Numerical Methods I

Root finding 3

Graziano Giuliani

/afs/ictp/public/g/ggiulian/WORLD

The Abdus Salam International Centre for Theoretical Physics

Secant method

- Like most of the root finding problems, origin can be traced back to Babylonians and Egyptian (Rhynd Papyrus)
- Today formulation a modification of the Newton method using a first order approximation to the first derivative

https://www.jstor.org/stable/pdf/10.4169/amer.math.monthly.120.06.500.pdf

Secant method: when

When does it work?

- **We can estimate an interval $[x_0,x_1]$ of the x axis where the root is expected to be**
- **The function f can be evaluated at any generic point in the interval and nowhere inside f(a) == f(b) for (a,b) in $[x_0,x_1]$**

Secant method: how

How does it work?

- Construct the line joining the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$

$$y(x) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_1) + f(x_1)$$

- Find the intersection with the x axis, x_2

$$y = 0 \rightarrow x_2 = x_1 - f(x_1)\frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

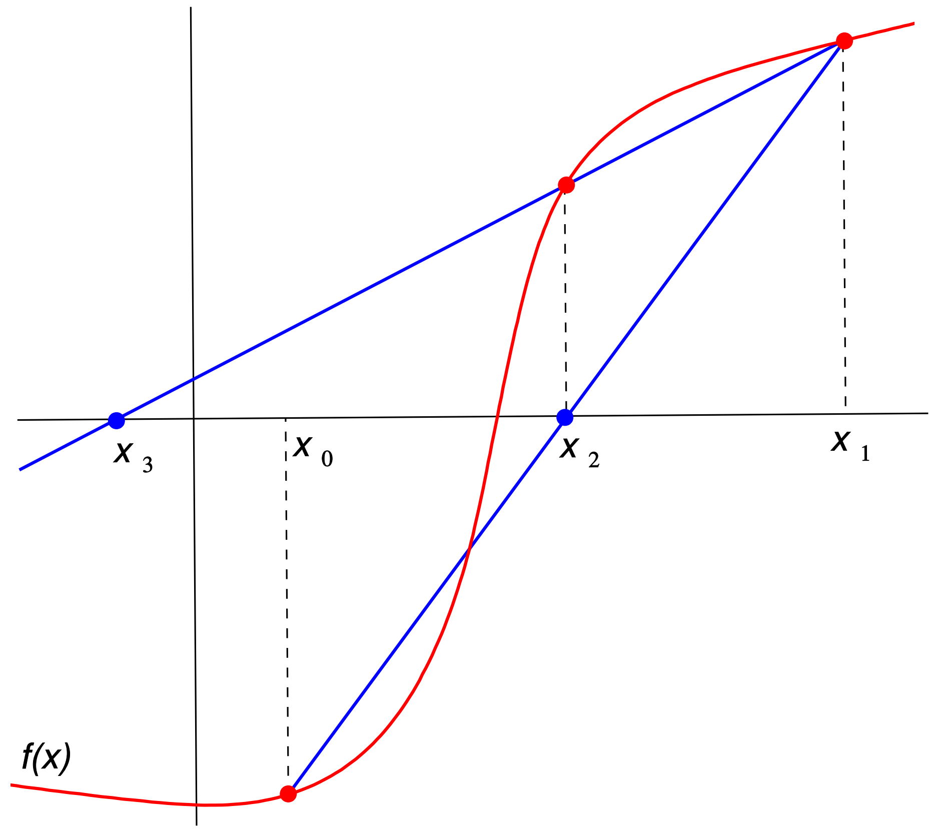
- Repeat the process using now x_1 and x_2 until convergence when the distance less ε

The Abdus Salam International Centre for Theoretical Physics

Secant algorithm

Given:

- The function f
- The initial estimate of x_0 and x_1
- The error tolerance $\varepsilon > 0$



1) Compute f in x_0 and x_1

2) If (abs($f(x_1) - f(x_0)$) < tiny(x_0)) exit for algorithm failure

3) Compute $x_2 = x_1 - f(x_1)\,(x_1 - x_0)/(f(x_1) - f(x_0))$

4) If (abs($x_2 - x_1$) < ε), stop and accept x_2 as root.

5) Set $x_0 = x_1$, $x_1 = x_2$ and iterate from point 1

The Abdus Salam International Centre for Theoretical Physics

Pros and Cons

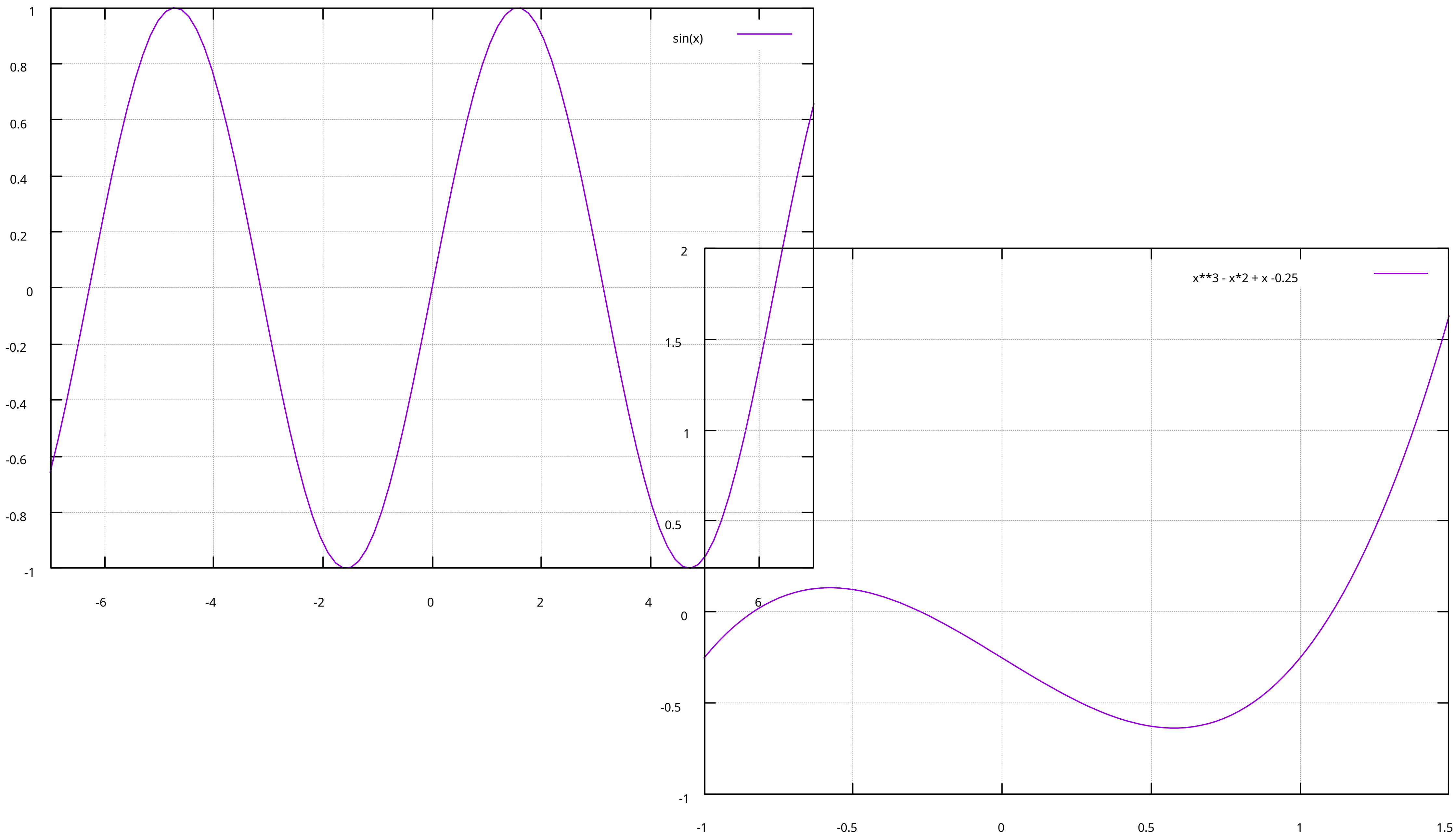
The advantage of this method is that

- The order of the convergence is … the golden ratio. (!)
- It does not require the computation of the derivative f′

However, there are also some disadvantages which are

- We need to bracket the root
- For the algorithm to converge, we need good estimates for the bracketing interval.
- It does not always converge as the bisection or false position and it is not as fast as the Newton method.

Pathological cases



Numerical Methods I

Root finding 3

Graziano Giuliani

/afs/ictp/public/g/ggiulian/WORLD

The Abdus Salam International Centre for Theoretical Physics

Secant method

- Like most of the root finding problems, origin can be traced back to Babylonians and Egyptian (Rhynd Papyrus)
- Today formulation a modification of the Newton method using a first order approximation to the first derivative

https://www.jstor.org/stable/pdf/10.4169/amer.math.monthly.120.06.500.pdf

The Abdus Salam International Centre for Theoretical Physics

Secant method: when

When does it work?

- **We can estimate an interval $[x_0,x_1]$ of the x axis where the root is expected to be**
- **The function f can be evaluated at any generic point in the interval and nowhere inside f(a) == f(b) for (a,b) in $[x_0,x_1]$**

The Abdus Salam International Centre for Theoretical Physics

Secant method: how

How does it work?

- Construct the line joining the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$

$$y(x) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_1) + f(x_1)$$

- Find the intersection with the x axis, x_2

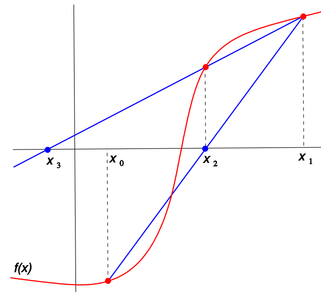
$$y = 0 \rightarrow x_2 = x_1 - f(x_1)\frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

- Repeat the process using now x_1 and x_2 until convergence when the distance less ε

Secant algorithm

Given:

- The function f
- The initial estimate of x_0 and x_1
- The error tolerance $\varepsilon > 0$



1) Compute f in x_0 and x_1

2) If (abs(f(x_1) – f(x_0)) < tiny(x_0)) exit for algorithm failure

3) Compute $x_2 = x_1 - f(x_1)\,(x_1 - x_0)/(f(x_1) - f(x_0))$

4) If (abs(x_2–x_1) < ε), stop and accept x_2 as root.

5) Set $x_0=x_1$, $x_1=x_2$ and iterate from point 1

The Abdus Salam International Centre for Theoretical Physics

Pros and Cons

The advantage of this method is that

- The order of the convergence is … the golden ratio. (!)
- It does not require the computation of the derivative f'

However, there are also some disadvantages which are

- We need to bracket the root
- For the algorithm to converge, we need good estimates for the bracketing interval.
- It does not always converge as the bisection or false position and it is not as fast as the Newton method.

The Abdus Salam International Centre for Theoretical Physics

Pathological cases