Problem 52-1

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1 Using the identity Var[X]=E[X^2]$E[X]^2, computeVar[X]ifXissamp$

$$Var[X] = E[X^2]E[X]^2$$

$$E[X^2] = \int_a^b x * k^2 dx$$

$$E[X]^2 = (\int_a^b x * k dx)^2$$

$$Var[X] = \int_a^b x * k^2 dx + (\int_a^b x * k dx)^2$$

$$bk^2 - ak^2 + (bk - ak)^2 = bk^2 - ak^2 + b^2k^2 - 2abk^2 + a^2k^2$$

$$k^2(b - a + b^2 - 2ab + a^2)$$

2 Using the identity $Var[X] = E[X^2]E[X]^2$, compute Var[X] if X is sampled from the exponential distribution p(x)= $\lambda e^{-\lambda x}$,x0.

$$Var[X] = E[X^2]E[X]^2$$

$$E[X^2] = \int_a^b x * (\lambda * e^{-\lambda x})^2 dx$$

$$E[X]^2 = (\int_a^b x * \lambda * e^{-\lambda x} dx)^2$$

$$Var[X] = \int_a^b x * (\lambda * e^{-\lambda x})^2 dx + (\int_a^b x * \lambda * e^{-\lambda x} dx)^2$$

$$\int_a^b x * \lambda^2 * e^{-2*\lambda x} dx + ((-x * e^{-\lambda x}|_a^b) + \int_a^b e^{-\lambda x} dx)^2$$

$$x * \frac{\lambda}{-2} e^{-2*\lambda x}|_a^b - \int_a^b \frac{\lambda}{-2} e^{-2*\lambda x} dx + ((-b * e^{-\lambda b} + a * e^{-\lambda a}) + (\frac{e^{-\lambda x}}{-\lambda}|_a^b))^2$$

$b*\frac{\lambda}{-2}e^{-2*\lambda b}-a*\frac{\lambda}{-2}e^{-2*\lambda a}-(\frac{1}{4}e^{-2*\lambda b}-\frac{1}{4}e^{-2*\lambda a})+((-b*e^{-\lambda b}+a*e^{-\lambda a})+(\frac{e^{-\lambda b}}{-\lambda}-\frac{e^{-\lambda a}}{-\lambda}))^2$

When worked out this probably comes to the solution when a=-∞ and b=∞:

$$\frac{1}{\lambda^2}$$

3 Using the identity Var[N]=E[N2]E[N]2, compute Var[N] if N is sampled from the Poisson distribution p(n)= $\frac{\lambda^n e^{-\lambda}}{n!}, n \in$0,1,2,. . . .

$$Var[X] = E[X^2]E[X]^2$$

$$E[X^2] = \int_a^b x*(\frac{\lambda^n e^{-\lambda}}{n!})^2 dx$$

$$E[X]^2 = (\int_a^b x*\frac{\lambda^n e^{-\lambda}}{n!}dx)^2$$

$$Var[X] = \int_a^b x*(\frac{\lambda^n e^{-\lambda}}{n!})^2 dx + (\int_a^b x*\frac{\lambda^n e^{-\lambda}}{n!}dx)^2$$

When worked out this probably comes to the solution when a=-∞ and b=∞:

$$\lambda$$