21-2

Nathan Reynoso

December 2020

$$p(x) = \begin{cases} k & x \in [3,7] \\ 0 & x \notin [3,7] \end{cases}$$

Problem a

Find the value of k such that $p(x)$ is a valid probability distribution.

$$\int_{-\infty}^{\infty} p(x)dx = 1$$
$$\int_{3}^{7} k \, dx = 1$$
$$7k - 3k = 1$$
$$k = \frac{1}{4}$$

Problem b

Given that $X \sim U[3,7]$, compute $\mathbf{E}[X]$.

$$E[X] = \int_{-\infty}^{\infty} xp(x)dx$$
$$= \int_{3}^{7} \frac{x}{4}dx$$
$$= \frac{49}{8} - \frac{9}{8}$$
$$= \frac{40}{8}$$
$$= 5$$

Problem c

Given that $X \sim U[3, 7]$, compute Var$[X]$.

$$Var[X] = \int_{-\infty}^{\infty} (x - E[X])^2 p(x) dx$$
$$= \int_{3}^{7} \frac{(x^2 - 10x + 25)}{4} dx$$
$$= \frac{133}{12} - \frac{117}{12}$$
$$= \frac{16}{12}$$
$$= \frac{4}{3}$$