











### Explaining Multivariate Time Series

# Temporal Feature Selection

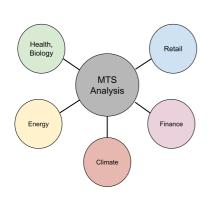
- 1. Context
- 2. Post-hoc xAI
- 3. Causality in MTS
- 4. Problem setting
- 5. Experiment setting
- 6. Results

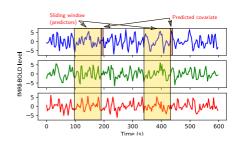
- 7. Context
- 8. Algorithm
- 9. Experiments
- 10. Baselines





### **Context - Multivariate Time Series (MTS)**

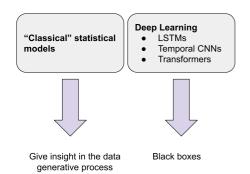




- MTS: Sequences of measured values at equally spaced instants
- Prediction task: classify/forecast next value of a variable given previous values over a window.



# Motivation - Explaining MTS modeling



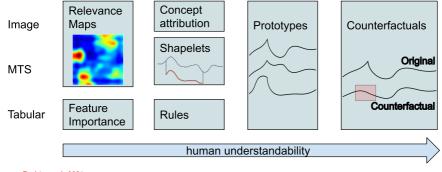
How does a model make decision?
 Nauta et al. 2023

Faithfulness Continuity Completeness

 How does the model relate to the data generative process? Nauta et al. 2023; Ismail et al. 2020



# Post-Hoc Deep Learning Explicability



see Bodria et al. 2021

Figure: Most common types of explanations



### Relevance attribution maps

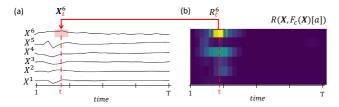


Figure: (a) Time series input, (b) Relevance Map

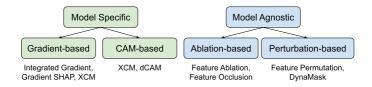


Figure: Nomenclature of relevance map methods



### **Causality in Multivariate Time Series**

Multivariate time series as **Stochastic processes**:  $X_t^i$  is a (probabilistic) function of previous values  $X_{t-}$ .

Cause, Effect: A causes B iff (A,B) is an

edge in the graph.

### Consistency through time

The generative function and causal graph are identical at all timesteps.

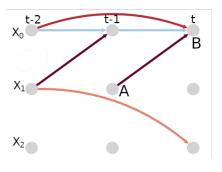


Figure: Example of a causal graph



### **Sufficiency of Causal Explanations**

Causation and Correlation can be distinguished (direct link vs indirect link).

# Sufficiency of the measured covariates

All variables that influence the process are observed.

### Causal Explanations

All observed information is contained in the set of causes.

The generative process is entirely explained by the causes: hence we speak of **causal explanations**.

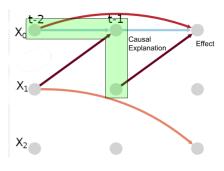


Figure: Example of a causal graph



# Key questions of our empirical study

How do relevance map post-hoc xAI methods compare to causal explanations?

- I) To what extent do xAI methods explain the same way different samples in a same MTS?
- II) How much do causal explanations and relevance maps overlap?
- III) We build surrogate models restricted to the most relevant / causal features. Which has the highest predictive performance?

03/07/2023



# **Experimental setting - Data, Models**

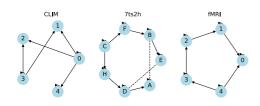


Figure: Exemple of causal graph

Dataset	Instances	Variables	Timestamps	Avg in-degree	Max lag
CLIM Runge et al. 2020	200	5	250	2.0 to 4.4	2
7ts2h Assaad, Devijver, and Gaussier 2022	10	7	4000	1.8	1
fMRI Huang and Kleinberg 2015	17	5	200 to 5000	2.0 to 2.6	1

#### Data properties:

CLIM: climate, linear

• 7ts2h: nonlinear

• fMRI: health, nonlinear

Models: we explain trained models above 0.7 AUROC.

	7ts2h	CLIM	fMRI
LSTM	0.912 (52)	0.775 (115)	0.807 (49)
XCM	0.904 (45)	0.820 (45)	0.893 (15)
DCAM	0.858 (38)	0.776 (61)	0.799 (25)
Transformer	0.899 (50)	0.784 (26)	0.915 (16)

Table: AUROC and number of explained models



# **Experimental setting - Metrics**

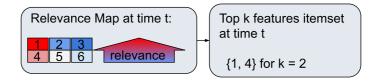


Figure: Extract itemsets

#### Metrics:

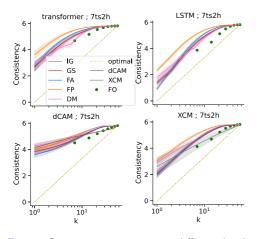
- Consistency is an entropy measure on the ensemble of explanations on many window.
- Precision, Recall of the itemsets compared to the causal explanation

Jacob et al. 2021

AUROC of surrogate models trained over masked data. The non-masked features
are those belonging to 1) the causal explanation or 2) the most frequently relevant
features.



### I - Consistency through time of explanations



#### Observations:

- Explanations become less consistent as we include less relevant terms.
- Smaller models are more consistent, xAl methods have similar behaviours despite individual differences.

Figure: Consistency over time at different levels of k.

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### II - Precision of explanations

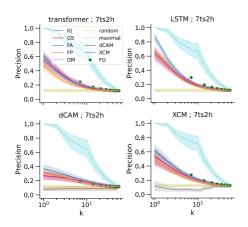


Figure: Precision over the causal graph at different levels of k

#### Observations

- High precision for low k, gets closer to random baseline (dashed yellow) as k increases.
- The xAI methods have similar behaviour.
- Model type affects the precision.

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### II - Recall of explanations

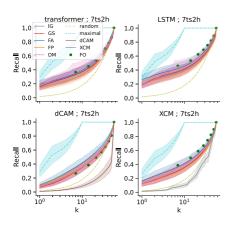


Figure: Recall over the causal graph at different levels of k

#### Observations

- Same observations as those for precision.
- The most salient features ( $k \le 10$ ) miss at least half of the causes (down to 20% in the linear CLIM dataset)



# **III** - Predictive performances of explanations

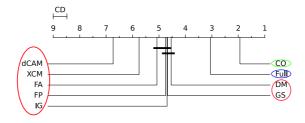


Figure: Critical Difference diagram of the AUROC of surrogate models

- Model specific and agnostic methods perform similarily (aside dCAM and XCM).
- xAI methods underperform compared to Full models.
- Causal explanations obtain significantly higher performances.



### **Conclusions**

First empirical study of explanation quality for relevance attribution maps and causal explanations.

- Post-hoc methods are based on associative mechanisms.
  - Not actionnable (hard to build good surrogate models)
  - Smaller explanation sizes are more consistent and precise.
- In the litterature, trade-off between explanation understandability and explanation performance.
  - Causal notions might go beyond this tradeoff.

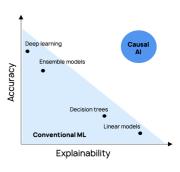


Figure: Explainability tradeoff



### **Future works**

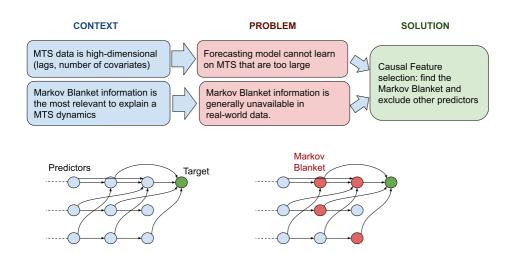
- Extend to other explanation methods (prototypes, conceptual, counterfactuals): we need novel metrics to quantify their quality.
- Extend to other prediction tasks and model types (regression, probabilistic).
- Select appropriate explanation methods depending on the dataset







### **Motivation - Causal feature selection**





### **Context - Traditional approaches**

#### Approaches:

- Filter: Fisher score, Correlation-based, RReliefF
- Information-theory: mRMR selection and/or clustering
- Wrapper: Recursive Feature Elimination
- Embedded: Random Forest gain, LASSO

#### Downsides:

- Rely on MTS vectorization: transform the considered MTS window into a single dimensional vector. This neglects the temporal structure of the data.
- Only filter approaches scale



### **Context - Modern approaches**

### **Deep Learning**

Temporal Fusion Transformer, Dual-stage Attention-based RNN, Neural Feature selector...

#### Downsides:

- Curse of dimensionality (problem of embedded methods)
- In most cases, feature selection weights are applied to extracted features.

#### Causal discovery

PCMCI, SVAR-FCI, Bivariate Granger, SyPI...

#### Downsides:

- Most techniques are not scalable to high dimensions due to combinatorial explosion
- The scalable Bivariate Granger method is a filter method.

Both approaches are generally not tested for MTS with more than 150 covariates.



# **Chronomp:** high level ideas

The Feature selection problem is *combinatorial*.

We build a feature set in a greedy, heuristic way:

- Forward phase heuristic: start from empty itemset, include variables one by one
- Multivariate: select the variable that explains best the residuals of an intermediate model
- Scalable: use correlation to select variables, to avoid building a model for each variable

Similar approach as Tsagris et al. 2022, Q. Wang and Qin 2013



### Algorithm

```
procedure CHRONOMP(MTS X, target variable i, lags L, stopping threshold \alpha) subset of selected features S \leftarrow \{i\} old model M' \leftarrow \text{None} new model M \leftarrow \text{MODEL}(\text{data: } X, \text{ target: } i, \text{ predictors: } S, \text{ lags: } L) repeat r \leftarrow \text{getResiduals}(M) \qquad \qquad \triangleright \text{ vector of size } T - L selected \leftarrow \arg\max_{c \in X \setminus S} \text{ASSOCIATION}(r, X^c, L) S \leftarrow S \cup \{\text{selected}\} M' \leftarrow M M \leftarrow \text{MODEL}(X, i, S, L) until STOPPING-CRITERION(M', M, S, \alpha) return S or S without last selected item depending on stopping criterion
```



### **Algorithm**

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```



### Model component

Auto Regressive Distributed Lags (ARDL) model (Pesaran, Shin, and Smith 2001)

$$y_t = \sum_{j \le p} a_j \cdot y_{t-j} + \sum_{j \le p} B_j \cdot X_{t-j} + \sum_{i \le s} g_i \cdot \mathbb{I}[t = i \mod s] + D \cdot (1, t, t^2) + \epsilon_t$$

 $y_t$  is the predicted covariate,  $y_{t-i}$  its lags,  $X_{t-i}$  the matrix of the other covariates at lag j.

- Linear size in function of lags and covariates
- Ordinary Least Squares estimation (fast)
- Robust to heteroscedasticity of noise
- Robust to autocorrelation

#### Assumptions

- Noise terms are independent
- Stationary, trend-stationary, season-stationary
- Need more observations than there are coefficients in the model (a usual working requirement is 10 times).
- Full rank data
- Finite 4th moment of each covariate



### **Algorithm**

```
procedure CHRONOMP(MTS X, target variable i, lags L, stopping threshold \alpha) subset of selected features S \leftarrow \{i\} old model M' \leftarrow \text{None} new model M \leftarrow \text{MODEL}(\text{data: } X, \text{ target: } i, \text{ predictors: } S, \text{ lags: } L) repeat r \leftarrow \text{getResiduals}(M) \Rightarrow \text{vector of size } T - L selected \leftarrow \text{arg max}_{c \in X \setminus S} \text{ ASSOCIATION}(r, X^c, L) S \leftarrow S \cup \{\text{selected}\} M' \leftarrow M M \leftarrow \text{MODEL}(X, i, S, L) until STOPPING-CRITERION(M', M, S, \alpha) return S or S without last selected item depending on stopping criterion
```



### **Association component**

Principle: measure the correlation between the residuals and lag 1 to  $\it L$  of a covariate. Then, return the maximal correlation, or the minimal p-value.

#### **Pearson Correlation**

- Linear relation
- Interval or ratio-level measurements
- Ideally, data is normally distributed
- No outlier

#### **Spearman Correlation**

- Monotonic relation
- Rank-order measurements
- Distribution-free
- No strong outlier



### **Algorithm**

```
procedure CHRONOMP(MTS X, target variable i, lags L, stopping threshold \alpha) subset of selected features S \leftarrow \{i\} old model M' \leftarrow \text{None} new model M \leftarrow \text{MODEL}(\text{data: } X, \text{target: } i, \text{predictors: } S, \text{lags: } L) repeat r \leftarrow \text{getResiduals}(M) \Rightarrow \text{vector of size } T - L selected \leftarrow \arg\max_{c \in X \setminus S} \text{ASSOCIATION}(r, X^c, L) S \leftarrow S \cup \{selected\} M' \leftarrow M M \leftarrow \text{MODEL}(X, i, S, L) until STOPPING-CRITERION(M', M, S, \alpha) return S or S without last selected item depending on stopping criterion
```



### **Stopping criterion**

```
procedure STOPPING-CRITERION(current model M, previous model M', selected S, stopping threshold \alpha)

if |S| \geq \max-to-select then

Decision \leftarrow Stop

if 10 \times |M| \geq |X| then

Decision \leftarrow Stop

if statTest(M, M').pvalue \leq \alpha then

Decision \leftarrow Continue

else

Decision \leftarrow Stop

return Decision
```

Statistical tests for nested models:

- F-test
- Wald-test
- Likelihood Ratio test



# **Stopping criterion**

F-test	Wald-test	LR-test	
Linear model	Linear model (variant for	Identifiable model by	
	some nonlinear model	Maximum Likelihood	
	exist)	Estimation	
Gaussian noise	Asymptotically normal	Asymptotically normal	
	noise in large samples	noise in large samples	
Independence of noise	Idem	Idem	
between timestamps and			
covariates			

Table: Stopping criterion assumptions



### Synthetic datasets



Figure: Linear dataset VAR10

- 10 (VAR10) 10000 (VAR10000) variables, 3500 timesteps
- Generated by a VAR model
- Up to lag 5 dependencies
- Autoregressive component (lags 1 to 5) plus 5 other direct causes (one lag each, between lag 1 and lag 5).

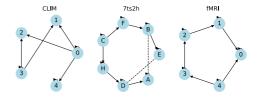


Figure: Causal datasets CLIM, 7ts2h, fMRI

Dataset	Instances	Variables	Timestamps	Avg in-degree	Max lag
CLIM	200	5	250	2.0 to 4.4	2
7ts2h	10	7	4000	1.8	1
fMRI	17	5	200 to 5000	2.0 to 2.6	1

- nonlinearity (7ts2h, fMRI)
- time aggregation (fMRI, CLIM)
- empirical causal graph (CLIM)



### **Metrics**

#### We evaluate:

ullet Predictive performance ( $R^2$ ) of an ARDL model trained on data corresponding to the selected itemset

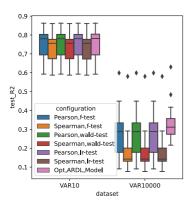
$$R^2=1-rac{{
m sum\ of\ squared\ residuals}}{{
m total\ variance\ of\ the\ predicted\ variable}}$$

- Overlap of the set of selected covariate S with the set of direct causes C:
  - Recall: how much of the direct causes are selected
  - Precision: how much of the selected variables are direct causes.
  - Recall is especially important, as we primarily seek to select all relevant variables.

$$Recall = \frac{|S \cap C|}{C}$$
 Precision =  $\frac{|S \cap C|}{S}$ 



### Results - VAR data - R2



#### Observations

- On linear data, Pearson correlation is better suited to the ARDL model
- The kind of stopping criterion has low impact on the final  $R^2$
- The Pearson correlation configurations are close to the optimal ARDL model (pink boxes)

Figure: R2 of each configuration

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### Results - VAR data - Recall, Precision

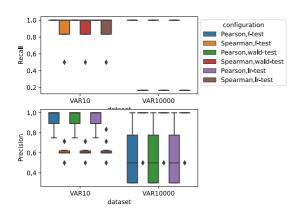


Figure: Recall of each FS configuration

#### Observations

- Pearson configurations select all direct causes
- Spearman misses most causes in large data
- Spearman high precision in VAR10000 indicate early stopping, while Pearson low precision indicate late stopping



### Results - VAR data - Graph building

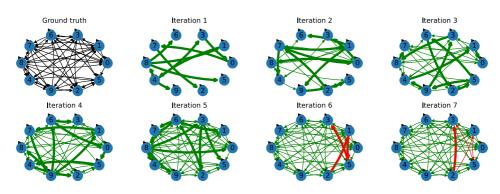
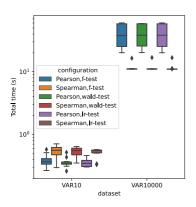


Figure: Evolution of graph built for all 10 variables, at each iteration of the algorithm. Green: direct causes, Red: other covariates, Bold: added at current step

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### Results - VAR data - Time

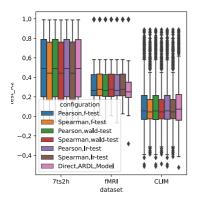


The difference in time spent can be explained by the differing number of iterations. For instance, on VAR10000, Spearman configurations select less variables than Pearson configurations.

Figure: Computing time (s) of each FS configuration



## Results - Causal Discovery data - R2



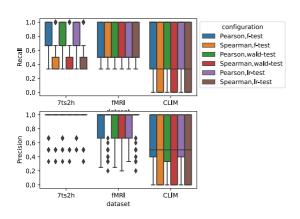
## Figure: R2 of each configuration

#### Observations

- The kind of stopping criterion has low impact on the final R<sup>2</sup>
- Pearson is better on 7ts2h, CLIM, while Spearman on fMRI
- On nonlinear data, direct causes might not necessarily be the best features for a linear model
- The Pearson correlation configurations are closer to the ARDL model built on direct causes (pink boxes)



## Results - Causal discovery data - Recall, Precision



#### Figure: Recall of each FS configuration

#### Observations

- Pearson configurations median recall is 1
- On fMRI, Spearman,f-test and Spearman,wald-test has a small edge in precision



### **Baselines**

Atemporal: Recursive Feature Elimination (Guyon et al. 2002)

- Backward selection method
- Vectorize time series
- Returns pairs (variable, lag)
- User-specified number of features (here set as the number of causes)
- Not scalable (wrapper approach)
- Wrapper FS: multivariate interactions

Choice made: transform the returned itemset from pairs (variable, lag) to singleton (variable,). Compute precision, recall on this itemset, compute  $R^2$  on an ARDL model for this itemset.

Temporal: **Bivariate Granger** (Sun et al. 2015)

### Algorithm:

- Fit VAR models on each pair (target, covariate)
- Test causality from each covariate to the target
- Select covariates that cause the target but not the opposite

### Properties:

- Linear interactions only
- Scalable (linear in number of covariates)
- Filter FS: bivariate interactions only



### **Baselines - Results**

	7ts2h	CLIM	VAR10	fMRI	VAR10000
bg	0.489999	0.102695	0.495436	0.351780	NaN
rfe	0.491199	0.108432	0.660708	0.340141	NaN
chronomp	0.493559	0.105842	0.747287	0.357854	0.273421

Table: Average R2 of ARDL model on selected features

	7ts2h	CLIM	VAR10	fMRI	VAR10000
ground truth	2.4	1.5	6	2.0	6
bg	2.142857	1.644031	1.133333	1.435294	484.70
rfe	1.828571	2.044907	4.000000	1.458824	NaN
chronomp	1.900000	1.685652	6.400000	1.623529	13.35

Table: Average size of selected set

- chronomp dominates on VAR10
- Similar R2 in other small datasets

 BG and RFE aren't practical for VAR10000



## **Baselines - Recall, Precision**

	7ts2h	CLIM	VAR10	fMRI	VAR10000
bg	0.730952	0.651406	0.188889	0.600000	0.575
rfe	0.745238	0.881763	0.666667	0.625490	NaN
chronomp	0.747619	0.670938	1.000000	0.643137	1.000000

Table: Average recall over the direct causes

	7ts2h	CLIM	VAR10	fMRI	VAR10000
bg	0.857381	0.613874	1.000000	0.871569	0.007119
rfe	0.940476	0.735542	1.000000	0.873529	NaN
chronomp	0.926190	0.619095	0.946429	0.849804	0.544552

Table: Average precision over the direct causes

- chronomp dominates on VAR
- chronomp recall higher otherwise

 RFE dominates on CLIM (translating to 3% increase in  $R^2$ )



## **Baselines - Results**

	7ts2h	CLIM	VAR10	fMRI	VAR10000
bg	0.092009	0.017455	0.139232	0.024697	134.649533
rfe	28.721915	0.026837	87.259455	2.465924	NaN
chronomp	0.084844	0.026936	0.385377	0.039340	41.073843

Table: Average time spent (s) in FS phase

#### Observations

- RFE method sklearn implementation scales badly with number of observations
- chronomp takes longer than BG, but stays within similar magnitudes.



### **Conclusion and future works**

#### Main conclusion:

- Promising results (high recall)
- Robust to high numbers of covariates

#### Improvements to be made:

- Include backward phase to increase Precision
- Explore branch-and-bound to create equivalent feature sets
- Use constrained linear models for data parsimony and time efficiency.

#### Experimental setup to put in place:

- Standardize  $R^2$  evaluation with Auto-ML selection of best final model for each FS method.
- Improve dataset diversity (real datasets, large datasets, nonlinear datasets, lag diversity)



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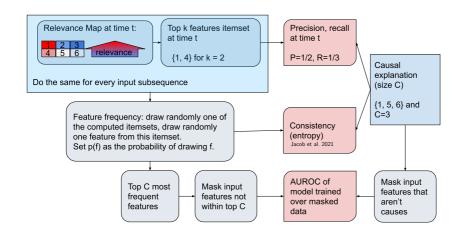
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# Thank you for your attention!

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### More about metrics





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