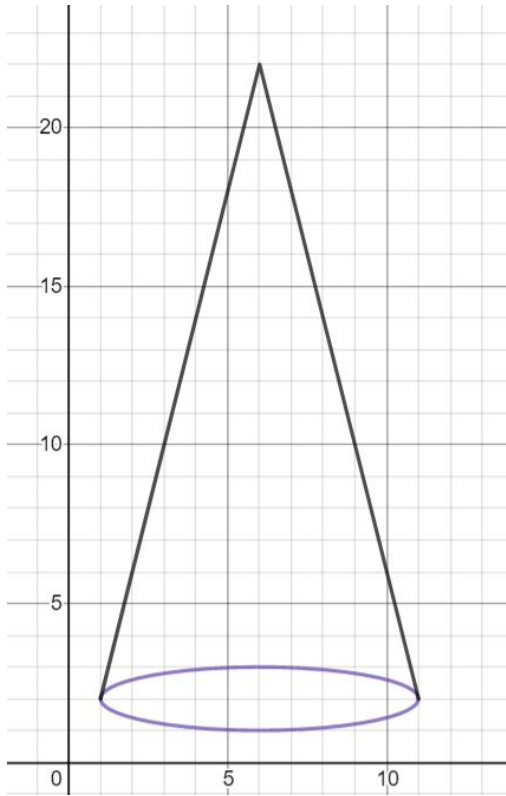$$\left(\frac{x-2}{2}\right)^2 + \left(\frac{y-3}{5}\right)^2 = 1$$

Demonstration - Shading. Observe the graph as you type each of the following inputs. In general, for large v, the graph of

$\left(\frac{x-a}{A}\right)^2 + \left(\frac{y+A\sin(vx)-b}{B}\right)^2 = 1$ shades vertically around the graph

of $\left(\frac{x-a}{A}\right)^2 + \left(\frac{y-b}{B}\right)^2 = 1$ with thickness $2A$.
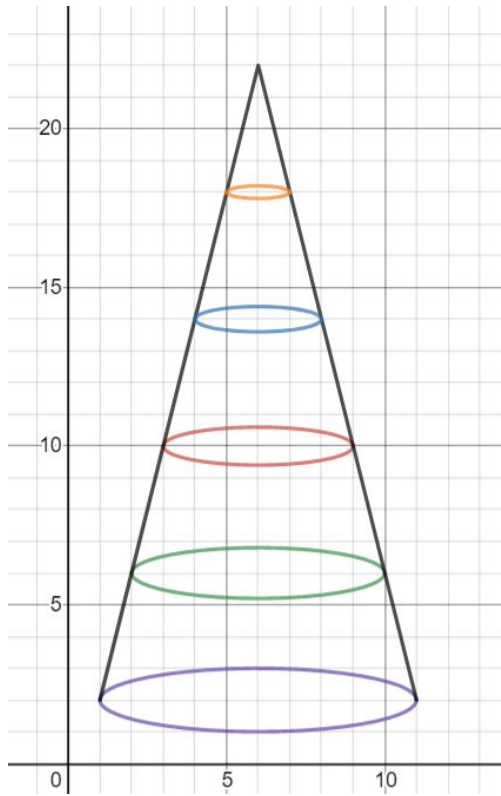
$$\left(\frac{x-0}{1}\right)^2 + \left(\frac{y+.1\sin(1000x)-0}{2}\right)^2 = 1$$

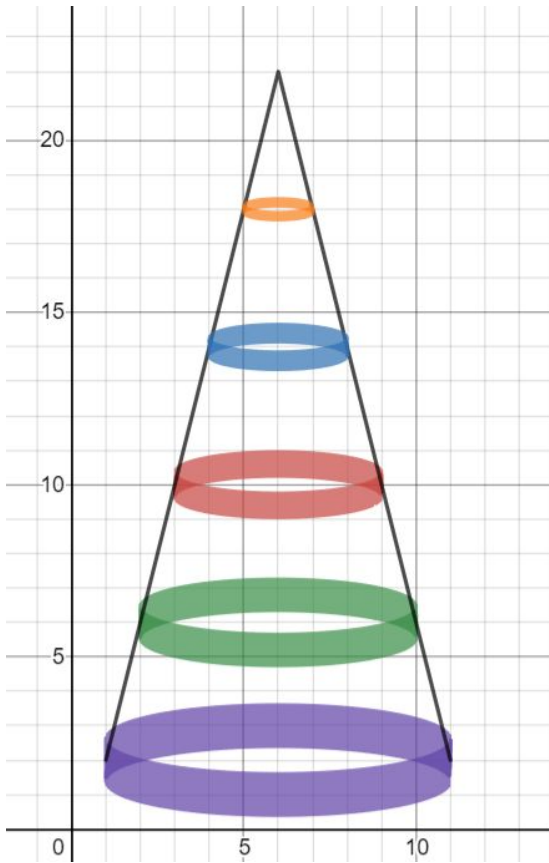$$\left(\frac{x-2}{2}\right)^2 + \left(\frac{y+.4\sin(1000x)-3}{5}\right)^2 = 1$$

Exercise. Use an absolute value function together with an ellipse to draw a cone.

Exercise. Stack ellipses vertically on the cone.

Exercise. Thicken the ellipses to form cylindrical shells.

Challenge. Try stacking cylindrical shells on the peaks of $y = \sin x$.