Machine Learning Assignment 63 Problem 1

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Problem A

(a)

$$P(T \leq 1) = \int_0^1 \frac{t}{16} \mathrm{d}t$$
$$= \frac{1}{32}$$

(b)

$$P(T \geq 2) = 1 - \int_0^2 \frac{t}{16} \mathrm{d}t$$
$$= 1 - \frac{1}{8} \qquad = \frac{7}{8}$$

(c)

$$P(1 \leq T \leq 3) = \int_1^3 \frac{t}{16} \mathrm{d}t$$
$$= \frac{1}{4}$$

Problem B

(a)

$$P(\text{not late}|\text{heavy traffic}|\text{not raining}) = \frac{2}{3} * \frac{1}{4} * \frac{1}{8}$$

(b)

$$\begin{aligned} P(\text{rainy}|\text{traffic}|\text{late}) &= \frac{1}{12} \\ P(\text{rainy}|\text{no traffic}|\text{late}) &= \frac{1}{24} \\ P(\text{not rainy}|\text{traffic}|\text{late}) &= \frac{1}{24} \\ P(\text{not rainy}|\text{no traffic}|\text{late}) &= \frac{1}{16} \\ P(\text{late}) &= \frac{11}{48} \end{aligned}$$

(c)

$$\begin{aligned} P(\text{rainy}|\text{late}) &= \frac{1}{3} * \frac{11}{48} \\ &= \frac{11}{144} \end{aligned}$$

Problem C

(a)

$$\begin{aligned} P(k) &= \sum_{n=1}^{\infty} \frac{c}{3^k} \\ \frac{1}{c} &= \sum_{n=1}^{\infty} \frac{1}{3^k} \\ &= \frac{\frac{1}{3}}{1-\frac{1}{3}} \\ &= \frac{1}{2} \\ c &= 2 \end{aligned}$$

(b)

$$\begin{aligned} P(2,4,6) &= \sum_{2,4,6} \frac{c}{3^k} \\ &= \frac{2}{9} + \frac{2}{81} + \frac{2}{729} \\ &= \frac{182}{729} \end{aligned}$$

(c)

$$\begin{aligned} P(3,4,5,...) &= \sum_{3,4,5,...} \left(\frac{c}{3^k}\right) \\ &= 2 * \frac{\frac{1}{27}}{1-\frac{1}{3}} \\ &= \frac{1}{18} \end{aligned}$$

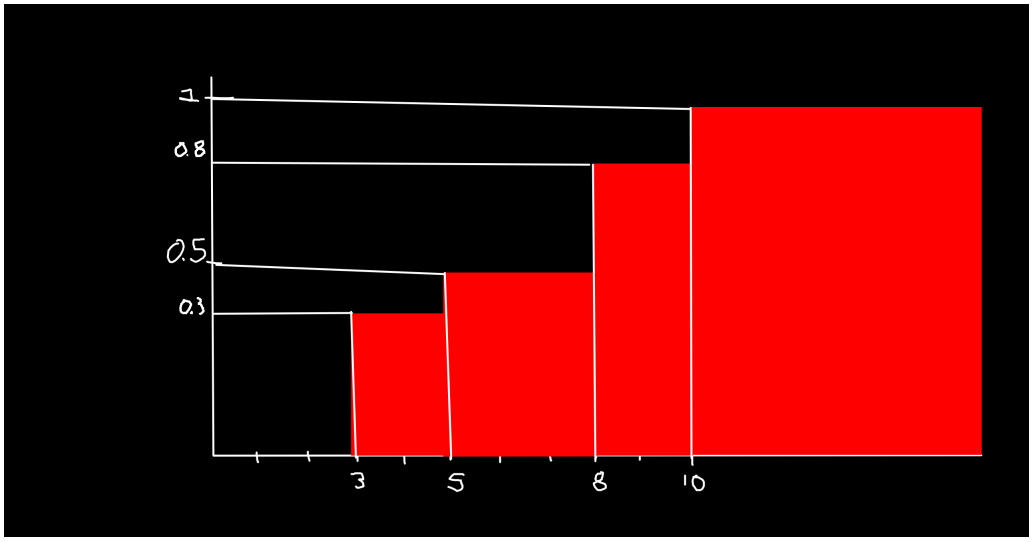
Problem D

$$P(\text{k red balls}) = \left(\frac{30}{100}\right)^k \left(\frac{70}{100}\right)^{20-k}$$

Problem E

$$P(\text{k red balls}) = \frac{30}{100} * \frac{29}{99} * \frac{28}{98} * \cdots * \frac{30-k}{100-k} * \frac{70}{99-k} * \frac{69}{98-k} * \cdots * \frac{50+k}{80}$$

Problem F



Problem G

$$\begin{cases} Var(2X - Y) = 6 \\ Var(X + 2Y) = 9 \end{cases}$$

$$=> \begin{cases} 4Var(X) - Var(Y) = 6 \\ Var(X) + 4Var(Y) = 9 \end{cases}$$

$$=> \left[\begin{array}{cc|c} 4 & -1 & 6 \\ 1 & 4 & 9 \end{array}\right]$$

$$=> \begin{cases} Var(X) = \dfrac{33}{17} \\ Var(Y) = \dfrac{30}{17} \end{cases}$$