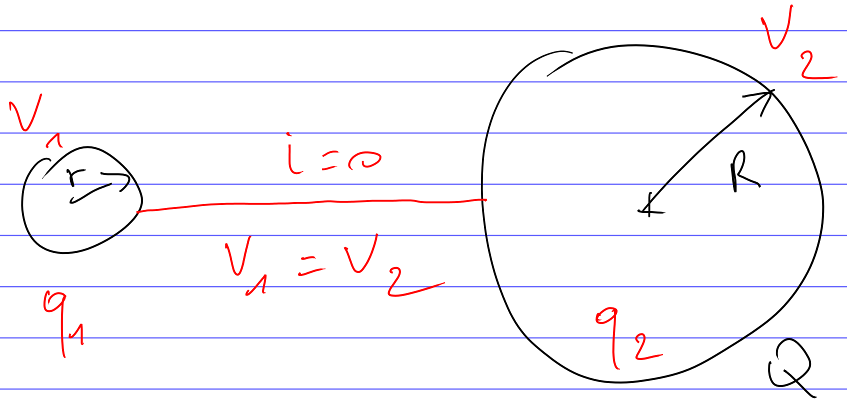
TD 2 :

Distributions de charges

Exercice 1 : Sphères chargées



1) petite sphère

$$V_1 = \frac{q_1}{4\pi\varepsilon_0 r}$$

Sphère chargée = charge ponctuelle

à la surface de la petite sphère

$$V_2 = \frac{q_2}{4\pi\varepsilon_0 R}$$

à la surface de la grande sphère

2) A l'équilibre $V_1 = V_2$

$$\frac{q_1}{\cancel{4\pi\varepsilon_0} r} = \frac{q_2}{\cancel{4\pi\varepsilon_0} R}$$

$$\frac{q_1}{r} = \frac{q_2}{R}$$

$$q_1 + q_2 = Q$$

$$\frac{q_1}{r} = \frac{Q - q_1}{R}$$

$$q_1 R = (Q - q_1) r$$

$$q_1 (R + r) = Q r$$

$$q_1 = \frac{Q r}{R + r}$$

$$\frac{Q - q_2}{r} = \frac{q_2}{R}$$

$$(Q - q_2) R = q_2 r$$

$$q_2 (r + R) = Q R$$

$$\begin{cases} q_2 = \dfrac{QR}{r+R} \\ \\ q_1 = \dfrac{Qr}{r+R} \end{cases}$$

$V_1 = V_2$

$$\Rightarrow \frac{q_1}{q_2} = \frac{r}{R}$$

3) $$E_1 = \frac{q_1}{4\pi \varepsilon_0 r^2}$$

$$E_2 = \frac{q_2}{4\pi \varepsilon_0 R^2}$$

$$E_1 = \frac{Qr}{(r+R)\, 4\pi \varepsilon_0 r^2}$$

$$E_2 = \frac{QR}{(r+R)\, 4\pi \varepsilon_0 R^2}$$

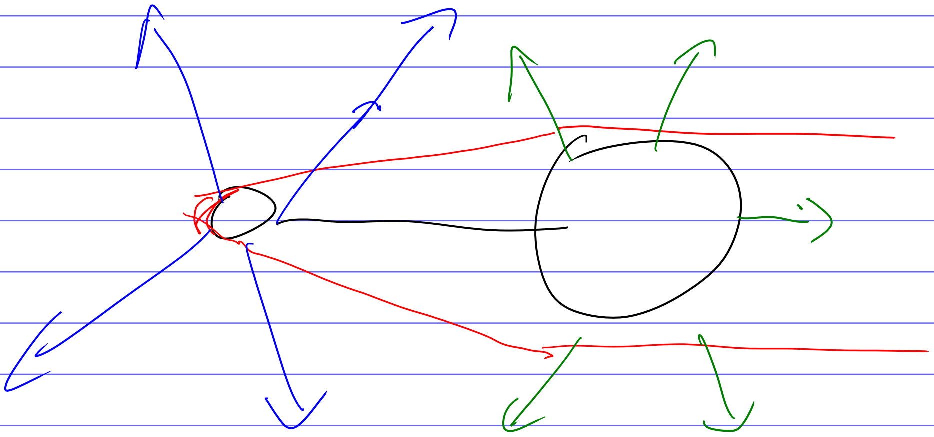
$$\frac{E_1}{E_2} = \frac{r}{r^2} \frac{R^2}{R} = \frac{R}{r}$$

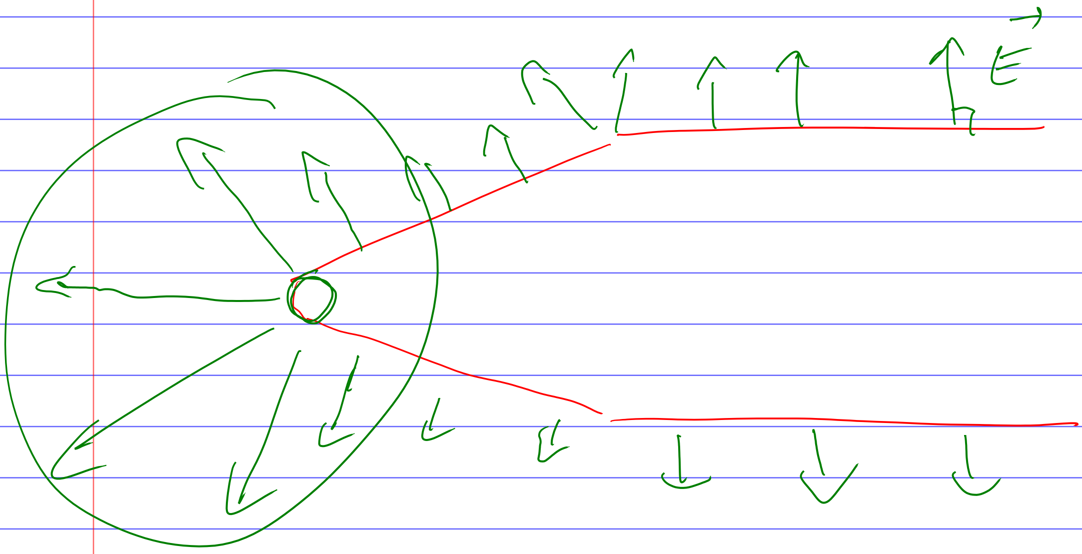
$$\frac{E_1}{E_2} = 10$$

$$\frac{V_1}{V_2} = 1$$

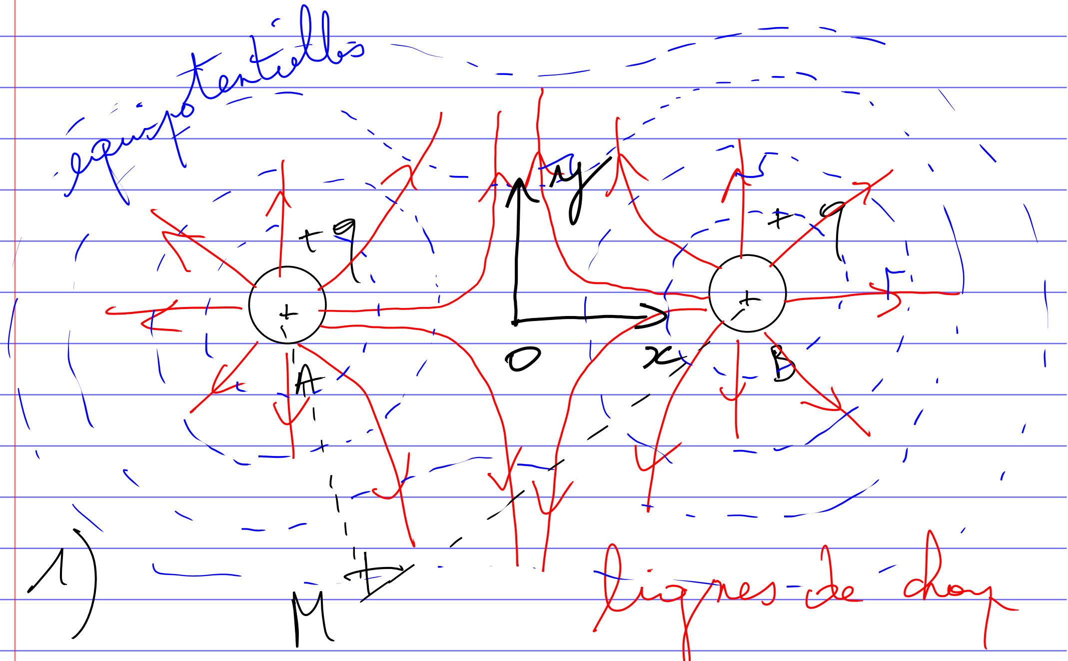
$$\frac{q_1}{q_2} = \frac{1}{10}$$

Effet de pointe





Ex 2 : 2 charges ponctuelles



1)

les équipotentielles $\perp$ lignes de champ

2) $M(x, y)$

$A(x_1, y_1)$ $\quad$ $B(x_2, y_2)$

$y_1 = y_2 = 0$

$$V_1 = \frac{q}{4\pi\varepsilon_0 AM} \qquad V_2 = \frac{q}{4\pi\varepsilon_0 BM}$$

$$\vec{AM} = \begin{pmatrix} x - x_1 \\ y - y_1 = y \end{pmatrix} \qquad \vec{BM} = \begin{pmatrix} x - x_2 \\ y - y_2 = y \end{pmatrix}$$

$$AM = \sqrt{(x-x_1)^2 + y^2}$$

$$BM = \sqrt{(x-x_2)^2 + y^2}$$

$$V(M) = V_1(M) + V_2(M)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{(x-x_1)^2+y^2}} + \frac{1}{\sqrt{(x-x_2)^2+y^2}}\right)$$

3) $E(M)$?

$$\vec{E}(M) = \frac{q\,\vec{PM}}{4\pi\varepsilon_0\, PM^3}$$

$$\vec{E}_1(M) = \frac{q\,\vec{AM}}{4\pi\varepsilon_0\, AM^3}$$

$$\vec{E}_2(M) = \frac{q\,\vec{BM}}{4\pi\varepsilon_0\, BM^3}$$

$$\vec{E}(M) = \vec{E}_1(M) + \vec{E}_2(M)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{\begin{pmatrix} x-x_1 \\ y \end{pmatrix}}{\left(\sqrt{(x-x_1)^2+y^2}\right)^3} + \frac{\begin{pmatrix} x-x_2 \\ y \end{pmatrix}}{\left(\sqrt{(x-x_2)^2+y^2}\right)^3}\right)$$

$$E_x(M) = \frac{q}{4\pi\varepsilon_0}\left(\frac{x-x_1}{\left((x-x_1)^2+y^2\right)^{3/2}} + \frac{x-x_2}{\left((x-x_2)^2+y^2\right)^{3/2}}\right)$$

$$E_y(M) = \frac{q}{4\pi\varepsilon_0}\left(\frac{y}{\left((x-x_1)^2+y^2\right)^{3/2}} + \frac{y}{\left((x-x_2)^2+y^2\right)^{3/2}}\right)$$

2^e méthode : $\vec{E}(M) = -\vec{\nabla} V(M)$

$$\vec{E}(M) = -\begin{pmatrix} \frac{\partial V(M)}{\partial x} & = E_x \\ \frac{\partial V(M)}{\partial y} & = E_y \end{pmatrix}$$

$$E_x(M) = -\frac{\partial}{\partial x} V(M)$$

$$= -\frac{q}{4\pi\varepsilon_0}\left(\frac{-\frac{1}{2}\,2(x-x_1)}{\left((x-x_1)^2+y^2\right)^{3/2}} + \right.$$

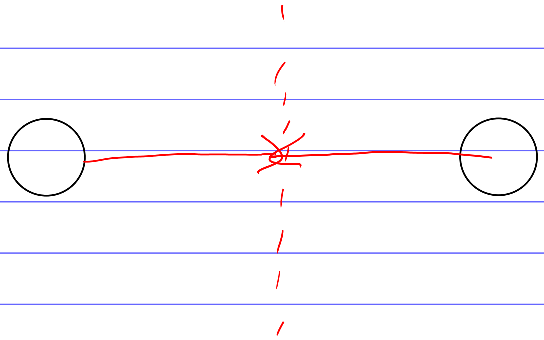
$$-\frac{\frac{1}{2}\,2(x-x_2)}{\left((x-x_1)^2+y^2\right)^{3/2}}\Bigg)$$

$$E_y(r) = -\frac{\partial}{\partial y}V(r)$$

$$= -\frac{1}{4\pi\varepsilon_0}\left(\frac{-\frac{1}{2}\,2y}{\left((x-x_1)^2+y^2\right)^{3/2}} +\right.$$

$$\left.\frac{-\frac{1}{2}\,2y}{\left((x-x_1)^2+y^2\right)^{3/2}}\right)$$

4)



$$\vec{E} = \vec{0} \implies E_x = 0$$

$$E_y = 0$$

$$E_y = 0 = \frac{1}{4\pi\varepsilon_0}\left(\frac{y}{(\quad)_1} + \frac{y}{(\quad)_2}\right)$$

$$y = 0$$

$$E_x = 0 = \frac{1}{4\pi\varepsilon_0}\left(\frac{x-x_1}{(\quad)_1} + \frac{x-x_2}{(\quad)_2}\right)$$

$$\frac{x-x_1}{\left((x-x_1)^2+y^2\right)^{3/2}} = \frac{x-x_2}{\left((x-x_2)^2+y^2\right)^{3/2}}$$

$$y=0$$

$$\frac{x-x_1}{\left((x-x_1)^2\right)^{3/2}} = \frac{x-x_2}{\left((x-x_1)^2\right)^{3/2}}$$

$$\frac{x-x_1}{|x-x_1|^3} = \frac{x-x_2}{|x-x_2|^3}$$

$$\frac{\text{signe}(x-x_1)}{(x-x_1)^2} = \frac{\text{signe}(x-x_1)}{(x-x_2)^2}$$

$$\pm(x-x_1)^2 = \pm(x-x_2)^2$$

$$\mp(x-x_1) = \mp(x-x_2)$$

$$\begin{matrix} - & - \\ + & + \end{matrix} \Rightarrow \cancel{x}-x_1 = \cancel{x}-x_2$$

pas possible

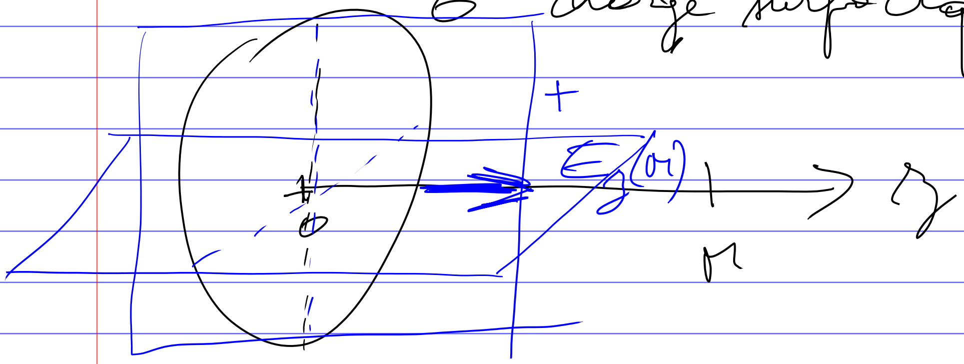
$\frac{- +}{+ -} \Rightarrow -x + x_1 = x - x_2$

$$2x = x_2 + x_1$$

$$x = \frac{x_2 + x_1}{2} \Rightarrow \text{le milieu de } AB$$

Ex 3 : Disque chargé

σ charge surfacique



1) symétries :

de révolution d'axe Oz

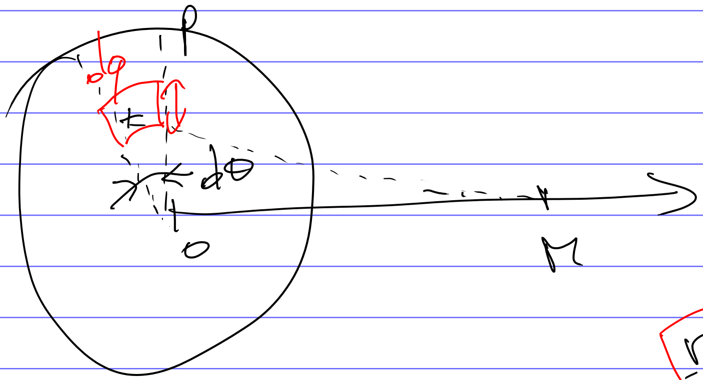
$\Rightarrow$ pas de composante θ

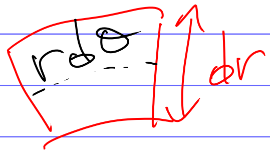
symétrie des plans $\perp$

↳ intersection $\Rightarrow \vec{E} = E\vec{u_z}$

2) Potentiel au point M

$$dV(M) = \frac{dq}{4\pi\varepsilon_0 PM}$$





$$dq = \sigma\, dS$$

$$dS = r\,d\theta\,dr$$

$$dV(M) = \frac{\sigma\, r\,d\theta\,dr}{4\pi\varepsilon_0\, PM}$$

$$\vec{PM} = \begin{pmatrix} -r \\ 0 \\ z \end{pmatrix} \qquad M\begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} \quad P\begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

$$PM = \sqrt{r^2 + z^2}$$

$$V(M) = \int_0^a \int_0^{2\pi} \frac{\sigma\, r\,d\theta\,dr}{4\pi\varepsilon_0 \sqrt{r^2+z^2}}$$

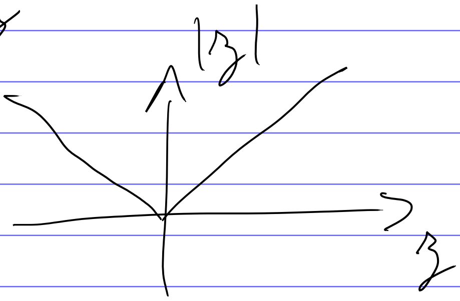
$$= \frac{\sigma\, 2\pi}{4\pi\varepsilon_0} \int_0^a \frac{r\,dr}{\sqrt{r^2+z^2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\sqrt{r^2+z^2}\right]_0^a$$

$$= \boxed{\frac{\sigma}{2\varepsilon_0}\left(\sqrt{a^2+z^2}-|z|\right)}$$

3) $\vec{E}(M) = -\vec{\nabla}V(M)$

$$= -\frac{\partial V(M)}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\left(\frac{\frac{1}{2}\,2z}{(a^2+z^2)^{3/2}} - \frac{z}{|z|}\right)\vec{u}_z$$

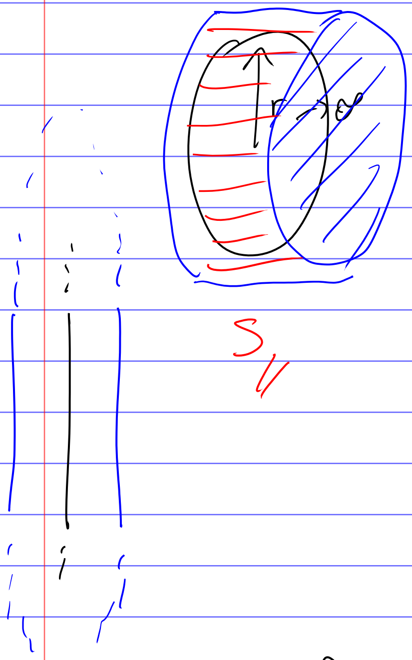


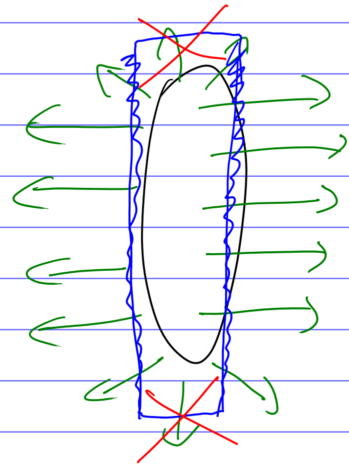
$$\vec{E}(M) = -\frac{\sigma}{2\varepsilon_0}\left(\frac{z}{(a^2+z^2)^{3/2}} - \frac{z}{|z|}\right)\vec{u}_z$$

Partie B :

4) Théorème de Gauss

$S_\perp \gg S_{\parallel}$





surface chargée infinie

$\Longrightarrow E_z$ uniforme

$\Longrightarrow$ négliger les bords

5) Th. de Gauss

$$\oiint_{(S_G)} \vec{E} \cdot \vec{dS} = \frac{Q_{int}}{\varepsilon_0}$$

choix de S_G tel que

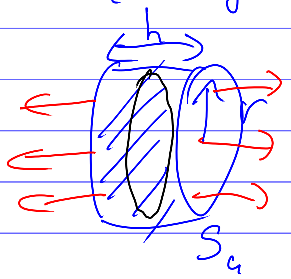
$E = \text{cste}$ sur S_G

et $\vec{E} \perp$ surface de Gauss

$\vec{E} \parallel \vec{dS}$

$\vec{dS} = \vec{n}\, dS$

$\Rightarrow$ S_G cylindre de rayon infini



$$\oiint_{(S_G)} \vec{E}\cdot d\vec{S} = E \oiint dS$$

$$= E\,\pi r^2 = \frac{Q_{int}}{\varepsilon_0}$$

champ selon E_z $\qquad \pi r^2 = S$

$$Q_{int} = \sigma S = \sigma \pi r^2$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\vec{u}_z \quad \text{pour } z > 0$$

$$\vec{E} = -\frac{\sigma}{\varepsilon_0}\vec{u}_z \quad \text{pour } z < 0$$

6) $V(z)$?

$$V(B) - V(A) = -\int_A^B \vec{E}\cdot d\vec{l}$$

$$-\int_0^z E\,dz = V(z) - V(0)$$

$$-\left[\frac{\sigma z}{\varepsilon_0}\right]_0^z = V(z) - V(0)$$

pour les $z > 0$

$$V(z) = V(0) - \frac{\sigma z}{\varepsilon_0}$$

pour les $z < 0$

$$V(z) = V(0) + \frac{\sigma z}{\varepsilon_0}$$

avec $V(0) = 0$