Applied Multivariate Analysis HW6

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Library

```
library(psych)
library(GPArotation)
library(corrplot)
library(ellipse)
library(nFactors)
```

Data

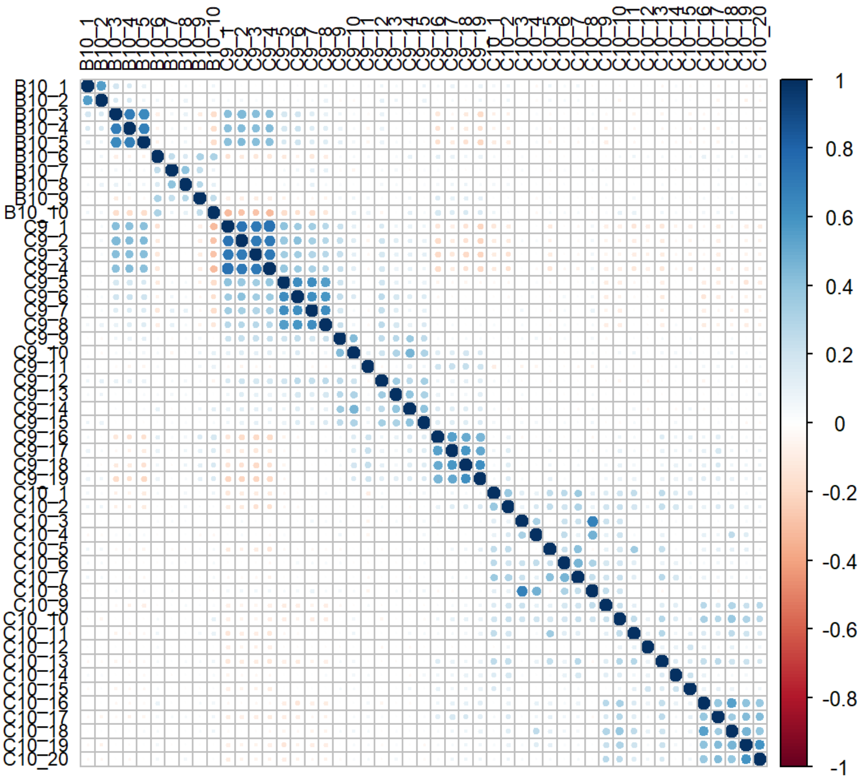
```
data<-read.csv("C:/Users/eva/Desktop/作業 上課資料(清大)/大四下/多變量/hw6/102年臺北10-18歲兒少問卷.csv", header=T)
dat<-with(data, cbind(B10_1, B10_2,B10_3,B10_4,B10_5,B10_6,B10_7,B10_8,B10_9,B10_10,
                 C9_1,C9_2,C9_3,C9_4,C9_5,C9_6,C9_7,C9_8,C9_9,C9_10,C9_11,C9_12,C9_13,C9_14,C9_15,C9_16,C9_17,C9_18,C9
_19,
                 C10_1,C10_2,C10_3,C10_4,C10_5,C10_6,C10_7,C10_8,C10_9,C10_10,C10_11,C10_12,C10_13,C10_14,C10_15,C10_1
6,C10_17,C10_18,C10_19,C10_20))
dat<-as.data.frame(dat)
dat1<-replace(dat, dat==9, 0) #Transform NA value(original=9) to 0
describe(dat1)
```

```
##        vars    n mean   sd median trimmed  mad min max range   skew kurtosis
## B10_1     1 1508 2.36 0.90      2    2.34 1.48   0   4     4  -0.06    -0.67
## B10_2     2 1508 2.45 0.93      3    2.45 1.48   0   4     4  -0.24    -0.63
## B10_3     3 1508 3.16 0.81      3    3.25 1.48   0   4     4  -1.02     1.44
## B10_4     4 1508 3.02 0.87      3    3.12 1.48   0   4     4  -0.91     0.89
## B10_5     5 1508 3.08 0.88      3    3.20 1.48   0   4     4  -1.03     1.09
## B10_6     6 1508 1.83 0.89      2    1.75 1.48   0   4     4   0.59    -0.31
## B10_7     7 1508 3.02 0.84      3    3.11 0.00   0   4     4  -1.05     1.66
## B10_8     8 1508 2.90 0.83      3    2.97 0.00   0   4     4  -0.90     1.28
## B10_9     9 1508 2.16 0.93      2    2.13 1.48   0   4     4   0.11    -0.57
## B10_10   10 1508 1.56 0.76      1    1.43 0.00   0   4     4   1.15     1.12
## C9_1     11 1508 3.28 0.74      3    3.37 1.48   0   4     4  -0.81     0.50
## C9_2     12 1508 3.11 0.86      3    3.18 1.48   0   4     4  -0.55    -0.57
## C9_3     13 1508 3.07 0.91      3    3.14 1.48   0   4     4  -0.54    -0.60
## C9_4     14 1508 3.24 0.85      3    3.34 1.48   0   4     4  -0.89     0.18
## C9_5     15 1508 3.37 0.73      3    3.49 1.48   0   4     4  -1.23     2.21
## C9_6     16 1508 3.17 0.82      3    3.25 1.48   0   4     4  -0.72     0.04
## C9_7     17 1508 3.47 0.73      4    3.61 0.00   0   4     4  -1.50     2.79
## C9_8     18 1508 3.25 0.87      3    3.37 1.48   0   4     4  -1.10     0.91
## C9_9     19 1508 1.98 0.95      2    1.86 1.48   0   4     4   0.72    -0.18
## C9_10    20 1508 1.44 0.79      1    1.26 0.00   0   4     4   1.70     2.47
## C9_11    21 1508 2.22 0.82      2    2.19 0.00   0   4     4   0.30     0.20
## C9_12    22 1508 2.42 0.86      2    2.41 1.48   0   4     4   0.21    -0.31
## C9_13    23 1508 1.78 0.86      2    1.68 1.48   0   4     4   0.80     0.14
## C9_14    24 1508 1.37 0.72      1    1.22 0.00   0   4     4   1.91     3.50
## C9_15    25 1508 1.93 0.95      2    1.82 1.48   0   4     4   0.67    -0.41
## C9_16    26 1508 1.79 0.92      2    1.65 1.48   0   4     4   0.98     0.21
## C9_17    27 1508 1.93 0.93      2    1.82 1.48   0   4     4   0.71    -0.17
## C9_18    28 1508 1.71 0.90      1    1.56 0.00   0   4     4   1.09     0.44
## C9_19    29 1508 1.53 0.79      1    1.37 0.00   0   4     4   1.50     1.84
## C10_1    30 1508 1.09 0.58      1    1.00 0.00   0   5     5   5.96    36.20
## C10_2    31 1508 1.32 0.86      1    1.09 0.00   0   5     5   3.07     9.33
## C10_3    32 1508 1.00 0.09      1    1.00 0.00   0   3     3   0.94   220.48
## C10_4    33 1508 1.00 0.13      1    1.00 0.00   0   5     5  17.38   547.79
## C10_5    34 1508 1.08 0.44      1    1.00 0.00   0   5     5   7.01    55.43
## C10_6    35 1508 1.01 0.20      1    1.00 0.00   0   5     5  16.29   326.88
## C10_7    36 1508 1.02 0.27      1    1.00 0.00   0   5     5  11.37   152.40
## C10_8    37 1508 1.00 0.06      1    1.00 0.00   0   1     1 -15.74   246.01
## C10_9    38 1508 1.03 0.27      1    1.00 0.00   0   5     5   9.70   123.62
## C10_10   39 1508 1.06 0.34      1    1.00 0.00   0   5     5   6.87    60.06
## C10_11   40 1508 1.04 0.27      1    1.00 0.00   0   5     5   7.78    84.35
## C10_12   41 1508 1.00 0.12      1    1.00 0.00   0   4     4   6.77   268.57
## C10_13   42 1508 1.19 0.64      1    1.03 0.00   0   5     5   4.02    18.22
## C10_14   43 1508 1.03 0.30      1    1.00 0.00   0   5     5   9.41   110.73
## C10_15   44 1508 1.00 0.14      1    1.00 0.00   0   5     5  15.52   447.76
## C10_16   45 1508 1.05 0.38      1    1.00 0.00   0   5     5   7.81    72.22
## C10_17   46 1508 1.38 0.93      1    1.14 0.00   0   5     5   2.75     7.24
## C10_18   47 1508 1.03 0.31      1    1.00 0.00   0   5     5   9.76   116.49
## C10_19   48 1508 1.17 0.63      1    1.00 0.00   0   5     5   4.44    21.71
## C10_20   49 1508 1.18 0.61      1    1.02 0.00   0   5     5   4.17    20.22
##          se
## B10_1  0.02
## B10_2  0.02
## B10_3  0.02
## B10_4  0.02
## B10_5  0.02
## B10_6  0.02
## B10_7  0.02
## B10_8  0.02
## B10_9  0.02
## B10_10 0.02
## C9_1   0.02
## C9_2   0.02
## C9_3   0.02
## C9_4   0.02
## C9_5   0.02
## C9_6   0.02
## C9_7   0.02
## C9_8   0.02
## C9_9   0.02
## C9_10  0.02
## C9_11  0.02
```

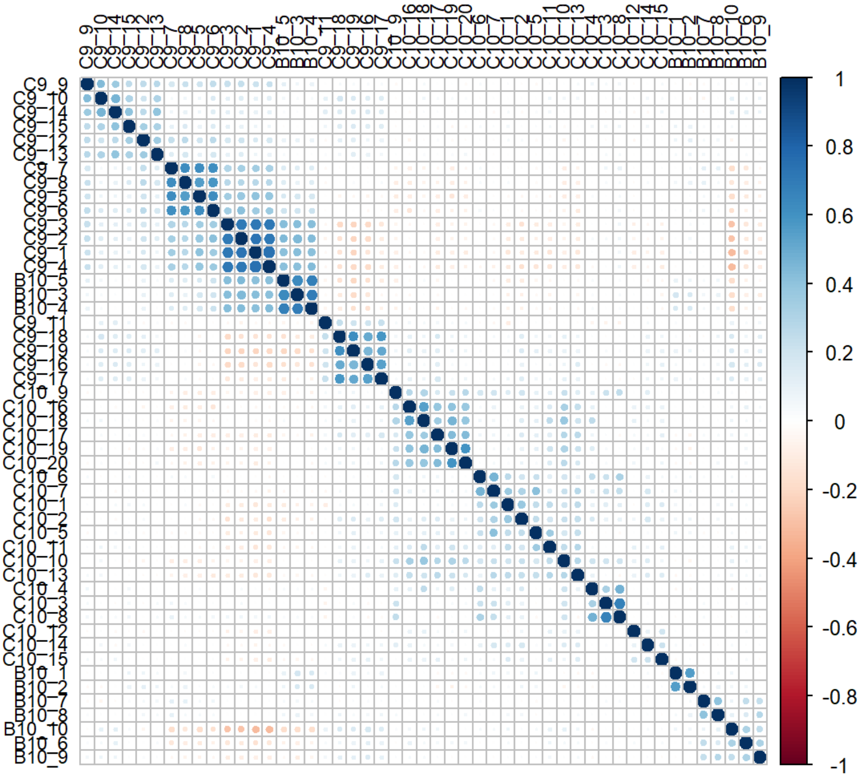
```
## C9_12  0.02
## C9_13  0.02
## C9_14  0.02
## C9_15  0.02
## C9_16  0.02
## C9_17  0.02
## C9_18  0.02
## C9_19  0.02
## C10_1  0.01
## C10_2  0.02
## C10_3  0.00
## C10_4  0.00
## C10_5  0.01
## C10_6  0.01
## C10_7  0.01
## C10_8  0.00
## C10_9  0.01
## C10_10 0.01
## C10_11 0.01
## C10_12 0.00
## C10_13 0.02
## C10_14 0.01
## C10_15 0.00
## C10_16 0.01
## C10_17 0.02
## C10_18 0.01
## C10_19 0.02
## C10_20 0.02
```

Correlation plot

```
# round(cor(dat1, use="complete.obs"),2)
corrplot(cor(dat1, use="complete.obs"), order = "original", tl.col='black', tl.cex=.75)
```



```
corrplot(cor(dat1, use="complete.obs"), order = "hclust", tl.col='black', tl.cex=.75)
```



We see that there are some “clumps” of items that are positively correlated - evidence of some common factors.

KMO Test

```
dat_corr<-cor(dat1)
KMO(dat_corr)
```

```
## Kaiser-Meyer-Olkin factor adequacy
## Call: KMO(r = dat_corr)
## Overall MSA =  0.83
## MSA for each item =
##  B10_1  B10_2  B10_3  B10_4  B10_5  B10_6  B10_7  B10_8  B10_9 B10_10   C9_1
##   0.59   0.58   0.87   0.85   0.88   0.74   0.62   0.64   0.75   0.88   0.91
##   C9_2   C9_3   C9_4   C9_5   C9_6   C9_7   C9_8   C9_9  C9_10  C9_11  C9_12
##   0.93   0.93   0.92   0.90   0.89   0.86   0.88   0.87   0.81   0.84   0.87
##  C9_13  C9_14  C9_15  C9_16  C9_17  C9_18  C9_19  C10_1  C10_2  C10_3  C10_4
##   0.84   0.79   0.84   0.86   0.84   0.81   0.85   0.81   0.85   0.68   0.65
##  C10_5  C10_6  C10_7  C10_8  C10_9 C10_10 C10_11 C10_12 C10_13 C10_14 C10_15
##   0.72   0.79   0.72   0.62   0.83   0.90   0.78   0.66   0.82   0.68   0.65
## C10_16 C10_17 C10_18 C10_19 C10_20
##   0.85   0.87   0.79   0.81   0.79
```

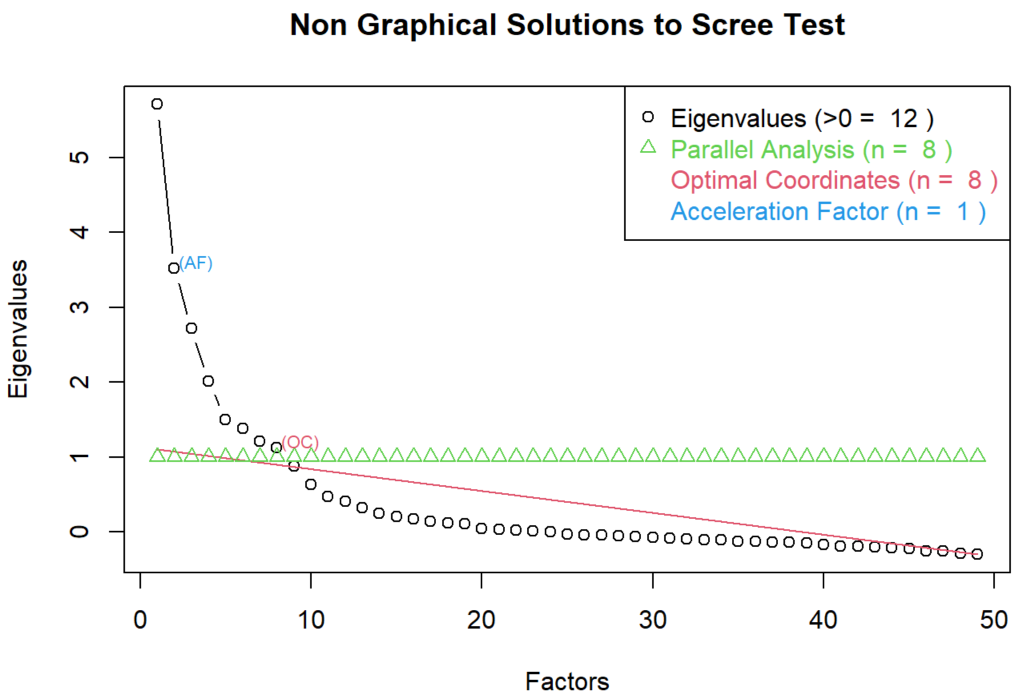
MSA (measure of sampling adequacy) is a measure for exclusion of variables. If MSA < 0.5 the variable should be dropped. Variables with MSA > 0.6 are suitable, variables with MSA > 0.8 very well suited for factor analysis. The result tells us that there is no variables to drop.

Scree plot

```
nScree(x=dat1,model="factors")
```

```
##   noc naf nparallel nkaiser
## 1   8   1         8      12
```

```
plot(nScree(x=dat1,model="factors"))
```



> According to the Scree plot, I choose nfactors=8.

Question 1: Use PC method, PF method and MLE method to derive the loading coefficients and rotate the results with both varimax and quartimax method.

```
#Varimax Rotated Principal Components Method
fitv <- principal(dat, nfactors=8, rotate="varimax")
fitv # print results
```

```
## Principal Components Analysis
## Call: principal(r = dat, nfactors = 8, rotate = "varimax")
## Standardized loadings (pattern matrix) based upon correlation matrix
##          RC1   RC2   RC3   RC6   RC4   RC7   RC5   RC8   h2    u2 com
## B10_1   0.04 -0.02  0.06  0.02  0.08 -0.01  0.06  0.83 0.70 0.302 1.0
## B10_2   0.00  0.00  0.04  0.06 -0.03  0.05  0.10  0.81 0.68 0.320 1.1
## B10_3  -0.06 -0.04  0.68 -0.04 -0.05  0.02  0.20  0.35 0.64 0.361 1.8
## B10_4  -0.05 -0.04  0.65 -0.03 -0.10  0.01  0.26  0.34 0.63 0.374 1.9
## B10_5  -0.06  0.07  0.65 -0.04 -0.06 -0.02  0.21  0.26 0.55 0.453 1.6
## B10_6   0.00  0.00 -0.03 -0.06 -0.06  0.04  0.67  0.04 0.46 0.543 1.0
## B10_7  -0.05  0.07  0.06  0.17  0.05 -0.11  0.67  0.02 0.50 0.502 1.3
## B10_8  -0.01  0.03  0.12  0.08  0.03 -0.01  0.67  0.10 0.49 0.513 1.1
## B10_9   0.01 -0.02  0.03 -0.07  0.08  0.09  0.69 -0.06 0.49 0.507 1.1
## B10_10  0.03 -0.01 -0.14 -0.12  0.12  0.12  0.64  0.12 0.49 0.514 1.4
## C9_1    0.03  0.01  0.73  0.29 -0.02  0.08 -0.13 -0.09 0.65 0.346 1.4
## C9_2    0.00 -0.03  0.77  0.25 -0.06  0.17 -0.11 -0.10 0.71 0.291 1.4
## C9_3    0.00 -0.01  0.73  0.25 -0.11  0.13 -0.10 -0.13 0.66 0.344 1.5
## C9_4    0.00  0.01  0.79  0.24 -0.04  0.11 -0.10 -0.16 0.72 0.276 1.4
## C9_5    0.04 -0.08  0.24  0.74  0.03  0.10  0.00 -0.01 0.63 0.371 1.3
## C9_6    0.04 -0.05  0.22  0.73  0.03  0.21  0.03  0.02 0.64 0.365 1.4
## C9_7    0.00 -0.06  0.15  0.80  0.03  0.11 -0.01  0.00 0.68 0.316 1.1
## C9_8    0.04 -0.04  0.08  0.74  0.03  0.18  0.00  0.07 0.60 0.404 1.2
## C9_9    0.04 -0.01  0.16  0.16  0.00  0.61  0.00 -0.03 0.43 0.573 1.3
## C9_10  -0.04  0.07  0.11 -0.05  0.15  0.64 -0.01 -0.09 0.46 0.539 1.3
## C9_11   0.03  0.03  0.07 -0.03  0.42  0.08  0.00 -0.06 0.20 0.804 1.2
## C9_12   0.00 -0.01  0.06  0.24  0.08  0.54 -0.05  0.08 0.37 0.632 1.5
## C9_13  -0.06  0.04 -0.02  0.10  0.05  0.61  0.04  0.03 0.39 0.605 1.1
## C9_14  -0.03  0.01  0.04  0.01  0.12  0.66  0.16 -0.01 0.48 0.523 1.2
## C9_15   0.04 -0.01  0.02  0.11  0.14  0.62 -0.01  0.06 0.42 0.576 1.2
## C9_16   0.05  0.06 -0.12  0.03  0.71  0.14  0.09 -0.01 0.56 0.440 1.2
## C9_17   0.04  0.03 -0.03  0.06  0.77  0.06  0.06  0.04 0.61 0.386 1.1
## C9_18   0.00  0.03 -0.10  0.01  0.82  0.10  0.00  0.01 0.70 0.298 1.1
## C9_19  -0.01  0.02 -0.19  0.09  0.77  0.13  0.06  0.10 0.67 0.327 1.3
## C10_1   0.72  0.05 -0.11  0.06  0.01  0.02  0.01  0.04 0.54 0.464 1.1
## C10_2   0.61  0.08 -0.14  0.03  0.09 -0.05 -0.04  0.11 0.42 0.579 1.3
## C10_3   0.86 -0.02  0.04  0.02 -0.02  0.00  0.02 -0.06 0.75 0.253 1.0
## C10_4   0.95 -0.01  0.06  0.01 -0.01  0.00 -0.01 -0.05 0.91 0.094 1.0
## C10_5   0.65  0.04 -0.04  0.03  0.03  0.01 -0.02  0.10 0.44 0.562 1.1
## C10_6   0.95 -0.01  0.03  0.03 -0.01  0.00  0.00 -0.02 0.90 0.101 1.0
## C10_7   0.93  0.01  0.00  0.02  0.00 -0.01 -0.01  0.01 0.87 0.125 1.0
## C10_8   0.96 -0.03  0.05  0.01 -0.01  0.00  0.00 -0.05 0.93 0.071 1.0
## C10_9   0.90  0.03  0.01 -0.04  0.01 -0.02  0.00 -0.03 0.82 0.177 1.0
## C10_10  0.88  0.07  0.02 -0.05  0.05  0.00  0.02 -0.06 0.78 0.218 1.0
## C10_11  0.02  0.82 -0.06  0.04 -0.04  0.04  0.02  0.04 0.68 0.323 1.0
## C10_12 -0.02  0.73  0.02  0.05 -0.06  0.01 -0.01  0.00 0.54 0.464 1.0
## C10_13  0.10  0.69 -0.04 -0.06  0.09  0.04  0.00 -0.02 0.50 0.495 1.1
## C10_14  0.00  0.78 -0.01  0.05 -0.01  0.00 -0.02  0.05 0.61 0.393 1.0
## C10_15 -0.02  0.90 -0.01  0.07 -0.01  0.02  0.01  0.00 0.81 0.185 1.0
## C10_16  0.02  0.85  0.01 -0.05  0.04 -0.01  0.01 -0.01 0.73 0.271 1.0
## C10_17  0.08  0.62 -0.03 -0.12  0.14  0.02  0.05 -0.02 0.43 0.566 1.2
## C10_18  0.01  0.89  0.05 -0.02  0.01  0.00  0.00  0.00 0.79 0.207 1.0
## C10_19  0.01  0.74  0.04 -0.12  0.04  0.02  0.03 -0.03 0.56 0.438 1.1
## C10_20  0.00  0.76  0.01 -0.08  0.04 -0.03  0.01 -0.04 0.59 0.415 1.0
##
##                        RC1  RC2  RC3  RC6  RC4  RC7  RC5  RC8
## SS loadings           7.28 6.17 3.94 2.80 2.75 2.53 2.49 1.82
## Proportion Var        0.15 0.13 0.08 0.06 0.06 0.05 0.05 0.04
## Cumulative Var        0.15 0.27 0.35 0.41 0.47 0.52 0.57 0.61
## Proportion Explained  0.24 0.21 0.13 0.09 0.09 0.09 0.08 0.06
## Cumulative Proportion 0.24 0.45 0.58 0.68 0.77 0.86 0.94 1.00
##
## Mean item complexity =  1.2
## Test of the hypothesis that 8 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.04
##  with the empirical chi square  4498.38  with prob <  0
##
## Fit based upon off diagonal values = 0.97
```

```
#Quartimax Rotated Principal Components Method
fitq <- principal(dat, nfactors=8, rotate="quartimax")
fitq # print results
```

```
## Principal Components Analysis
## Call: principal(r = dat, nfactors = 8, rotate = "quartimax")
## Standardized loadings (pattern matrix) based upon correlation matrix
##          RC1   RC2   RC3   RC4   RC6   RC5   RC7   RC8   h2    u2 com
## B10_1   0.02 -0.02  0.03  0.09  0.02  0.03 -0.03  0.83 0.70 0.302 1.0
## B10_2  -0.01  0.00  0.01 -0.01  0.07  0.08  0.04  0.82 0.68 0.320 1.0
## B10_3  -0.05 -0.04  0.66 -0.05 -0.07  0.21 -0.01  0.39 0.64 0.361 1.9
## B10_4  -0.05 -0.03  0.63 -0.10 -0.06  0.27 -0.01  0.37 0.63 0.374 2.1
## B10_5  -0.06  0.07  0.63 -0.06 -0.07  0.22 -0.04  0.29 0.55 0.453 1.8
## B10_6   0.00  0.00 -0.06 -0.05 -0.05  0.67  0.05  0.05 0.46 0.543 1.1
## B10_7  -0.05  0.08  0.05  0.05  0.15  0.67 -0.12  0.03 0.50 0.502 1.2
## B10_8  -0.01  0.04  0.10  0.04  0.07  0.67 -0.02  0.12 0.49 0.513 1.1
## B10_9   0.02 -0.02  0.01  0.08 -0.06  0.69  0.09 -0.04 0.49 0.507 1.1
## B10_10  0.02 -0.01 -0.16  0.13 -0.10  0.63  0.14  0.14 0.49 0.514 1.5
## C9_1    0.04  0.01  0.76 -0.02  0.25 -0.11  0.02 -0.07 0.65 0.346 1.3
## C9_2    0.02 -0.03  0.79 -0.07  0.22 -0.09  0.12 -0.07 0.71 0.291 1.3
## C9_3    0.01  0.00  0.76 -0.11  0.22 -0.08  0.08 -0.10 0.66 0.344 1.3
## C9_4    0.01  0.01  0.81 -0.05  0.21 -0.07  0.06 -0.13 0.72 0.276 1.2
## C9_5    0.04 -0.08  0.29  0.04  0.73  0.00  0.03 -0.01 0.63 0.371 1.3
## C9_6    0.04 -0.04  0.27  0.05  0.73  0.03  0.14  0.03 0.64 0.365 1.4
## C9_7    0.01 -0.06  0.20  0.05  0.80 -0.01  0.04  0.01 0.68 0.316 1.1
## C9_8    0.04 -0.04  0.13  0.05  0.75  0.00  0.11  0.07 0.60 0.404 1.1
## C9_9    0.04 -0.01  0.20  0.03  0.19  0.00  0.59 -0.02 0.43 0.573 1.5
## C9_10  -0.03  0.07  0.15  0.17 -0.02 -0.01  0.63 -0.08 0.46 0.539 1.3
## C9_11   0.03  0.03  0.08  0.42 -0.04  0.00  0.06 -0.06 0.20 0.804 1.2
## C9_12   0.00  0.00  0.09  0.10  0.28 -0.05  0.51  0.09 0.37 0.632 1.8
## C9_13  -0.06  0.05  0.02  0.08  0.14  0.04  0.60  0.03 0.39 0.605 1.2
## C9_14  -0.03  0.01  0.07  0.15  0.05  0.16  0.65  0.00 0.48 0.523 1.3
## C9_15   0.03  0.00  0.06  0.17  0.16 -0.02  0.60  0.06 0.42 0.576 1.3
## C9_16   0.04  0.06 -0.11  0.72  0.03  0.08  0.11 -0.02 0.56 0.440 1.1
## C9_17   0.04  0.03 -0.02  0.78  0.05  0.05  0.03  0.03 0.61 0.386 1.0
## C9_18   0.00  0.03 -0.08  0.83  0.01 -0.01  0.07 -0.01 0.70 0.298 1.0
## C9_19  -0.02  0.02 -0.17  0.78  0.10  0.04  0.10  0.08 0.67 0.327 1.2
## C10_1   0.71  0.05 -0.12  0.01  0.07  0.00  0.03  0.05 0.54 0.464 1.1
## C10_2   0.60  0.08 -0.15  0.09  0.04 -0.05 -0.05  0.11 0.42 0.579 1.3
## C10_3   0.86 -0.02  0.03 -0.02  0.02  0.02  0.00 -0.04 0.75 0.253 1.0
## C10_4   0.95 -0.01  0.05 -0.01  0.01 -0.01  0.00 -0.03 0.91 0.094 1.0
## C10_5   0.65  0.04 -0.06  0.04  0.04 -0.03  0.01  0.11 0.44 0.562 1.1
## C10_6   0.95 -0.01  0.01 -0.01  0.03  0.00  0.00  0.01 0.90 0.101 1.0
## C10_7   0.93  0.01 -0.01  0.00  0.02 -0.01 -0.01  0.03 0.87 0.125 1.0
## C10_8   0.96 -0.03  0.04 -0.01  0.01  0.00  0.00 -0.02 0.93 0.071 1.0
## C10_9   0.91  0.03 -0.01  0.01 -0.04 -0.01 -0.01 -0.01 0.82 0.177 1.0
## C10_10  0.88  0.07  0.00  0.05 -0.05  0.02  0.00 -0.04 0.78 0.218 1.0
## C10_11  0.02  0.82 -0.06 -0.03  0.04  0.02  0.04  0.04 0.68 0.323 1.0
## C10_12 -0.02  0.73  0.02 -0.06  0.04 -0.01  0.00  0.00 0.54 0.464 1.0
## C10_13  0.10  0.69 -0.04  0.09 -0.06  0.00  0.04 -0.03 0.50 0.495 1.1
## C10_14  0.00  0.78 -0.01 -0.01  0.05 -0.02  0.00  0.05 0.61 0.393 1.0
## C10_15 -0.02  0.90  0.00 -0.01  0.06  0.00  0.01  0.00 0.81 0.185 1.0
## C10_16  0.02  0.85  0.00  0.04 -0.06  0.01 -0.01 -0.01 0.73 0.271 1.0
## C10_17  0.08  0.62 -0.04  0.14 -0.12  0.05  0.02 -0.02 0.43 0.566 1.2
## C10_18  0.01  0.89  0.04  0.01 -0.03  0.00 -0.01  0.00 0.79 0.207 1.0
## C10_19  0.01  0.74  0.03  0.03 -0.13  0.03  0.02 -0.03 0.56 0.438 1.1
## C10_20  0.00  0.76  0.00  0.04 -0.09  0.01 -0.03 -0.04 0.59 0.415 1.0
##
##                        RC1  RC2  RC3  RC4  RC6  RC5  RC7  RC8
## SS loadings           7.28 6.17 4.09 2.84 2.78 2.47 2.30 1.87
## Proportion Var        0.15 0.13 0.08 0.06 0.06 0.05 0.05 0.04
## Cumulative Var        0.15 0.27 0.36 0.42 0.47 0.52 0.57 0.61
## Proportion Explained  0.24 0.21 0.14 0.10 0.09 0.08 0.08 0.06
## Cumulative Proportion 0.24 0.45 0.59 0.68 0.78 0.86 0.94 1.00
##
## Mean item complexity =  1.2
## Test of the hypothesis that 8 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.04
##  with the empirical chi square  4498.38  with prob <  0
##
## Fit based upon off diagonal values = 0.97
```

```
#Varimax Rotated Principal factor Method
fit2v <-  fa(dat1, nfactors = 8, rotate = 'varimax', fm = 'pa', scores = "regression")
fit2v
```

```
## Factor Analysis using method =  pa
## Call: fa(r = dat1, nfactors = 8, rotate = "varimax", scores = "regression",
##     fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##          PA1   PA5   PA2   PA3   PA6   PA7   PA4   PA8    h2   u2 com
## B10_1   0.23 -0.05 -0.12  0.13  0.20 -0.01 -0.07  0.13 0.152 0.85 4.2
## B10_2   0.23 -0.01 -0.13  0.08  0.19 -0.01 -0.05  0.18 0.150 0.85 3.9
## B10_3   0.78  0.04 -0.01 -0.05 -0.02 -0.02  0.00  0.04 0.613 0.39 1.0
## B10_4   0.78  0.06 -0.03 -0.06 -0.01 -0.01 -0.01  0.06 0.623 0.38 1.0
## B10_5   0.72  0.06  0.02 -0.13 -0.06  0.02 -0.01  0.04 0.538 0.46 1.1
## B10_6  -0.08 -0.13  0.07  0.05 -0.01  0.10  0.00  0.46 0.252 0.75 1.4
## B10_7   0.05  0.14  0.02 -0.03 -0.04 -0.04  0.04  0.53 0.311 0.69 1.2
## B10_8   0.09  0.10  0.04  0.04  0.01 -0.01  0.03  0.52 0.289 0.71 1.2
## B10_9  -0.07 -0.05  0.02  0.08 -0.01  0.09 -0.01  0.46 0.239 0.76 1.2
## B10_10 -0.22 -0.22  0.01  0.14  0.10  0.09 -0.02  0.36 0.260 0.74 3.2
## C9_1    0.60  0.36 -0.04 -0.12 -0.20  0.18  0.05 -0.22 0.634 0.37 2.6
## C9_2    0.62  0.36  0.00 -0.16 -0.16  0.23  0.07 -0.19 0.654 0.35 2.6
## C9_3    0.60  0.32 -0.01 -0.18 -0.18  0.23  0.06 -0.19 0.618 0.38 2.6
## C9_4    0.59  0.35 -0.03 -0.15 -0.23  0.20  0.05 -0.21 0.637 0.36 2.8
## C9_5    0.16  0.73 -0.09 -0.04 -0.02  0.10  0.00 -0.05 0.577 0.42 1.2
## C9_6    0.18  0.71 -0.09 -0.02  0.01  0.20  0.00  0.00 0.591 0.41 1.3
## C9_7    0.10  0.80 -0.06  0.00  0.01  0.07  0.04  0.04 0.667 0.33 1.1
## C9_8    0.08  0.71 -0.11  0.02  0.05  0.15  0.00  0.03 0.553 0.45 1.2
## C9_9    0.09  0.16  0.03 -0.02  0.00  0.55  0.04 -0.01 0.341 0.66 1.3
## C9_10   0.05 -0.01  0.05  0.13  0.02  0.62  0.00 -0.01 0.409 0.59 1.1
## C9_11  -0.02 -0.04  0.05  0.30 -0.06  0.07 -0.05  0.01 0.107 0.89 1.4
## C9_12   0.10  0.23 -0.05  0.09  0.05  0.41 -0.04  0.05 0.249 0.75 2.0
## C9_13   0.01  0.11  0.01  0.05  0.00  0.51  0.02  0.08 0.285 0.72 1.2
## C9_14   0.01 -0.02 -0.01  0.11  0.00  0.71 -0.01  0.05 0.523 0.48 1.1
## C9_15   0.01  0.07 -0.05  0.13  0.07  0.52  0.03  0.03 0.298 0.70 1.2
## C9_16  -0.13 -0.02  0.04  0.63  0.06  0.11 -0.04  0.10 0.446 0.55 1.2
## C9_17  -0.01  0.00  0.06  0.73  0.10  0.07  0.00  0.06 0.564 0.44 1.1
## C9_18  -0.09  0.00  0.08  0.77  0.08  0.10  0.03  0.03 0.621 0.38 1.1
## C9_19  -0.16  0.02  0.07  0.68  0.12  0.12  0.03  0.08 0.533 0.47 1.3
## C10_1  -0.08  0.00  0.00 -0.04  0.56  0.05  0.12  0.03 0.339 0.66 1.2
## C10_2  -0.06  0.00  0.12  0.10  0.49 -0.02  0.11  0.01 0.283 0.72 1.3
## C10_3   0.00  0.02  0.00 -0.02  0.08  0.01  0.69  0.05 0.487 0.51 1.0
## C10_4   0.02  0.00  0.15 -0.03  0.09  0.02  0.48 -0.02 0.261 0.74 1.3
## C10_5  -0.02 -0.03  0.09  0.01  0.53  0.00 -0.06 -0.04 0.299 0.70 1.1
## C10_6   0.01  0.04  0.01 -0.04  0.42  0.00  0.34  0.00 0.293 0.71 2.0
## C10_7   0.01  0.01  0.02 -0.04  0.62 -0.01  0.25 -0.05 0.454 0.55 1.3
## C10_8   0.02  0.04 -0.03 -0.02  0.04  0.04  0.94  0.02 0.886 0.11 1.0
## C10_9  -0.04 -0.07  0.32  0.02  0.18 -0.03  0.26  0.01 0.211 0.79 2.7
## C10_10  0.00 -0.08  0.44  0.04  0.29  0.01  0.22  0.01 0.335 0.66 2.4
## C10_11 -0.06 -0.02  0.25 -0.02  0.38  0.08 -0.05  0.06 0.228 0.77 2.0
## C10_12 -0.03 -0.02  0.11 -0.01  0.08 -0.04 -0.03  0.04 0.023 0.98 2.8
## C10_13 -0.04 -0.07  0.31  0.03  0.39  0.03 -0.03  0.01 0.263 0.74 2.0
## C10_14 -0.01  0.02  0.07  0.09  0.25  0.01  0.02  0.00 0.075 0.93 1.5
## C10_15 -0.07  0.02  0.11  0.06  0.17  0.04 -0.03  0.00 0.052 0.95 2.8
## C10_16  0.01 -0.07  0.63  0.08  0.07 -0.02  0.03  0.01 0.419 0.58 1.1
## C10_17 -0.05 -0.03  0.57  0.14  0.10  0.01  0.00  0.03 0.360 0.64 1.2
## C10_18  0.05 -0.03  0.68  0.04  0.11  0.04  0.05  0.02 0.481 0.52 1.1
## C10_19 -0.02 -0.06  0.70  0.01  0.04 -0.01  0.03 -0.01 0.501 0.50 1.0
## C10_20 -0.03 -0.02  0.66  0.04  0.01 -0.01  0.02  0.03 0.435 0.57 1.0
##
##                        PA1  PA5  PA2  PA3  PA6  PA7  PA4  PA8
## SS loadings           3.54 2.92 2.74 2.38 2.25 2.25 1.95 1.38
## Proportion Var        0.07 0.06 0.06 0.05 0.05 0.05 0.04 0.03
## Cumulative Var        0.07 0.13 0.19 0.24 0.28 0.33 0.37 0.40
## Proportion Explained  0.18 0.15 0.14 0.12 0.12 0.12 0.10 0.07
## Cumulative Proportion 0.18 0.33 0.47 0.60 0.71 0.83 0.93 1.00
##
## Mean item complexity =  1.6
## Test of the hypothesis that 8 factors are sufficient.
##
## The degrees of freedom for the null model are  1176  and the objective function was  16.63 with Chi Square of  2477
9.4
## The degrees of freedom for the model are 812  and the objective function was  2.75
##
## The root mean square of the residuals (RMSR) is  0.03
## The df corrected root mean square of the residuals is  0.04
##
```

```
## The harmonic number of observations is  1508 with the empirical chi square  3573.54  with prob <  0
## The total number of observations was  1508  with Likelihood Chi Square =  4084.65  with prob <  0
##
## Tucker Lewis Index of factoring reliability =  0.798
## RMSEA index =  0.052  and the 90 % confidence intervals are  0.05 0.053
## BIC =  -1858.01
## Fit based upon off diagonal values = 0.96
## Measures of factor score adequacy
##                                                    PA1  PA5  PA2  PA3  PA6  PA7
## Correlation of (regression) scores with factors   0.93 0.92 0.90 0.90 0.86 0.87
## Multiple R square of scores with factors          0.86 0.84 0.81 0.81 0.74 0.76
## Minimum correlation of possible factor scores     0.72 0.68 0.62 0.62 0.49 0.53
##                                                    PA4  PA8
## Correlation of (regression) scores with factors   0.95 0.81
## Multiple R square of scores with factors          0.91 0.65
## Minimum correlation of possible factor scores     0.82 0.30
```

```
#Quartimax Rotated Principal factor Method
fit2q <-  fa(dat1, nfactors = 8, rotate = 'quartimax', fm = 'pa', scores = "regression")
fit2q
```

```
## Factor Analysis using method =  pa
## Call: fa(r = dat1, nfactors = 8, rotate = "quartimax", scores = "regression",
##     fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##          PA1   PA2   PA5   PA3   PA6   PA7   PA4   PA8    h2   u2 com
## B10_1   0.18 -0.11 -0.06  0.16  0.22 -0.03 -0.07  0.15 0.152 0.85 4.8
## B10_2   0.18 -0.12 -0.02  0.12  0.21 -0.02 -0.04  0.21 0.150 0.85 4.3
## B10_3   0.76 -0.01 -0.04  0.01  0.07 -0.08 -0.02  0.13 0.613 0.39 1.1
## B10_4   0.77 -0.03 -0.02 -0.01  0.09 -0.08 -0.03  0.14 0.623 0.38 1.1
## B10_5   0.72  0.02 -0.01 -0.08  0.03 -0.03 -0.02  0.12 0.538 0.46 1.1
## B10_6  -0.13  0.06 -0.11  0.06 -0.03  0.13  0.01  0.45 0.252 0.75 1.6
## B10_7   0.01  0.03  0.14 -0.01 -0.04 -0.04  0.05  0.53 0.311 0.69 1.2
## B10_8   0.04  0.04  0.09  0.06  0.00 -0.01  0.04  0.52 0.289 0.71 1.1
## B10_9  -0.11  0.02 -0.04  0.10 -0.04  0.11  0.00  0.45 0.239 0.76 1.4
## B10_10 -0.28  0.01 -0.18  0.15  0.06  0.14 -0.01  0.32 0.260 0.74 3.6
## C9_1    0.69 -0.03  0.31 -0.09 -0.12  0.09  0.04 -0.15 0.634 0.37 1.7
## C9_2    0.71  0.00  0.31 -0.12 -0.08  0.15  0.06 -0.12 0.654 0.35 1.7
## C9_3    0.69  0.00  0.27 -0.15 -0.10  0.15  0.05 -0.13 0.618 0.38 1.7
## C9_4    0.69 -0.03  0.30 -0.12 -0.16  0.12  0.04 -0.15 0.637 0.36 1.7
## C9_5    0.24 -0.07  0.72 -0.02 -0.02  0.02  0.00 -0.03 0.577 0.42 1.2
## C9_6    0.26 -0.06  0.71  0.01  0.02  0.12  0.01  0.01 0.591 0.41 1.3
## C9_7    0.18 -0.04  0.79  0.02  0.00 -0.01  0.05  0.04 0.667 0.33 1.1
## C9_8    0.15 -0.08  0.72  0.05  0.05  0.07  0.00  0.03 0.553 0.45 1.2
## C9_9    0.16  0.03  0.20  0.02  0.01  0.52  0.05 -0.02 0.341 0.66 1.5
## C9_10   0.09  0.05  0.03  0.16  0.01  0.61  0.01 -0.02 0.409 0.59 1.2
## C9_11  -0.03  0.04 -0.03  0.30 -0.09  0.06 -0.06 -0.01 0.107 0.89 1.5
## C9_12   0.14 -0.04  0.26  0.12  0.05  0.38 -0.03  0.05 0.249 0.75 2.5
## C9_13   0.06  0.02  0.15  0.07 -0.01  0.50  0.03  0.06 0.285 0.72 1.3
## C9_14   0.06 -0.01  0.04  0.14 -0.01  0.71  0.00  0.03 0.523 0.48 1.1
## C9_15   0.05 -0.04  0.11  0.16  0.06  0.50  0.04  0.02 0.298 0.70 1.4
## C9_16  -0.17  0.04  0.00  0.63 -0.01  0.10 -0.04  0.06 0.446 0.55 1.2
## C9_17  -0.06  0.06  0.00  0.74  0.04  0.05 -0.01  0.03 0.564 0.44 1.0
## C9_18  -0.13  0.08  0.00  0.77  0.00  0.08  0.02 -0.01 0.621 0.38 1.1
## C9_19  -0.21  0.07  0.04  0.68  0.04  0.11  0.02  0.02 0.533 0.47 1.3
## C10_1  -0.14  0.03  0.03  0.00  0.55  0.06  0.12  0.03 0.339 0.66 1.3
## C10_2  -0.12  0.15  0.02  0.13  0.47 -0.02  0.11  0.00 0.283 0.72 1.7
## C10_3   0.00  0.03  0.02 -0.01  0.08  0.00  0.69  0.04 0.487 0.51 1.0
## C10_4   0.03  0.17  0.00 -0.01  0.08  0.01  0.47 -0.03 0.261 0.74 1.3
## C10_5  -0.08  0.11 -0.01  0.04  0.52  0.01 -0.06 -0.04 0.299 0.70 1.2
## C10_6  -0.03  0.04  0.05 -0.01  0.42 -0.01  0.34  0.00 0.293 0.71 2.0
## C10_7  -0.05  0.06  0.03  0.00  0.62 -0.02  0.25 -0.05 0.454 0.55 1.4
## C10_8   0.04  0.00  0.03  0.00  0.05  0.02  0.94  0.01 0.886 0.11 1.0
## C10_9  -0.07  0.34 -0.07  0.04  0.16 -0.03  0.25  0.00 0.211 0.79 2.6
## C10_10 -0.03  0.46 -0.09  0.07  0.26  0.01  0.21  0.01 0.335 0.66 2.2
## C10_11 -0.11  0.27  0.00  0.01  0.36  0.09 -0.05  0.05 0.228 0.77 2.3
## C10_12 -0.04  0.11 -0.02 -0.01  0.07 -0.03 -0.03  0.04 0.023 0.98 2.8
## C10_13 -0.09  0.33 -0.06  0.06  0.37  0.04 -0.04  0.01 0.263 0.74 2.3
## C10_14 -0.04  0.08  0.02  0.11  0.23  0.00  0.02 -0.01 0.075 0.93 1.8
## C10_15 -0.09  0.11  0.04  0.07  0.15  0.04 -0.03 -0.01 0.052 0.95 3.8
## C10_16 -0.01  0.63 -0.09  0.09  0.04 -0.02  0.01  0.01 0.419 0.58 1.1
## C10_17 -0.07  0.57 -0.04  0.15  0.05  0.01 -0.02  0.02 0.360 0.64 1.2
## C10_18  0.03  0.68 -0.05  0.05  0.07  0.03  0.03  0.02 0.481 0.52 1.1
## C10_19 -0.02  0.70 -0.08  0.02  0.01  0.00  0.01 -0.01 0.501 0.50 1.0
## C10_20 -0.04  0.65 -0.04  0.04 -0.03 -0.01  0.00  0.02 0.435 0.57 1.0
##
##                        PA1  PA2  PA5  PA3  PA6  PA7  PA4  PA8
## SS loadings           4.21 2.80 2.78 2.41 2.00 1.98 1.94 1.28
## Proportion Var        0.09 0.06 0.06 0.05 0.04 0.04 0.04 0.03
## Cumulative Var        0.09 0.14 0.20 0.25 0.29 0.33 0.37 0.40
## Proportion Explained  0.22 0.14 0.14 0.12 0.10 0.10 0.10 0.07
## Cumulative Proportion 0.22 0.36 0.50 0.63 0.73 0.83 0.93 1.00
##
## Mean item complexity =  1.7
## Test of the hypothesis that 8 factors are sufficient.
##
## The degrees of freedom for the null model are  1176  and the objective function was  16.63 with Chi Square of  2477
9.4
## The degrees of freedom for the model are 812  and the objective function was  2.75
##
## The root mean square of the residuals (RMSR) is  0.03
## The df corrected root mean square of the residuals is  0.04
##
```

```
## The harmonic number of observations is  1508 with the empirical chi square  3573.54  with prob <  0
## The total number of observations was  1508  with Likelihood Chi Square =  4084.65  with prob <  0
##
## Tucker Lewis Index of factoring reliability =  0.798
## RMSEA index =  0.052  and the 90 % confidence intervals are  0.05 0.053
## BIC =  -1858.01
## Fit based upon off diagonal values = 0.96
## Measures of factor score adequacy
##                                                    PA1  PA2  PA5  PA3  PA6  PA7
## Correlation of (regression) scores with factors   0.94 0.90 0.92 0.90 0.85 0.86
## Multiple R square of scores with factors          0.89 0.81 0.84 0.81 0.73 0.75
## Minimum correlation of possible factor scores     0.78 0.63 0.68 0.63 0.46 0.50
##                                                    PA4  PA8
## Correlation of (regression) scores with factors   0.95 0.80
## Multiple R square of scores with factors          0.91 0.65
## Minimum correlation of possible factor scores     0.82 0.29
```

```
# Varimax Rotated Maximum Likelihood Method
fit3v<-factanal(dat1, factors=8, scores = "regression",rotation="varimax")
print(fit3v, digits=2, cutoff=.3, sort=TRUE)
```

```
##
## Call:
## factanal(x = dat1, factors = 8, scores = "regression", rotation = "varimax")
##
## Uniquenesses:
##  B10_1  B10_2  B10_3  B10_4  B10_5  B10_6  B10_7  B10_8  B10_9 B10_10   C9_1
##   0.89   0.91   0.34   0.26   0.37   0.93   0.97   0.95   0.95   0.81   0.24
##   C9_2   C9_3   C9_4   C9_5   C9_6   C9_7   C9_8   C9_9  C9_10  C9_11  C9_12
##   0.28   0.31   0.26   0.41   0.39   0.32   0.42   0.67   0.60   0.89   0.75
##  C9_13  C9_14  C9_15  C9_16  C9_17  C9_18  C9_19  C10_1  C10_2  C10_3  C10_4
##   0.69   0.48   0.70   0.55   0.43   0.37   0.47   0.67   0.72   0.53   0.74
##  C10_5  C10_6  C10_7  C10_8  C10_9 C10_10 C10_11 C10_12 C10_13 C10_14 C10_15
##   0.69   0.69   0.49   0.02   0.78   0.67   0.80   0.97   0.73   0.93   0.96
## C10_16 C10_17 C10_18 C10_19 C10_20
##   0.58   0.64   0.52   0.48   0.54
##
## Loadings:
##        Factor1 Factor2 Factor3 Factor4 Factor5 Factor6 Factor7 Factor8
## C9_1    0.83
## C9_2    0.77
## C9_3    0.77
## C9_4    0.80
## C10_16          0.62
## C10_17          0.57
## C10_18          0.68
## C10_19          0.72
## C10_20          0.68
## C9_16                   0.65
## C9_17                   0.74
## C9_18                   0.77
## C9_19                   0.68
## C9_5    0.31                    0.69
## C9_6                            0.69
## C9_7                            0.78
## C9_8                            0.72
## C9_9                                    0.54
## C9_10                                   0.61
## C9_13                                   0.54
## C9_14                                   0.71
## C9_15                                   0.53
## C10_1                                           0.56
## C10_5                                           0.54
## C10_7                                           0.68
## C10_3                                                   0.68
## C10_8                                                   0.99
## B10_3   0.39                                                    0.69
## B10_4   0.38                                                    0.76
## B10_5   0.38                                                    0.66
## B10_1
## B10_2
## B10_6
## B10_7
## B10_8
## B10_9
## B10_10 -0.36
## C9_11                   0.30
## C9_12                                   0.41
## C10_2                                           0.49
## C10_4                                                   0.47
## C10_6                                           0.47
## C10_9           0.33
## C10_10          0.43                            0.31
## C10_11                                          0.34
## C10_12
## C10_13                                          0.41
## C10_14
## C10_15
##
##                Factor1 Factor2 Factor3 Factor4 Factor5 Factor6 Factor7 Factor8
## SS loadings       3.56    2.74    2.40    2.37    2.23    2.22    1.92    1.80
## Proportion Var    0.07    0.06    0.05    0.05    0.05    0.05    0.04    0.04
## Cumulative Var    0.07    0.13    0.18    0.23    0.27    0.32    0.36    0.39
```

```
##
## Test of the hypothesis that 8 factors are sufficient.
## The chi square statistic is 3493.36 on 812 degrees of freedom.
## The p-value is 0
```

```
# Quartimax Rotated Maximum Likelihood Method
fit3q<-factanal(dat1, factors=8, scores = "regression",rotation="quartimax")
print(fit3q, digits=2, cutoff=.3, sort=TRUE)
```

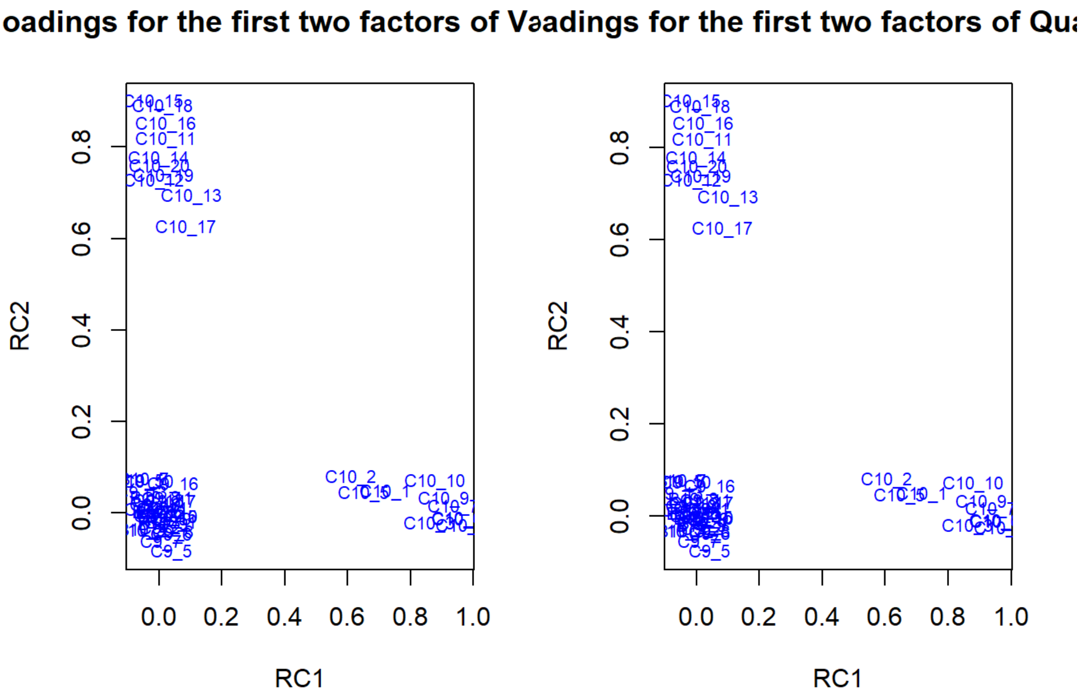
```
##
## Call:
## factanal(x = dat1, factors = 8, scores = "regression", rotation = "quartimax")
##
## Uniquenesses:
##  B10_1  B10_2  B10_3  B10_4  B10_5  B10_6  B10_7  B10_8  B10_9 B10_10   C9_1
##   0.89   0.91   0.34   0.26   0.37   0.93   0.97   0.95   0.95   0.81   0.24
##   C9_2   C9_3   C9_4   C9_5   C9_6   C9_7   C9_8   C9_9  C9_10  C9_11  C9_12
##   0.28   0.31   0.26   0.41   0.39   0.32   0.42   0.67   0.60   0.89   0.75
##  C9_13  C9_14  C9_15  C9_16  C9_17  C9_18  C9_19  C10_1  C10_2  C10_3  C10_4
##   0.69   0.48   0.70   0.55   0.43   0.37   0.47   0.67   0.72   0.53   0.74
##  C10_5  C10_6  C10_7  C10_8  C10_9 C10_10 C10_11 C10_12 C10_13 C10_14 C10_15
##   0.69   0.69   0.49   0.02   0.78   0.67   0.80   0.97   0.73   0.93   0.96
## C10_16 C10_17 C10_18 C10_19 C10_20
##   0.58   0.64   0.52   0.48   0.54
##
## Loadings:
##        Factor1 Factor2 Factor3 Factor4 Factor5 Factor6 Factor7 Factor8
## C9_1    0.84
## C9_2    0.80
## C9_3    0.79
## C9_4    0.83
## C10_16          0.63
## C10_17          0.57
## C10_18          0.68
## C10_19          0.72
## C10_20          0.67
## C9_5    0.32            0.69
## C9_6                    0.71
## C9_7                    0.78
## C9_8                    0.73
## C9_16                           0.65
## C9_17                           0.75
## C9_18                           0.77
## C9_19                           0.68
## C10_1                                   0.54
## C10_5                                   0.53
## C10_7                                   0.68
## C9_9                                            0.50
## C9_10                                           0.60
## C9_13                                           0.52
## C9_14                                           0.70
## C9_15                                           0.51
## C10_3                                                   0.68
## C10_8                                                   0.99
## B10_3   0.39                                                    0.71
## B10_4   0.37                                                    0.77
## B10_5   0.38                                                    0.68
## B10_1
## B10_2
## B10_6
## B10_7
## B10_8
## B10_9
## B10_10 -0.35
## C9_11
## C9_12                                           0.38
## C10_2                                   0.46
## C10_4                                                   0.47
## C10_6                                   0.46
## C10_9           0.34
## C10_10          0.44
## C10_11                                  0.32
## C10_12
## C10_13          0.31                    0.39
## C10_14
## C10_15
##
##                Factor1 Factor2 Factor3 Factor4 Factor5 Factor6 Factor7 Factor8
## SS loadings       3.86    2.78    2.46    2.42    1.98    1.95    1.94    1.86
## Proportion Var    0.08    0.06    0.05    0.05    0.04    0.04    0.04    0.04
## Cumulative Var    0.08    0.14    0.19    0.24    0.28    0.32    0.35    0.39
```

```
##
## Test of the hypothesis that 8 factors are sufficient.
## The chi square statistic is 3493.36 on 812 degrees of freedom.
## The p-value is 0
```

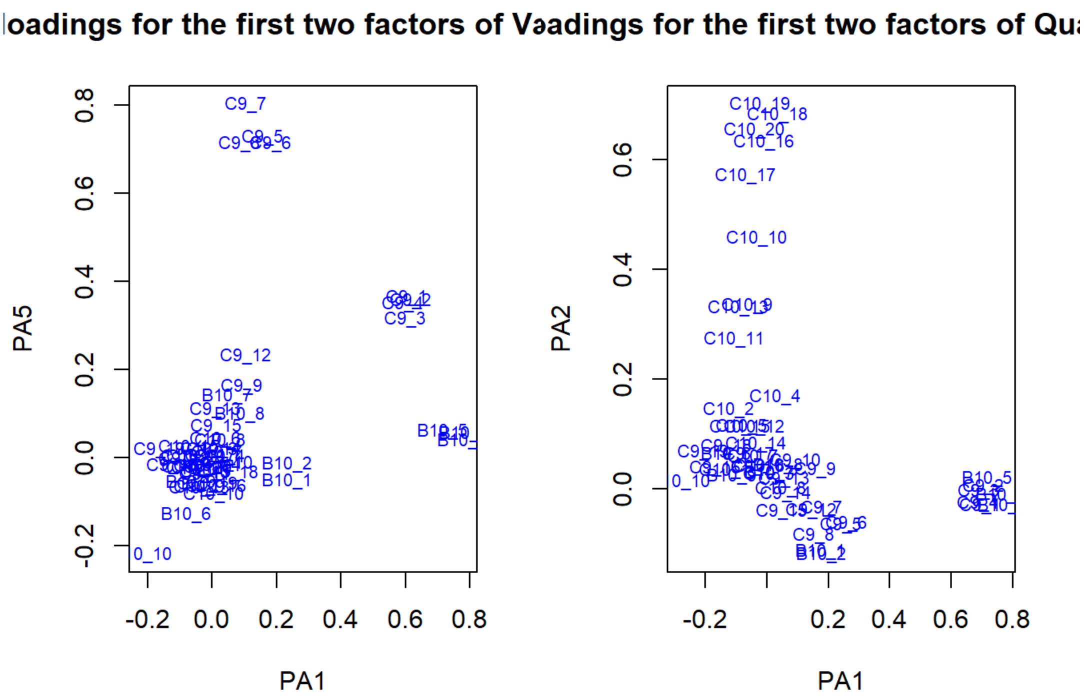
> The sums of squared (SS) loadings are the eigenvalues, or the variance in all variables which is accounted for by that factor. If a factor has a “high” SS loading, then it is helping to explain the variances in the variables. A general rule-of-thumb called the Kaiser Rule, suggest that a factor is important if its eigenvalue is greater than 1. Here, factors 1-8 appear to be important.

Question 2: Compare the results and select one combination to conclude your analysis.

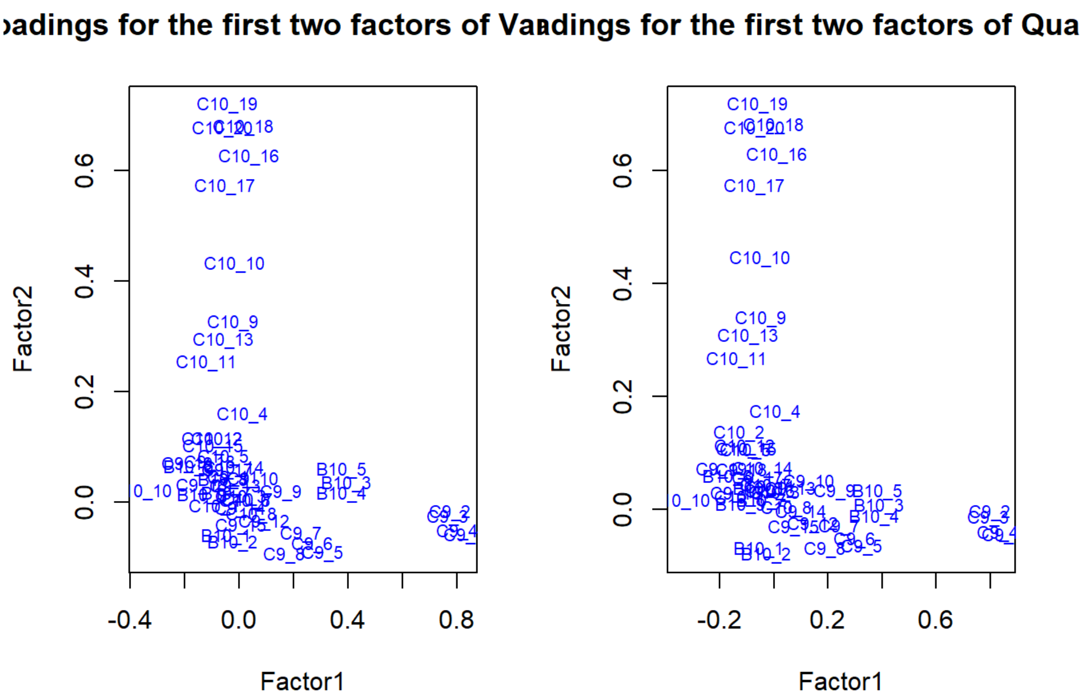
```
par(mfrow=c(1,2))
{load = fitv$loadings[,1:2]
plot(load, type="n", main="New loadings for the first two factors of Varimax PC") # set up plot
text(load,labels=names(dat1),cex=.7, col="blue") # add variable names

load2 = fitq$loadings[,1:2]
plot(load2, type="n", main="New loadings for the first two factors of Quartimax PC") # set up plot
text(load2,labels=names(dat1),cex=.7, col="blue") # add variable names
}
```




```
par(mfrow=c(1,2))
{load3 = fit2v$loadings[,1:2]
plot(load3, type="n", main="New loadings for the first two factors of Varimax PF") # set up plot
text(load3,labels=names(dat1),cex=.7, col="blue") # add variable names

load4 = fit2q$loadings[,1:2]
plot(load4, type="n", main="New loadings for the first two factors of Quartimax PF") # set up plot
text(load4,labels=names(dat1),cex=.7, col="blue") # add variable names
}
```



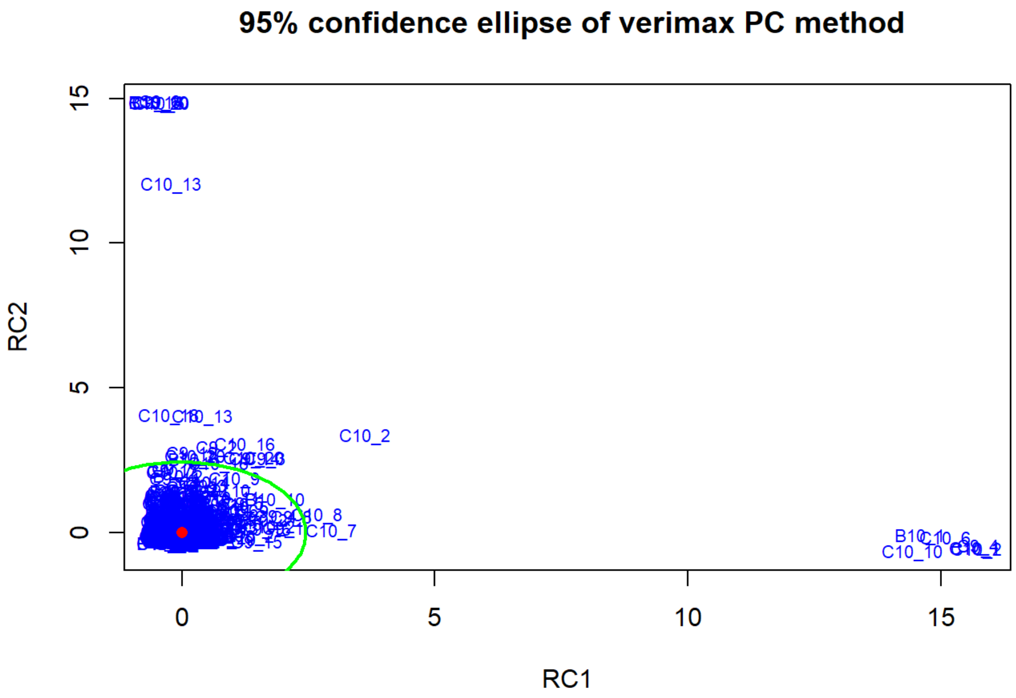

```
par(mfrow=c(1,2))
{load5 <- fit3v$loadings[,1:2]
plot(load5, type="n", main="New loadings for the first two factors of Varimax MLE") # set up plot
text(load5,labels=names(dat1),cex=.7, col="blue") # add variable names

load6 <- fit3q$loadings[,1:2]
plot(load6, type="n", main="New loadings for the first two factors of Quartimax MLE") # set up plot
text(load6,labels=names(dat1),cex=.7, col="blue") # add variable names
}
```



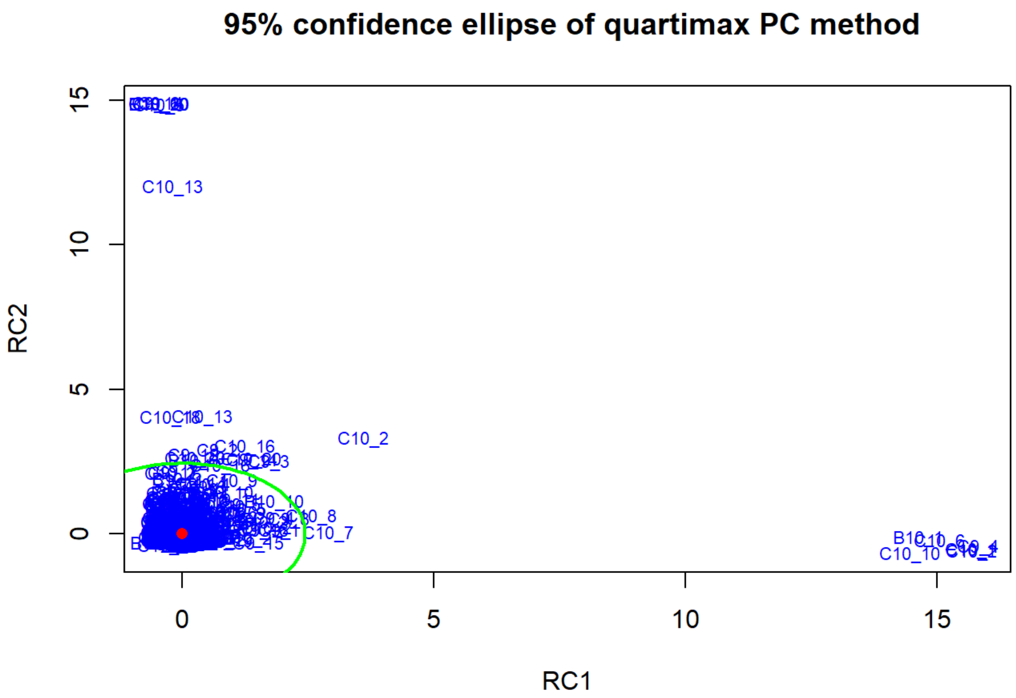
- Comparing to the plot above, varimax and quartimax Rotated Principal Components Method can better explain the data, as in other methods, the description of one factor might overlap with a description of another factor(Like MLE method). Or the output did not show the obvious classification(Principal method).
- According to the loading plot of varimax and quartimax Rotated Principal Components Method, we can see that now all the C_1 to C_10 variables load heavily on Factor 1, but have very low loadings on Factor 2. In the vertical direction, we see that the C_11 to C_20 variable load heavily on Factor 2 but less so on Factor 1.
- As a result, we can define or label the factors using those terms, e.g., Factor 1 might be labeled adolesence personal delinquency, and Factor 2 might be labeled adolesence group delinquency.

Question 3: Use regression method to derive factor scores. Make a scatter plot for the first two factors and select the potential outliers according to the 95% confidence ellipse. Check the raw data and describe the special characteristics of outliers.

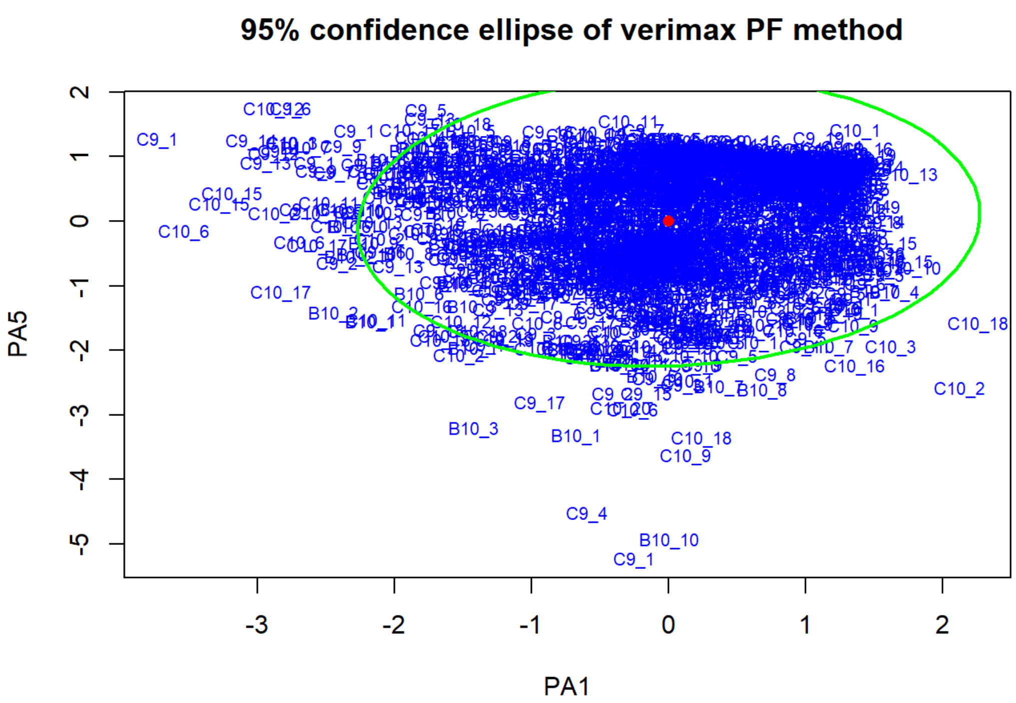
```
par(mfrow=c(1,1))
scv1<-fitv$scores
plot(scv1, col='blue', pch=19, type="n", main=' 95% confidence ellipse of verimax PC method')
text(scv1,labels=names(dat1),cex=.7, col="blue")
m_score1<-c(mean(scv1[,1]),mean(scv1[,2]))
vv1<-var(scv1)
conf.ellipse<-ellipse(vv1,centre=m_score1, level=0.95)
{lines(conf.ellipse, type="l", lwd=2, col='green')}
points(x=m_score1[1],y=m_score1[2],pch=16, col="red")
```



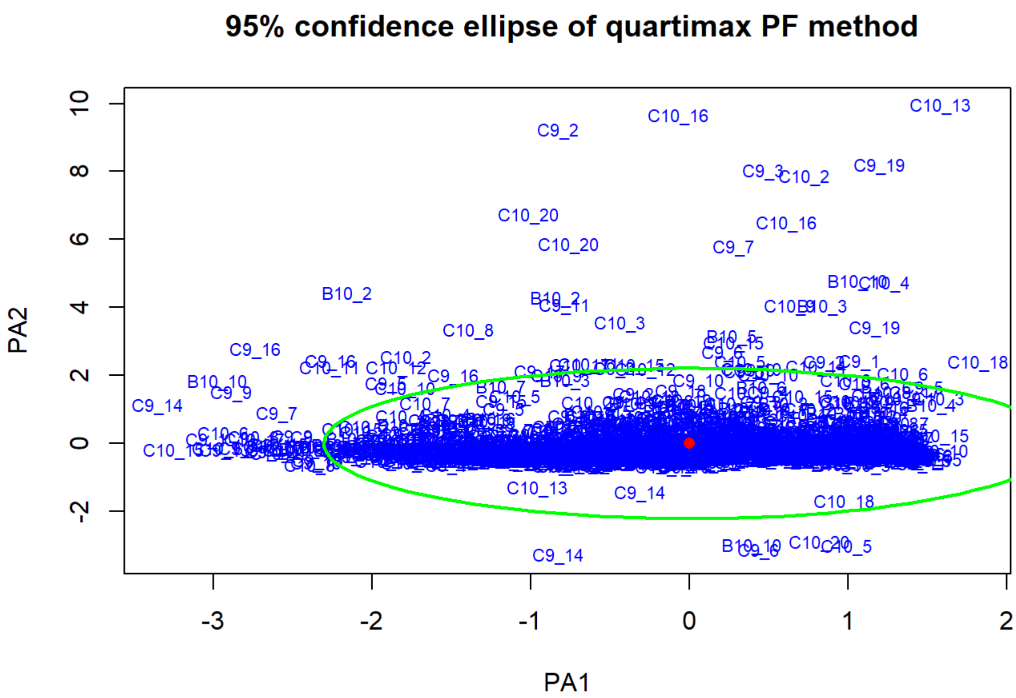

```
scq1<-fitq$scores
plot(scq1, col='blue', pch=19, type="n", main=' 95% confidence ellipse of quartimax PC method')
text(scq1,labels=names(dat1),cex=.7, col="blue")
m_score2<-c(mean(scq1[,1]),mean(scq1[,2]))
vq1<-var(scq1)
conf.ellipse2<-ellipse(vq1,centre=m_score2, level=0.95)
{lines(conf.ellipse2, type="l", lwd=2, col='green')}
points(x=m_score2[1],y=m_score2[2],pch=16, col="red")
```



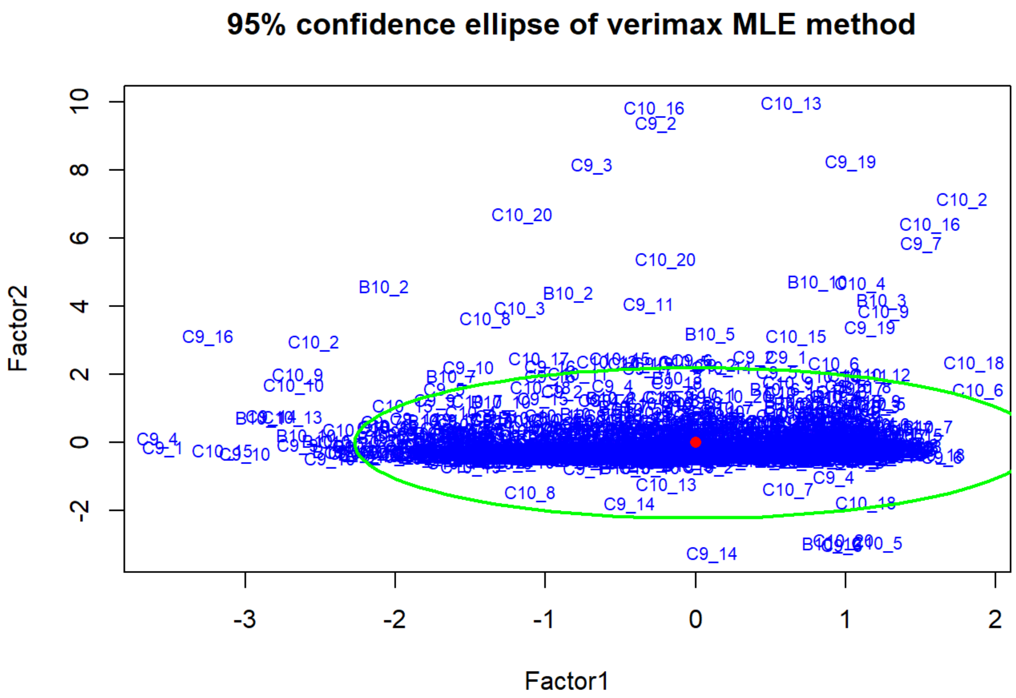

```
scv2<-fit2v$scores
plot(scv2, col='blue', pch=19, type="n", main=' 95% confidence ellipse of verimax PF method')
text(scv2,labels=names(dat1),cex=.7, col="blue")
m_score3<-c(mean(scv2[,1]),mean(scv2[,2]))
vv2<-var(scv2)
conf.ellipse3<-ellipse(vv2,centre=m_score3, level=0.95)
{lines(conf.ellipse3, type="l", lwd=2, col='green')}
points(x=m_score3[1],y=m_score3[2],pch=16, col="red")
```



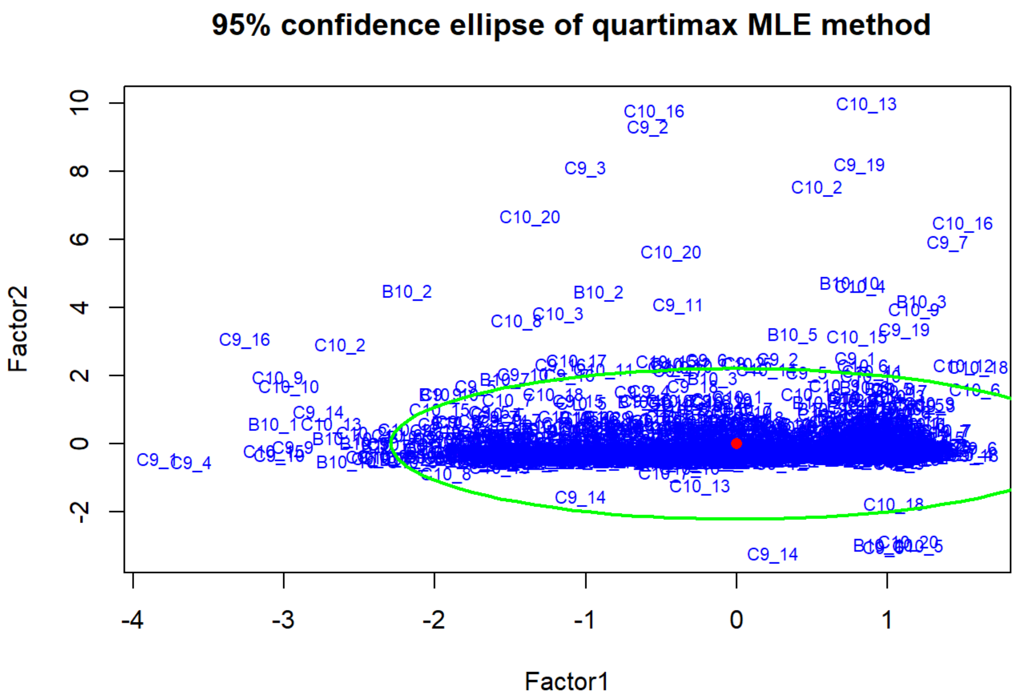
```
scq2<-fit2q$scores
plot(scq2, col='blue', pch=19, type="n", main=' 95% confidence ellipse of quartimax PF method')
text(scq2,labels=names(dat1),cex=.7, col="blue")
m_score4<-c(mean(scq2[,1]),mean(scq2[,2]))
vq2<-var(scq2)
conf.ellipse4<-ellipse(vq2,centre=m_score4, level=0.95)
{lines(conf.ellipse4, type="l", lwd=2, col='green')}
points(x=m_score4[1],y=m_score4[2],pch=16, col="red")
```




```
scv3<-fit3v$scores
plot(scv3, col='blue', pch=19, type="n", main=' 95% confidence ellipse of verimax MLE method')
text(scv3,labels=names(dat1),cex=.7, col="blue")
m_score5<-c(mean(scv3[,1]),mean(scv3[,2]))
vv3<-var(scv3)
conf.ellipse5<-ellipse(vv3,centre=m_score5, level=0.95)
{lines(conf.ellipse5, type="l", lwd=2, col='green')}
points(x=m_score5[1],y=m_score5[2],pch=16, col="red")
```




```
scq3<-fit3q$scores
plot(scq3, col='blue', pch=19, type="n", main=' 95% confidence ellipse of quartimax MLE method')
text(scq3,labels=names(dat1),cex=.7, col="blue")
m_score6<-c(mean(scq3[,1]),mean(scq3[,2]))
vq3<-var(scq3)
conf.ellipse6<-ellipse(vq3,centre=m_score6, level=0.95)
{lines(conf.ellipse6, type="l", lwd=2, col='green')}
points(x=m_score6[1],y=m_score6[2],pch=16, col="red")
```



- I still choose the verimax and quartimax PC method to do the analysis. The outliers are C_1 to C_20, and B10_1.
- 根據原始資料，B10_1代表爸媽/主要照顧者讓我感覺到我有權利發脾氣，而C_1至C_20代表著青少年的不良行為，我認為這兩者之間是有關連的，若是青少年認為自己不開心時有權利發脾氣，很可能導致這一些不良行為的發生。