Numerical Methods I

Root finding 2

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Newton-Raphson method

- First described in the work of Sir Isaac Newton as a method for finding the root of polynomial
- Generalized by Thomas Simpson (1740) as an iterative method for solving nonlinear equations using calculus

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Newton method: when

When does it work?

- **We have a first estimate of the root**
- **We can evaluate BOTH the function f and its first derivative at generic points in an interval of the x axis where the root is expected to be**

Newton method: how

How does it work?

- Start with initial estimate of x_0 such that

$$y = f(x_0)$$
$$y' = f'(x_0)$$

- Compute the new estimate x_1 as

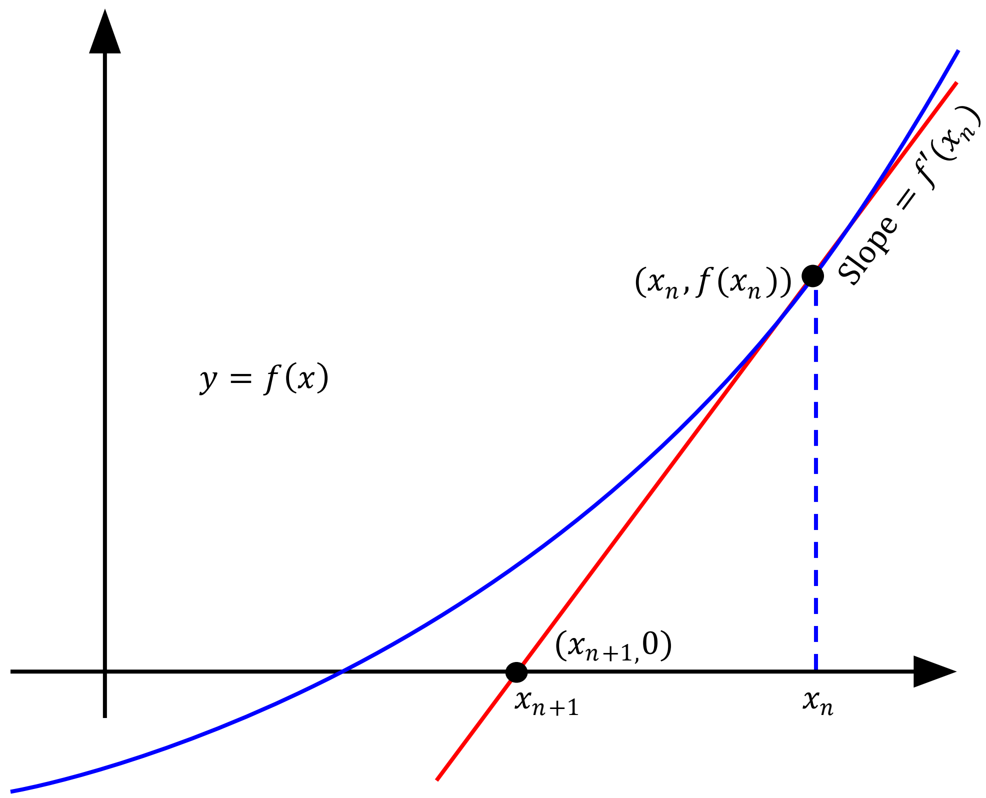
$$x_1 = x_0 - \frac{y}{y'}$$

- Iterate until the distance between x_0 and x_1 is less than ε

Newton algorithm

Given:

- The function f and derivative f′
- The initial estimate x_0
- The error tolerance $\varepsilon > 0$



1) Compute f, f′ in x_0

2) If $(f'(x_0) < \text{tiny}(x_0))$, stop for algorithm failure.

3) Compute $x_1 = x_0 - f(x_0)/f'(x_0)$

4) If $\text{abs}(x_1 - x_0) < \varepsilon$, accept x_1 as the solution, else iterate with $x_0 = x_1$

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Pros and Cons

The advantage of this method is that

- We need only a single estimate x_0
- Fast convergence nearby the root.

However, there are also some disadvantages which are

- It requires the analytical form of both f, f'
- We must place initial estimate near the expected root
- The derivative cannot be zero near the root

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Pathological cases