Machine Learning Assignment 99

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1 Problem 99

1.1 Compute $\frac{\partial E}{\partial w_{35}}$

Calculating the derivative of the squared sum of errors with respect to the w_{35} edge in the given neural network:

$$\begin{aligned}
\frac{\partial E}{\partial w_{35}} &= \frac{\partial}{\partial w_{35}} \left[(y_{\text{predicted}} - y_{\text{actual}})^2 \right] \\
&= \frac{\partial}{\partial w_{35}} \left[(a_5 - y_{\text{actual}})^2 \right] \\
&= 2 (a_5 - y_{\text{actual}}) \frac{\partial}{\partial w_{35}} [a_5 - y_{\text{actual}}] \\
&= 2 (a_5 - y_{\text{actual}}) \frac{\partial}{\partial w_{35}} [a_5] \\
&= 2 (a_5 - y_{\text{actual}}) \frac{\partial}{\partial w_{35}} [f_5(i_5)] \\
&= 2 (a_5 - y_{\text{actual}}) f_5(i_5) \frac{\partial}{\partial w_{35}} [i_5] \\
&= 2 (a_5 - y_{\text{actual}}) f_5(i_5) \frac{\partial}{\partial w_{35}} [a_3 * w_{34} + a_4 * w_{45}] \\
&= 2 (a_5 - y_{\text{actual}}) f_5(i_5) \frac{\partial}{\partial w_{35}} [a_3 * w_{35} + 0] \\
&= 2 (a_5 - y_{\text{actual}}) f_5'(i_5) a_3
\end{aligned}$$

To check my work:

$$\begin{aligned}
\frac{\partial E}{\partial w_{35}} &= 2 (a_5 - y_{\text{actual}}) f_5'(i_5) a_3 \\
&= 2(7 - 1) \cdot 13 \cdot 5 \\
&= 780
\end{aligned}$$

1.2 Compute $\frac{\partial E}{\partial w_{45}}$

Using similar reasoning to the last problem, we will find:

$$\frac{\partial}{\partial w_{45}} E = 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) a_4$$

To check my work:

$$\begin{aligned}\frac{\partial}{\partial w_{45}} E &= 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) a_4 \\ &= 2(7-1) \cdot 13 \cdot 6 \\ &= 936\end{aligned}$$

1.3 Compute $\frac{\partial E}{\partial w_{13}}$

Using the same reasoning as the problems above, we can skip to:

$$\begin{aligned}\frac{\partial E}{\partial w_{13}} &= 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) \frac{\partial}{\partial w_{13}} \left[a_3 \cdot w_{35} + 0\right] \\ &= 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) w_{35} \frac{\partial}{\partial w_{13}} \left[a_3\right] \\ &= 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) w_{35} \frac{\partial}{\partial w_{13}} \left[f_3(i_3)\right] \\ &= 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) w_{35} f_3'(i_3) \frac{\partial}{\partial w_{13}} \left[i_3\right] \\ &= 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) w_{35} f_3'(i_3) \frac{\partial}{\partial w_{13}} \left[a_1 \cdot w_{13} + a_2 \cdot w_{23}\right] \\ &= 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) w_{35} f_3'(i_3) \frac{\partial}{\partial w_{13}} \left[a_1 \cdot w_{13} + 0\right] \\ &= 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) w_{35} f_3'(i_3) a_1\end{aligned}$$

To check my work:

$$\begin{aligned}\frac{\partial E}{\partial w_{13}} &= 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) w_{35} f_3'(i_3) a_1 \\ &= 2(7-1) \cdot 13 \cdot 21 \cdot 11 \cdot 3 \\ &= 108108\end{aligned}$$

1.4 Compute $\frac{\partial E}{\partial w_{23}}$

Using similar reasoning as shown above, we will find:

$$\frac{\partial E}{\partial w_{23}} = 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) w_{35} f_3'(i_3) a_2$$

To check my work:

$$\begin{aligned}\frac{\partial}{\partial w_{23}} E &= 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) w_{35} f_3'(i_3) a_2 \\ &= 2(7-1) \cdot 13 \cdot 21 \cdot 11 \cdot 4 \\ &= 144144\end{aligned}$$

1.5 Compute $\frac{\partial E}{\partial w_{14}}$

Using similar reasoning as shown above, we will find:

$$\frac{\partial E}{\partial w_{14}} = 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) w_{45} f_4'(i_4) a_1$$

To check my work:

$$\begin{aligned}\frac{\partial E}{\partial w_{14}} &= 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) w_{45} f_4'(i_4) a_1 \\ &= 2(7-1) \cdot 13 \cdot 22 \cdot 12 \cdot 3 \\ &= 123552\end{aligned}$$

1.6 Compute $\frac{\partial E}{\partial w_{24}}$

Using similar reasoning as shown above, we will find:

$$\frac{\partial E}{\partial w_{24}} = 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) w_{45} f_4'(i_4) a_2$$

To check my work:

$$\begin{aligned}\frac{\partial E}{\partial w_{24}} &= 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5) w_{45} f_4'(i_4) a_2 \\ &= 2(7-1) \cdot 13 \cdot 22 \cdot 12 \cdot 4 \\ &= 164736\end{aligned}$$

1.7 Compute $\frac{\partial E}{\partial w_{01}}$

$$
\begin{aligned}
\frac{\partial E}{\partial w_{01}} &= \frac{\partial}{\partial w_{01}}\left[\left(y_{\text{predicted}} - y_{\text{actual}}\right)^2\right] \\
&= \frac{\partial}{\partial w_{01}}\left[\left(a_5 - y_{\text{actual}}\right)^2\right] \\
&= 2\left(a_5 - y_{\text{actual}}\right)\frac{\partial}{\partial w_{01}}\left[a_5 - y_{\text{actual}}\right] \\
&= 2\left(a_5 - y_{\text{actual}}\right)\frac{\partial}{\partial w_{01}}\left[a_5\right] \\
&= 2\left(a_5 - y_{\text{actual}}\right)\frac{\partial}{\partial w_{01}}\left[f_5(i_5)\right] \\
&= 2\left(a_5 - y_{\text{actual}}\right)f_5'(i_5)\frac{\partial}{\partial w_{01}}\left[i_5\right] \\
&= 2\left(a_5 - y_{\text{actual}}\right)f_5'(i_5)\frac{\partial}{\partial w_{01}}\left[a_3 \cdot w_{35} + a_4 \cdot w_{45}\right] \\
&= 2\left(a_5 - y_{\text{actual}}\right)f_5'(i_5)\left(w_{35}\frac{\partial}{\partial w_{01}}\left[a_3\right] + w_{45}\frac{\partial}{\partial w_{01}}\left[a_4\right]\right) \\
&= 2\left(a_5 - y_{\text{actual}}\right)f_5'(i_5)\left(w_{35}\frac{\partial}{\partial w_{01}}\left[f_3(i_3)\right] + w_{45}\frac{\partial}{\partial w_{01}}\left[f_4(i_4)\right]\right) \\
&= 2\left(a_5 - y_{\text{actual}}\right)f_5'(i_5)\left(w_{35}f_3'(i_3)\frac{\partial}{\partial w_{01}}\left[i_3\right] + w_{45}f_4'(i_4)\frac{\partial}{\partial w_{01}}\left[i_4\right]\right) \\
&= 2\left(a_5 - y_{\text{actual}}\right)f_5'(i_5)\left(w_{35}f_3'(i_3)\frac{\partial}{\partial w_{01}}\left[a_1 \cdot w_{13}\right] + w_{45}f_4'(i_4)\frac{\partial}{\partial w_{01}}\left[a_1 \cdot w_{14}\right]\right) \\
&= 2\left(a_5 - y_{\text{actual}}\right)f_5'(i_5)\left(w_{35}f_3'(i_3)w_{13}\frac{\partial}{\partial w_{01}}\left[a_1\right] + w_{45}f_4'(i_4)w_{14}\frac{\partial}{\partial w_{01}}\left[a_1\right]\right) \\
&= 2\left(a_5 - y_{\text{actual}}\right)f_5'(i_5)\left(w_{35}f_3'(i_3)w_{13} + w_{45}f_4'(i_4)w_{14}\right)\frac{\partial}{\partial w_{01}}\left[a_1\right] \\
&= 2\left(a_5 - y_{\text{actual}}\right)f_5'(i_5)\left(w_{35}f_3'(i_3)w_{13} + w_{45}f_4'(i_4)w_{14}\right)\frac{\partial}{\partial w_{01}}\left[f_1(i_1)\right] \\
&= 2\left(a_5 - y_{\text{actual}}\right)f_5'(i_5)\left(w_{35}f_3'(i_3)w_{13} + w_{45}f_4'(i_4)w_{14}\right)f_1'(i_1)\frac{\partial}{\partial w_{01}}\left[i_1\right] \\
&= 2\left(a_5 - y_{\text{actual}}\right)f_5'(i_5)\left(w_{35}f_3'(i_3)w_{13} + w_{45}f_4'(i_4)w_{14}\right)f_1'(i_1)\frac{\partial}{\partial w_{01}}\left[a_0 \cdot w_{01}\right] \\
&= 2\left(a_5 - y_{\text{actual}}\right)f_5'(i_5)\left(w_{35}f_3'(i_3)w_{13} + w_{45}f_4'(i_4)w_{14}\right)f_1'(i_1)a_0
\end{aligned}
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To check my work:

$$\begin{aligned}\frac{\partial E}{\partial w_{01}} &= 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5)\left(w_{35} f_3'(i_3) w_{13} + w_{45} f_4'(i_4) w_{14}\right) f_1'(i_1) a_0 \\ &= 2(7-1)\cdot 13(21\cdot 11\cdot 16 + 22\cdot 12\cdot 17)\cdot 9\cdot 2 \\ &= 22980672\end{aligned}$$

1.8 Compute $\frac{\partial E}{\partial w_{02}}$

Using similar reasoning as shown above, we will find:

$$\frac{\partial E}{\partial w_{02}} = 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5)\left(w_{35} f_3'(i_3) w_{23} + w_{45} f_4'(i_4) w_{24}\right) f_2'(i_2) a_0$$

To check my work:

$$\begin{aligned}\frac{\partial E}{\partial w_{02}} &= 2\left(a_5 - y_{\text{actual}}\right) f_5'(i_5)\left(w_{35} f_3'(i_3) w_{23} + w_{45} f_4'(i_4) w_{24}\right) f_2'(i_2) a_0 \\ &= 2(7-1)\cdot 13(21\cdot 11\cdot 18 + 22\cdot 12\cdot 19)\cdot 10\cdot 2 \\ &= 28622880\end{aligned}$$