Siemens-Nixdorf Case Report

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1 Introduction & Problem Definition

In the manufacturing of printed circuit boards (PCBs), one of the key steps is drilling holes into the boards. This step is causing delays, especially when producing large quantities of PCBs. The holes being drilled vary in size and location, depending on the specific type of PCB being produced.

To drill these varying hole sizes, the drill's head needs to be adjusted. This adjustment is done by moving the drill head to a toolbox where the changes to the drill diameter can be made. The drill head is controlled by two separate drives that move it in the x-direction (left and right) and the y-direction (up and down), with different speeds.

The path the drill head takes is the only aspect of this process that we can control to improve efficiency. By optimizing this movement, we can reduce the time it takes to drill the holes and consequently speed up the PCB production process.

To summarize, here are the characteristics of the problem:

- Control over the Drilling Process: The only aspect we can control in the drilling process is the path the drill head takes.
- Drilling Sequence: For any given PCB, holes with smaller diameters are drilled first, followed by holes with larger diameters. This sequence is a standard procedure.
- Speed Difference Between Drives: The motor that moves the drilling head along the x-axis operates 10% slower than the motor for the y-axis.
- Movement Duration: The time it takes for the drilling head to move from one coordinate to another is determined by the slower of the two movements. This means the overall movement speed is constrained by the slower axis.
- Starting Position: The drill always begins its operations from the starting position at coordinate (0,0).
- Toolbox Location: The station where the drill size can be changed, known as the toolbox, is the last point of the path of the drill.

2 Model Formulation & Model Analysis

To develop a mathematical model for optimizing the path of a drill head during the manufacturing of printed circuit boards (PCBs), we need to focus on minimizing the total distance the drill head travels while adhering to the constraints of the process. The primary goal is to find an efficient sequence to visit the given pairs of x and y coordinates, which represent the locations where holes need to be drilled.

The drill head moves in a two-dimensional plane, controlled by separate drives for the x-axis (horizontal) and y-axis (vertical) movements, with the x-axis drive operating at a speed 10 percent slower than the y-axis drive. The sequence of drilling mandates that holes with smaller diameters are drilled before those with larger diameters, therefore we will consider holes of each diameter as different instances of the same problem. The starting point for the drill head is at the origin (0,0), and it must return to the toolbox (the final point in the path) for adjustments, which is also considered in the total distance. The objective is to formulate an algorithm that, given a set of coordinate pairs, calculates the sequence that minimizes the drill head's travel distance while considering the speed differential between the two axes. To adjust for the different motor speeds we increase the distance on the axis of the slower engine by 10 percent.

Therefore we consider the following variables:

x_i The x-coordinate of the i^{th} hole to be drilled on the PCB.

y_i The y-coordinate of the i^{th} hole to be drilled on the PCB.

d_{ij} The distance between hole i and hole j.

D The total distance traveled by the drill head during the drilling process.

P The path taken by the drill head.

Hence this problem can be modelled as an instance of the Travelling salesman problem. The Travelling Salesman Problem is a classic optimization problem where the goal is to find the shortest possible route that visits a set of locations once and returns to the origin point demonstrated in a study by Dantzig et al. [1]

2.0.1 Decision Variables

p_{ij} A binary variable that equals 1 if the drill moves from hole i to hole j, and 0 otherwise.

2.0.2 Objective Function

The objective is to minimize the total distance traveled by the drill head, represented by D. Mathematically, this can be expressed as:

$$\min D = \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} d_{ij} \cdot p_{ij}$$

where n is the total number of holes to be drilled.

2.0.3 Constraints

Several constraints must be considered, including:

- Each hole must be visited exactly once:

$$\sum_{i=1, i\neq j}^{n} p_{ij} = 1 \quad \forall j$$

- After drilling at a hole, the drill must move to another hole:

$$\sum_{j=1, j\neq i}^{n} p_{ij} = 1 \quad \forall i$$

- The drilling process must start at the origin point (0,0):

$$\sum_{j=1}^{n} p_{0j} = 1$$

- To prevent any sub-tours from creation, a dummy variable u_i is introduced that keeps track of the order in which the holes are drilled, counting from hole 0, and the following set of constraints is imposed:

$$u_i - u_j + 1 \leq (n-1)(1 - x_{ij}) \quad \forall \text{ distinct } i, j \in \{1, ..., n\}$$

With the model and its constraints well-defined, the next step involves implementing an algorithm that gives a solution that satisfies these constraints and optimizes the total travel distance.

3 Method of Approach

3.1 Distance calculation

The engine moves slower along the x-axis than the y-axis by 90%. This difference implies that for the same time, the x-axis engine covers 10% less distance as compared to the y-axis engine on average. Hence, for the model to reflect this variation, we need to re-scale the distance measurements made along the x-axis. That is, we stretch the x-axis distance vector by a factor of 1.1. This correction ensures that our model captures the fact of relative speed differences between the two engines and makes our analysis more accurate.

3.2 Siemens solution

Currently, holes of equal diameter are drilled in lexicographical order. Thus points are drilled in order of increasing x-coordinates. Points with the same x-coordinates are drilled in order of increasing y-coordinates. The following picture shows the route taken by a drill in one of the specified example data.

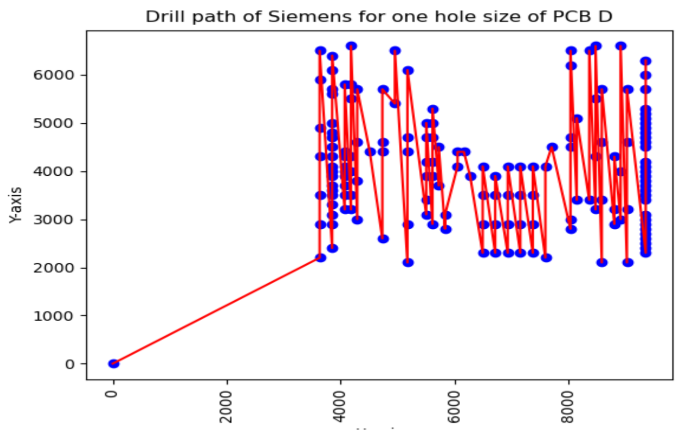

Figure 1: Siemens' solution drilling path

With this approach, the total Euclidean distance traveled on the re-scaled plan, is 142825.1 machine units.

4 First Results

4.1 Optimal path

We have developed an initial solution by creating an algorithm that addresses the previously mentioned linear programming model. To satisfy Siemens's requirements, we imposed a time limit of 5 minutes on the solver. Our primary objective was to minimize the total distance traveled. The graph illustrates the outcomes of our approach.

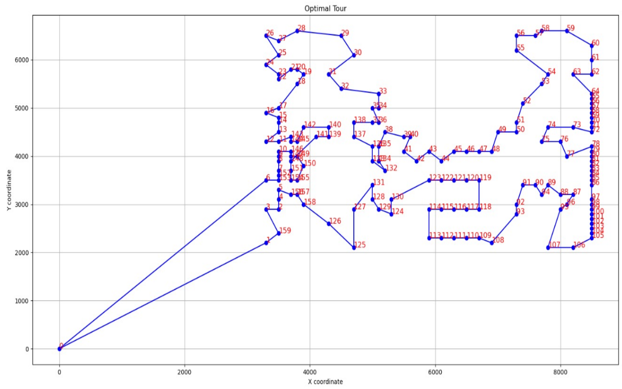

Figure 2: Better drilling path

The total Euclidean distance traveled equals 53344.687 machine units which is approximately 62.65% lower than the distance traveled by the drill using the current Siemens solution.

4.2 The lower bound

Given our calculations, we have derived that the lower bound is within 4.1619 percent of the true optimum. This indicates that letting the algorithm run for a longer amount of time will only lead to negligible improvements and that the previous solution is close to the optimum.

4.3 Next steps

The current algorithm is effective for printed circuit boards (PCBs) that require a smaller number of holes to be drilled. However, there's a need to explore a more time-efficient solution to account for PCBs with thousands of holes. This new solution should be able to generate feasible paths that are shorter in distance compared to those produced by the existing Siemens solution. While this might mean a compromise on the optimality of the solution, the aim is to enhance overall production efficiency.

Our next steps would be to investigate heuristic approaches using the Nearest Neighbour algorithm, Greedy algorithm, or others and compare their efficiency for our problem.

References

[1] Dantzig, G., Fulkerson, R., & Johnson, S. (1954). Solution of a large-scale traveling-salesman problem. *Journal of the Operations Research Society of America, 2*(4), 393-410.