1

$\mathbb{F} = \{ \underset{1}{\underset{\top}{f_1}}, \dots, \underset{0}{\underset{\top}{f_n}} \}$

$\mathbb{P} = \{ \underline{p_1}, \dots, p_n \}$ $\qquad\qquad T$

$$T ::= x \quad (x \in \mathbb{X}) \mid f(\underbrace{T, \dots, T}_{\#f})$$

$$F ::= p(\underbrace{T, \dots, T}_{\#p}) \mid F \wedge F \mid F \vee F \mid \neg F \mid \dots$$

$$\mid \forall x . F \mid \exists x . F$$

②

$F = \{+/2, +1/1, 0/0\}$
$\mathbb{P} = \{\doteq/2\}$

$\doteq(+(+1(x), 0), 0) \approx (x+1)+0 \doteq 0$

$\forall y(\forall x((<(x,y) \vee <(y,x)) \vee =(x,y)))$

$\mathbb{P} = \{=/2, </2\}$
$\mathbb{F} = \emptyset$

③

VOLNÉ / VÁZANÉ PROMĚNNÉ

$$\varphi \equiv \forall x \left(p(f(x), y) \rightarrow \forall y \left(p(f(x), y) \right) \right)$$

VÁZANÁ   VOLNÁ   VÁZANÁ   VÁZANÁ

④ Realizace $\mathcal{M}$ jazyka

$$\mathcal{M} = (M, \mathcal{L})$$

$$\mathcal{L}(f) : M^{\#f} \to M$$

$$\mathcal{L}(p) \subseteq M^{\#p}$$

(5) $M = \mathbb{N}$

$\alpha(+) = +_{\mathbb{N}}$

$\alpha(+1) = \underline{\lambda x \;\; x +_{\mathbb{N}} 1} \qquad \approx \qquad +1(x) = x +_{\mathbb{N}} 1$

$\alpha(0) = 0_{\mathbb{N}}$

$\alpha(\doteq) = \;=_{\mathbb{N}}$

(6) $M =$ list

struct list {
list *next;
}

$\alpha(0) =$ list *zero {
return null;
}

$\alpha(+1) =$ list * ~~zero~~ plus_one (list *val) {
list *x = new;
new → next = val;
return new
}

$\alpha(+) =$ list *concat (list *a, *b) {
list *x = ~~new~~ null;
copy (x, a);
copy (x, b);
return x;
}

$\alpha(\stackrel{\circ}{=}) =$ bool equal (list *a, list *b);

7

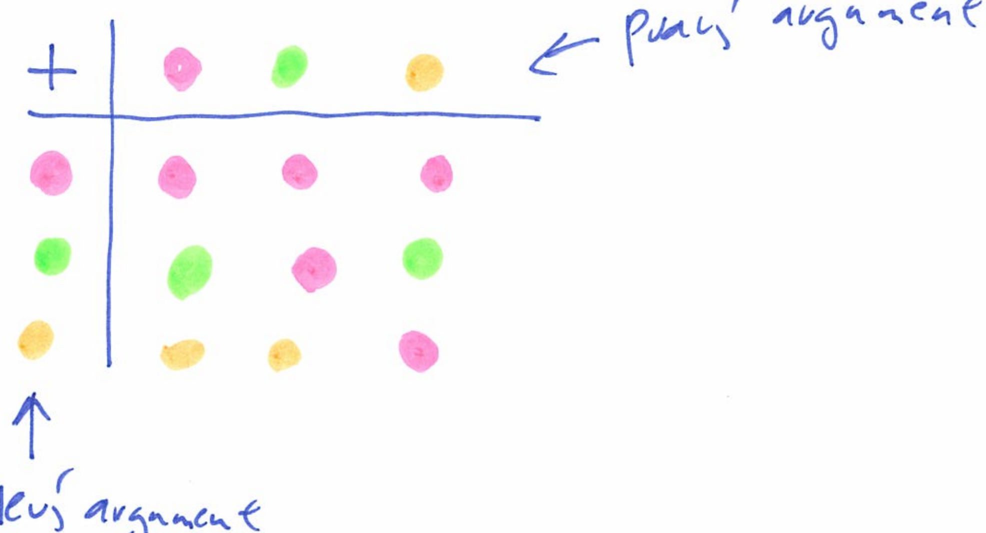
$\mathbb{F} = \{+/2,\ 0/0\}$

$\mathbb{P} = \{n/2\}$

Realizace:

$M = \{$  $\}$

$\alpha(0) =$ 



$n(x,y) \iff x = y$

(8)

$$(x+y)+z\,\left[\{x\mapsto {\color{magenta}\bullet},\ y\mapsto {\color{orange}\bullet},\ z\mapsto {\color{orange}\bullet}\}\right]$$

$$({\color{magenta}\bullet}+{\color{orange}\bullet})+{\color{orange}\bullet} = {\color{magenta}\bullet}+{\color{orange}\bullet} = {\color{magenta}\bullet}$$

Formule φ je pravdivá při ohodnocení e

$$\varphi \equiv n(((x+y)+z),0) \equiv n((x+y)+z,0)$$

$e_1: [x\mapsto {\color{magenta}\bullet},\ y\mapsto {\color{orange}\bullet},\ z\mapsto {\color{orange}\bullet}]$

$\varphi[e_1] \equiv n({\color{magenta}\bullet},{\color{magenta}\bullet}) \equiv \text{True}$

$$\boxed{\mathcal{M}^B \models \varphi[e_1]}$$

$e_2: [x\mapsto {\color{green}\bullet},\ y\mapsto {\color{magenta}\bullet},\ z\mapsto {\color{magenta}\bullet}]$

$\varphi[e_2] \equiv n({\color{green}\bullet},{\color{magenta}\bullet}) \equiv \text{False}$

$$\boxed{\mathcal{M}^B \not\models \varphi[e_2]}$$

1. φ je splněná v realizaci $\mathcal{M}$, pokud je pravdivá pro $\forall e$

$\mathcal{M} \models \varphi$

2. φ je pravdivá v realizaci $\mathcal{M}$ pokud je uzavřená (a vyhodnocena na TRUE)

$\mathcal{M} \models \varphi$

3. φ je splnitelná $\Leftrightarrow \exists \mathcal{M}.\ \mathcal{M} \models \varphi$

$\cancel{\varphi \equiv p(X) \to \neg p(X)}$ $\quad \varphi \equiv p(x) \wedge \neg p(x) \Leftarrow$ NESPLNITELNÁ

4. φ je log. platná $\Leftrightarrow \forall \mathcal{M}.\ \mathcal{M} \models \varphi$

7)

$\boxed{1}$ $\varphi_1 \equiv \exists x, y.\ n((x+y)+z, 0)$ je splnitelná v $\mathcal{M}^B$ ← BARVIČKY

$e_1 : \{ z \mapsto \color{orange}\bullet \}$ $\qquad (\color{magenta}\bullet \color{black}+ \color{magenta}\bullet\color{black}) + \color{orange}\bullet \color{black}= \color{magenta}\bullet$

$e_2 : \{ z \mapsto \color{magenta}\bullet \}$ $\qquad (\color{magenta}\bullet \color{black}+ \color{magenta}\bullet\color{black}) + \color{magenta}\bullet \color{black}= \color{magenta}\bullet$

$e_3 : \{ z \mapsto \color{green}\bullet \}$ $\qquad (\color{magenta}\bullet \color{black}+ \color{magenta}\bullet\color{black}) + \color{green}\bullet \color{black}= \color{magenta}\bullet$

$$\mathcal{M}^B \models \varphi_1$$

$\boxed{2}$ $\varphi_2 \equiv \exists x, y, z.\ n((x+y)+z, 0)$ je pravdivá v $\mathcal{M}^B$

$$\mathcal{M}^B \models \varphi_2$$

$\varphi_2^A \equiv \forall x, y, z.\ n((x+y)+z, 0)$ v realizaci $\mathcal{M}^B$ není pravdivá

$$\mathcal{M}^B \not\models \varphi_2^A$$

(10)

$\boxed{3}$ $\varphi_3 \equiv \exists x,y .\ x \neq y \wedge n(x,y)$

- splněna v libovolní realizaci $\mathcal{I}(n) = TRUE \wedge |M| \geq 2$

=> proto je splnitelná

$\boxed{4}$ $\varphi_4 \equiv p(x) \vee \neg p(x)$ je log. platná

Mějme jazyk L s binárním pred. symbolem p a následující formule

$$\Phi \equiv \exists x.\ p(x,x)$$

$$\chi \equiv p(x,y) \to \exists z.\ p(z,x)$$

$$\Psi \equiv \exists x \exists y.\ p(x,y)$$

Uvažujme realizaci $\mathcal{M} = (\mathbb{Z}, \alpha)$

$$\alpha: \quad (a,b) \in p_{\mathcal{M}} \overset{def}{\iff} nsd(a,b) = 1$$

---

a) $\mathcal{M} \overset{?}{\models} \Phi$

$$\mathcal{M} \models \exists x.\ p(x,x) \overset{\mathcal{M}}{\iff} \exists m \in \mathbb{Z} : \mathcal{M} \models \boxed{p(x,x)[m/x]}$$

$$\overset{\mathcal{M}}{\iff} \exists m \in \mathbb{Z} : \mathcal{M} \models nsd(x,x) = 1\ \cancel{[x/m]}\,[m/x]$$

$$\overset{\mathcal{M}}{\implies} \text{PRAVDA} \quad e = \{x \mapsto 1\}$$

$$\overset{\mathcal{M}}{\implies} \Phi \text{ je splněná v } \mathcal{M}$$

a protože je uzavřená, tak je PRAVDIVÁ

Nechť $e = \{x \mapsto 2, y \mapsto 1, z \mapsto 4\}$. Rozhodněte zda $\mathcal{M} \models X[e]$

$$\mathcal{M} \models X[e] \overset{\mathcal{M}}{\Longleftrightarrow} p(2,1) \rightarrow p(4,2)$$

$$\overset{\mathcal{M}}{\Longleftrightarrow} \text{TRUE} \rightarrow \text{FALSE} \equiv \text{FALSE}$$

Formule X není ~~splněna~~ pravdivá v realizaci $\mathcal{M}$ s ohodnocením e.

$$\mathcal{M} \not\models X[e]$$

Nechť $e_1 = \{x \mapsto 2, y \mapsto 1\}$. Rozhodněte zda $\mathcal{M} \models X[e_1]$

$$\mathcal{M} \models X[e_1] \overset{\mathcal{M}}{\Longleftrightarrow} p(2,1) \rightarrow \exists z.\, p(z,2)$$

$$\overset{\mathcal{M}}{\Longleftrightarrow} \text{TRUE} \rightarrow \exists z.\, p(z,2)$$

$\overset{\mathcal{M}}{\Longleftrightarrow}$ pro $\{z \mapsto 1\}$ formule platí

$\overset{\mathcal{M}}{\Longrightarrow}$ Formule $\boxed{X \text{ je pravdivá v ohodnocení } e_1}$

$$\underline{\mathcal{M} \models X[e_1]}$$

Nechť L je jazyk s =, binárním funkčním symbolem f, nulárním funkčním symbolem e, a bin. pred. symbolem p.

$\mathbb{F} = \{f, e\}$ $\mathbb{P} = \{p, =\}$

Mějme realizaci $\mathcal{M} = (\Sigma^*, \alpha)$

α: $e = \varepsilon$

$f(a,b) = a ++ b$ ++ ≈ konkatenace

$p(a,b) \Leftrightarrow a <_{lex} b$

$$\boxed{\begin{array}{l} ab <_{lex} cf \\ ab <_{lex} ac \\ abc <_{lex} af \end{array}}$$

Rozhodněte, zda pro $\mathcal{M} \models T_4$

$T_4 = \{ \neg p(x,x),\ p(x,y) \to (p(y,z) \to p(x,z)) \}$

a) $\mathcal{M} \overset{?}{\models} \neg p(x,x)$

$$\mathcal{M} \models \neg p(x,x) \overset{\mathcal{M}}{\iff} \mathcal{M} \models \neg(x<x) \overset{\mathcal{M}}{\iff} \forall x.\ \neg(x<x)$$

Neexistuje $x.\ x<x$

$$\Rightarrow \underline{\underline{\mathcal{M} \models \neg p(x,x)}}$$

b) $\mathcal{M} \overset{?}{\models} p(x,y) \to (p(y,z) \to p(x,z))$

$$\mathcal{M} \models x<y \to (y<z \to x<z) \iff \forall x,y,z \in \Sigma^*.\ x<y \to (y<z \to x<z)$$

vyplývá z transitivity $\boxed{<_{lex}}$

$$\mathcal{M} \models p(x,y) \to (p(y,z) \to p(x,z))$$

c) $M \overset{?}{\models} \neg\psi$

$M \models \neg\psi \overset{\mathcal{M}}{\Longleftrightarrow} M \not\models \psi \overset{\mathcal{M}}{\Longleftrightarrow} M \not\models \exists x \exists y \; p(x,y)$

$\overset{\mathcal{M}}{\Longleftrightarrow} M \not\models \exists x \exists y \;\; nsd(x,y) = 1$

NEPLATÍ – PŘÍKLADEM JE $\{x \mapsto 1, y \mapsto 1\}$

ZÁVĚR $\mathcal{M} \not\models \neg\psi$

---

d) $\models \Phi$

NEPLATÍ: $\mathcal{M} = (M, \mathcal{I})$

$M = \{0, 1\}$

$\mathcal{I}(p) = FALSE$

$\mathcal{M}_1 = (\mathbb{N}, <)$

Rozhodněte zda $\mathcal{M} \models \varphi$

$$\varphi \equiv f(s_1, f(c, s_2)) = f(s_1, s_2)$$

$$\mathcal{M} \models \varphi \iff \forall s_1, s_2 .\ s_1 +\!\!+ (\varepsilon +\!\!+ s_2) = s_1 +\!\!+ s_2$$

$$\iff \forall s_1 s_2 .\ s_1 +\!\!+ s_2 = s_1 +\!\!+ s_2$$

PRAVDA

$$\underline{\underline{\mathcal{M} \models \varphi}}$$

Najděte realizaci $\mathcal{N}$ jazyka L takovou, že

$$\mathcal{N} \models T_{\mathcal{M}} \cup \{\underline{\varphi}\}$$

$\mathcal{N} = \mathbb{N}_0$ $f(a,b) = a+b$, $c = 0$, $p(a,b) \iff a < b$

Najděte realizaci $\mathcal{N}'$.

$$\mathcal{N}' \models T_{\mathcal{M}} \wedge \mathcal{N}' \not\models \varphi$$

$\mathcal{N} = \mathbb{N}_0$, $c = 0$, $p(a,b) \iff a < b$, $f(a,b) = \underline{a \cdot b}$