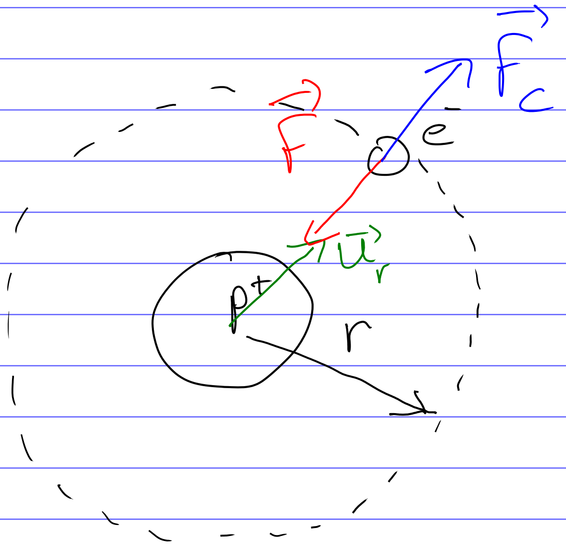
TD1 : Charges ponctuelles

Exercice 1 : Atome d'hydrogène



$$F = \frac{q_1 q_2}{4\pi \varepsilon_0 r^2} = \frac{-e^2}{4\pi \varepsilon_0 r^2} < 0$$

$\Rightarrow$ attraction

la force sur l'e^-

$$\vec{F} = \frac{-e^2}{4\pi \varepsilon_0 r^2} \vec{u_r}$$

2) Force centrifuge

$$\vec{F_c} = m \vec{a_c}$$

$$\vec{a}_c = \frac{v^2}{r}\vec{u}_r$$

3) Bilan des forces

$$\Sigma \vec{F} = m\vec{\gamma}$$

dans la direction radiale $r = \text{cste}$

$$\dot{r} = 0$$

$$\gamma_r = 0$$

$$\vec{F} + \vec{F}_c = \vec{0}$$

$$-\frac{e^2}{4\pi\varepsilon_0 r^{\cancel{2}}} + \frac{mv^2}{\cancel{r}} = 0$$

$$mv^2 = \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow r = \frac{e^2}{4\pi\varepsilon_0 m v^2}$$

m : masse de l'e^-

v ?

4) $mrv = n\hbar$

$$\hbar = \frac{h}{2\pi}$$

$$h = 6{,}62 \cdot 10^{-34}$$

$$v = \frac{n\hbar}{mr}$$

$$\cancel{r} = \frac{e^2}{4\pi\varepsilon_0 m v^2} = \frac{e^2 m^{\cancel{2}} r^{\cancel{2}}}{4\pi\varepsilon_0 \cancel{m} n^2 \hbar^2}$$

$$4\pi\varepsilon_0 n^2\hbar^2 = e^2 m r$$

$$r = \frac{4\pi\varepsilon_0 n^2 \hbar^2}{e^2 m}$$

$$= \frac{\cancel{4\pi}\varepsilon_0 n^2 h^2}{\cancel{4}\pi^{\cancel{2}} e^2 m}$$

$$= \boxed{\frac{\varepsilon_0 n^2 h^2}{\pi e^2 m}}$$

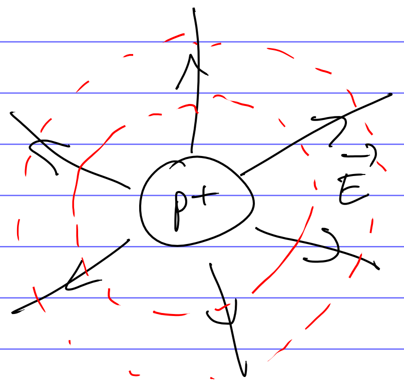
pour $n = 1 \Rightarrow r = a_0$

$$a_0 = \frac{\varepsilon_0 h^2}{\pi e^2 m} = 53 \text{ pm}$$

$$= 52,994 \cdot 10^{-12} \text{ m}$$

$$= 0,53 \text{ Å}$$

5)



$$\vec{E}(r) = \frac{e}{4\pi\varepsilon_0 r^2}\vec{u}_r$$

$$E(a_0) = \frac{e}{4\pi\varepsilon_0 a_0^2} = 5,13 \cdot 10^{11} \text{ V/m}$$

6) Le potentiel

$$V(a_0) = \frac{e}{4\pi\varepsilon_0 a_0} = 27,1 \text{ V}$$

7) $E_{\text{ionisation}} = E_p + E_c$

E_p : énergie potentielle électrostatique

E_c : " cinétique de l'e^-

$$E_p = qV(r) = -eV(a_0)$$

$$= \boxed{-27,1 \text{ eV}}$$

$$= -4,34 \cdot 10^{-18} \text{ J}$$

$$E_c = \frac{1}{2} m v^2 = \frac{1}{2} \cancel{m} \left(\frac{\hbar^2}{m^{\cancel{2}} r^2} \right)$$

$$m r v = n \hbar$$

$$n = 1 \quad v = \frac{\hbar}{mr}$$

$$E_c(a_0) = \frac{1}{2} \frac{\hbar^2}{m a_0^2} = 13,57 \text{ eV}$$

$$E_{ion} = E_p + E_c = -27,1 + 13,57$$

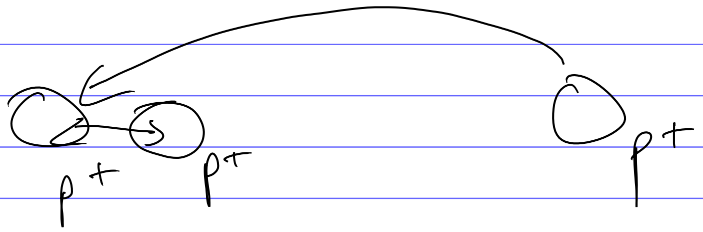
$$= -13,53 \text{ eV}$$

$$\Rightarrow \boxed{-13,6 \text{ eV}}$$

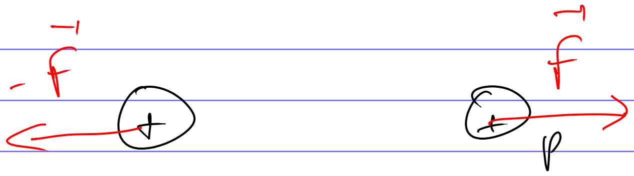
à l'état fondamental

Exercice 2 : Fusion nucléaire

$$\Delta E = \Delta m \, c^2$$



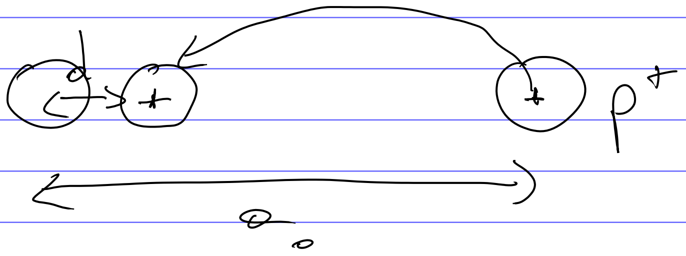
1) si $a_0 = 1 \text{ Å} = 10^{-10}$ m



$$F = \frac{e^2}{4\pi \varepsilon_0 a_0^2} = 2{,}3 \cdot 10^{-8} \text{ N}$$

2) Énergie potentielle :

$\Delta U = q \, \Delta V$ $\qquad$ $d = 10$ fm $= 10^{-14}$ m



$$\Delta U = q\,\Delta V = q\,(V(d) - V(a_0))$$

$$= q\left(\frac{q}{4\pi\varepsilon_0 d} - \frac{q}{4\pi\varepsilon_0 a_0}\right)$$

$q = e$

$$= \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{d} - \cancel{\frac{1}{a_0}}\right)$$

$$d \ll a_0$$

$$\Delta U = \frac{e^2}{4\pi\varepsilon_0 d} = 2{,}3 \cdot 10^{-14}\ J$$

$$= 143869\ eV$$

$$= 144\ keV$$

3) $E_c = \Delta U = \frac{1}{2} m v^2$

$m = m_p$

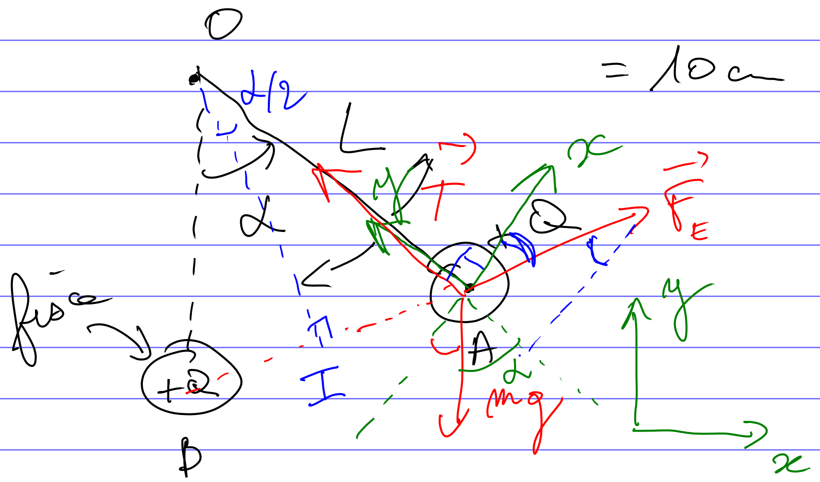
v ? $\quad v^2 = \dfrac{2\Delta U}{m}$

$$v = \sqrt{\frac{2\Delta U}{m}} = 5{,}25 \cdot 10^{6}\ m/s$$

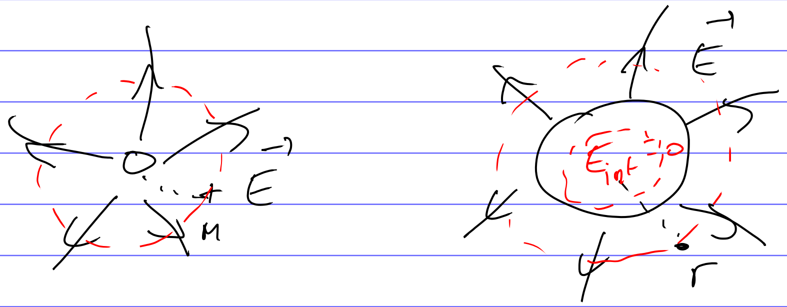
$$< c = 3 \cdot 10^{8}\ m/s$$

Exercice 3 :

$OA = L = 10\,cm$



1)



$$E_p = \frac{q}{4\pi\varepsilon_0 r^2} \qquad E_s = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$q = Q$$

$$E_p(M) = E_s(M)$$

2) $m = 1g$

$\alpha = 30°$

Choisir 1 repère $\sum \vec{F} = \vec{0}$

$\sum F_x = 0$

$\sum F_y = 0$

$$\sum F_x = F_E \cos\frac{\alpha}{2} - mg\sin\alpha = 0$$

$$\sum F_y = -mg\cos\alpha + T - F_E \sin\alpha/_2 = 0$$

$$F_E = \frac{Q^2}{4\pi\varepsilon_0 (AB)^2}$$

$$AB = 2L\sin\frac{\alpha}{2}$$

$$\rightarrow \frac{Q^2}{4\pi\varepsilon_0 4L^2\sin^2\frac{\alpha}{2}}\cos\frac{\alpha}{2} - mg\sin\alpha = 0$$

$$\sin(a+b) = \sin a\cos b + \sin b\cos a$$

$$\sin\left(2\times\frac{\alpha}{2}\right) = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

$$\frac{Q^2}{4\pi\varepsilon_0 4L^2\sin^2\frac{\alpha}{2}}\cancel{\cos\frac{\alpha}{2}} - mg\,2\sin\frac{\alpha}{2}\cancel{\cos\frac{\alpha}{2}} = 0$$

$$Q^2 = \left(2mg\sin\frac{\alpha}{2}\right)4\pi\varepsilon_0 4L^2\sin^2\frac{\alpha}{2}$$

$$Q = \sqrt{32\,mg\,\varepsilon_0 L^2\sin^3\frac{\alpha}{2}}$$

$$= 39\ nC$$

Charge d'un condensateur :

$Q = CU$ $\qquad$ $C = 10^{-6}$ F

$U = 10$ V

$Q = 10^{-6} \times 10 = 10^{-5}$ C

$= 10\ \mu C$