TD 2 - Math

Ex 1.

1) $\iint_{[0,1]\times[-2,0]} \sin(3x-2y)\,dx\,dy$

$= \int_0^1 \left( \int_{-2}^0 \sin(3x-2y)\,dy \right) dx$

$= \int_0^1 \left( \left[ \frac{1}{2}\cos(3x-2y) \right]_{-2}^0 \right) dx$

$= \int_0^1 \frac{1}{2}\cos(3x) - \frac{1}{2}\cos(3x+4)\,dx$

$= \left[ \frac{1}{6}\sin(3x) - \frac{1}{6}\sin(3x+4) \right]_0^1$

$= \frac{1}{6}\sin(3) - \frac{1}{6}\sin(7) - \frac{1}{6}\sin(0) + \frac{1}{6}\sin(4)$

$= \frac{1}{6}\left(\sin(3) - \sin(7) + \sin(4)\right)$

2) $\iint_{[0,2]\times[0,1]} e^{x+y}$

$= \int_0^2 \left( \int_0^1 e^{x+y}\,dy \right) dx$

$= \int_0^2 \left[ \frac{1}{y} e^{x+y} \right]_0^1 dx$

$= \int_0^2 e^{x+1}\,dx$

$= \left[ \frac{1}{x} e^{x+1} \right]_0^2 =$

5- $\iint_{[3,4]\times[1,2]} e^x \cos(e^x+y)$

$= \int_3^4 \left( \int_1^2 e^x \cos(e^x+y)\, dy \right) dx$

$= \int_3^4 \left[ \sin(e^x+y) \right]_1^2 dx$

$= \int_3^4 \sin(e^x+2) - \sin(e^x+1)\, dx$

$= \left[ -e^x \cos(e^x+2) + e^x \cos(e^x+1) \right]_3^4$

$= e^4(-\cos(e^4+2) + \cos(e^4+1))$

Ex 2-

1- $\iint xy^2\, dx\, dy$ , $D = \{(x,y) \in \mathbb{R}, x \in [0,1] \text{ et } x^2 \leq y \leq \sqrt{x}\}$

$= \int_0^1 \left( \int_{x^2}^{\sqrt{x}} xy^2\, dy \right) dx$

$= \int_0^1 \left[ \frac{x}{3} y^3 \right]_{y=x^2}^{y=\sqrt{x}} dx$

$= \int_0^1 \frac{x^2}{3}\sqrt{x} - \frac{x^7}{3}\, dx$

$= \frac{2}{21} - \frac{1}{24}$

Solveur