Back-casting Principal European Economic Indicators

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1 Introduction

The goal of this report is to provide methodology for the improved back-calculation, or back-casts, for Principal European Economic Indicators (PEEIs). The following requests were made as part of the methodological enquiry:

- All the available actual observations of all time-series must be used.
- The use of statistically sound, sufficiently detailed documentation, publically available and easily replicable methodology.
- The methodology should be applicable both to seasonally adjusted and non-seasonally adjusted data.
- The methodology should be flexible and enough easily to incorporate new time-series produced by Member States.
- The horizons for the back-calculation must be defined accordingly to the amount of the available data, acceptable back-forecasting (back-casting/back-calculation) error and interpretability of the results.

Back-casting is not dissimilar to forecasting. Suppose we have time series data $\mathbf{y} = (y_1, \ldots, y_n)$ and a horizon h. In a forecasting setting, the goal is to predict future values of the time series $y_{n+h} = (y_{n+1}, \ldots, y_{n+h})$. In a back-casting setting, we instead wish to predict past values of the time series $y_{1-h} = (y_0, \ldots, y_{-h+1})$.

The current approach taken to back-casting the PEEIs is reversing the time series and forecasting using the **auto.arima()** function from the **forecast** package available on CRAN. Under the assumptions of stationarity, and assuming a Gaussian ARMA model, forecasting $\mathbf{y} = (y_1, \ldots, y_n)$ forwards in time, using an ARMA model, is different but equivalent to forecasting backwards in time (Shumway and Stoffer 2017). Therefore, this current approach is sound.

Despite the current approach being sound for each individual PEEI time series, as a whole, it has it's drawbacks. In particular, it does not guarantee that the forecasts make sense over the entire structure of all the PEEI time series. For example, for each of the indicators, forecasts for each constituent member of an EU region, will not sum to the forecast for the region. Additionally, forecasts for quarterly data, will not sum to the corresponding forecasts for yearly data.

As such, in order to improve this existing approach, we wish to overlay a hierarchical adjustment, such that the forecasts at all levels of the hierarchy of the data structure coincide, this includes temporal consistency too. This idea will be outlined in Section 2.

2 Hierarchical Approach

The idea behind taking a hierarchical approach to back-casting is that we want to achieve complete reconciliation of the PEEI time series. This means that we have hierarchical consistency both cross-sectionally and also temporally.

For each of the indicators, we have data for the European area, but also for each of the constituent countries in the area. See Figure 1 for an illustration. So, practically, it makes sense that the aggregated back-casts for each of the countries amounts to the back-cast for the corresponding European area. In addition to this, some of the indicators have different time frequencies, see Figure 2, we additionally want the forecasts for the monthly, quarterly and annual time series to be consistent. This amounts to having both cross-sectional and also temporal consistency, i.e. cross-temporal hierarchical forecasts.

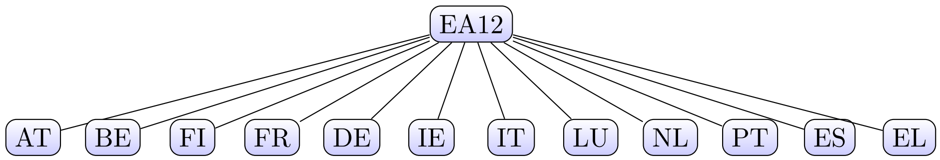

Figure 1: The cross-sectional hierarchy for EA-12 (1 November 1993 - 31 December 1994):

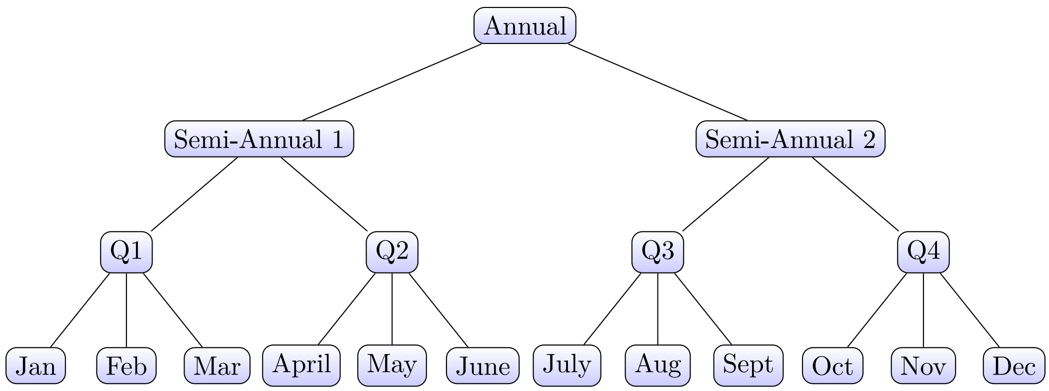

Figure 2: Temporal hierarchy for monthly time series

The general hierarchical approach we choose to adopt is that of (Hyndman et al. 2011) and (Athanasopoulos et al. 2017). In each, the method is similar, however in the former it is applied to cross sectional hierarchies and in the latter it is applied to the temporal hierarchies within the data. We outline the method in the following section.

2.1 Method Outline

Often when we have a hierarchy of time series, the most common approach is to apply a `bottom up'' or`top down'‘method. (Hyndman et al. 2011) propose an alternative approach to hierarchical forecasting which performs better than either of these top-down or bottom-up approaches, and even so called "middle-out" approaches. In summary, their approach is based upon independently forecasting all series at all levels of the hierarchy and then using a regression model to optimally combine and reconcile these forecasts.

To formalise this idea, following (Hyndman et al. 2011), suppose we have a multi-level hierarchy where level 0 represents the completely aggregated series and level K represents the most disaggregated time series.

Then if we define Y_t to be the aggregate of all time series at time t then we can write

$$Y_t = \sum_i Y_i, t, \quad Y_{i,t} = \sum_j Y_{ij,t}, \quad Y_{ij,t} = \sum_k Y_{ijk,t} \quad \text{etc} \ ,$$

representing that observations at higher levels are obtained by summing the observations at lower levels.

Now, let m_i denote the total number of series at level i, $i = 1, \ldots, K$, where the m_i are ordered such that $m_i > m_j$ if and only if $i > j$. Then the total number of series in the hierarchy is given by

$$m = m_0 + m_1 + \ldots + m_K.$$

Define $\mathbf{Y}_{i,t}$ to be the vector of all observations at level i and time t and $\mathbf{Y}_t = [Y_t, \mathbf{Y}_{1,t}, \ldots, \mathbf{Y}_{K,t}]'$. We can write

$$\mathbf{Y}_t = \mathbf{S}\mathbf{Y}_{K,t}$$

where $\mathbf{S}$ is a summing matrix of order $m \times m_K$ used to aggregate the lowest level of the time series. For the hierarchies in Figures 1 and 2, the summation matrices would be,

$$\begin{bmatrix} 1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1 \\ \mathbf{I}_{12} \end{bmatrix} \text{ and } \begin{bmatrix} 1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1 \\ 0\,0\,0\,0\,0\,0\,1\,1\,1\,1\,1\,1 \\ 1\,1\,1\,1\,1\,1\,0\,0\,0\,0\,0\,0 \\ 1\,1\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,1\,1\,1\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,1\,1\,1\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,1\,1 \\ \mathbf{I}_{12} \end{bmatrix},$$

respectively.

Once the summation matrix has been specified, the next step is to produce base forecasts, or in our case, back-casts, for each of the time series. These can be based upon any forecasting method as long as the model is stationary and Gaussian.

Suppose then, that we have generated m independent back-casts for each of the aggregate time series for times $0, \ldots, -h+1$, and denote these backcasts as $\hat{Y}_{X,n}(h)$ where X denotes the series being back-cast. Define $\hat{\mathbf{Y}}_n(h) = [\hat{Y}_n(h), \hat{\mathbf{Y}}_{1,n}(h), \ldots, \hat{\mathbf{Y}}_{K,n}(h)]'$.

(Hyndman et al. 2011), express the base forecasts in terms of the following regression equation

$$\hat{\mathbf{Y}}_n(h) = \mathbf{S}\beta_n(h) + \epsilon_h,$$

where $\beta_n(h) = \mathbb{E}[\mathbf{Y}_{K,n+h}|\mathbf{Y}_1, \ldots, \mathbf{Y}_n]$ is the unknown mean of the bottom level K and ϵ_h has zero mean and covariance matrix Σ_h. The ϵ_h represent the "reconciliation error", the difference between the base forecasts $\hat{\mathbf{Y}}_n(h)$ and their expected value if they were reconciled (Athanasopoulos et al. 2017).

If $\mathbf{\Sigma}_h$ was known, then the generalised least squares estimator of $\beta_n(h)$ would lead to the following revised forecasts

$$\tilde{\mathbf{Y}}_n(h) = \mathbf{S}\hat{\beta}_n(h) = \mathbf{S}\mathbf{P}\hat{\mathbf{Y}}_n(h),$$

where $\mathbf{P} = (\mathbf{S}'\Sigma_h^{\dagger}\mathbf{S})^{-1}\mathbf{S}'\Sigma_h^{\dagger}$. However, generally, $\mathbf{\Sigma}_h$ is unknown and requires estimation.

(Hyndman et al. 2011) avoid this estimation by approximating $\mathbf{\Sigma}_h$ by $\sigma^2\mathbf{I}$. Later, (Wickramasuriya et al. 2015) show that actually it is impossible to estimate $\mathbf{\Sigma}_h$ in practice due to identifiability conditions.

To overcome this, (Wickramasuriya et al. 2015) propose an estimator based on minimising the variances of the reconciled forecast errors, this results in unbiased reconciled forecasts given by

$$\tilde{\mathbf{Y}}_n(h) = \mathbf{S}(\mathbf{S}'\mathbf{W}^{-1}\mathbf{S})^{-1}S'\mathbf{W}^{-1}\hat{\mathbf{Y}}_n(h)$$

where $\mathbf{W} = Var(\mathbf{Y}_n(h) - \hat{\mathbf{Y}}_n(h))$ is the covariance matrix of the base forecast errors.

(Athanasopoulos et al. 2017) propose three diagonal estimators which approximate the sample covariance estimator of $\mathbf{W}$, each leading to alternative weighted least squares (WLS) estimators. In our hierarchical approach, we will consider variance scaling, where we weight the combinations according to the in-sample mean square errors of each of the forecasts. The benefit of this approach will be further described in Section 2.2.2.

In the following, we describe some additional considerations and amendments for the methodology.

2.2 Considerations and Amendments

For the PEEIs, there are some features of the data structure which must be considered. These include:

- For a set of indicators there may be more than one hierarchical structure which needs to be reconciled.

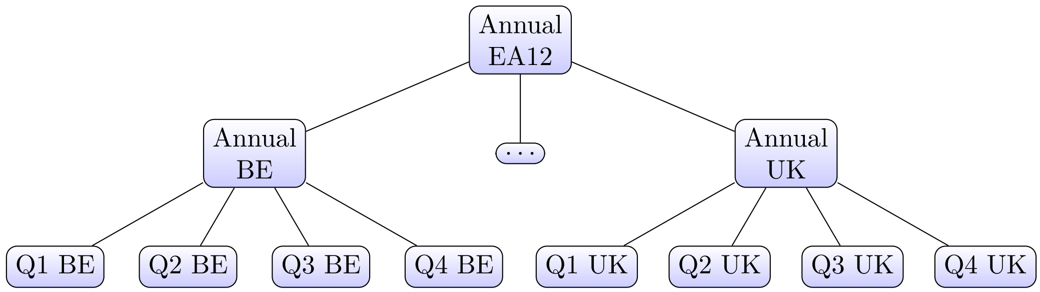

Figure 3: Hierarchical structure branching from the annual European Region time series - first by country and then by quarter.

- The start dates for each of the time series are not consistent. We need to ensure that the known values are fixed during the reconciliation process.

Each of the above will lead to some amendments of the methodology described in Section 2. We address each of these in further detail in Section 2.2.1 and 2.2.2 respectively.

In addition to these, in Section 2.2.3, we address the question of how long a horizon is suitable to backcast over, and in Section 2.2.4 we make a note regarding the availability of time series.

2.2.1 Complete Reconciliation

Suppose we have the hierarchical structure as in Figure 1 for both annual and quarterly frequencies. These two hierarchies could be expressed in a single hierarchy where we branch both up, Figure 3, and down, Figure 4, from the annual European Region time series.

Appending each of the hierarchies in Figures 4 and 3 gives the following $[114 \times 48]$ summation matrix:

$$\begin{bmatrix} \mathbf{I}_{48} \\ 0\,0\,0\,0\,0\,0\,0 \ldots 0\,0\,1\,1\,1\,1\,1\,1 \\ \vdots \\ 1\,1\,1\,1\,1\,1\,0 \ldots 0\,0\,0\,0\,0\,0\,0\,0 \\ 1\,1\,1\,1\,1\,1\,1 \ldots 1\,1\,1\,1\,1\,1\,1\,1 \\ 1\,1\,1\,1\,0\,0\,0 \ldots 0\,0\,0\,0\,0\,0\,0\,0 \\ \vdots \\ 0\,0\,0\,0\,0\,0\,0 \ldots 0\,0\,0\,0\,1\,1\,1\,1 \\ \mathbf{I}_{48} \end{bmatrix},$$

By using summation matrices like the one in equation 2.2.1, we will be able to obtain complete cross-temporal consistency for back-casts. This will ensure our back-casts make practical sense when we have more than one hierarchical structure to consider.

2.2.2 Inconsistent Start Times

The PEEI time series have varying start times. This means we can exploit the hierarchical framework to gain accuracy in our back-casts. The hierarchical reconciliation method, described in Section 2, will need to be adjusted to account for this.

There is no existing methodology for this in the literature because the hierarchical approach is usually applied in a forecasting setting, as such, it is unlikely we would observe future values for any of the time series.

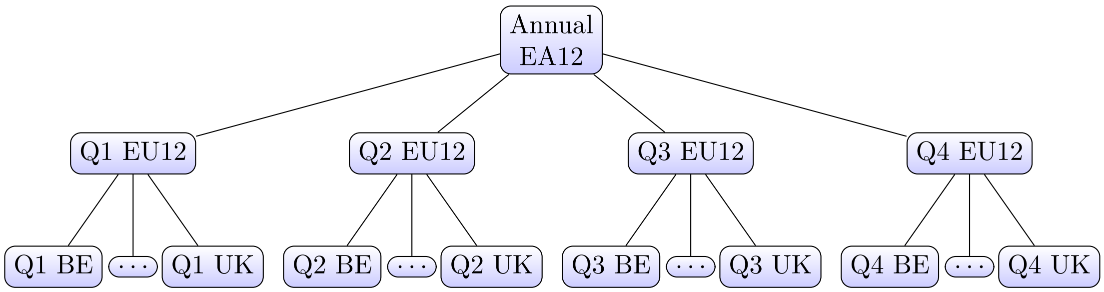

Figure 4: Hierarchical structure branching from the annual European Region time series - first by quarter and then by country.

So, in order to gain power from the available time series, we will adjust the combination weights accordingly. The combination weights are based on the mean squared errors for the in-sample backcasts of each of the time series. Therefore, in the case where we already observe the backcast at the specified horizon, we can replace the backcast with the truth and set the weight for that backcast to be large. This means that we can gain extra power for the other backcasts as we are including additional true information. In the final steps of the procedure, we replace any reconciled lowest level backcasts with the truth, and apply the appropriate summation matrix to obtain the higher level time series.

It is at this point we make a comment regarding data quality. All higher level basckcasts depend upon the lowest level backcasts. Currently, in the database, there is some aggregation error - that is, the higher level time series are not always a sum of the lower level time series. As such, any aggregation error present in the backcast results, is a consequence of existing descrepencies and the methodological approach is not accountable.

2.2.3 Back-casting Horizon

For each of the Principal Economic Indicators (PEEIs), in any instance, the earliest date to which we will back-cast will be the earliest observed value for the indicator. So for example, for Employment, the earliest observed date is for Denmark and France in 1975. Therefore, for any of the Employment time series, this is the furthest back we will go. The code, however, is written in such a way that the back-cast horizon can be extended - but this is at the discretion of the user.

In addition to the above requirement, for each of the individual time series, the back-casting horizon will be dependent upon the time series model fit and it's location within the hierarchical structure. These will be pruned accordingly.

2.2.4 Data Availability

It has been noted by Sogeti that the some time series for the aggregated EU and EA regions are not available in the database, despite being backcast by previous contractors. Sogeti used other EU and EA region as proxies for the ones requiring backcasting.

Having looked at the previous contractors results, it is clear that they inferred EU15 and EA12 from the lowest level time series in the hierarchy. We will do the same.

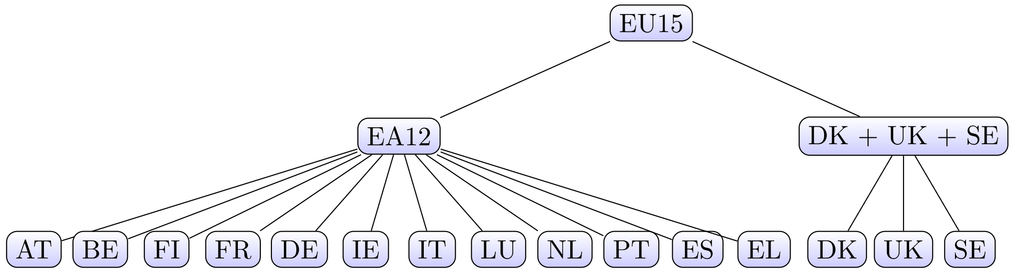

Figure 5: Cross sectional hierarchy for employment time series.

2.3 Summary of Method

To summarise, the backcasting procedure is as follows.

For each of the economic indicators:

1. Infer any required time series;
2. Identify the earliest date for which we have an observation and set this to be our earliest backcast date, $t_{\min}$;
3. For any time series which are not observed as far back as the earliest backcast date:
    - Reverse the time series
    - Fit the best ARMA as selected by **auto.arima()** in the forecast package;
    - Backcast to the earliest backcast date $t_{\min}$.
4. Define the hierarchical structure of the economic indicator in which we set κ to be the number of temporal frequencies we observe and m to be the number of time series in the cross-sectional hierarchy;
5. Identify the earliest date for which we have an observation for every time series within the economic indicator's hierarchy, and set these as our reconciliation starting points, call these $t_j^{\min}$ for $j = 1, \ldots, m \times \kappa$;
6. For t in $\left(\max_j \left\{t_j^{\min}\right\}, t_{\min}\right]$:
    - Reconcile any backcasts and true observations available at time t, if for series j we have a backcast, then the weight is proportional to the in-sample mean squared error for that series, otherwise, if an observation is available, we set the weight to be large.
7. Prune any backcasts where necessary.

In Sections 3, 4, 5 and 6 we apply the above procedure to Employment, Unemployment, Industrial Production and Retail Trade respectively. Note that for Industrial Production and Retail Trade we only backcast the calander adjusted data. In rder to provide complete and consistent backasts for both the calandar adjusted and seasonally adjusted, we need to be provided with details of the seasonal adjustment procedure.

3 Employment

For the Employment indicator we have time series for EU15, EA12 and each of the 15 EU countries and these are available at annual and quarterly temporal levels. The cross sectional hierarchy is depicted in Figure 5.

Inferred time series: Time series which are not directly available from the database for Employment include:

1. annual PT;
2. quarterly EL, FR and PT.

The annual PT time series was inferred from the other annual time series, i.e.

$$\mathrm{PT} = \mathrm{EA12} - \sum_{\text{countries}} \mathrm{EA12} \setminus \mathrm{PT},$$

and for the quarterly time series, we inferred the sum from the total and then weighted them according to the annual weights. This constitutes a top-down approach to data inference.

These inferred time series may be omitted from the final backcasts, however at this stage they are required for completion of the hierarchy. Annual PT may be considered the 'truth' however the quarterly inferred time series should be considered as (good) proxy's for the truth, because for each year, the weighting per country per quarter is fixed.

In addition to this, for the countries Finland and Denmark, the annual observations go back further than the quarterly. This is problematic because all higher level forecasts are aggregated from the lowest level, and during the hierarchical reconciliation process all level forecasts are amended and we are only able to recover the truth in the instances we know the true lowest level observations. For this reason, these cases need to be treated individually.

To this end, in the cases of Employment for Finland and Denmark, we first perform a top-down inference as a pre-processing step. In contrary to the above inference (for the quarterly EL,FR and PT), we estimate the proportions following the approach of (Hyndman et al. 2011).

We applied the methodology described in Section 2.3 and compared it to the backcast provided by Sogeti and the previous contractor's results. The following Figure shows the results for annual and quarterly Employment for the EU15 and EA12 regions.

```
## [1] 1 2
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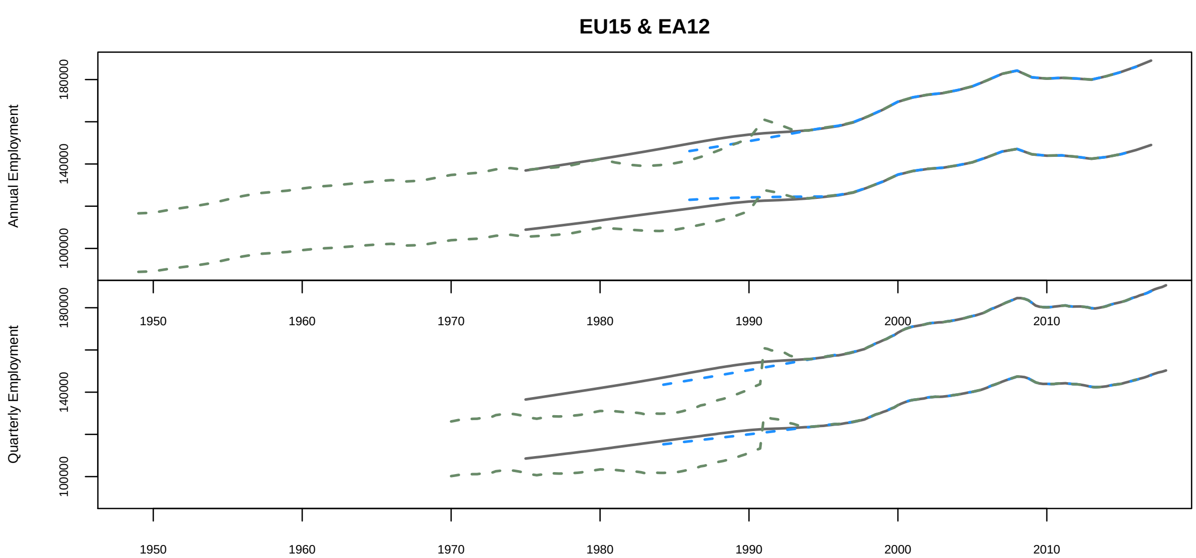

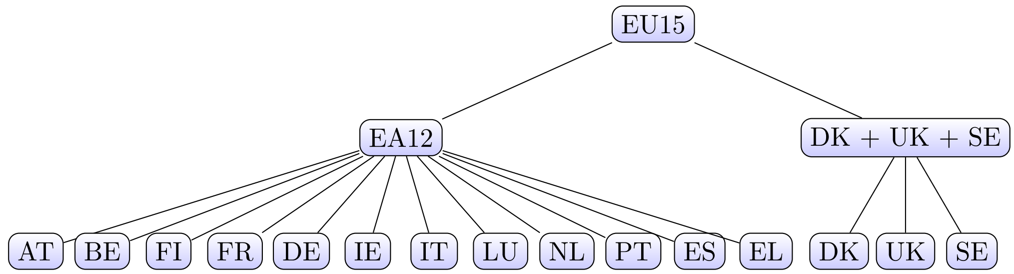

Figure 6: Cross sectional hierarchy for Unemployment time series.

4 Unemployment

For the Unemployment indicator we have time series for EU15, EA12 and each of the 15 EU countries and these are available at annual and monthly temporal levels. The cross sectional hierarchy is depicted in Figure 6.

Inferred time series: Time series which are not directly available from the database for Unemployment include:

1. annual EU15 and EA12;
2. monthly EU15 and EA12.

These can all be inferred from the aggregated individual countries data.

We applied the methodology described in Section 2.3 and compared it to the backcast provided by the previous contractor. Note that we do not compare with Sogeti's results here as EU28 and EA19 were used as proxys for EU15 and EA12 respectively. The following Figure shows the results for annual and quarterly Unemployment for the EU15 and EA12 regions.

```
## [1] 1 2
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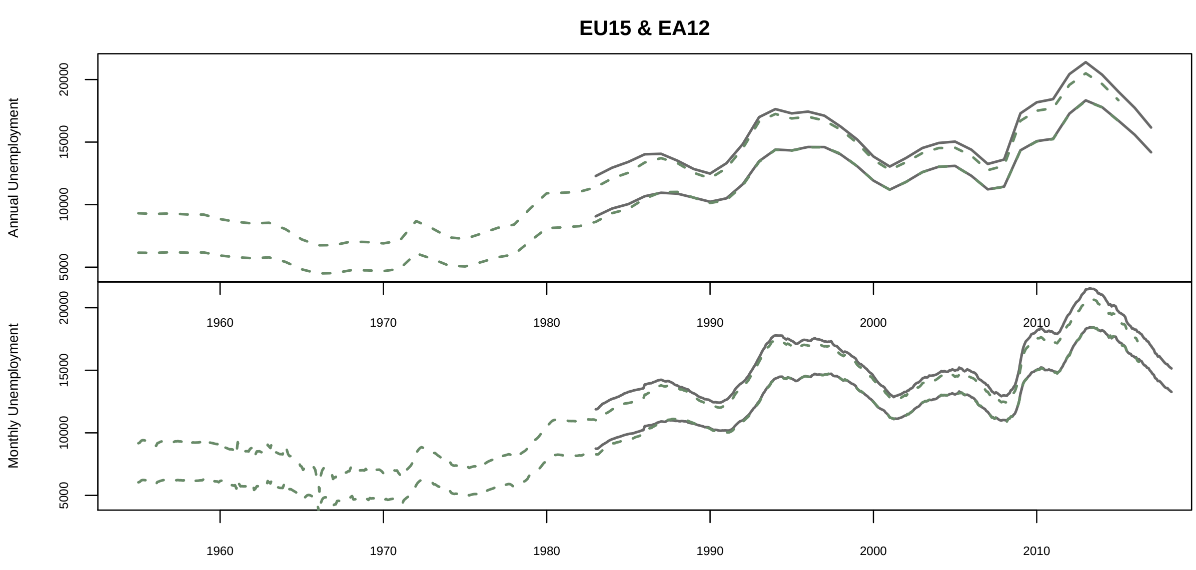


We can see that our backcasts are similar to that provided by the previous contractors, but we do not backcast as far back as they have.

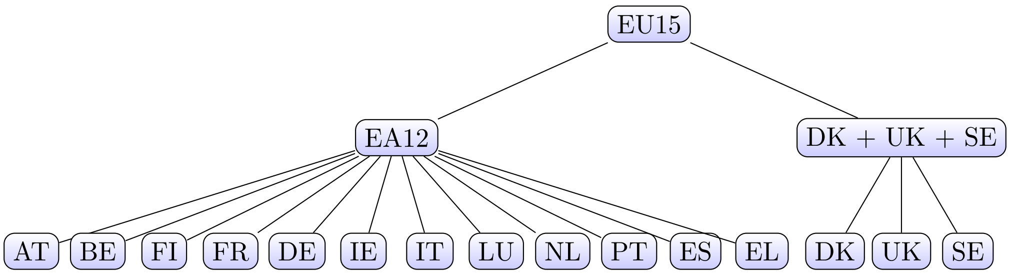


Figure 7: Cross sectional hierarchy for Industrial Production time series.

5 Industrial Production

For the Industrial Production index we have monthly time series for EU15, EA12 and each of the 15 EU countries. For this indicator we have only a cross sectional hierarchy, depicted in Figure 7, and no temporal hierarchy to consider.

Inferred time series: Time series which are not directly available from the database for Industrial Production include:

1. monthly EU15 and EA12.

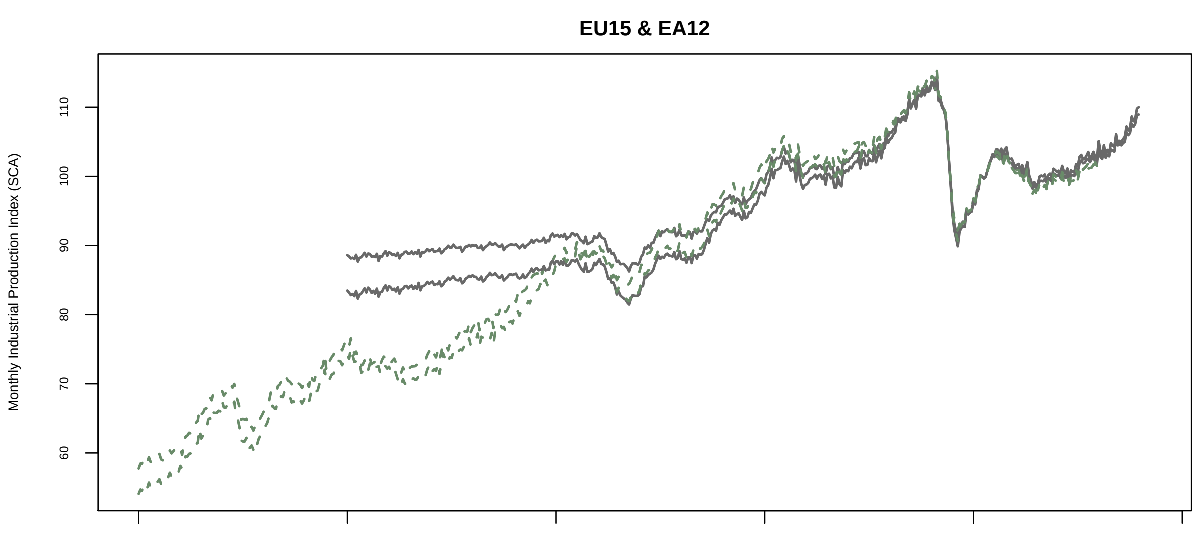
These can all be inferred from the aggregated individual countries data.

For Industrial Production, each country does not contribute equally to the EU or EA region. The weights we use in the reconciliation process are as follows.

Table 1: Weights used during the reconciliation process for Industrial Production Index

	BE	DK	DE	IE	EL	ES	FR	IT	LU	NL	AT	PT	FI	SE	UK
EU15	3.2	2.1	30.2	2.0	1.2	7.4	12.9	13.8	0.2	4.4	3.0	1.3	1.8	3.4	13.2
EA12	4.0	0.0	37.1	2.4	1.5	9.1	15.8	16.9	0.2	5.4	3.7	1.6	2.2	0.0	0.0

We applied the methodology described in Section 2.3 and compared it to the backcast provided by the previous contractor. Note that we do not compare with Sogeti's results here as EU28 and EA19 were used as proxys for EU15 and EA12 respectively. The following Figure shows the results for monthly Industrial Production for the EU15 and EA12 regions.



We can see that after 1990 our backcasts diverge from that of the previous contractors. In order to improve forecasts prior to this time, we would require explanatory variables for the Industrial Production indicator.

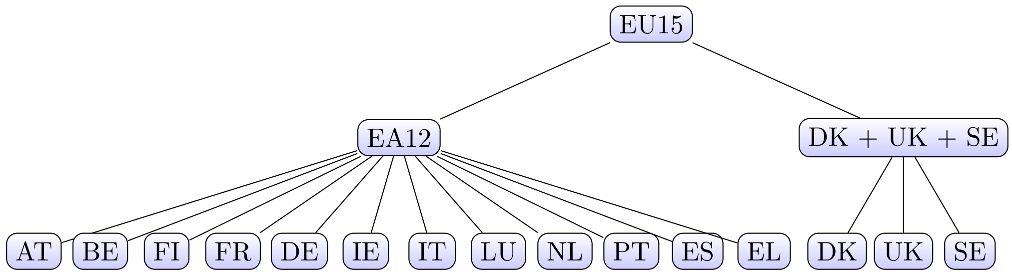


Figure 8: Cross sectional hierarchy for Retail Trade time series.

6 Retail Trade

For the Retail Trade index we have monthly time series for EU15, EA12 and each of the 15 EU countries. For this indicator we have only a cross sectional hierarchy, depicted in Figure 8, and no temporal hierarchy to consider.

Inferred time series: Time series which are not directly available from the database for Retail Trade include:

1. monthly EU15 and EA12.

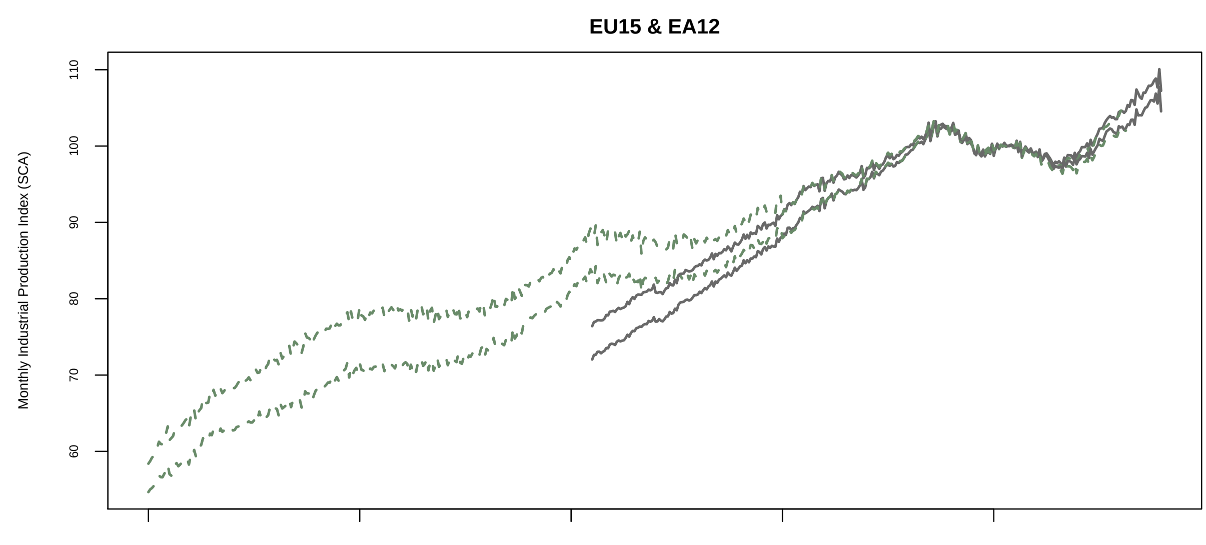
These can all be inferred from the aggregated individual countries data.

For Retail Trade, each country does not contribute equally to the EU or EA region. The weights we use in the reconciliation process are as follows.

Table 2: Weights used during the reconciliation process for Retail Trade Index

	BE	DK	DE	IE	EL	ES	FR	IT	LU	NL	AT	PT	FI	SE	UK
EU15	3.6	1.7	21.1	1.4	2.3	9.4	17.8	12.5	0.7	4.3	2.4	1.9	1.6	2.7	16.8
EA12	4.5	0.0	26.8	1.8	2.9	11.9	22.5	15.8	0.8	5.4	3.0	2.4	2.0	0.0	0.0

We applied the methodology described in Section 2.3 and compared it to the backcast provided by the previous contractor. Note that we do not compare with Sogeti's results here as EU28 and EA19 were used as proxys for EU15 and EA12 respectively. The following Figure shows the results for monthly Retail Trade for the EU15 and EA12 regions.



7 Quality of the Backcast Data

The backcasts we have provided are consistent over both the cross-sectional and temporal hierarchies of the data.

In addition to this, by exploiting the hierarchical nature of the data, we can gain additional insight from those time series which date back further than others - this means that true observations are informing our backcasts whenever possible.

In (Hyndman et al. 2011) it is shown that combining information from forecasts at different hierarchies produces better forecasts. This makes sense because at different levels of the hierarchy we are better able to capture particular features of the data. This illustrates that a hierarchical approach not only provides more interpretable forecsts, but it also produces better results than the approach taken by Sogeti.

We are unable to compare our approach to that of the previous contractors, in terms of in-sample or out-of-sample forecast errors, because details of their methodology is unavailable.

We also note that the previous contractors have backcast further back than we have, whilst still retaining structure in the time series. This must have been attributed to the use of explanatory variable in the model. If we were to be provided with the appropriate explanatory variables for each of the PEEIs, then these could be incorporated into the methodology to produce better forecasts.

We can incorporate this into our methodology by including the explanatory variables into the fitting of our ARMA model. We note however than although **auto.arima()** can include explanatory variables, it does not perform model selection on these variables.

8 Summary

To summarise, we have provided backcasts for four Principal Economic European Indicators namely, Employment, Unemployment, Industrial Production and Retail Trade. We use all available observations for each indicator, and more importantly, share information accross the time series.

The methodology is statistically proven to be an effective way to produce consistent forecasts and we adapt this methodology for a backcasting exercise.

Additionally, the methodology is flexible, it can model any temporal or cross-sectional hierarchy, it can also be extended to include external explanatory variables. Not only this, but it is also flexible enough to be provided with any individual forecasts and reconcile these. This means that the use of an ARMA model for the individual time series is also flexible.

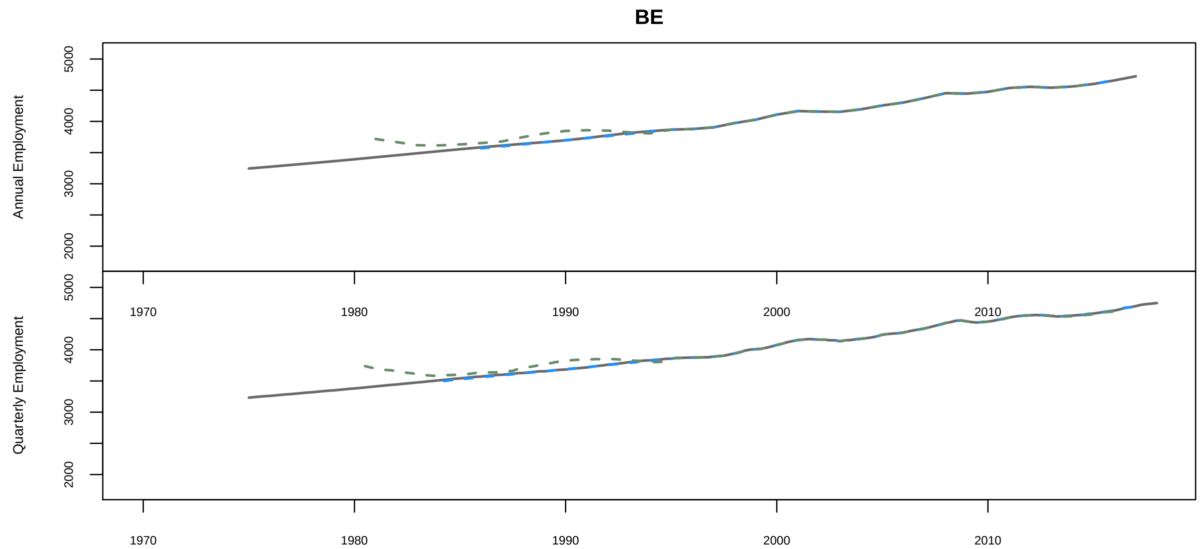
We have defined the backcasts horizons accoding to the amount of available data, in such a way that it is preferable to only go as far back as the earliest observed time series. The motivation for this, is that in this case we are always including some kind of "truth" to guide our backacsts. We are unable to provide a general rule of thumb, which specifies a suitable backcasting horizon based on the number of observations available, as this is not a sound thing to do. In each case, this is model dependent, and if we implement an ARMA model, a suitable backcasting horizon is dependent on the number of parameters we need to estimate, and the quality of, the model fit. We have pruned our backcasts accordingly.

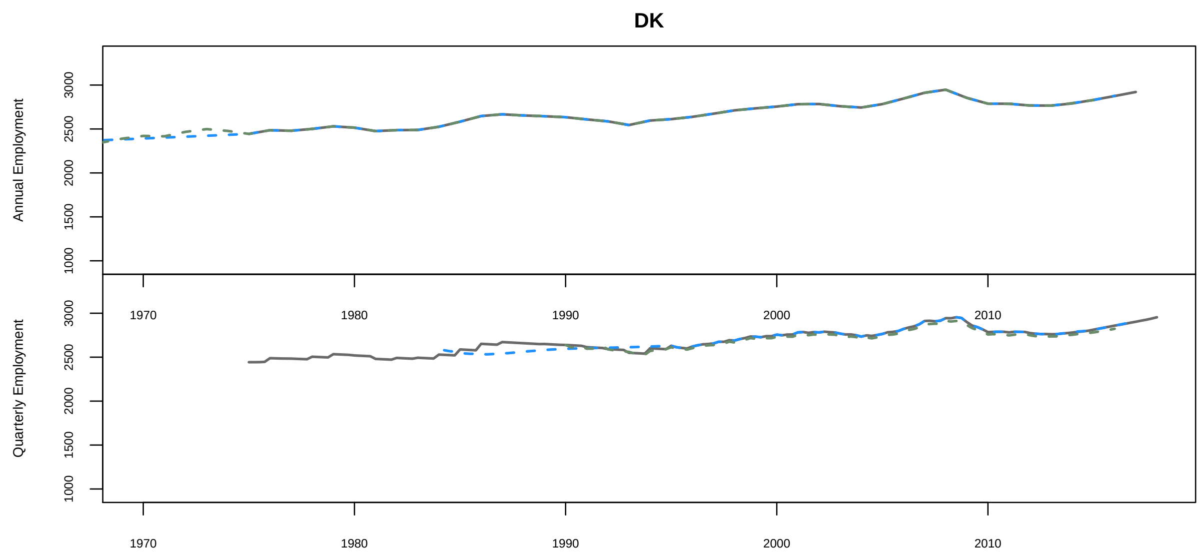
If we were to be provided with explanatory variables for the time series, then the quality of the backcasts would improve and there would be the potential to increase the backcasting horizon.

Finally, our methodology is fully consistent over the hierarchical structure of the data and therefore provides interpretable results.

9 Appendix

9.1 Employment Results





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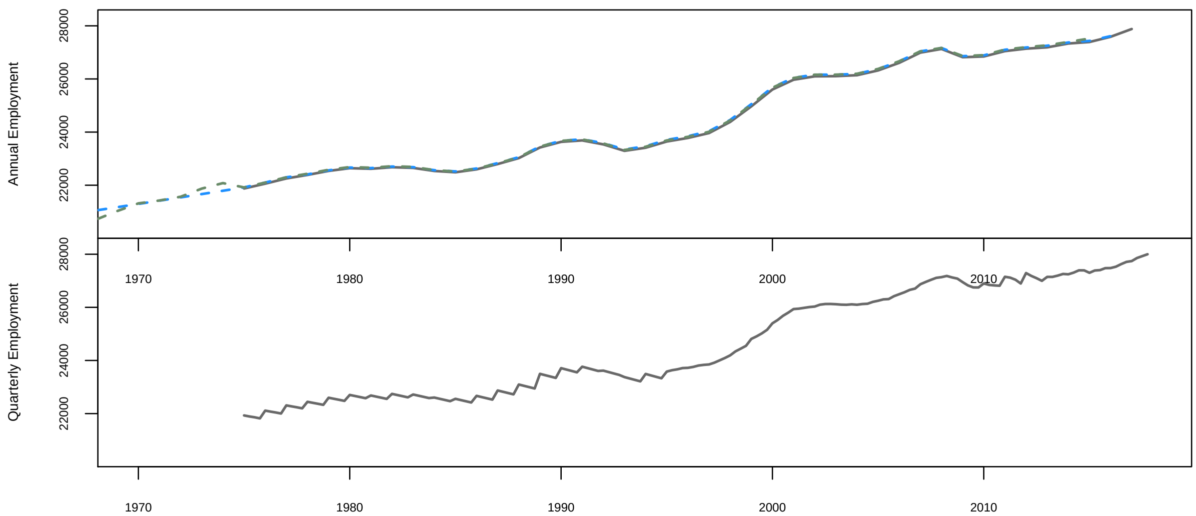
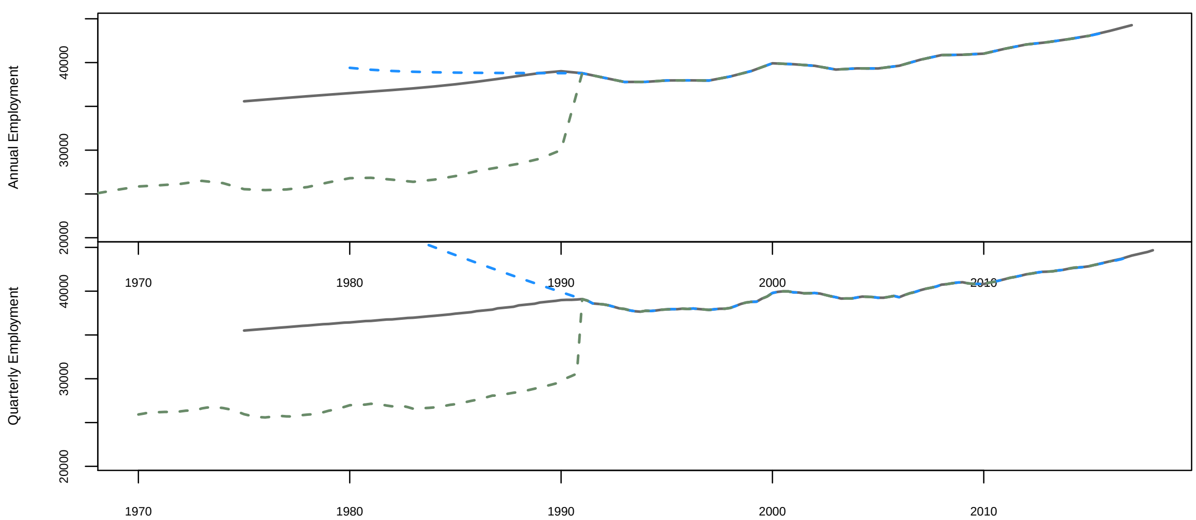


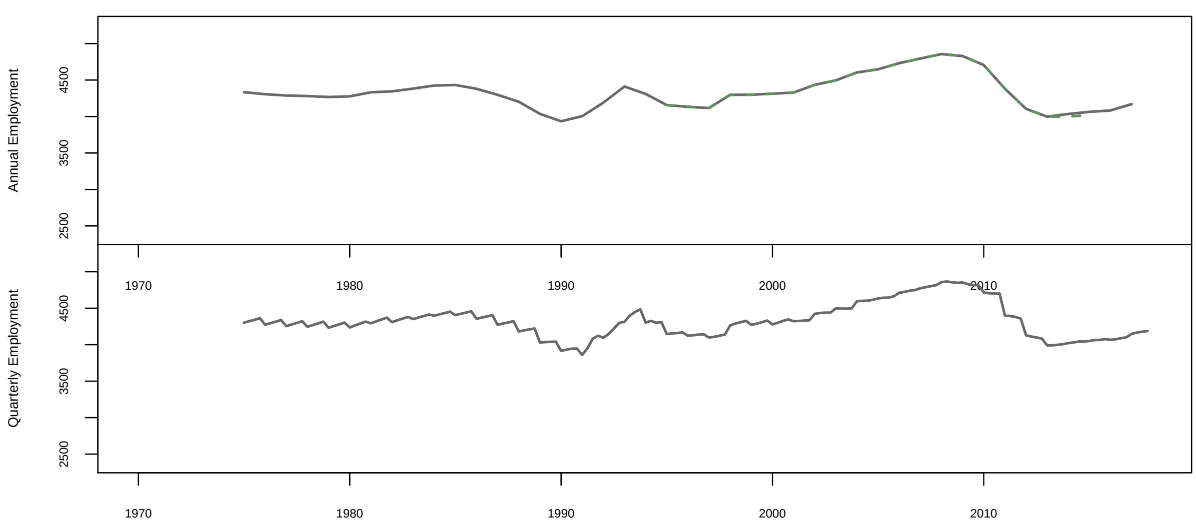
Annual Employment
22000
24000
26000
28000
Quarterly Employment
22000
24000
26000
28000
1970
1980
1990
2000
2010

DE

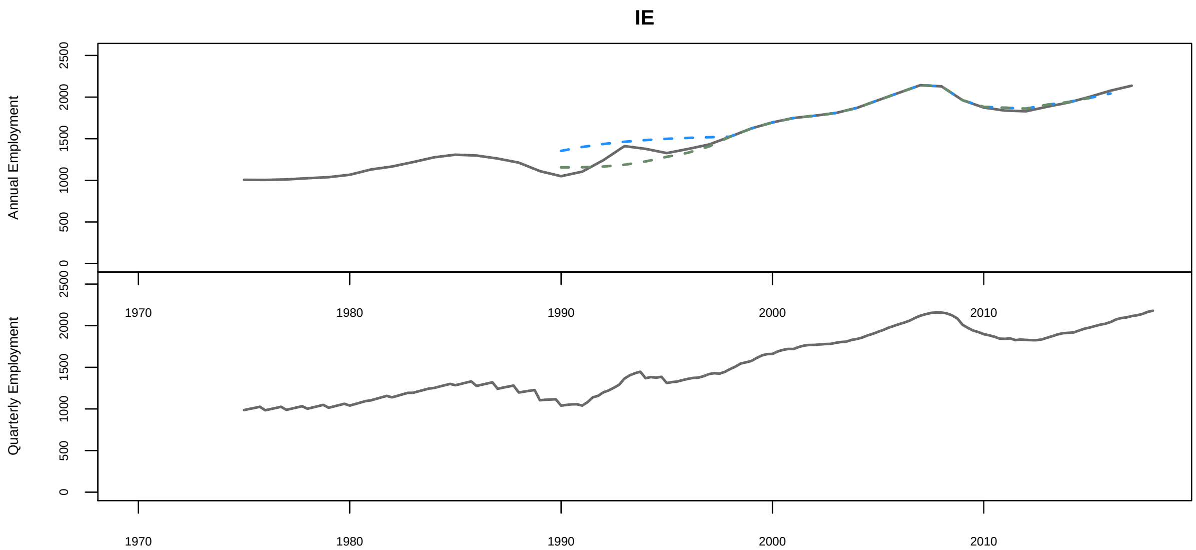


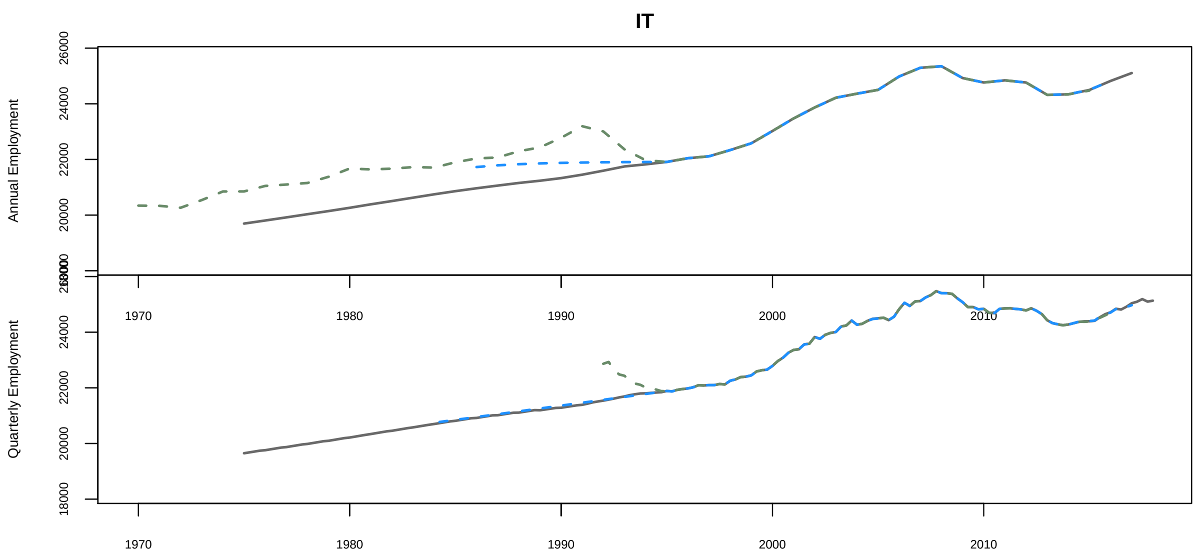
Annual Employment
20000
30000
40000
Quarterly Employment
20000
30000
40000
1970
1980
1990
2000
2010

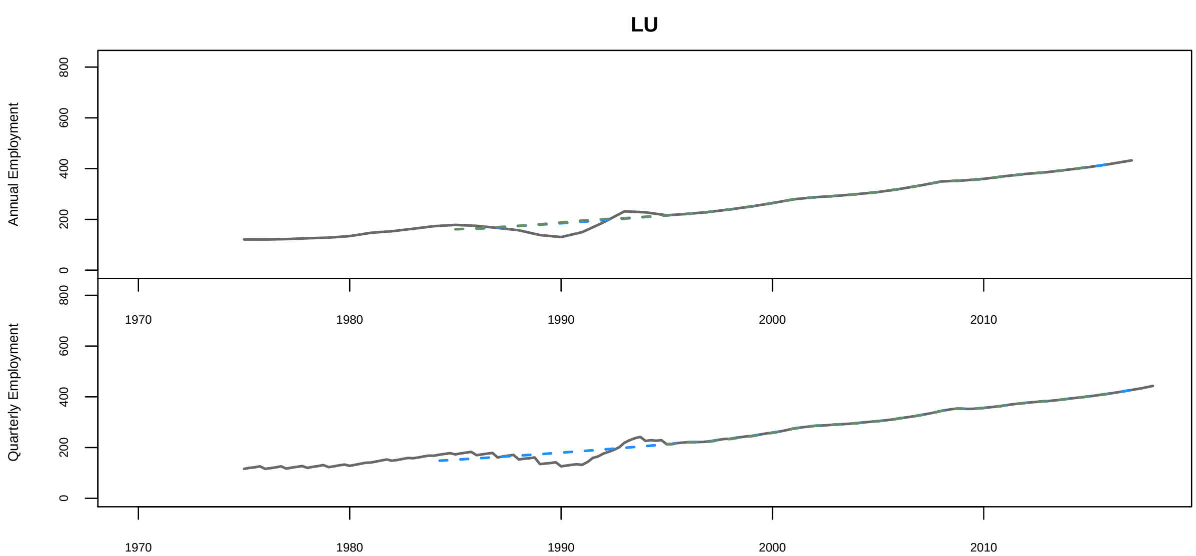
EL

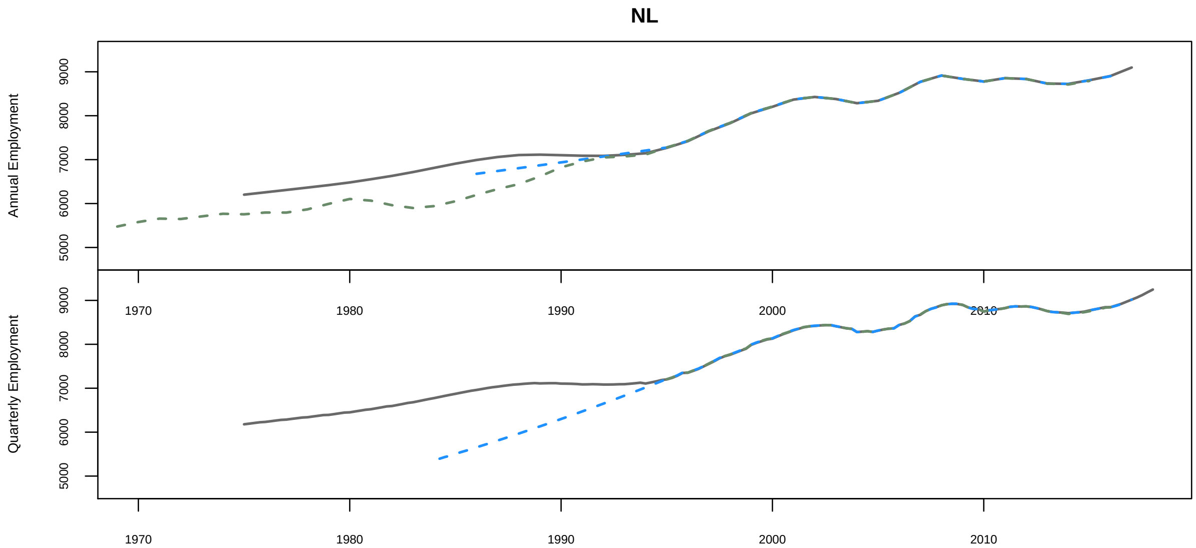


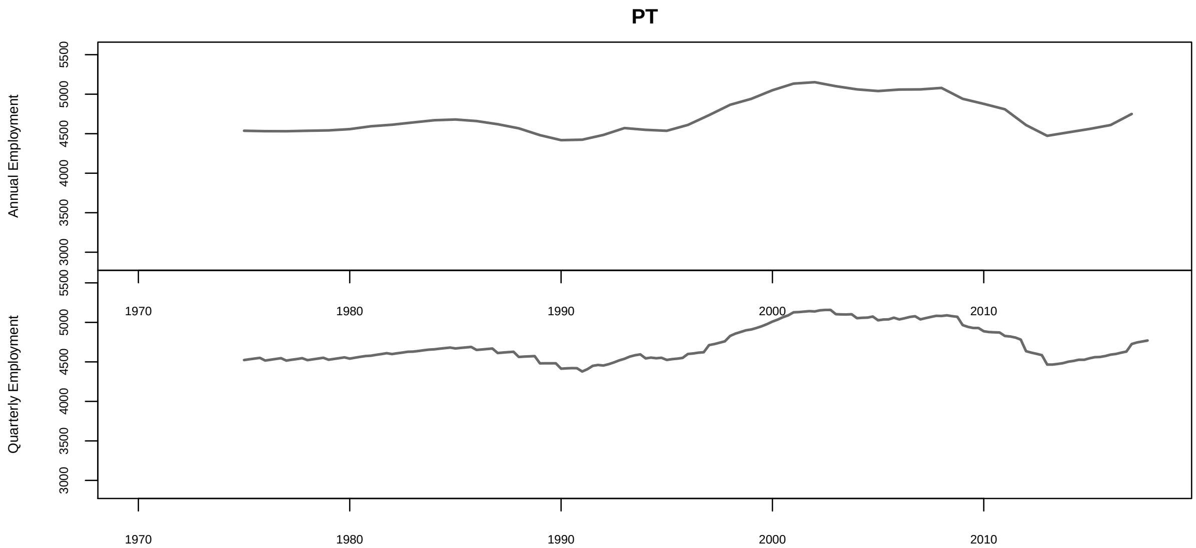
Annual Employment
2500
3500
4500
Quarterly Employment
2500
3500
4500
1970
1980
1990
2000
2010

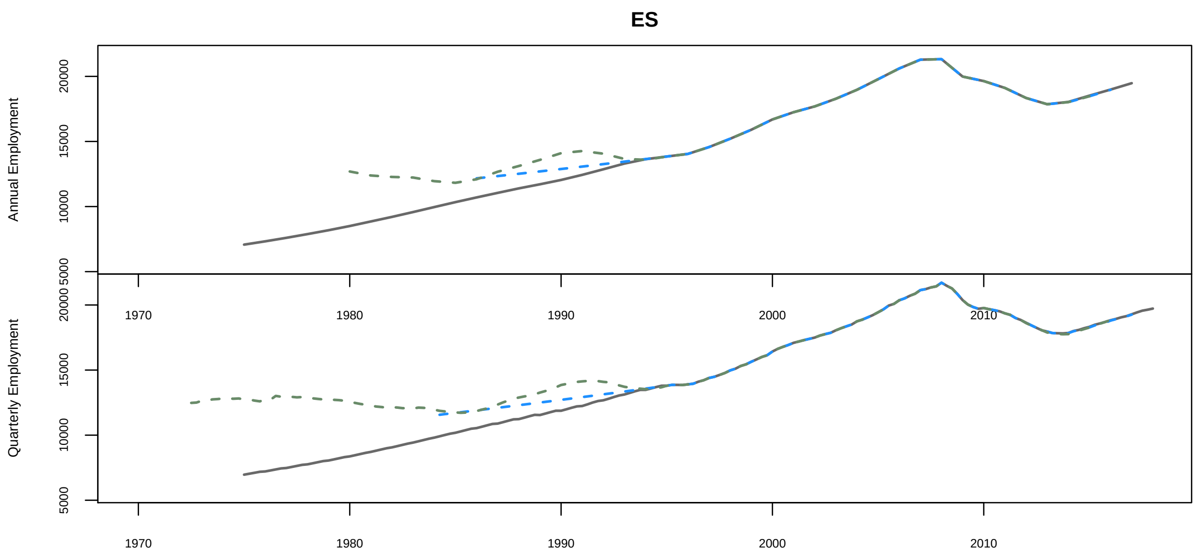

IE
Annual Employment
Quarterly Employment
0
500
1000
1500
2000
2500
1970
1980
1990
2000
2010


IT
Annual Employment
Quarterly Employment
18000
20000
22000
24000
26000
1970
1980
1990
2000
2010

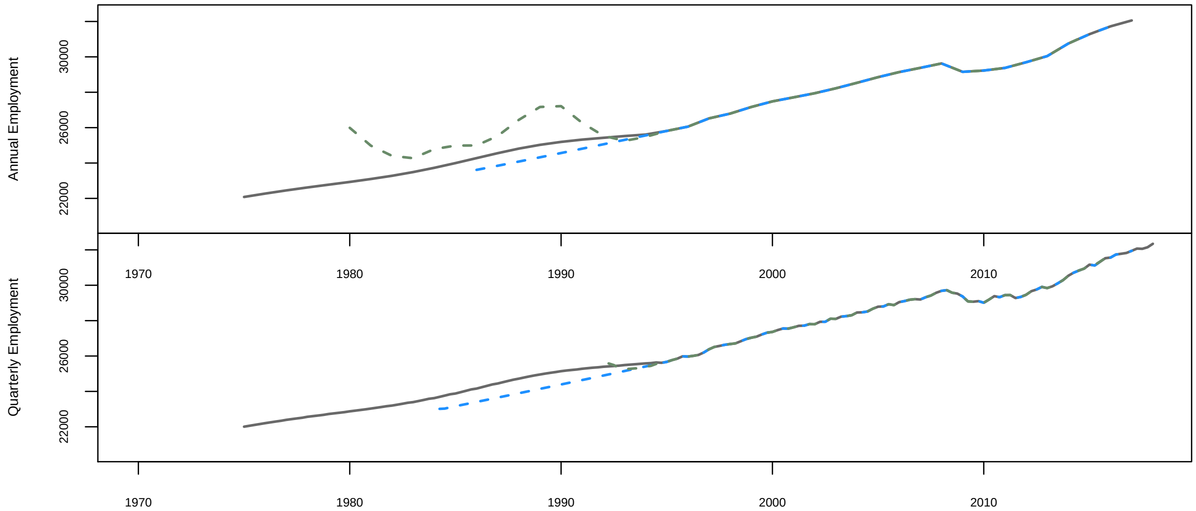

LU
Annual Employment
Quarterly Employment
0
200
400
600
800
1970
1980
1990
2000
2010


NL
Annual Employment
Quarterly Employment
5000
6000
7000
8000
9000
1970
1980
1990
2000
2010


PT
Annual Employment
Quarterly Employment
3000
3500
4000
4500
5000
5500
1970
1980
1990
2000
2010

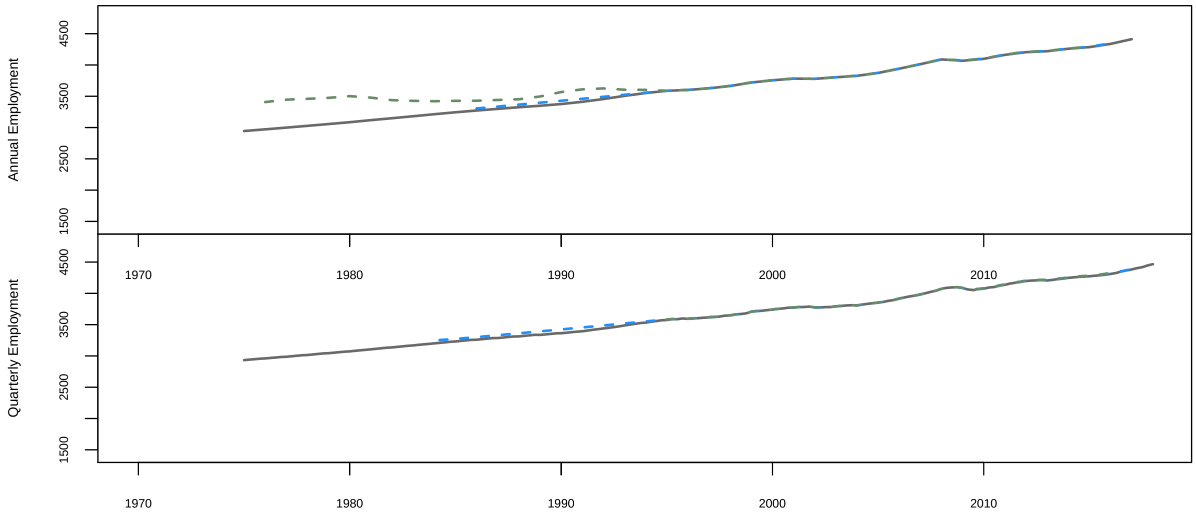

ES
Annual Employment
Quarterly Employment
5000
10000
15000
20000
1970
1980
1990
2000
2010

UK



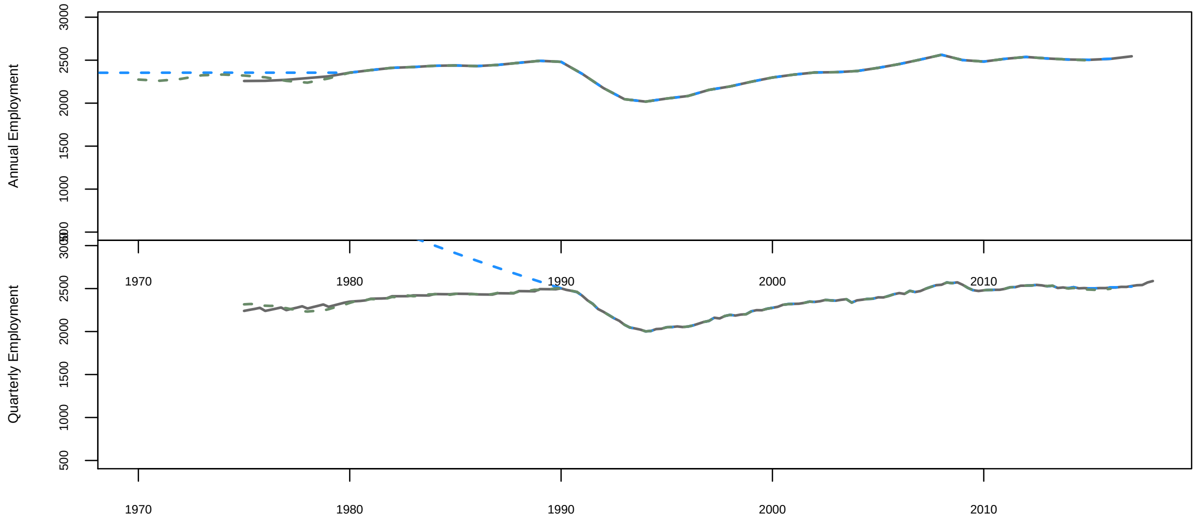
Annual Employment
Quarterly Employment
22000
26000
30000
1970
1980
1990
2000
2010

AT

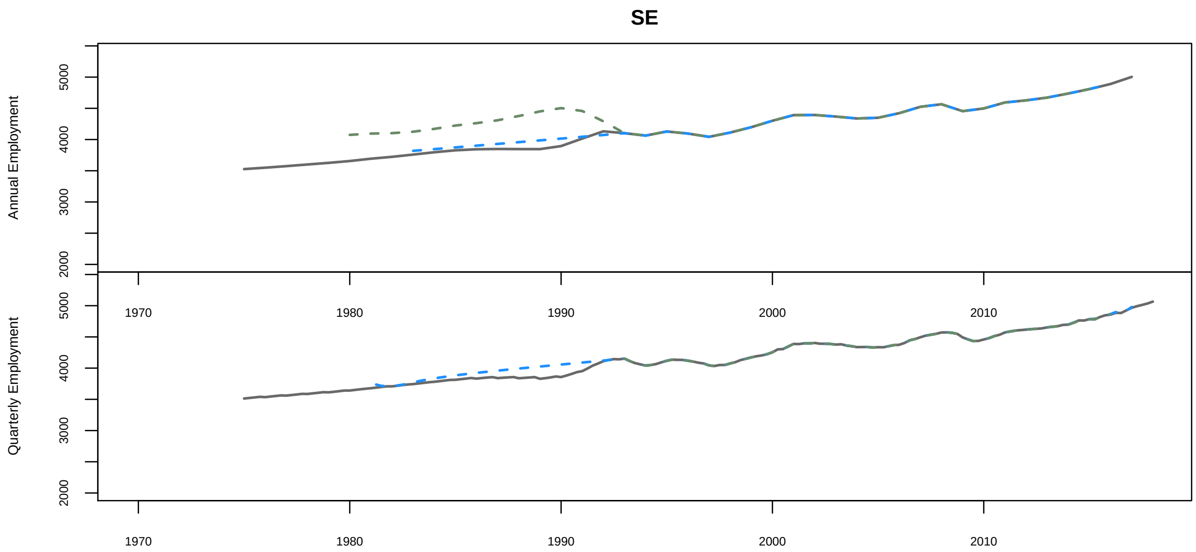


Annual Employment
Quarterly Employment
1500
2500
3500
4500
1970
1980
1990
2000
2010

FI

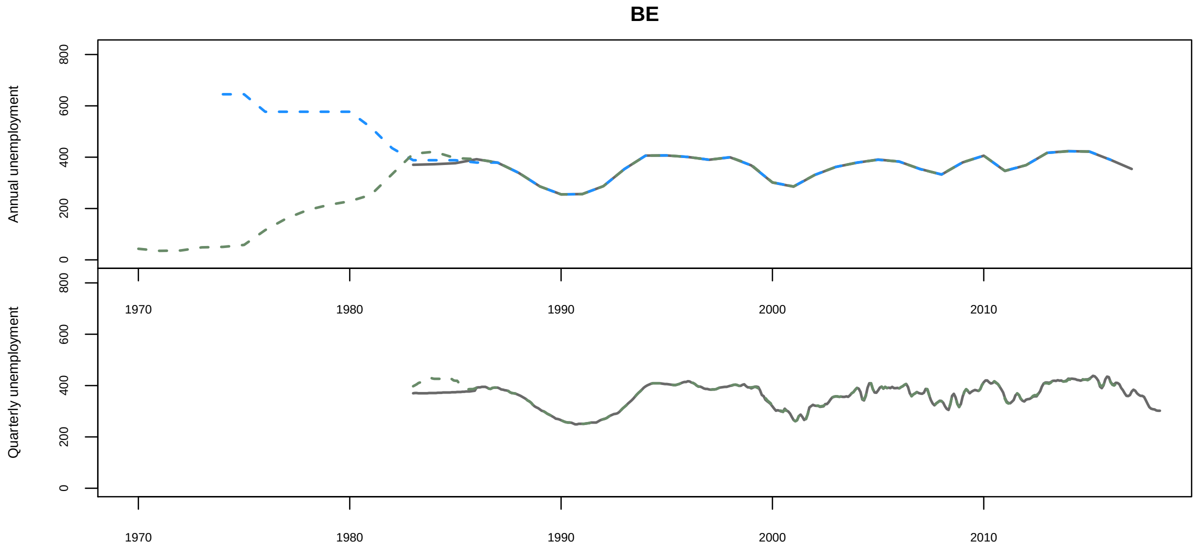


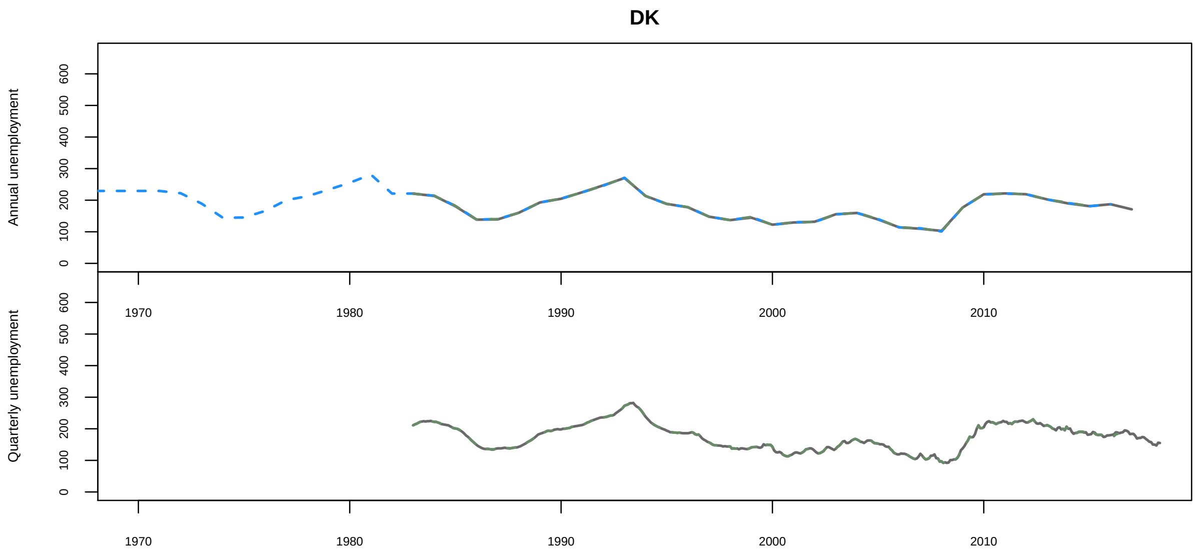
Annual Employment
Quarterly Employment
500
1000
1500
2000
2500
3000
1970
1980
1990
2000
2010



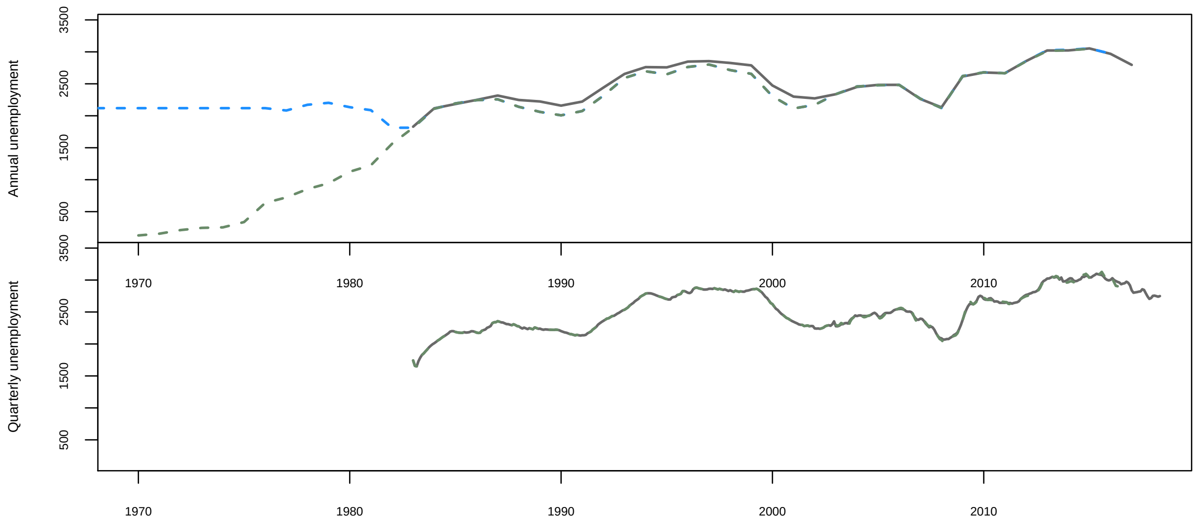
SE
Annual Employment
Quarterly Employment
2000
3000
4000
5000
1970
1980
1990
2000
2010

9.2 Unemployment Results

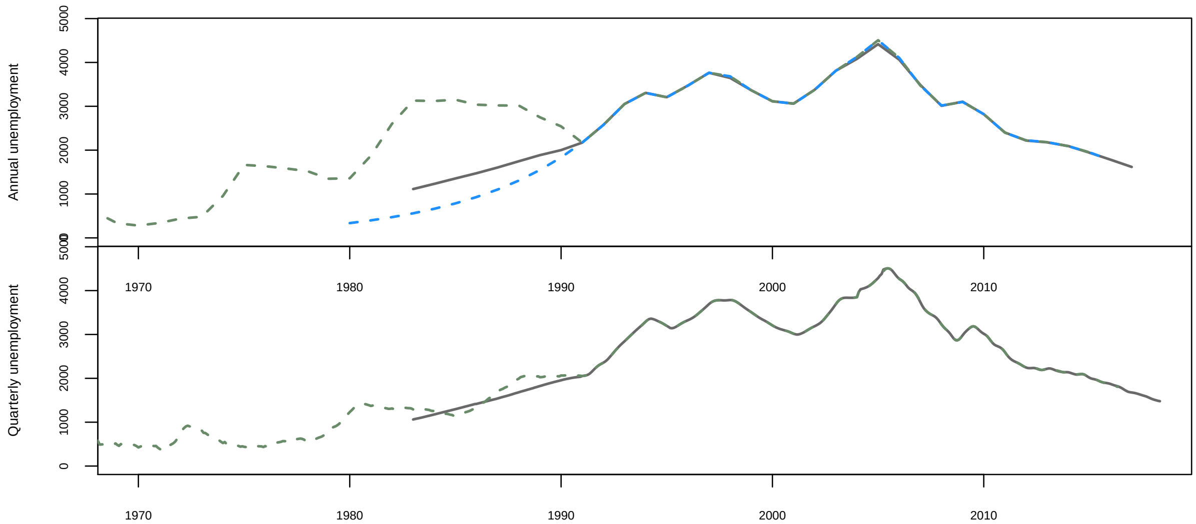




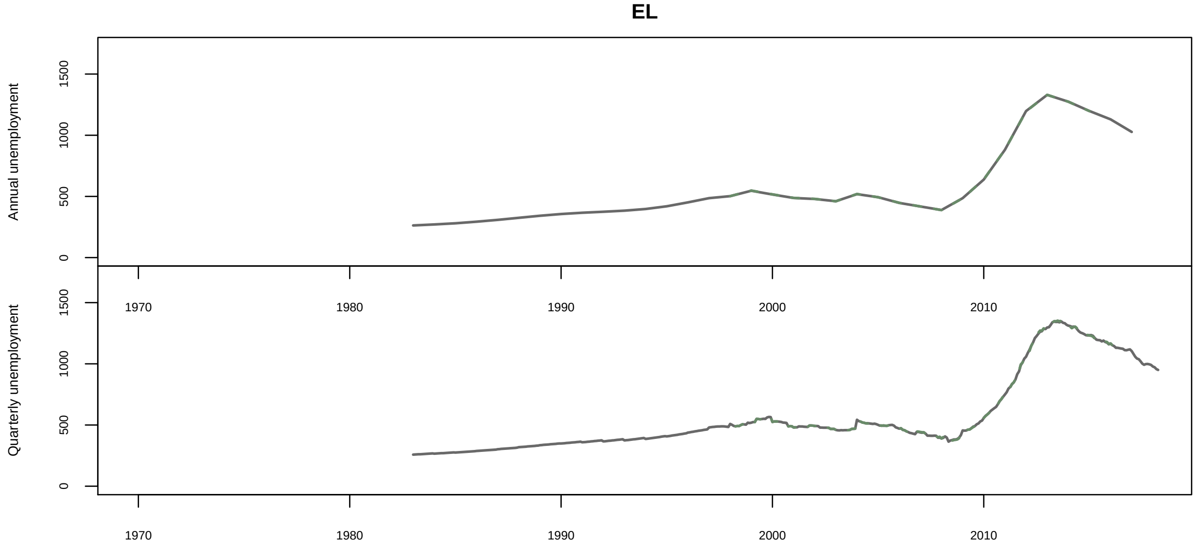
FR


Annual unemployment
Quarterly unemployment
500
1500
2500
3500
1970
1980
1990
2000
2010

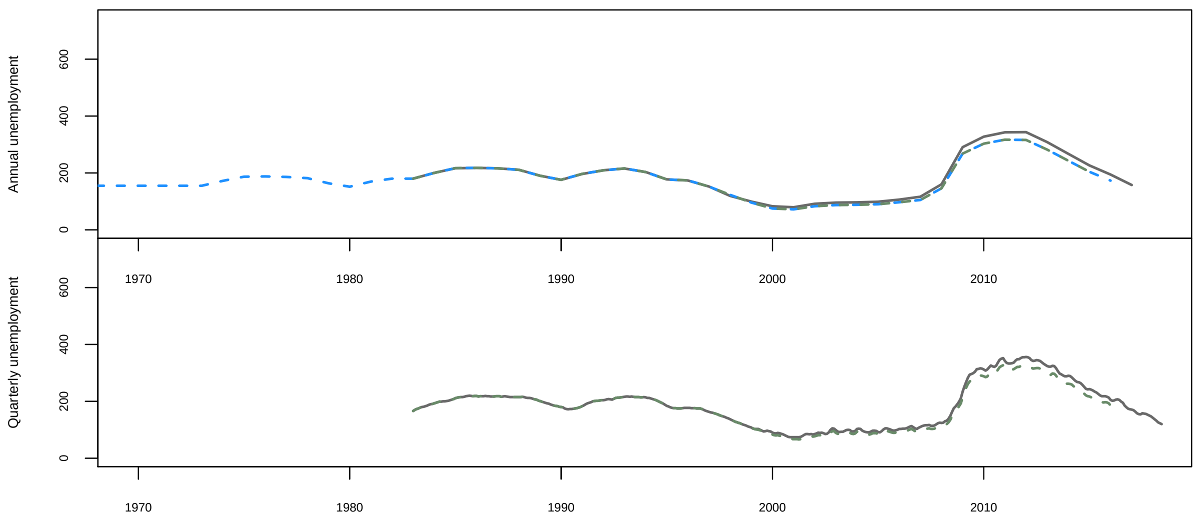
DE


Annual unemployment
Quarterly unemployment
0
1000
2000
3000
4000
5000
1970
1980
1990
2000
2010

EL

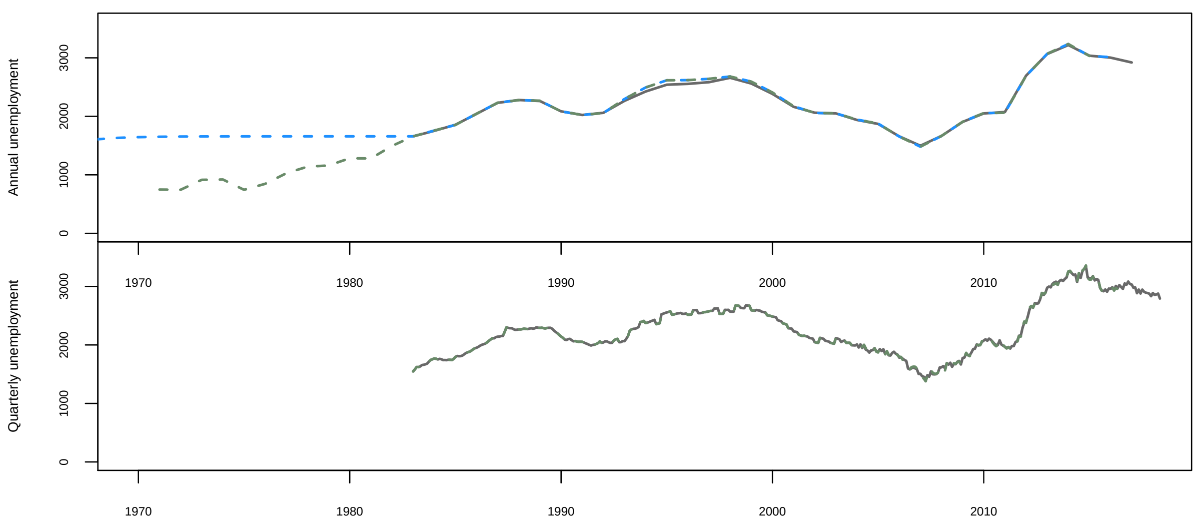

Annual unemployment
Quarterly unemployment
0
500
1000
1500
1970
1980
1990
2000
2010

IE



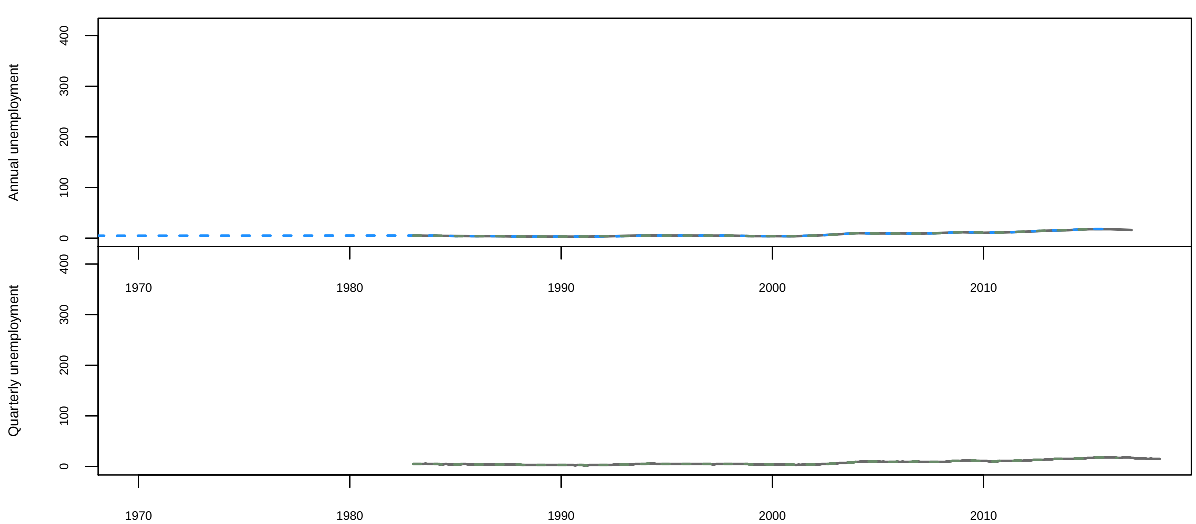
Annual unemployment
0
200
400
600
Quarterly unemployment
0
200
400
600
1970
1980
1990
2000
2010

IT



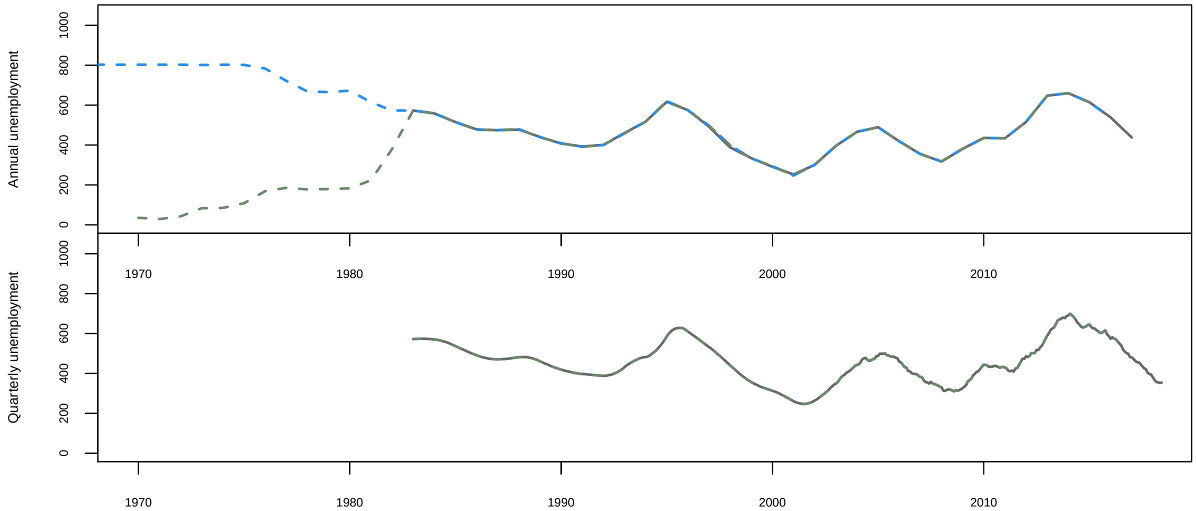
Annual unemployment
0
1000
2000
3000
Quarterly unemployment
0
1000
2000
3000
1970
1980
1990
2000
2010

LU

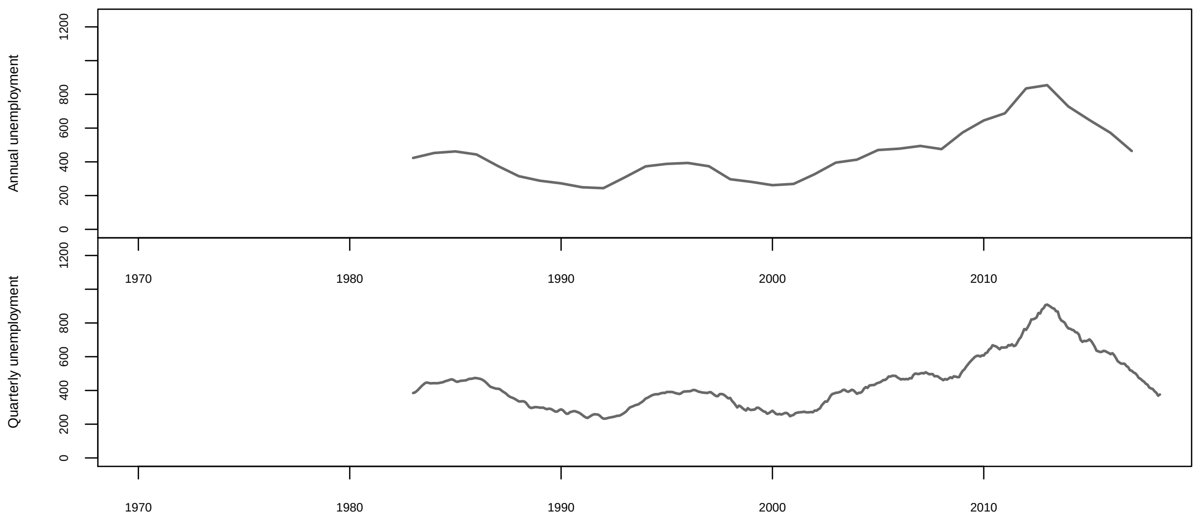


Annual unemployment
0
100
200
300
400
Quarterly unemployment
0
100
200
300
400
1970
1980
1990
2000
2010

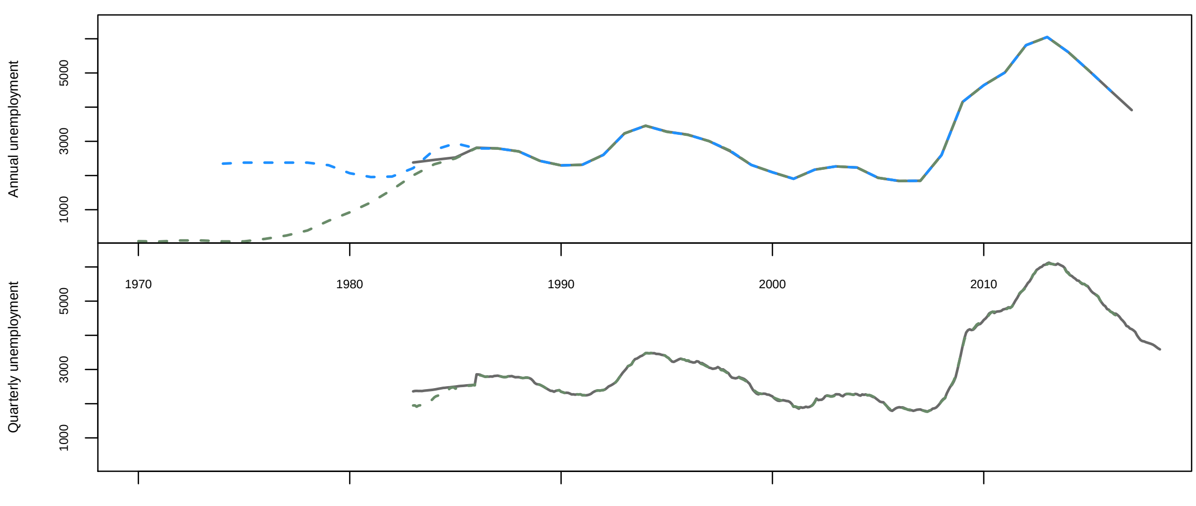
NL

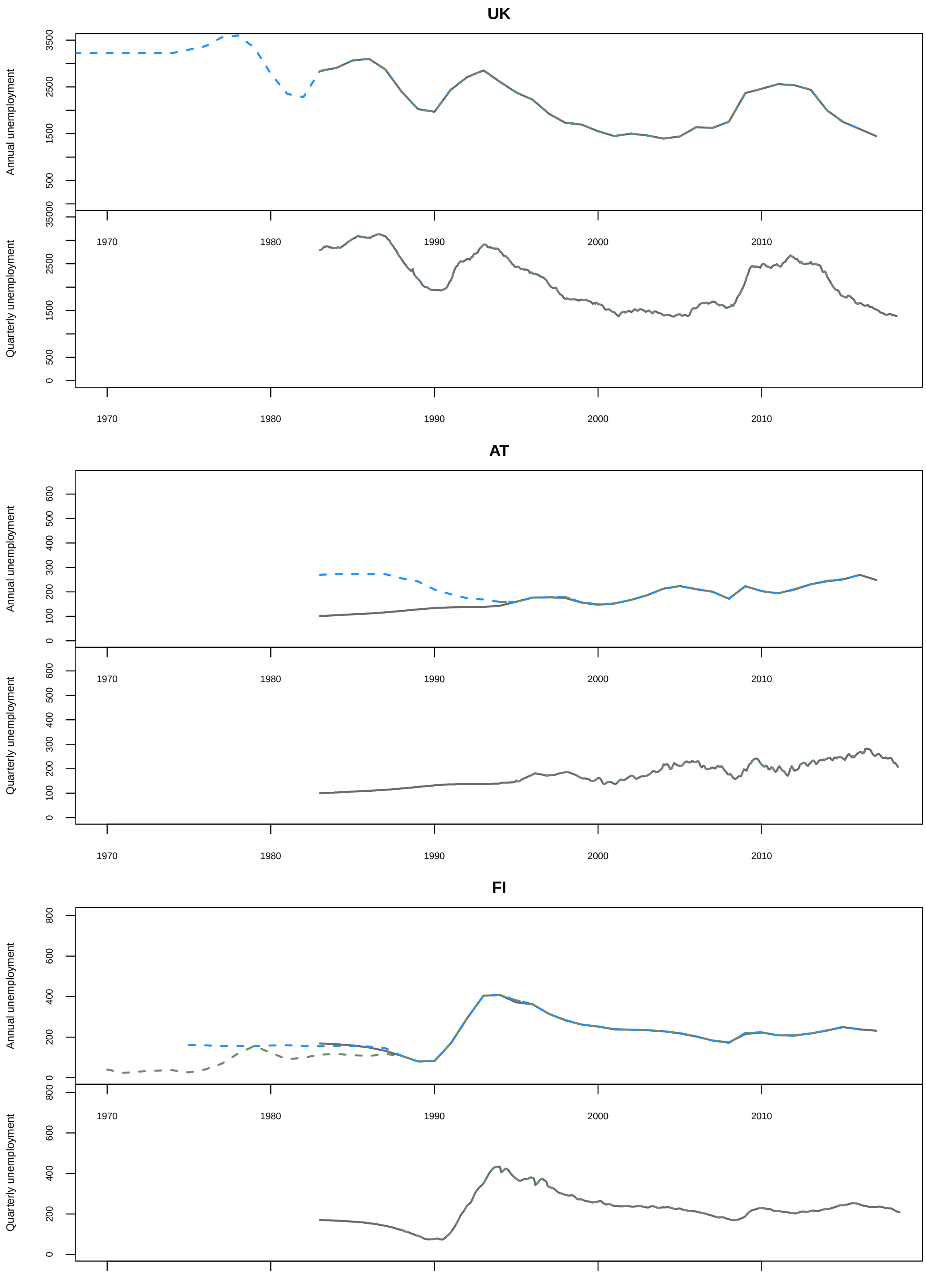

Annual unemployment
Quarterly unemployment
0
200
400
600
800
1000
1970
1980
1990
2000
2010

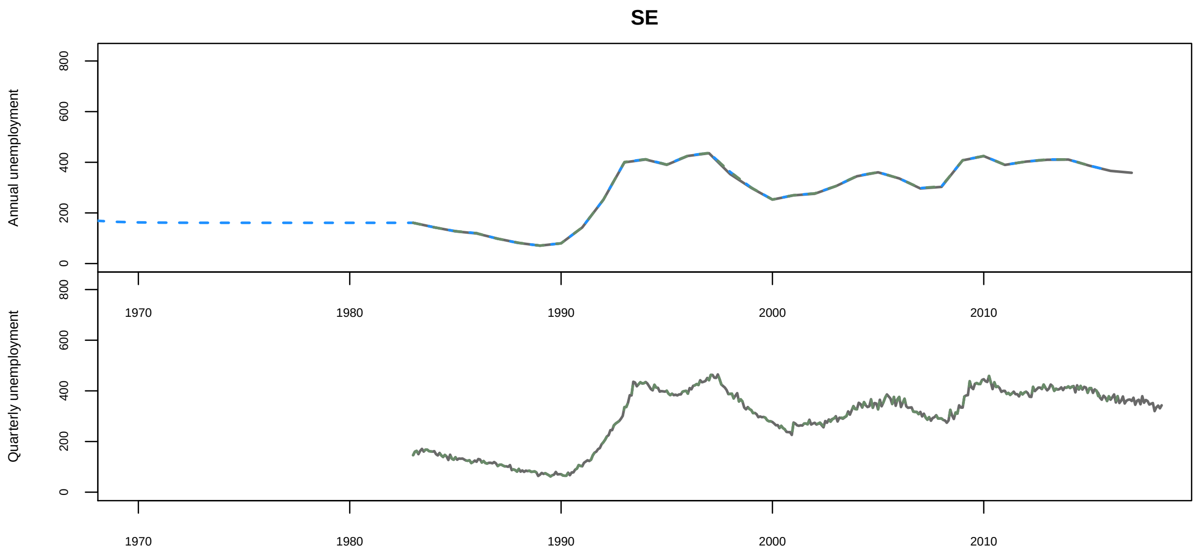
PT


Annual unemployment
Quarterly unemployment
0
200
400
600
800
1200
1970
1980
1990
2000
2010

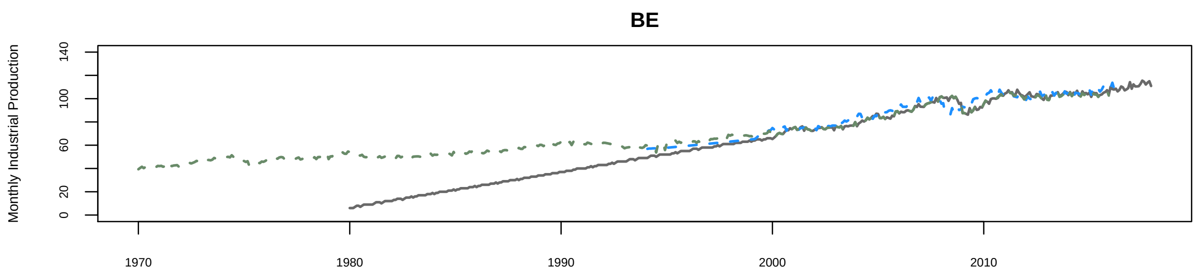
ES

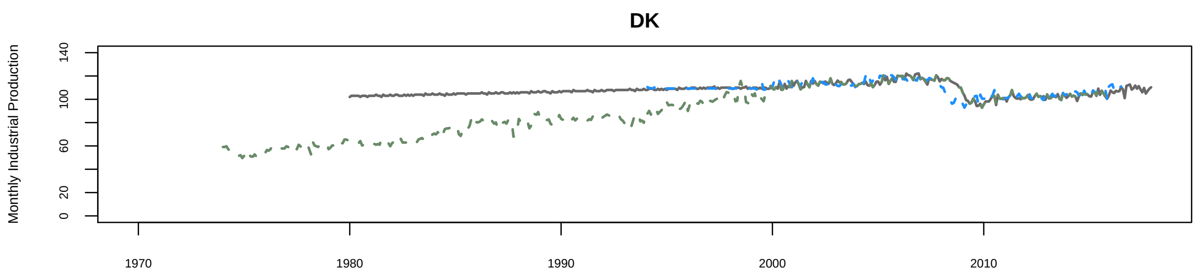

Annual unemployment
Quarterly unemployment
1000
3000
5000
1970
1980
1990
2000
2010

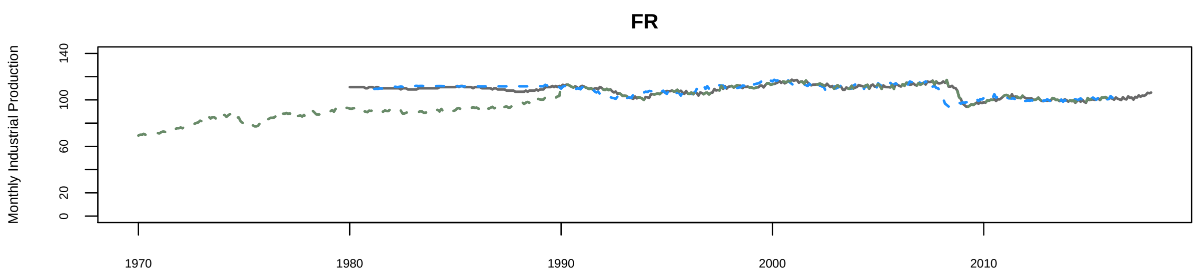

UK
Annual unemployment
Quarterly unemployment
AT
Annual unemployment
Quarterly unemployment
FI
Annual unemployment
Quarterly unemployment
1970
1980
1990
2000
2010

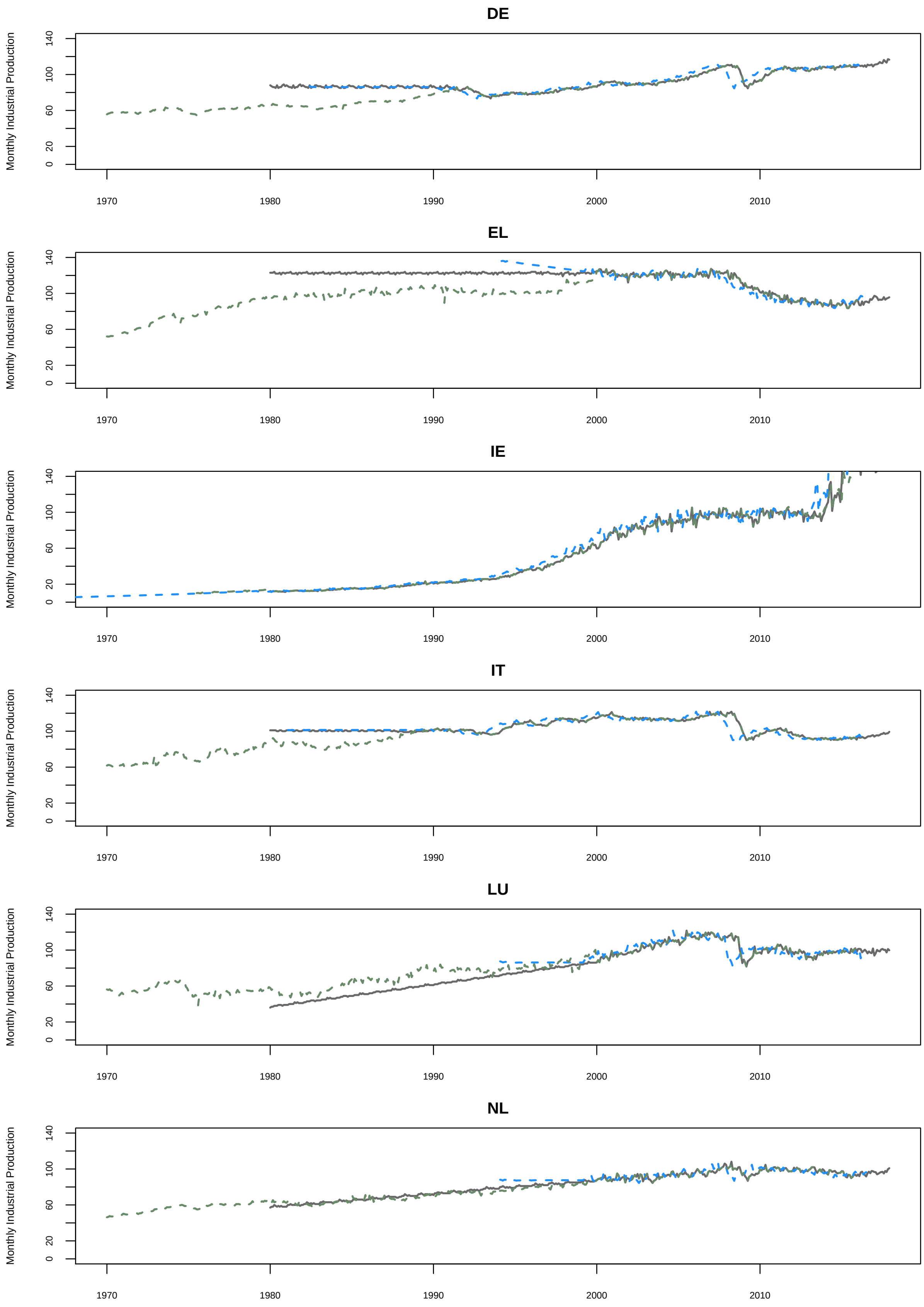


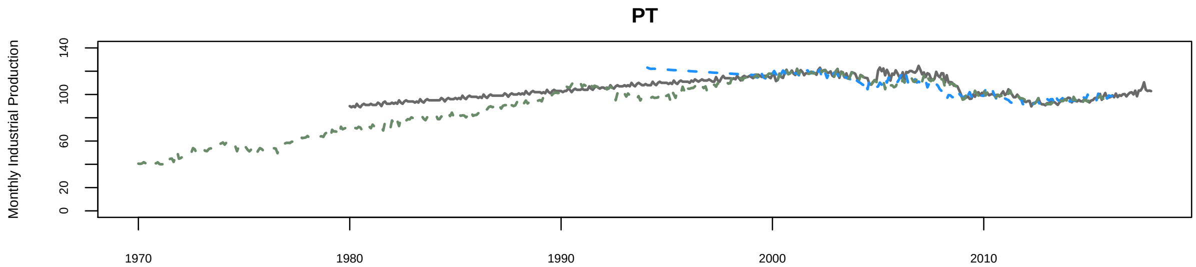
9.3 Industrial Production Results

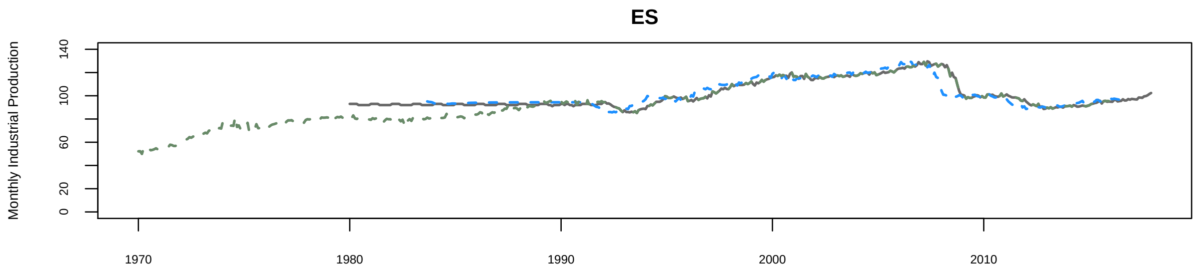


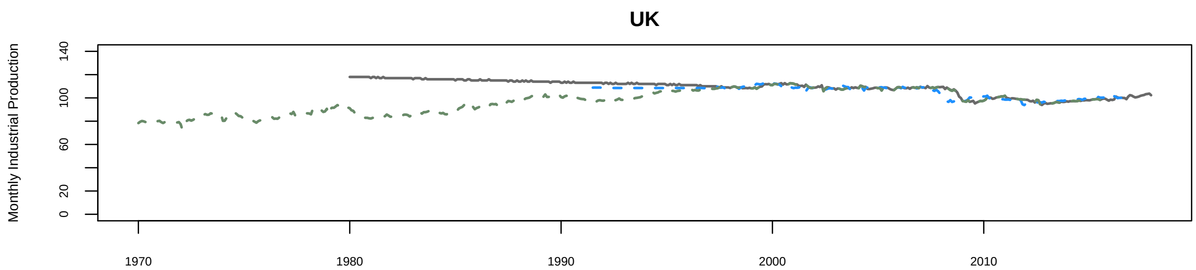


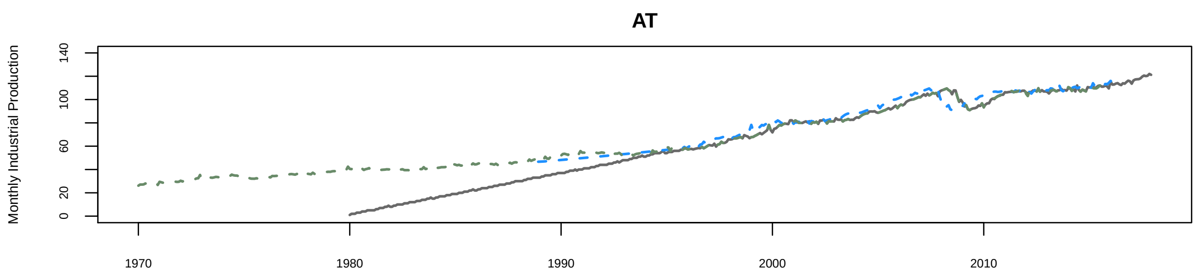


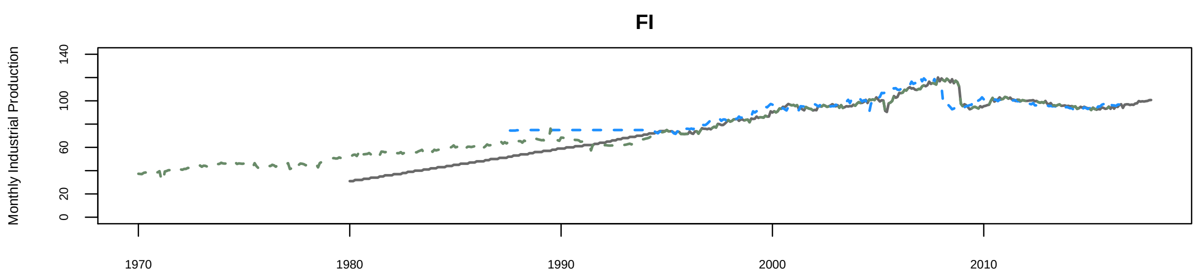

DE
EL
IE
IT
LU
NL
Monthly Industrial Production
0
20
60
100
140
1970
1980
1990
2000
2010

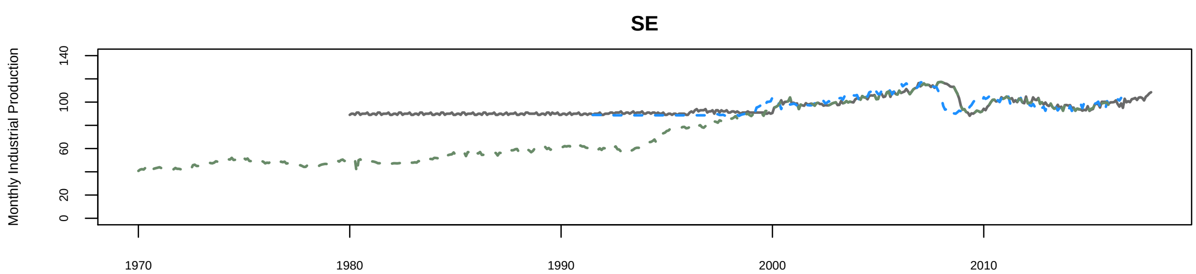

PT
Monthly Industrial Production
0
20
60
100
140
1970
1980
1990
2000
2010


ES
Monthly Industrial Production
0
20
60
100
140
1970
1980
1990
2000
2010

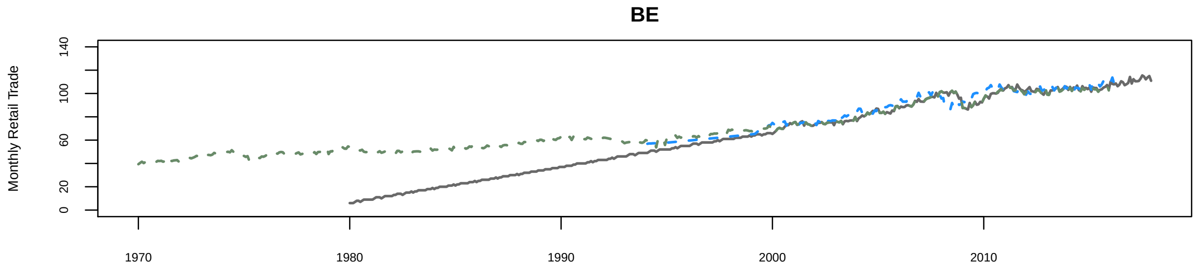

UK
Monthly Industrial Production
0
20
60
100
140
1970
1980
1990
2000
2010

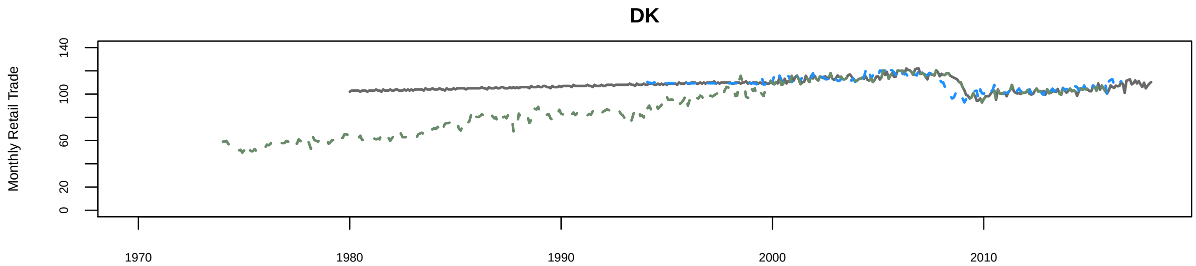

AT
Monthly Industrial Production
0
20
60
100
140
1970
1980
1990
2000
2010

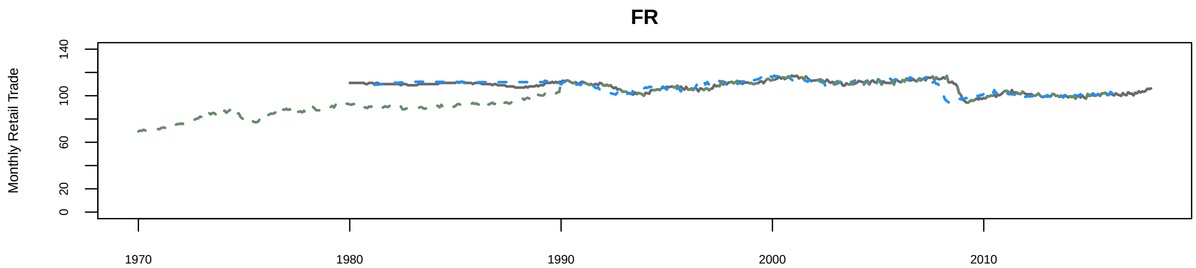

FI
Monthly Industrial Production
0
20
60
100
140
1970
1980
1990
2000
2010

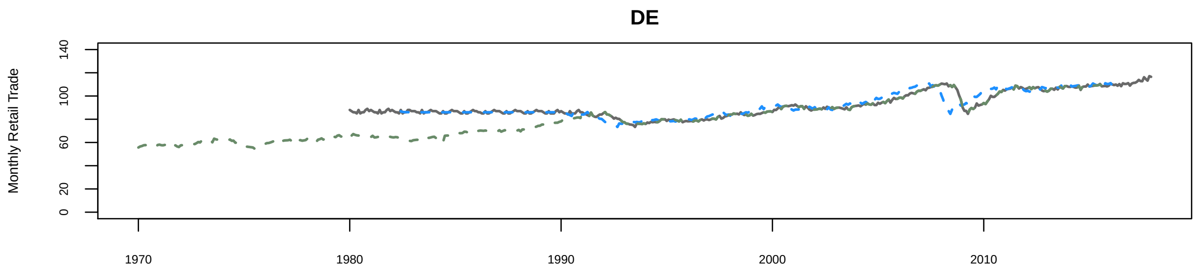

SE
Monthly Industrial Production
0
20
60
100
140
1970
1980
1990
2000
2010

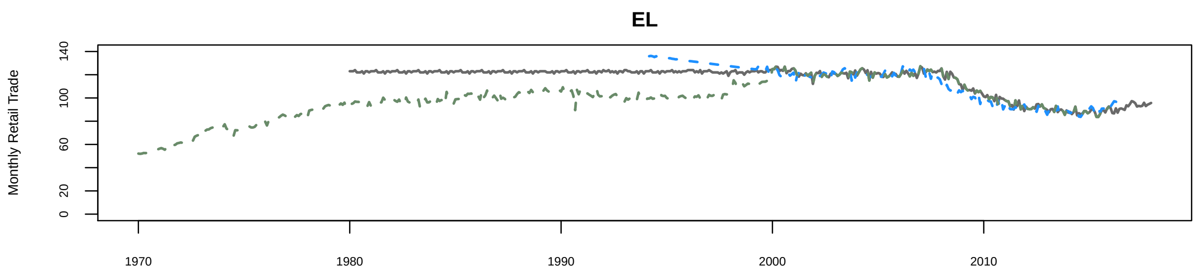
9.4 Retail Trade Results

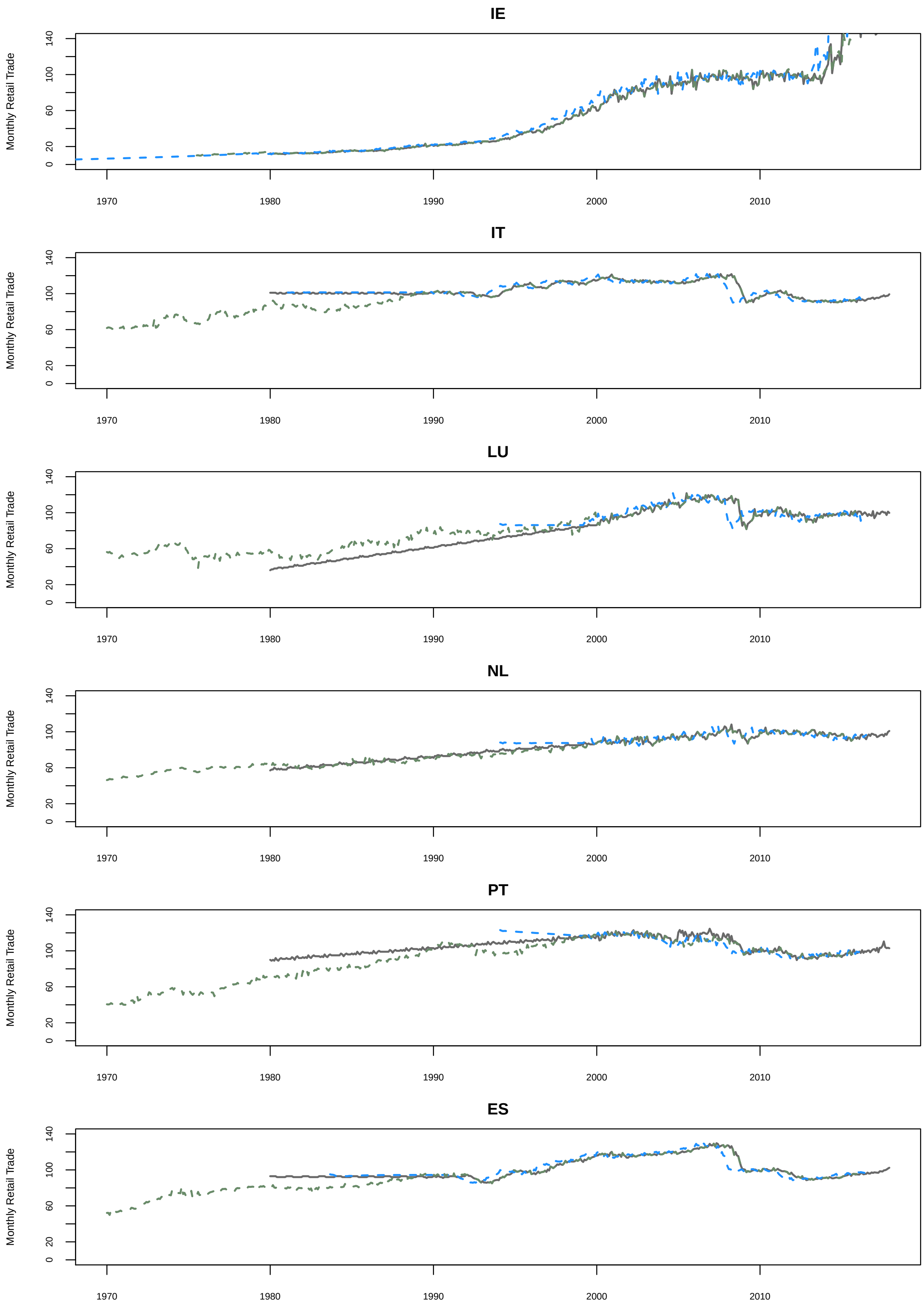


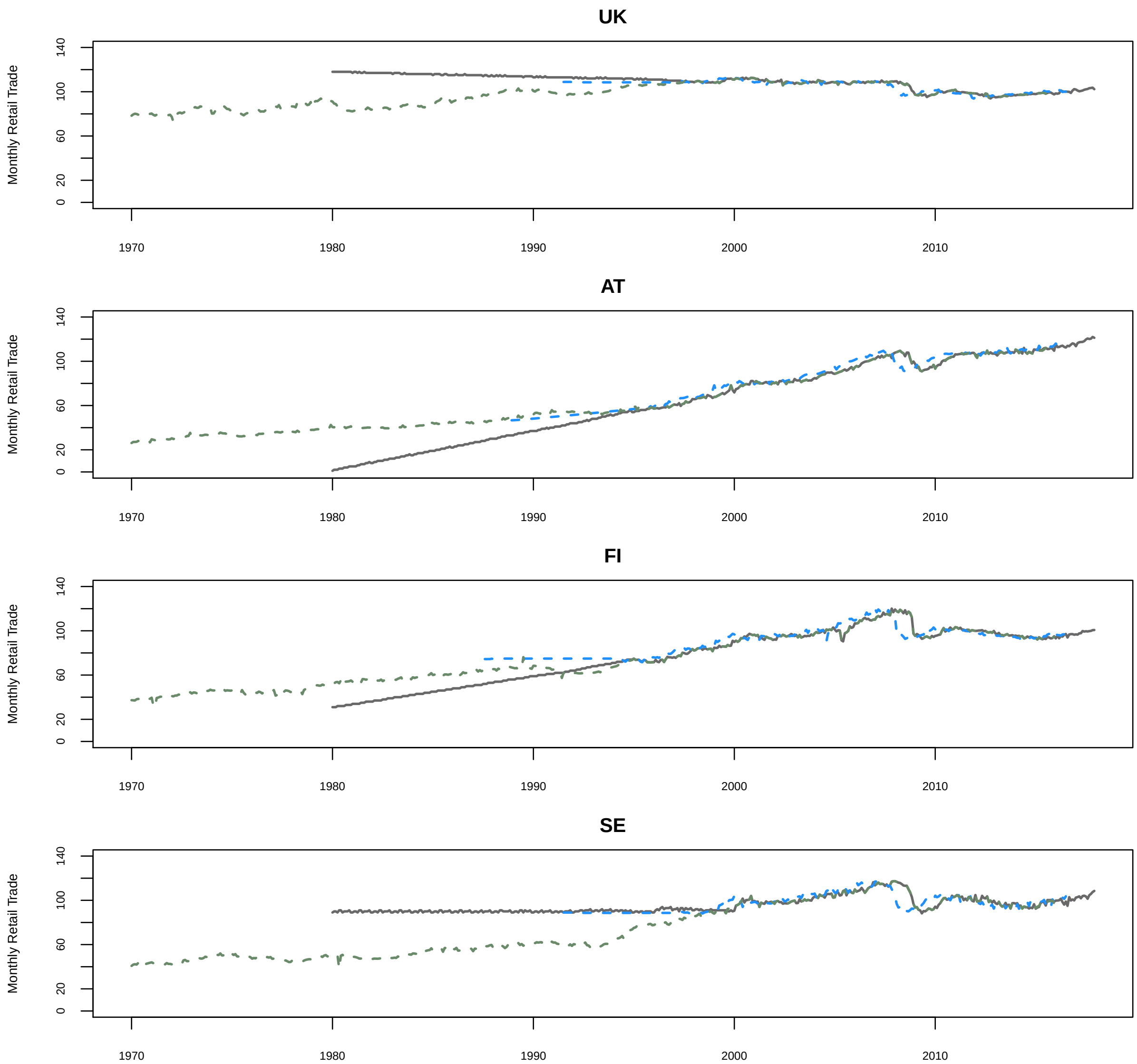









IE
IT
LU
NL
PT
ES
Monthly Retail Trade
0
20
60
100
140
1970
1980
1990
2000
2010



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