53-1

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1 Compute $Cov[X, X^2]$.

$$E[X] = \int_0^1 xkdx = \frac{kx^2}{2}\Big|_0^1 = \frac{k}{2}$$

$$E[X^2] = \int_0^1 x^2 * kdx = \frac{kx^3}{3}\Big|_0^1 = \frac{k}{3}$$

$$Cov[X, X^2] = E[(X - \frac{k}{2})(X^2 - \frac{k}{3})]$$

$$\int_0^1 x^3 * k - \frac{k}{2}x^2 * k - \frac{k}{3}x * k + \frac{k^2}{6} * kdx =$$

$$\frac{x^4 * k}{4} - \frac{x^3 * k^2}{6} - \frac{x^2 * k^2}{6} + \frac{x * k^3}{6}\Big|_0^1$$

$$\frac{k}{4} - \frac{k^2}{6} - \frac{k^2}{6} + \frac{k^3}{6} =$$

$$\frac{1}{4} - \frac{1}{6} = \frac{3}{12} - \frac{2}{12} = \frac{1}{12}$$

2 Compute cov$[\mathbf{X}_1, X_2]$.

For simplicity, I'll be calling X_1 X and X_2 Y

$$E[X] = \int_0^1 \int_0^1 x * kdxdy = \int_0^1 \frac{kx^2}{2}\Big|_0^1 dy = \frac{k}{2}$$

$$E[Y] = \int_0^1 \int_0^1 y * kdxdy = \int_0^1 xy * k\Big|_0^1 dy = \frac{k * y^2}{2}\Big|_0^1 = \frac{k}{2}$$

$$Cov[X, Y] = E[(X - \frac{k}{2})(Y - \frac{k}{2})] =$$

$$\int_0^1 \int_0^1 x*y*k - \frac{y*k^2}{2} - \frac{x*k^2}{2} + \frac{k^3}{4} dxdy =$$

$$\int_0^1 \frac{y*k*x^2}{2} - \frac{y*k^2}{2}*x - \frac{x^2*k^2}{4} + \frac{k^3}{4}*x\Big|_0^1 dy =$$

$$\int_0^1 \frac{y*k}{2} - \frac{y*k^2}{2} - \frac{k^2}{4} + \frac{k^3}{4} dy =$$

$$\frac{y^2*k}{4} - \frac{y^2*k^2}{4} - \frac{k^2}{4}*y + \frac{k^3}{4}*y\Big|_0^1$$

$$\frac{k}{4} - \frac{k^2}{4} - \frac{k^2}{4} + \frac{k^3}{4} = \frac{1}{4} - \frac{1}{4} - \frac{1}{4} + \frac{1}{4} = 0$$

3 Prove Var[X+Y]=Var[X]+Var[Y]+2Cov[X,Y].

$$E[X+Y] = \int_0^1 \int_0^1 x*k + y*kdxdy = k$$

$$Var[X+Y] = E[X+Y+E[X+Y]] = E[X+Y+k]$$

Because of what we proved in 49-1.b

$$E[X+Y+k] = E[X+\frac{k}{2}Y+\frac{k}{2}] = E[x+\frac{k}{2}] + E[Y+\frac{k}{2}]$$

We know that E[X] = $\frac{k}{2}$ and E[Y] = $\frac{k}{2}$, so

$$E[x+\frac{k}{2}] + E[Y+\frac{k}{2}] = Var[X] + Var[Y]$$

And this proves what we were aiming for because Coc[X,Y]=0

4 Prove Cov[X,Y]=E[X*Y]-E[X]*E[Y].

$$Cov[X,Y] = E[(X-E[X])(Y-E[Y])] =$$

$$E[(X-E[X])(Y-E[Y])] = E[X*Y - XE[Y] - YE[X] + E[X]E[Y]]$$

$$E[X*Y] - 2E[Y][X] + E[Y]E[X] = E[X*Y] = E[X]E[Y]$$