Assignment 60-1

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1 Introduction

For two positive functions $f(n)$ and $g(n)$, we say that $f = O(g)$ if

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} < \infty \text{ or } f(n) < c \cdot g(n)$$

for all n.

Using the definition above, prove the following:
a. $3n^2 + 2n + 1 = O(n^2)$.

$$\lim_{n\to\infty} \frac{3n^2 + 2n + 1}{n^2} < \infty$$

$$\lim_{n\to\infty} \frac{6n + 2}{2n} < \infty$$

$$\lim_{n\to\infty} \frac{6}{2} < \infty$$

$$3 < \infty$$

b. $O(f + g) = O(\max(f, g))$.

$$h = O(f + g) \to f(x) < c \cdot (f(n) + g(n))$$

$$h(n) < c \cdot (\max(f(n), g(n)) + \min(f(n), g(n))$$

Because f(x) is less than the max plus the min we can say that:

$$h(n) < c \cdot (\max(f(n), g(n)) + \max(f(n), g(n))$$

$$h(n) < 2c \cdot (\max(f(n), g(n)))$$

$$h(n) < d \cdot (\max(f(n), g(n)))$$

$$h = O(max(f, g))$$

c. $O(f) \cdot O(g) = O(f \cdot g)$.

$$h = O(f) \cdot O(g)$$

$$h(n) < cf(n) * dg(n)$$

$$h(n) < c * d * f(n) * g(n)$$

$$h(n) < e * (f(n) * g(n))$$

$$h = O(f \cdot g)$$

d. If $f = O(g)$ and $g = O(h)$ then $f = O(h)$.

$$f = O(g) \rightarrow \lim_{n \to \infty} \frac{f(n)}{g(n)} < \infty$$

$$g = O(h) \rightarrow \lim_{n \to \infty} \frac{g(n)}{h(n)} < \infty$$

Because they are both less than infinity we can say:

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} \cdot \lim_{n \to \infty} \frac{g(n)}{h(n)} < \infty$$

$$\lim_{n \to \infty} \frac{f(n) \cdot g(n)}{g(n) \cdot h(n)} < \infty$$

$$\lim_{n \to \infty} \frac{f(n)}{h(n)} < \infty$$

$$f = O(h)$$