Gyroscope

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Abstract

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2.1 Introduction to general rotations

Similar to translations, there are equivalent equations for rotations:

- *Velocity Addition / Angular Velocity Addition*: $\boldsymbol{\omega}_{1,3} = \boldsymbol{\omega}_{1,2} + \boldsymbol{\omega}_{2,3}$
- *Second Newton's Law*: $\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$
- *Momentum / Angular Momentum*: $\mathbf{L} = \int \mathbf{r} \times (\boldsymbol{\omega} \times \mathbf{r}) dm = \mathbf{I}\boldsymbol{\omega}$
- *Mass / Inertia Moment*: $\mathbf{I} = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix}$
- *Kinetic Energy*: $T = \frac{1}{2}\boldsymbol{\omega} \cdot \mathbf{L}$

Thus, one must calculate each component in the inertia moment tensor ($\mathbf{I}$) depending on the coordinate system[1] to compute the net torque ($\boldsymbol{\tau}$). However, there always exists a set of principal axes[2] (see Section 9.3 [**1**]) for which all the off-diagonal components of $\mathbf{I}$ tensor are zero. Thus, I reduces to *diag* $\begin{pmatrix} I_{xx} & I_{yy} & I_{zz} \end{pmatrix}$. There are two important consequences:

- $\mathbf{L}$ is aligned with the net $\boldsymbol{\omega}$, because the principal inertia tensor only scales $\boldsymbol{\omega}$.[3]
- A rigid body *likes* rotating around principal axes (that is, $\boldsymbol{\tau} = \mathbf{0}$).

If one calculated quantities associated with rotations around CM, equations shown below relate those quantities with respect to other origins as discussed in pp. 380-383 [**1**].

- *Angular Momentum*: $\mathbf{L} = M(\mathbf{R} \times \mathbf{V}) + \mathbf{L}_{CM}$
- *Kinetic Energy*: $T = \frac{1}{2}MV^2 + \frac{1}{2}\boldsymbol{\omega} \cdot \mathbf{L}_{CM}$
- *Parallel-axis theorem*: $\mathbf{L} = (\mathbf{I}_R + \mathbf{I}_{CM})\,\boldsymbol{\omega}$

2.2 Precession

In precession, a heavy symmetric top (as shown in figure....) keeps a constant angle θ with the vertical $\hat{z}$ by rotating around this axis with angular velocity $\boldsymbol{\Omega} = \Omega\hat{z}$, and spinning around its symmetry axis $\hat{x_3}$ with angular velocity $\boldsymbol{\omega}_3 = \omega_3\hat{x_3}$. Since a heavy symmetric top is symmetrical around $\hat{x_3}$, this axis is its principal axis. All other axes (e.g., $\hat{x_2}$ and $\hat{x_1}$) orthogonal to $\hat{x_3}$ are also principal if the origin lies on the $\hat{x_3}$ axis (see Section 9.6 and Section 9.3 Theorem 9.5-9.6 [**1**] for more details).

For a system in figure..., the precession frequency Ω and the spinning ω are related by:

$$\Omega_\pm = \frac{I_3\omega_3}{2I\cos(\theta)} \left(1 \pm \sqrt{1 - \frac{4MIgl\cos(\theta)}{I_3^2\omega_3^2}} \right)$$

where

[1] The origin of the coordinate system is arbitrary.
[2] For every origin, there exist a set of principal axes
[3] In general, angular momentum is not collinear with angular velocity.

- I: inertia moment of the heavy symmetric top around $\hat{x_2}$ or $\hat{x_1}$ ($I_1 = I_2 = I$)
- I_3: inertia moment of the heavy symmetric top around $\hat{x_3}$
- M: mass of the top
- l: distance from the pivot to CM of the top

However, in this lab report an approximate relation for large ω_3 is used.

$$\Omega \approx \frac{Mgl}{I_3\omega_3} \tag{1}$$

Then, precession period is

$$T_p = \frac{2\pi}{\Omega} = \frac{2\pi I_3\omega_3}{Mgl} = \frac{4\pi^2 I_3}{MglT_3} \tag{2}$$

where $\omega_3 = \frac{2\pi}{T_3}$.

2.3 Nutation

Nutation is a phenomenon where θ is not constant but it fluctuates around some value with angular velocity ω_n. Since nutation is often coupled with precession, the top precesses in an ellipse around $\hat{z}$ axis. Nutation angular velocity ω_n and θ are given by:

$$\omega_n = \frac{I_3\omega_3}{I} \tag{3}$$

$$\theta(t) = B + (\frac{A}{\omega_n}\sin\theta_0)\cos(\omega_n t + \gamma), \{A, B, \theta_0, \gamma\} \subset \mathbb{R} \tag{4}$$

2.4 Gyroscope Equations

3 Methods

4 Results

5 Discussion

6 Conclusions

References

[1] Morin, D. (n.d.). Introduction to Classical Mechanics. Cambridge University Press.