Machine Learning Assignment 63
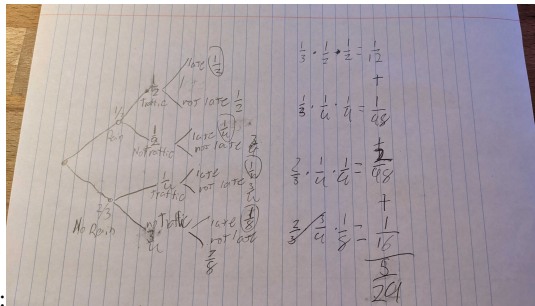
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63-1

A

$$A : P(1) = \frac{1^2}{16} = \frac{1}{16}$$

$$B : P(t > 2) = 1 - P(t < 2) = 1 - \frac{2^2}{16} = 1 - 1/4 = 3/4$$

$$C : P(1 \le t \le 3) = P(t \ge 3) - P(t \le 1) = P(t \le 3) - (1 - P(t \le 1)) = \frac{3^2}{16} - (1 - \frac{1^2}{16}) = \frac{9}{16} - \frac{15}{16} = -\frac{6}{16}$$

B

$$A : \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$$B :$$

$$C : \frac{1}{12} + \frac{1}{48} = \frac{5}{48}$$

C

$$A : \sum_{k=1}^{\infty} \frac{c}{3^k} = c\frac{1}{2} = 1 \implies c = 2$$

$$B : P(\{2, 4, 6\}) = \frac{2}{3^{(2)}} + \frac{2}{3^{(4)}} + \frac{2}{3^{(6)}} = \frac{182}{729}$$

$$C : P(\{3, 4, 5, \dots\}) = \sum_{k=1}^{\infty} \frac{2}{3^k} = \frac{2}{3^{(3)}} + \frac{2}{3^{(4)}} + \frac{2}{3^{(5)}} + \dots + \frac{2}{3^{(\infty)}} = \frac{2}{18}$$
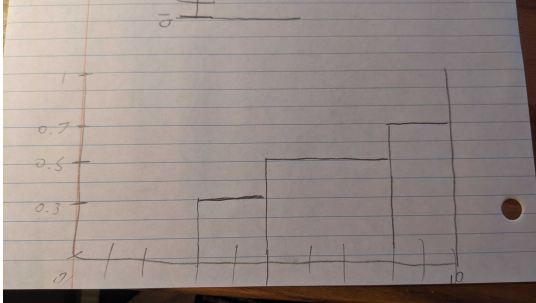
D

P(getting exactly k red balls) $= \binom{20}{k} \cdot 0.3^k \cdot 0.7^{20-k}$

E

P(getting exactly k red balls without replacement) $= \dfrac{\binom{30}{k}\cdot\binom{70}{20-k}}{\binom{100}{20}}$

F

G

$$Var(2X - Y) = 6 \ \& \ Var(X + 2Y) = 9$$
$$4\cdot Var(X) - Var(Y) = 6 \ \& \ Var(X) + 4\cdot Var(Y) = 9$$
$$Var(X) = \frac{Var(Y)+6}{4} \ \& \ Var(X) = 9 - 4\cdot Var(Y)$$
$$\implies 9 - 4\cdot Var(Y) = \frac{Var(Y)+6}{4} \qquad\qquad \implies 36 - 16\cdot Var(Y) = Var(Y) + 6$$
$$\implies 30 - 15\cdot Var(Y) = 0$$
$$\implies Var(Y) = \frac{30}{15} = 2$$
$$\implies Var(X) = 9 - 4\cdot(2) = 9 - 8 = 1$$