Outline

- Phenomenological description of ferromagnets: Ginzburg-Landau theory
- Anisotropic ferromagnetic materials: ferromagnetic domains, hysteresis loops, engeneering domain structures
- Origin of domains: energetics of domain walls. Single-domain particles.

Entropy of a partially polarized state

$$\begin{cases} N = N_\uparrow + N_\downarrow \\ \langle s_z \rangle = \dfrac{1}{2}\dfrac{N_\uparrow - N_\downarrow}{N} \end{cases} \Rightarrow \begin{cases} \dfrac{N_\uparrow}{N} = \dfrac{1}{2} + \langle s_z \rangle \\ \dfrac{N_\downarrow}{N} = \dfrac{1}{2} - \langle s_z \rangle \end{cases}$$

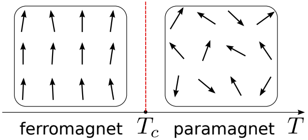
Entropy (relative to the $\langle s_z \rangle = 0$ state)

$$S = k_B \ln \mathbb{P} \qquad \mathbb{P} = \frac{1}{2^N} C_N^{N_\uparrow} = \frac{1}{2^N} \frac{N!}{N_\uparrow!\, N_\downarrow!}$$

If $\langle s_z \rangle \to -\langle s_z \rangle$, then $N_\uparrow \longleftrightarrow N_\downarrow$ and $S = \mathrm{inv}$.
Entropy is thus an even function of $\langle s_z \rangle$, and for $\langle s_z \rangle \to 0$

$$S = a\langle s_z \rangle^2 + b\langle s_z \rangle^4 + \ldots$$

Spontaneous symmetry breaking in magnetic phase transitions



Order parameter:

"Paramagnetic" phase: $\mathbf{M} = 0$

Ferromagnetic phase: $\mathbf{M} \neq 0$

Thermodynamically, "PM" phase is *more* symmetric then FM phase: having finite $\mathbf{M}$ breaks *rotational invariance* and *time-inversion symmetry*.
Indeed, as $t \to -t$, $\mathbf{v} \to -\mathbf{v}$, and ($\mathbf{M} = n_{\rm at}\mathbf{m}$)

$$\mathbf{m} \equiv \frac{e}{2c} \sum_i \mathbf{r}_i \times \mathbf{v}_i \to -\mathbf{m}$$

But the free energy (as a function of the order parameter) must respect these symmetries.

GINZBURG-LANDAU THEORY I

Free energy =

interaction energy − temperature × entropy

For $|T - T_c| \ll T_c$ magnetization is small, $\mathbf{M}^2 \ll 1$

Ginzburg-Landau ansatz: $F = F_0(T) + a\,\mathbf{M}^2 + b\,\mathbf{M}^4 + \dots$

Here $F_0(T) \equiv F(T; \mathbf{M} = 0)$
NB: F and $\mathbf{M}$ are the free energy and magnetization *per unit volume*.

Require that: $\begin{cases} T > T_c & \mathbf{M} = 0 \text{ is stable} \\ T < T_c & \mathbf{M} \neq 0 \text{ is stable} \end{cases} \Rightarrow$ at $T = T_c$: $\begin{cases} a = 0 \\ b > 0 \end{cases}$

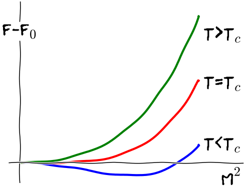
Thus, assume $a = \alpha(T - T_c)$ and $b = \mathrm{const}$.

Ginzburg-Landau theory II

$$F(T) = F_0(T) + \alpha(T - T_c)\,\mathbf{M}^2 + b\,\mathbf{M}^4$$

$$\alpha > 0\,, \qquad b > 0$$

The *phenomenological constants* α and b are input parameters in the GL theory. To fix them, use experiment or microscopic theory.



Equilibrium value of spontaneous magnetization for $T < T_c$:

$$0 = \frac{\partial F}{\partial M} = 2M\left[\alpha(T - T_c) + 2b\,M^2\right]$$

$\Rightarrow M_{\mathrm{eq}} = \sqrt{\alpha/2b}\,(T_c - T)^{1/2}$ —in agreement with microscopics!

Ginzburg-Landau theory III

Magnetic susceptibility for $T > T_c$ (PM phase)

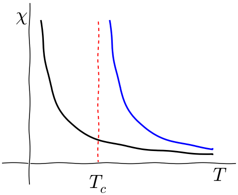
In the PM phase expect $\mathbf{M} \upuparrows \mathbf{H}$ and M small (neglect the M^4 term):

$$\begin{aligned} F &= F_0 - \mathbf{H}\cdot\mathbf{M} + \alpha(T - T_c)\,\mathbf{M}^2 + b\,\mathbf{M}^4 \\ &\approx F_0 - H\,M + \alpha(T - T_c)\,M^2 \end{aligned}$$

$$0 = \frac{\partial F}{\partial M} = -H + 2\alpha(T - T_c)M \qquad \Rightarrow \qquad M = \frac{1}{2\alpha}\frac{H}{T - T_c} \equiv \chi H$$

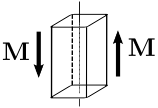
$$\chi_{\text{ferro}}(T > T_c) \propto \frac{1}{T - T_c}$$

$$\chi_{\text{Curie}} \propto \frac{1}{T}$$



Anisotropy of ferromagnetic materials

Crystals have lattice symmetry (not rotationally invariant)

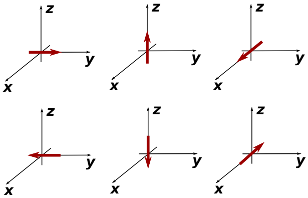


$$F - F_0 = \alpha(T - T_c)\,\mathbf{M}^2 + b\,\mathbf{M}^4 + \kappa M_z^2$$

$\kappa < 0$: easy axis $\kappa > 0$: easy plane

($\kappa \neq 0$ due to *eg* spin-orbit coupling)

Cubic crystals (*eg*, Fe): several anisotropy axes

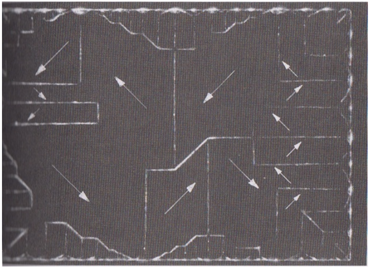


$$F - F_0 = \alpha(T - T_c)\,\mathbf{M}^2 + b\,\mathbf{M}^4 + \kappa\left(M_x^4 + M_y^4 + M_z^4\right)$$

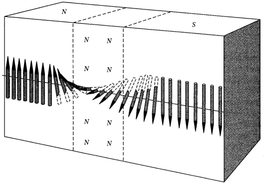
(since $M_x^2 + M_y^2 + M_z^2 = \mathbf{M}^2$)

Ferromagnetic domains

Upon cooling a ferromagnet, spontaneous magnetization forms below T_c.



Since there are several energetically equivalent magnetization directions, specimen splits into *domains*: distant parts of the crystal are essentially independent, and phase transition develops simultaneously.

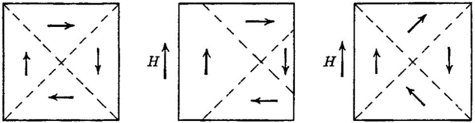


Domain walls (DWs)—the boundaries between domains—are *topological defects*: they cannot be eliminated by a smooth local changes of $\mathbf{M}(\mathbf{r})$.

Domain walls

DWs are topological defects. The way of re-magnetizing the sample is to move the DWs so that unfavorably oriented domains shrink [$\Leftrightarrow$ domain walls annihilate.]

In a field, magnetization can proceed by two mechanisms: DW displacement and/or domain rotation (usually: strong fields).



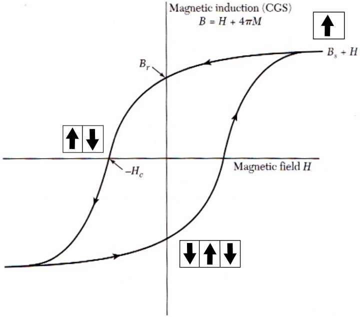
Unmagnetized

Magnetized by domain growth (boundary displacement)

Magnetized by domain rotation

Hysteresis loop

Crystalline defects pin the DWs. $\Rightarrow$ moving the DWs dissipates energy. The area of the hysteresis loop (in the M-H) plane gives the total work agains the friction due to defects.



Coercivity H_c: reduces the induction B to zero starting from saturation B_s . Or, H_{ci}: reduces magnetization to zero starting from saturation M_s . Remanence B_r: the value of B at $H = 0$.

Representative values of T_c and M_s

	Magnetization M_s, gauss			
substance	$(T = 300\,\mathrm{K})$	$(T = 0\,\mathrm{K})$	T_c, K	T_{melt}, K
Fe	1707	1740	1043	1810
Co	1400	1446	1388	1768
Ni	485	510	627	1726
MnAs	670	870	318	1209

Halbach arrays

J.C. Mallinson, 1973; K. Halbach, 1980: "one-sided" magnetic flux structures

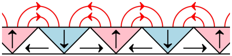


image (c) Wikipedia

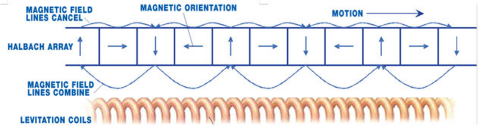


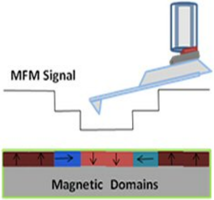
Figure 1. A Basic Halbach Array

image (c) NASA

Used in

- fridge magnets
- focusing electron beams in accelerators
- maglev levitating trains (google for "inductrack")
- axial magnetic bearings (NASA)

Magnetic force microscopy (MFM)



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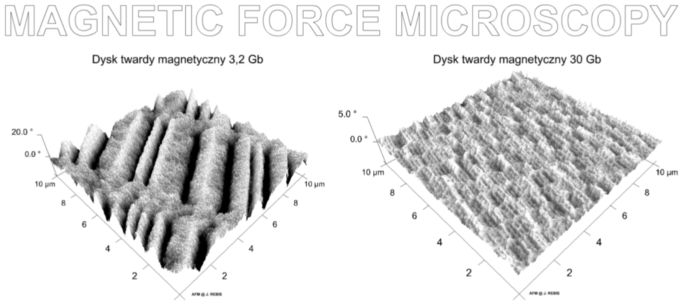
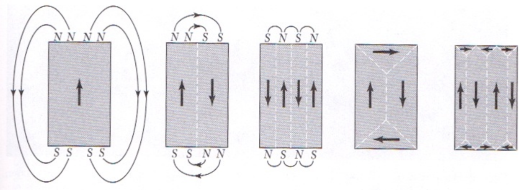


Origin of domains

Domain structure is a consequence of an interplay between several contributions to the energy— exchange, anisotropy and magnetic — of a magnetic body.

Magnetic energy



magn energy

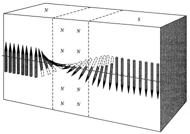
$$= \frac{1}{8\pi} \int B^2 \mathrm{d}\Omega$$

Magnetic energy of a sample with N domains is roughly $1/N$ of an energy of a saturated sample. For two rightmost sketches: magnetic energy is zero due to the *domains of closure* .

Origin of domains: exchange energy

expensive

Bloch wall: spins tilt out-of-plane



much cheaper

$$\varepsilon_{12} = -J\mathbf{S}_1 \cdot \mathbf{S}_2 = -JS^2 \cos\varphi \approx -JS^2 \left(1 - \varphi^2/2\right)$$

$\Rightarrow$ energy cost per pair $w_{\mathrm{ex}} = JS^2\varphi^2/2$

For $N \gg 1$ spins along a chain: $\varphi = \pi/N$

$$Nw_{\mathrm{ex}} = NJS^2(\pi/N)^2/2 = (\pi^2/2)JS^2/N \to 0 \qquad \text{as} \quad N \to \infty$$

The wall would thicken without limit. Limiting the thickness is the *anisotropy energy*.

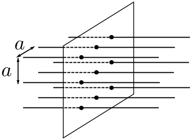
Origin of domains: Anisotropy energy

As a rough estimate, the anisotropy energy is proportional to the number of spins tilted away from an easy axis:

$$U_{\rm anis} \sim K \times \text{area} \times \text{wall thickness}$$

Here K has units of energy per unit volume, $\mathrm{erg/cm^3}$.

Consider a wall parallel to the face of a sc lattice and separating the domains of the opposite magnetization.



Energy per unit area: $\sigma_{\rm wall} = \sigma_{\rm exch} + \sigma_{\rm anis}$

$$\sigma_{\rm anis} \approx KNa$$

$$\sigma_{\rm exch} = Nw_{\rm ex} \times 1/a^2 = (\pi^2/2)JS^2/N \times 1/a^2$$

(there are $1/a^2$ chains per unit area)

Origin of domains: Anisotropy energy II

The wall energy per unit area:

$$\sigma_{\mathrm{wall}} = \frac{\pi^2}{2}\frac{JS^2}{a^2}\frac{1}{N} + KNa$$

Minimize σ_{wall} with respect to N:

$$0 = \frac{\partial \sigma_{\mathrm{wall}}}{\partial N} = -\frac{\pi^2}{2}\frac{JS^2}{a^2}\frac{1}{N^2} + Ka$$

At a minimim

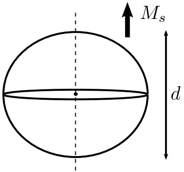
$$N = \left(\frac{\pi^2}{2}\frac{JS^2}{Ka^3}\right)^{1/2} \qquad \sigma_{\mathrm{wall}} = \left(2\pi^2\frac{JS^2K}{a}\right)^{1/2}$$

Representative values: Fe: $N \approx 300$ $\qquad \sigma_{\mathrm{wall}} \approx 1\ \mathrm{erg/cm^2}$
Co: $\sigma_{\mathrm{wall}} \approx 3\ \mathrm{erg/cm^2}$ (due to higher anisotropy const)

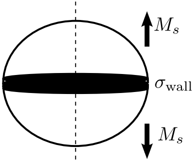
Single-domain particles

Consider a small spherical particle of a uniaxial ferromagnet.

Single domain:



Two domains, a DW in the equatorial plane:



As a *very* rough estimate: magnetic energy $\sim M_s^2 \times$ volume $\sim M_s^2\, d^3$

wall energy $\sim \sigma_{\rm wall} \times$ area $\sim \sigma_{\rm wall} d^2$

Two-domain structure has twice lower magn energy.

$\Rightarrow$ the *critical* diameter is s.t. $M_s^2 d_c^3 \sim \sigma_{\rm wall} d_c^2$.

For $d < d_c = \sigma_{\rm wall}/M_s^2$ single domain is favourable.