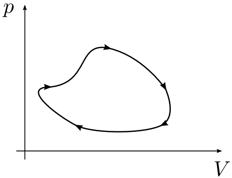
Overview

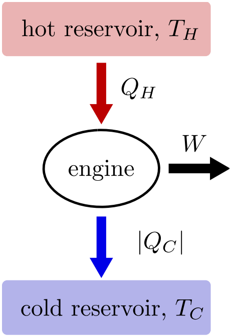
- Review: Thermal engines and refrigerators
- Carnot engine
- Kelvin temperature scale
- Real-world engines and refrigerators

Heat engines

A heat engine operates using a *working fluid*, which undergoes a cyclic process.

$$\Delta U = 0 \qquad \Rightarrow \qquad Q_{\mathrm{net}} = W_{\mathrm{net}}$$



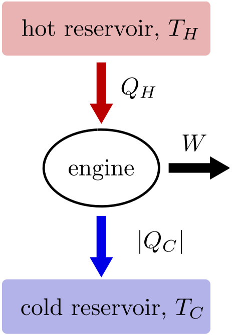


- *Hot reservoir:* a heat source, transfers to the working fluid Q_H of heat per cycle
- *Cold reservoir:* absorbs rejected heat, Q_C per cycle

Sign convention: Q_H and Q_C are quantities of heat transferred *to* the working fluid:

$$Q_H > 0 \qquad Q_C < 0$$

Heat engines



Net heat absorbed (by the engine), per cycle

$$Q_{\text{net}} = Q_H + Q_C = Q_H - |Q_C|$$

First law of thermodynamics:

$$W = Q_{\text{net}} = Q_H - |Q_C|$$

NB: The heat rejected to a cold reservoir is gone for good.

Thermal efficiency of an engine:

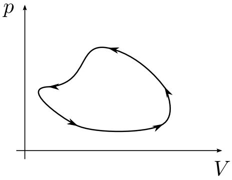
$$e = \frac{\text{what we get}}{\text{what we pay for}}$$

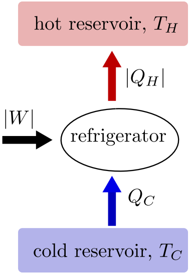
$$= \frac{W}{Q_H} = 1 - \frac{|Q_C|}{Q_H}$$

Refrigerators

The cycle of the working fluid is run backwards, so that

$$W < 0$$





Each cycle, a refrigerator

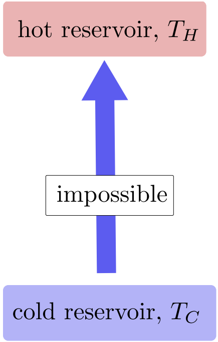
- absorbs $Q_C > 0$ of heat from the cold reservoir
- and rejects $|Q_H|$ of heat to the hot reservoir ($Q_H < 0$)

Which requires a net *input* of work.

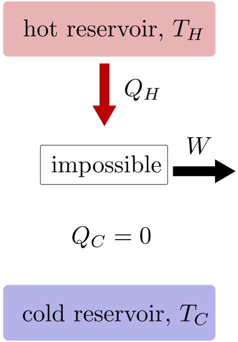
Additional problem: Given a heat machine operating with efficiency e, find the coefficient of performance of a corresponding refrigerator.

Second law of thermodynamics

“Refrigerator” formulation:



“Engine” formulation:



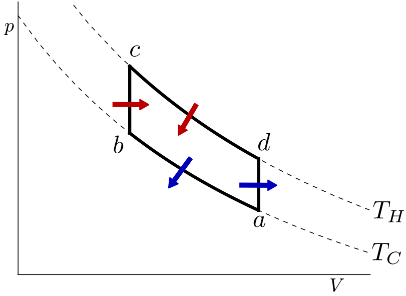
Closed system statement:

The entropy of a *closed system* does not decrease with time.

$$\Delta S \geqslant 0 \qquad \text{(for a closed system)}$$

Stirling engine

A cycle of an (idealized) Stirling engine:



$a \to b$: isothermal compression
$b \to c$: isochoric heating
$c \to d$: isothermal expansion
$d \to a$: isochoric cooling

More information:

Young and Freedman, Problem 20.52

`http://www.robertstirlingengine.com`

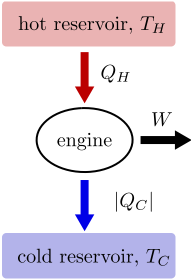
`http://www.stirlingengine.co.uk/ks90t-black-twin-ltd-476-p.asp`

Maximizing the efficiency of an engine

The Second Law prohibits having a 100% efficiency.
Q: What is the maximum efficiency attainable?

Carnot cycle

To minimize entropy production, need to avoid irreversible processes. Heat transfer over a finite temperature difference is irreversible, $\Rightarrow$ need to avoid that.

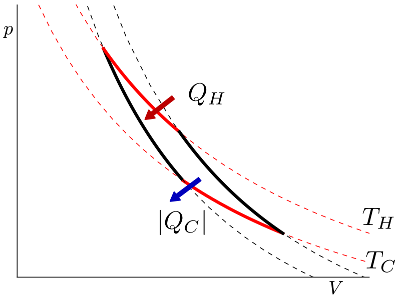


- While absorbing heat from the hot reservoir, the working fluid must have temperature T_H; while transferring heat to the cold reservoir, it must be at T_C
- At intermediate stages of the cycle there must be no heat transfer $\Rightarrow$ adiabatic processes

Carnot engine

A hypothetical, idealized engine; maximum possible efficiency (given T_H and T_C). — developed by Sadi Carnot, 1824

Carnot cycle (plotted for the ideal gas):



- isothermal expansion at T_H, absorbing Q_H of heat
- adiabatic expansion until $T = T_C$
- isothermal compression at T_C, rejecting Q_C of heat
- adiabatic compression back to $T = T_H$

Efficiency of the Carnot engine

First law of thermodynamics: $W = Q_H - |Q_C|$. Hence

$$e = \frac{W}{Q_H} = 1 - \frac{|Q_C|}{Q_H}$$

The cycle is reversible, hence

$$\begin{aligned} 0 &= \Delta S \qquad \text{(entropy change of the working fluid)} \\ &= \frac{Q_H}{T_H} - \frac{|Q_C|}{T_C} \qquad \text{(heat transfers are isothermal)} \end{aligned}$$

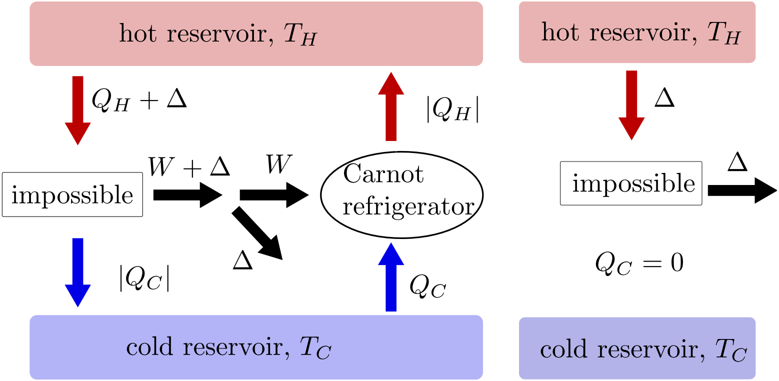
Therefore, $\dfrac{|Q_C|}{Q_H} = \dfrac{T_C}{T_H}$, and

$$e = 1 - \frac{T_C}{T_H} \qquad \text{(efficiency of the Carnot engine)}$$

(An alternative derivation: Young and Freedman, Chapter 20.6)

Efficiency of the Carnot engine

No machine operating between two temperatures T_H and T_C can be more efficient than a Carnot machine operating between the same temperatures.



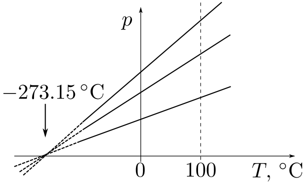
Efficiency of the Carnot engine

- *No engine can be more efficient than the Carnot engine operating between the same two temperatures* — one more equivalent formulation of the Second law of thermodynamics
- Efficiency of the Carnot engine does not depend on the working fluid used.

Use the latter fact for establishing the *absolute temperature scale*.

(Four weeks ago) Gas thermometers

Take a fixed-volume vessel of a gas. At constant volume, the pressure, p, increases with temperature, T.
Measure the pressure: A mixture of ice and water gives $0\,^\circ$C, a mixture of water and steam gives $100\,^\circ$C. Draw a straight line between the two. Repeat for different quantities of different gases.



Experimentally: extrapolated to zero pressure, all curves cross at $T \approx -273.15\,^\circ$C.

Use this fact for establishing the *Kelvin temperature scale* a.k.a. *absolute temperature scale*.

Absolute (Kelvin) temperature scale

NB: Fixed-volume gas thermometers rely on the ideal gas law ($p \propto T$)

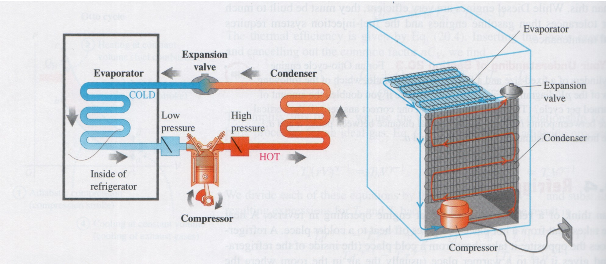
Use instead the Carnot engine to establish the temperature scale: *Define* the ratio of temperatures

$$\frac{T_H}{T_C} = -\frac{Q_C}{Q_H}$$

Since the efficiency of the Carnot engine does not depend on the working substance, neither does the temperature ratio defined.

Complete the definition of the Kelvin scale by assigning the (arbitrary) value of 273.16 K to the triple point of water.

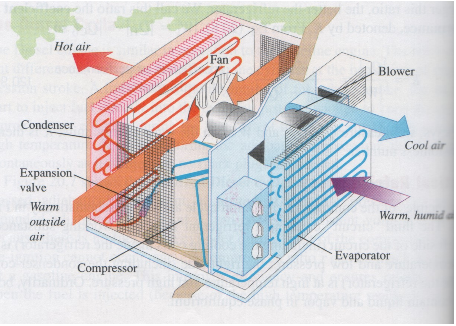
Real-world refrigerators



Compressor takes in the working fluid, compresses it adiabatically, and delivers into the *condenser* coil. The fluid has T higher than that of the air, it cools down, partially liquefies, and gives off heat Q_H. Then it expands (adiabatically) into the *evaporator* coil, where it absorbs heat Q_C, partially vaporizes, and enters the compressor again.

Air conditioners

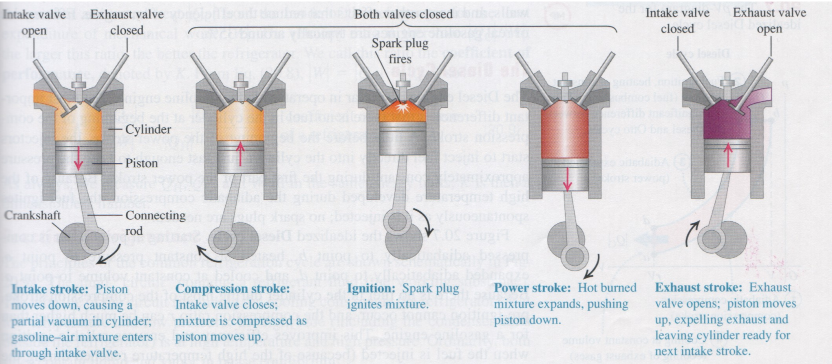
Here the refrigerator box is the room to be cooled. Evaporator coils are inside, condenser coils are outside.



Q: What if an air conditioner is flipped: evaporator coils are outside, condenser coils are inside?
A: It's then a *heat pump*: can be used to heat buildings by cooling the outside air.

Internal combustion engines

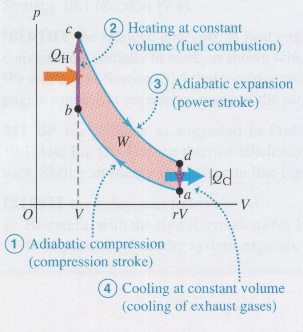
Working cycle of a typical four-stroke internal combustion engine:



intake—compression—ignition—power stroke—exhaust stroke

Otto cycle

An idealized cycle of an internal combustion engine:



Efficiency is governed by the *expansion ratio* r. Treating the air/fuel mixture as an ideal gas ($\gamma = C_p/C_V$),

$$e = 1 - \frac{1}{r^{\gamma-1}}$$

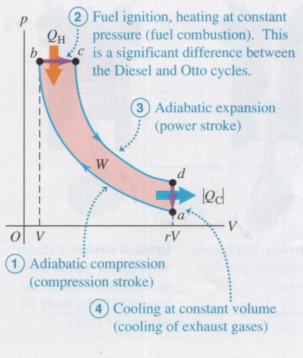
[for derivation see Young and Freedman Chap. 20.3]

For air (diatomic gas) $\gamma = 7/5$. Taking $r = 10$, efficiency would be $e \approx 60\%$. For real engines: it's about 30%.

Increasing r is limited by *detonation* of the mixture.

Diesel cycle

The difference to the Otto cycle is that there is no fuel in the cylinder during the compression stroke.



During the compression stroke, the air is being compressed; the fuel is injected fast enough so that $p \approx \mathrm{const}$ during the first part of the power stroke. Ignition is spontaneous due to the high temperatures developed during the compression stroke.

This way, values of $r = 15$–20 are reachable.