



How collaborative forecasting can reduce forecast accuracy



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ARTICLE INFO

Article history:

Received 29 July 2014

Received in revised form

18 April 2015

Accepted 19 April 2015

Available online 27 April 2015

ABSTRACT

In this paper, we provide an analytical perspective on the link between supply chain collaboration and forecast accuracy, showing that collaborative forecasting can lead to less accurate demand forecasts over a wide range of cost and demand parameters. The result is explained by the decision maker's relative preference for investing in forecasting vs. order quantities to manage demand uncertainty.

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Keywords:

Collaborative forecasting

Information sharing

Supply chain coordination

1. Introduction

The goal of this paper is to provide an analytical perspective on the link between supply chain coordination and forecast accuracy. We consider a supply chain model that captures two key elements – endogenous forecast accuracy and retailer-supplier collaboration – while keeping all other model components as simple as possible. First, we consider a non-collaborative scenario where the retailer and supplier invest in forecasting independently and do not share any information. Next, we consider a collaborative scenario where a central decision maker decides on the retailer's and supplier's investments into forecasting and pools the demand forecasts to obtain a single shared forecast that forms the basis of the retailer's ordering and the supplier's capacity decisions. We show that the move from non-collaborative to collaborative forecasting can have the unexpected impact of decreasing demand forecast accuracy. This is despite the complete collaboration among supply chain partners. The driver of this result is the complex interplay of two levers for managing demand uncertainty in a supply chain—investing in better forecasts and adjusting order quantities. The collaborative supply chain will sometimes emphasize the order quantity lever to an extent that its demand forecast accuracy is actually worse than what would be chosen by a retailer acting independently in a non-collaborative supply chain.

Our paper's main contribution is at the intersection of the literatures on collaborative forecasting in supply chains [1,2]

and the vast literature on selling to the newsvendor (e.g. [7]). A number of papers, such as [8,12,13], have focused on how the downstream party's forecasting accuracy in a selling to the newsvendor setting plays a role in firm performance. These papers investigate contracts that could improve system performance or how forecasting accuracy impacts firm performance. While there are papers in the selling to the newsvendor literature that show that the *retailer* might overinvest in demand forecasting (e.g. [12]), to our knowledge the problem of forecasting overinvestment in a *collaborative* setting, where both the supplier and the retailer can improve the accuracy of the demand forecast, has never been addressed in the academic literature. In all of the papers mentioned above, only the retailer can acquire information and improve the demand forecast accuracy. In contrast, in our model the upstream party (supplier) can also improve the forecast accuracy and can share its forecast with the retailer. The final demand forecast in such a collaborative forecasting model is a combination of the downstream and upstream party's forecasts. This idea is motivated by the recent popularity of retailing initiatives such as Collaborative Forecasting, Planning, and Replenishment (CPFR), which originated in the consumer goods sector with a specific focus on sharing and reconciling demand forecasts between supplier and retailer.

2. Model

The supply chain decisions taken by the retailer, R , and the supplier, S , are the order quantity Q (determined before the demand is realized), and the capacity K (determined before receipt of the retailer's order), respectively. Sales revenue is given by

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$r \min\{Q, D\}$, where r is the unit sales price of the product and D is a random variable that denotes demand. The price paid by the retailer to the supplier is wQ . We assume that terms of trade consist of a simple wholesale price only contract, which is a common business model in many supply chains (see [7,9]). We assume that w is exogenous—see Section 5 for a discussion of the impact of the negotiated value of w on our results. Let also c denote the unit cost of capacity for the supplier.

We employ a forecasting model based on [15,4]. Demand, D , is normally distributed with mean μ . The supplier and the retailer can each invest to privately obtain an imperfect demand signal ψ_i that is a realization of $\Psi_i = D + \varepsilon_i$, $i \in \{S, R\}$, where ε_i 's are error terms distributed according to the bivariate normal distribution with unconditional mean $\mathbb{E}[\varepsilon_i] = 0$ (ensuring that the signals are unbiased), variance $\mathbb{V}[\varepsilon_i] = \sigma_i^2$, and correlation ρ , $0 \leq \rho < 1$. Correlation allows for a dependence between the information that the two signals carry, capturing the fact that the supplier and the retailer might both utilize some common data or share common assumptions. Observing the signal ψ_i allows party i to generate a more precise forecast, $D|\psi_i$.

To isolate the roles of forecasting investment and joint forecasting from the role of the prior information available, we assume that the distribution of D is diffuse, though our results readily extend to general normal distributions. We denote the density of the standard normal distribution by $\phi(\cdot)$ and its cumulative distribution by $\Phi(\cdot)$.

Let $A_i \doteq \frac{1}{\mathbb{V}[\varepsilon_i]}$, $i \in \{S, R\}$ be the accuracy of the signal observed by party i . We assume that the cost of achieving accuracy A_i is κA_i^q , $\kappa > 0$, $q > 0$, where κ is a scaling parameter and q is the “forecasting technology” parameter. Values of q above 1 imply that forecasting costs are convex in accuracy, while values below 1 imply concave costs. Given that we are dealing with normal distributions, the idea of buying some amount of variance reduction directly is akin to buying additional demand signals as in [6]. Effectively, n signals provide the same information as one signal equal to the mean of those n but drawn from a distribution with $(\frac{1}{n})$ th the variance.

The sequence of events is as follows. In Stage 1, both the supplier and the retailer simultaneously invest into forecasting (either separately in the non-collaborative case or jointly in the collaborative case) and observe signals ψ_S and ψ_R . The demand forecast is updated. In Stage 2, supplier capacity and retailer order quantity are determined (again, either separately or jointly). Finally, demand is observed and profits are realized.

3. Non-collaborative forecasting

We begin by analyzing the non-collaborative scenario in which the supplier and retailer invest in forecasting independently and do not share any information [6]. The retailer uses only its own signal to forecast demand and the supplier uses its own signal to forecast the retailer's order. To determine their respective investment levels, the two parties solve independent optimization problems, weighing forecasting cost against the benefits of greater accuracy about demand (for the retailer) or about the retailer's order (for the supplier). They then make ordering and capacity decisions, respectively.

We focus on the retailer's strategy, as the retailer is solely responsible for forecasting final demand in a non-collaborative context. For a given accuracy level A_R and signal realization ψ_R , the retailer decides on an order quantity, Q . The retailer maximizes Stage 2 expected profit, $\pi_R^{non}(Q; A_R, \psi_R) = \mathbb{E}[r \min(Q, D|\psi_R)] - wQ$, where w is the wholesale price at which the retailer acquires the product from the supplier, and the superscript *non* denotes the non-collaborative model.

The order quantity is given by $Q^{non}(\psi_R) = \mathbb{E}[D|\psi_R] + \Phi^{-1}\left(1 - \frac{w}{r}\right)\sqrt{\mathbb{V}[D|\psi_R]}$, where $\mathbb{E}[D|\psi_R] = \psi_R$ and $\mathbb{V}[D|\psi_R] = 1/A_R$. All derivations may be found in the Online Appendix (available at <http://www.mikeshor.com/research/operations/ORL2015Appendix.pdf>). Substituting, the retailer's expected profit for a given signal realization, ψ_R is given by

$$\pi_R^{non}(Q^{non}; A_R, \psi_R) = (r - w)\psi_R - \frac{x_R}{\sqrt{A_R}}, \quad (1)$$

where $x_R \doteq r\phi\left(\Phi^{-1}\left(1 - \frac{w}{r}\right)\right)$ is the cost of uncertainty and can be interpreted as the retailer's loss per unit of standard deviation. The first term in the profit expression is the profit in the absence of uncertainty and the second term is the loss due to uncertainty.

At stage 1, the retailer invests in forecasting to maximize its expected profit over all possible signals, which is $\Pi_R^{non}(A_R) = \mathbb{E}_{\psi_R} [\pi_R^n(Q^{non}; \psi_R, A_R)] - \kappa A_R^q = (r - w)\mu - \frac{x_R}{\sqrt{A_R}} - \kappa A_R^q$. Let A_R^{non} be the retailer's optimal accuracy in the non-collaborative scenario.

Lemma 1. *In the non-collaborative model, the accuracy of the forecast is $A_R^{non} = \left(\frac{x_R}{2\kappa}\right)^{\frac{2}{1+2q}}$.*

The retailer's optimal accuracy depends on the loss per unit of standard deviation, x_R , which is symmetric and obtains its maximum at $w = \frac{r}{2}$. To see why, note that the retailer's order quantity trades off the risk of ordering too much (at a cost of w per unit) with the opportunity cost of lost sales resulting from ordering too little (at a cost of $r - w$ per unit). When w is very low relative to r , over-ordering is relatively cheap, which is reflected in the optimal order quantity that is much larger than the signal. Since it is inexpensive for the retailer to over-order, the cost of uncertainty is low. When w is close to r , inventory is expensive and the optimal ordering quantity is much smaller than the signal. Here, too, as the retailer is likely to under-order, reducing uncertainty has little value in improving profits. In cases where the cost of over- and under-ordering are similar, the retailer's order is close to his expected demand, and therefore an accurate signal is most valuable.

In sum, the retailer has two levers to manage demand uncertainty: investment in forecasting, which directly reduces the uncertainty, and adjustment of order quantity, which mitigates the impact of uncertainty. At the extremes, the retailer finds it more profitable to adjust the inventory level than to invest in a more precise forecast of demand. The retailer's incentive to invest in forecasting is maximized when the costs of over- and under-ordering are equal ($w = \frac{r}{2}$). This interplay between the two managerial levers of forecast accuracy and order quantity will play a central role in our insights in this paper.

4. Collaborative forecasting

We assume that in the collaborative supply chain, the supplier and the retailer engage in centralized decision making. The central decision maker selects forecasting investments and pools the demand signals to form a single shared demand forecast (see [1–3]). Based on this single shared forecast, the central decision maker determines the order quantity Q that maximizes expected supply chain profit and sets $K = Q$ so that the cost of capacity-order mismatch is eliminated.

For given signal realizations ψ_S and ψ_R and accuracy levels A_S and A_R , the central decision maker decides on order quantity Q that maximizes the expected Stage 2 supply chain profit $\pi_{SC}(Q; \psi_S, \psi_R, A_S, A_R) = \mathbb{E}[r \min(Q, D|\psi_S, \psi_R)] - cQ$. The optimal order quantity is given by $Q^{col}(\psi_S, \psi_R) = \mathbb{E}[D|\psi_S, \psi_R] + \Phi^{-1}\left(1 - \frac{c}{r}\right)\sqrt{\mathbb{V}[D|\psi_S, \psi_R]}$, where $\mathbb{E}[D|\psi_S, \psi_R]$ is a convex combination of the two signals, $\mathbb{V}[D|\psi_S, \psi_R] = \frac{1-\rho^2}{A_R+A_S-2\rho\sqrt{A_R A_S}}$, and the

superscript *col* is used to denote the collaborative model. Substituting, the expected supply chain profit at Stage 1 (denoted by Π_{SC}) as a function of both accuracy levels is given by

$$\begin{aligned} \Pi_{SC}(A_R, A_S) = & (r - c)\mu - x_J \sqrt{\frac{1 - \rho^2}{A_R + A_S - 2\rho\sqrt{A_R A_S}}} \\ & - \kappa A_R^q - \kappa A_S^q, \end{aligned} \quad (2)$$

where $x_J = r\phi(\Phi^{-1}(1 - \frac{c}{r}))$.

The central decision maker then solves $\max_{A_R, A_S} \Pi_{SC}(A_S, A_R)$ to obtain A_S^{col} and A_R^{col} . Let A_J^{col} denote the accuracy of the demand forecast using A_S^{col} and A_R^{col} . Depending on the underlying parameters, the following lemma demonstrates that maximizing supply chain profit involves either *joint* forecasting investment, characterized by equivalent contributions to the accuracies of the supplier and retailer, or *targeted* forecasting investment, in which only one of the two signals is informative.

Lemma 2. (i) If $1 + \rho^{2q} > 2^{1-q}(1 + \rho)^q$, then joint investment is optimal, with $A_R^{col} = A_S^{col} = \left(\frac{x_J}{2\kappa} \left(\frac{\sqrt{1+\rho}}{2\sqrt{2}}\right)\right)^{\frac{2}{1+2q}}$ and $A_J^{col} = \frac{2}{1+\rho} \left(\frac{x_J}{2\kappa} \left(\frac{\sqrt{1+\rho}}{2\sqrt{2}}\right)\right)^{\frac{2}{1+2q}}$.

(ii) Otherwise, targeted investment is optimal with either A_R^{col} or $A_S^{col} = \left(\frac{x_J}{2\kappa} \left(\frac{1}{1+\rho^{2q}}\right)\right)^{\frac{2}{1+2q}}$ and $A_J^{col} = \left(\frac{x_J}{2\kappa} \left(\frac{1}{1+\rho^{2q}}\right)\right)^{\frac{2}{1+2q}}$.

Paralleling Lemma 1, the optimal forecast accuracy is a function of x_J , which is maximized at $c = \frac{r}{2}$. Note that for a concave cost function ($q < 1$), an extra unit of overall accuracy is obtained at lower cost by incrementing whichever firm's accuracy level is higher, and thus targeted investment is always optimal under concave costs. For convex costs ($q > 1$), an extra unit of forecast accuracy is less expensively obtained by raising the lower accuracy level, but this is attenuated by the degree of correlation. When signals are highly correlated, a positive level of investment in both signals is redundant. Targeted investment is optimal under convex costs provided that the correlation of the demand signals is sufficiently high.

5. Forecast accuracy comparison

A common expectation among managers is that the accuracy of demand forecasts will increase as suppliers and retailers collaborate [5]. In this section we assess this idea by comparing the accuracies of the collaborative and non-collaborative models derived in the preceding section. As shown in the following proposition, we find that the forecast accuracy of the non-collaborative approach exceeds that of the collaborative approach over a wide range of the parameter space.

Proposition 1. $A_R^{non} > A_J^{col}$ iff

$$\frac{x_J}{x_R} = \frac{\phi(\Phi^{-1}(1 - \frac{c}{r}))}{\phi(\Phi^{-1}(1 - \frac{w}{r}))} < \min\{2^{1-q}(1 + \rho)^q, (1 + \rho^{2q})\}.$$

Certainly, for signals of a given accuracy, better estimates are obtained by using both signals rather than one, but this does not account for the change in the central decision maker's incentives to make forecasting investments (which must be traded off against the alternative of adjusting order quantities) when forecasts will be combined. Proposition 1 establishes that even a fully-coordinated supply chain may have a poorer demand forecast than what would be obtained without any coordination or information sharing.

The relative accuracy of demand forecasts depends on the ratio of x_J to x_R . Since the condition in the proposition is perhaps not very intuitive, we express it in the following corollary in terms of the cost of production c , the retail price r , and the wholesale price, w .

Corollary 1.1. i. If targeted investment is optimal in the collaborative supply chain and if $\frac{w}{r} + \frac{c}{r} < 1$, then $A_R^{non} > A_J^{col} \forall \rho$ and q .
ii. If joint investment is optimal in the collaborative supply chain and if $(1 - \frac{c}{r})\frac{c}{r} \approx \beta(1 - \frac{w}{r})\frac{w}{r}$ where $\beta = 2^{1-q}(1 + \rho)^q$, then $A_R^{non} > A_J^{col}$.
iii. If $\frac{w}{r} + \frac{c}{r} \approx 1 + \frac{wc}{r(2w-c)}$, then $A_R^{non} < A_J^{col} \forall \rho$ and q .

The corollary presents conditions under which the accuracy of the demand forecast is higher in the non-collaborative model. The first part of the corollary considers the case where targeted investment is optimal in the collaborative context (left panel of Fig. 1). In this case, both collaborative and non-collaborative supply chains estimate final demand from a single informative signal. Yet, the retailer invests more in forecast accuracy in the non-collaborative model, for the following reasons. Recall that the forecast accuracy of the profit-maximizing solution is symmetric around $w = \frac{r}{2}$ in the non-collaborative case, around $c = \frac{r}{2}$ in the collaborative case, and that accuracy is maximized at these values, respectively. For $\frac{w}{r} + \frac{c}{r} < 1$, we have $|\frac{r}{2} - w| < |\frac{r}{2} - c|$, that is, the relative costs of under- or over-ordering are more balanced in the non-collaborative model. Hence, the value of uncertainty reduction is larger, and the optimal accuracy is higher.

The second part of the corollary compares the single forecast obtained by the retailer in the non-collaborative model to the pooled forecast in the case where joint forecasting is optimal in the collaborative model. Even though the collaborative approach forms a joint forecast from two signals, these signals are not necessarily of a higher quality than in the non-collaborative model, since the central decision-maker is balancing forecast improvements against her other lever for managing demand uncertainty, order quantity adjustments. The result is that many cases exist where the central decision maker chooses a lower forecast investment, and a better forecast accuracy is obtained from the single (but higher quality) demand signal obtained by the retailer in the non-collaborative case (right panel of Fig. 1).

The third part of the Corollary (using an approximation of the normal distribution in [11]) defines the region where the accuracy of the final demand forecast is always higher in the collaborative model regardless of whether targeted or joint forecasting is optimal (the crescent-shaped region in both panels of Fig. 1). This region exists because when w is very close to r , the optimal forecast accuracy is very low in the non-collaborative model.

A discussion of the parameter w is appropriate at this point. As noted earlier, w is typically the result of a negotiation process that is exogenous to our context in this paper. However, from Fig. 1 we can see that the outcome of this negotiation does have an impact on the relative accuracies of non-collaborative vs. collaborative forecasting. Specifically, in the classic selling-to-newsvendor setting where the manufacturer offers a 'take it or leave it' wholesale price, our main result (that forecast accuracy is higher in the non-collaborative model) is unlikely to hold. The reason is as follows. Since the classic selling-to-newsvendor model focuses on one end of the power spectrum, where the supplier is powerful, the supplier can squeeze retailer margins by setting a very high wholesale price, and this weak position leaves the retailer with little incentive to invest in forecasting. Thus, in this setting the non-collaborative forecast accuracy is low. For example, consider our model with $c = 3$ and $r = 10$. The supplier's optimal wholesale price is approximately $w = 9.8$, and it is not sensitive to the choice of other parameters (K, q , etc.). Given $c = 3, r = 10$, and

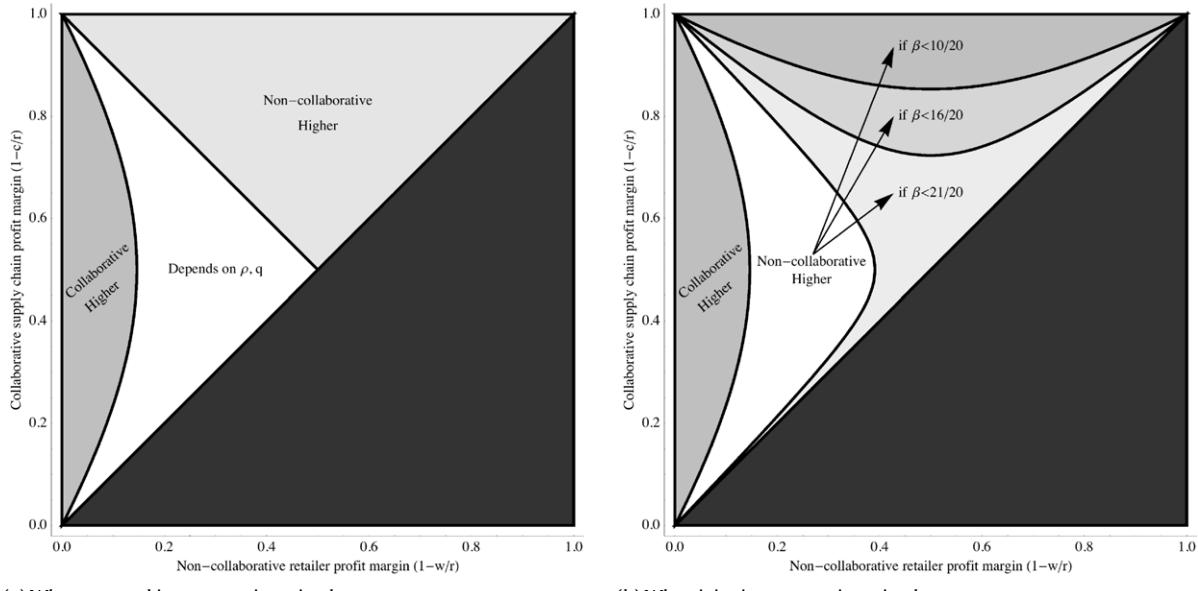


Fig. 1. Comparison of final accuracies in the non-collaborative and collaborative models as a function of $1 - c/r$ (supply chain profit margin in the collaborative case) and w/r (retailer profit margin in the non-collaborative case). The region below the diagonal is not feasible as $c \leq w$. When joint investment is optimal in the collaborative case, the comparison depends on the value of $\beta = 2 \left(\frac{1+\rho}{2} \right)^q$.

$w = 9.8$, from Fig. 1 it is clear that the accuracy in collaborative forecasting is higher compared to the non-collaborative model (i.e., these parameters would fall in the crescent area to the left). On the other hand, when the wholesale price is negotiated rather than dictated by the supplier, our main result would hold for cases where the retailer's margin is reasonable. Although the specific w that is agreed upon in the negotiation depends entirely on the bargaining power of the players, for our purposes it suffices to note that different power structures will yield different w values, and there are many examples in which our result will hold. For example, if the negotiation terms were such that a wholesale price of 7 were agreed upon in the above example, resulting in a more equitable split of the profit margin (i.e. $w = 7$ when $c = 3$ and $r = 10$), then from Fig. 1 we see that forecast accuracy under non-collaboration is in fact higher.

6. Discussion

The results in this paper are driven by the interplay between order quantity and forecasting as two levers to manage demand uncertainty. From a supply chain perspective, the relative attractiveness of these two levers can be understood by considering the role of unit cost (w in the non-collaborative model, c in the collaborative one). First, consider the case in which unit cost is zero. Clearly, in this case it makes sense to ignore forecast accuracy and instead simply use the (free) order quantity lever to manage uncertainty by placing a large order. At the other extreme, when unit cost is equal to r , units are unprofitable and order quantity will be zero regardless of the forecast accuracy. In other words, at both extremes of unit cost, forecasting investment is not justified. The incentive to invest in forecast accuracy is maximized when unit cost is exactly $\frac{r}{2}$. At this point, the costs of over-ordering and under-ordering are the same and thus the supply chain selects an order quantity equal to forecasted demand. The further away unit costs are from this critical value in either direction, the more incentive there will be to manage demand uncertainty using order quantity rather than forecast accuracy.

Our results show that the incentive to invest in forecasting is not necessarily higher in a collaborative context. In fact, as

described in Corollary 1.1 and visualized in Fig. 1, we show that it is quite possible for forecast accuracy in a collaborative supply chain to be lower than in a non-collaborative one. By proving this analytically, we contribute to the academic literature on the impact of centralized supply chain optimization on demand forecasting.

In practice, collaborative forecasting partners anticipate the primary benefit to be improved forecast accuracy [5,10]. Indeed, the sponsor of the CPFR standards cites higher forecast accuracy as the primary value of and key metric for evaluating CPFR [14]. This is based on the belief that two forecasts are better than one, which is certainly true in our model for given levels of forecast investment. However, forecast accuracy is *endogenously* determined by the investment decisions of the parties, which will change from their non-collaborative levels when forecast collaboration is implemented. We find that with collaborative forecasting, final forecast accuracy drops over a large set of parameters. This finding could help explain disappointments such as the one experienced by the Grocery Manufacturers of America, in which two-thirds of member firms initiated some level of CPFR by 2002, with 86% citing the *expected* improvement in forecast accuracy as the primary reason for CPFR, but only a minority reporting improved accuracy as a *realized* benefit [5]. In contrast, our research shows that the final forecast accuracy can be lower with collaborative forecasting, even though profit is always higher. The implication of our results is that forecast accuracy is a misleading measure of the success of collaborative forecasting practices. The reliance on improved forecast accuracy as a success metric may explain why less than 20% of these initiatives proceeded beyond pilot studies [5]. Our research suggests that forecast accuracy should neither be the aim of nor the success criterion for CPFR. Instead, such initiatives should be framed as not necessarily improving forecasting accuracy, which can be a misleading indicator of the benefits of collaborative initiatives such as CPFR. Instead, firms would do well to maintain a focus on profits as the appropriate measure of success.

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