Statistical Computing HW1

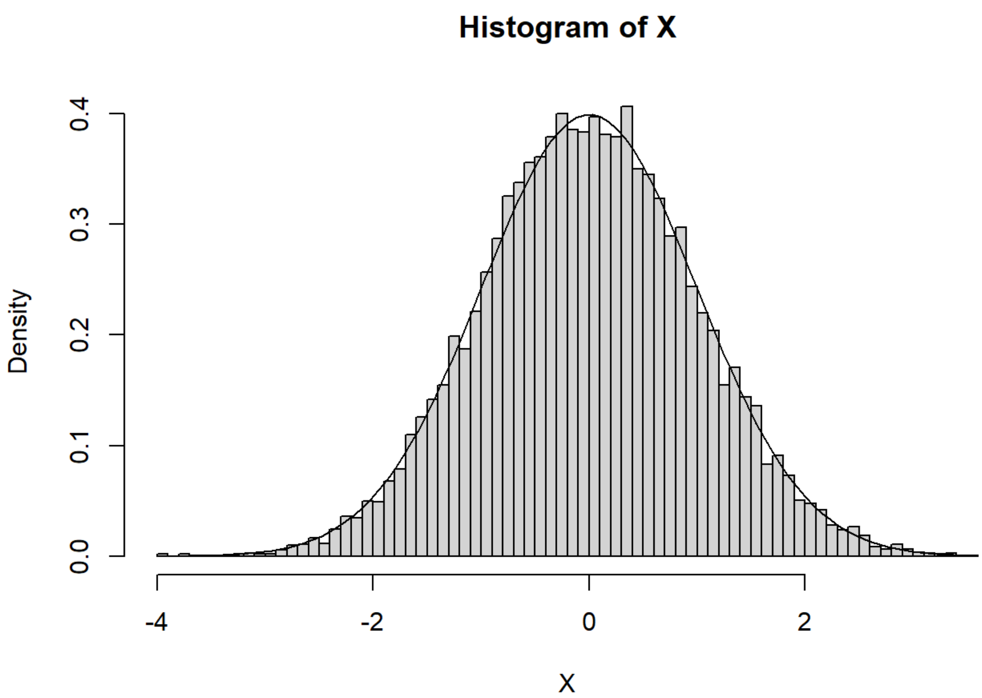
106070020

2021-03-27

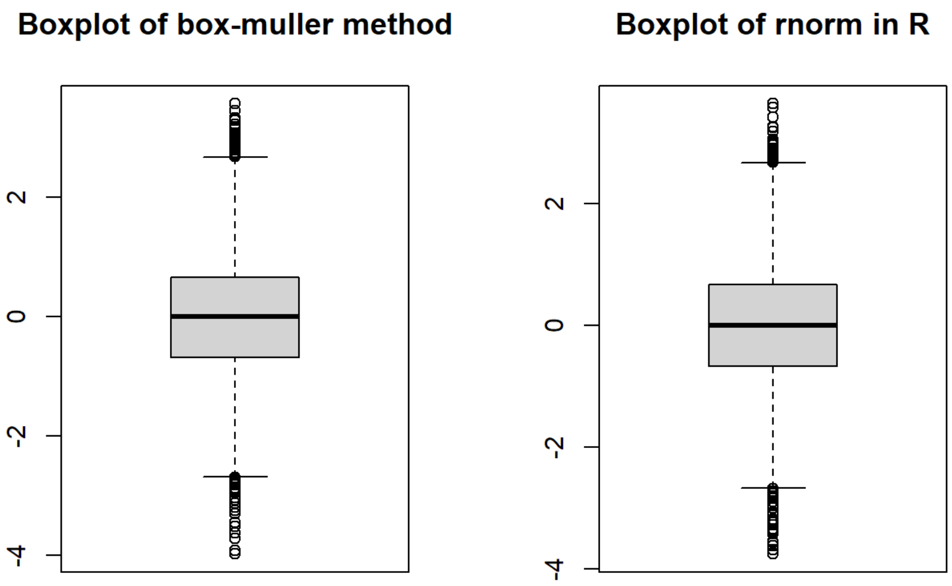
Problem 1

(a)

```
#Q1(a)

set.seed(0404)
t0 <- Sys.time()
U1<-runif(10000,0,1)
U2<-runif(10000,0,1)
X<-sqrt(-2*log(U1))*cos(2*pi*U2)
t1 <- Sys.time()
time_box_muller<-t1-t0
t2 <- Sys.time()
Y<-rnorm(10000,0,1)
t3 <- Sys.time()
time_rnorm<- t3-t2
hist(X, prob=T, breaks = 100)
curve(dnorm(x,0,1), add=T)
```




```
#Y<-sqrt(-2*log(U1))*sin(2*pi*U2)
#plot(density(Y))
op=par(mfrow=c(1,2))
boxplot(X, main="Boxplot of box-muller method")
boxplot(rnorm(10000,0,1),main="Boxplot of rnorm in R")
```




```
par(op)
time_box_muller
```

```
## Time difference of 0.007994175 secs
```

```
time_rnorm
```

```
## Time difference of 0.003997087 secs
```

(b)

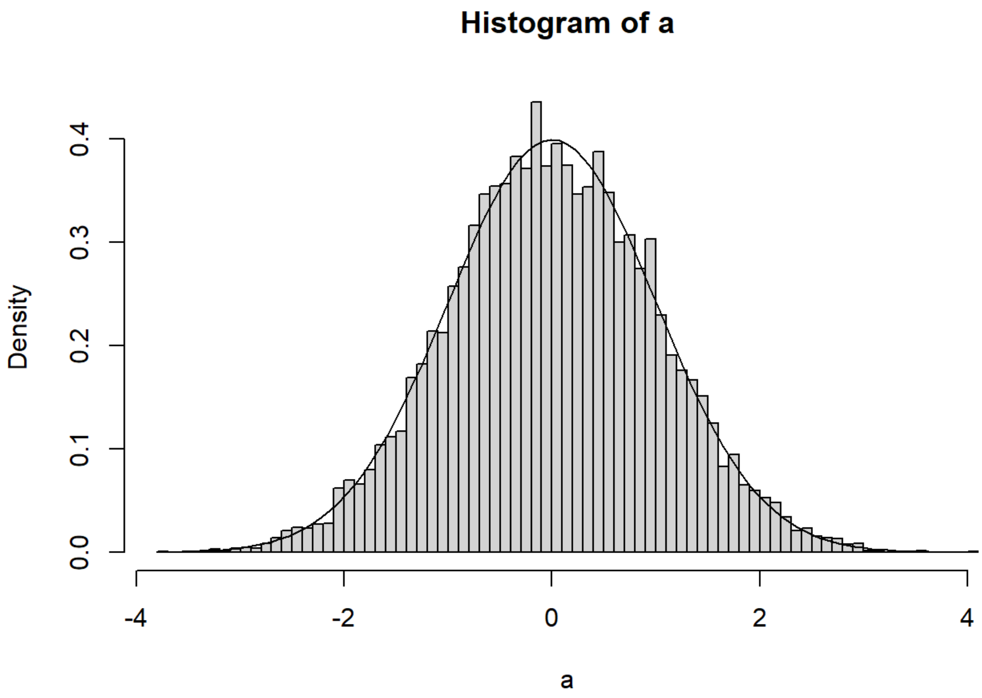
```
#(b)
set.seed(1106)
acfuc<-function(f_x){
  f <- function(x){
    sqrt(2/pi)*exp(-x^2/2)
  }
  g <- function(x){
    exp(-x)
  }
  for(i in 1:length(f_x)){
    repeat{
      x<- -log(runif(1))#generate exp(1) using inverse CDF
      u <- runif(1)
      c=sqrt(2/pi)*exp(1/2)
      if(u<f(x)/(c*g(x))){break} #accept until U1<=f(abs(x))/c*g(x)
    }
    # flip a coin(using abs(x)generate normal(0,1))
    if(runif(1)<0.5){
      Z = x
    }
    else Z = -x
    f_x[i] <- Z
  }
  return(f_x)
}
f_x <- rep(0,10000)
c=sqrt(2/pi)*exp(1/2)
c
```

```
## [1] 1.315489
```

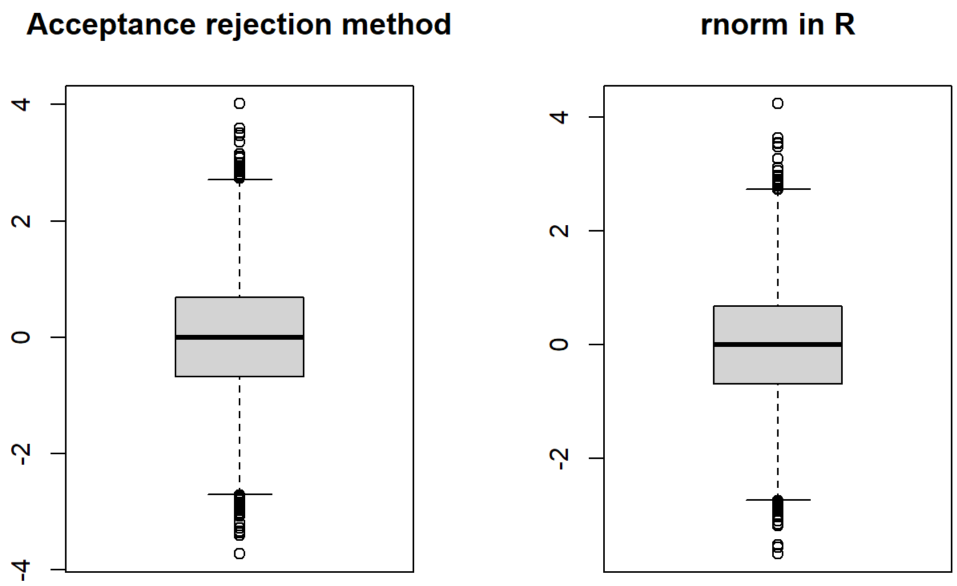
```
t0 <- Sys.time()
a<-acfuc(f_x)

t1 <- Sys.time()
time_acc_rej<-t1-t0
time_acc_rej
```

```
## Time difference of 0.3098221 secs
```

```
hist(a,prob=T,breaks = 100)
curve(dnorm(x,0,1),add=T)
```




```
op=par(mfrow=c(1,2))
boxplot(a, main="Acceptance rejection method")
boxplot(rnorm(10000,0,1),main="rnorm in R")
```



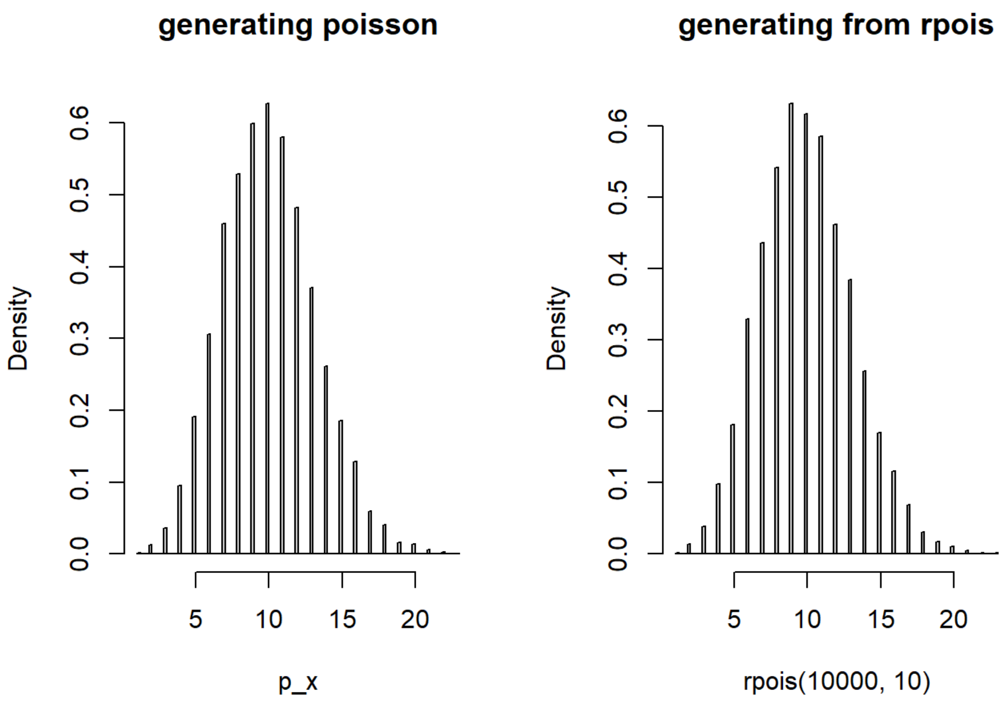

```
par(op)
paste("The acceptance rate is",1/c)
```

```
## [1] "The acceptance rate is 0.760173450533314"
```

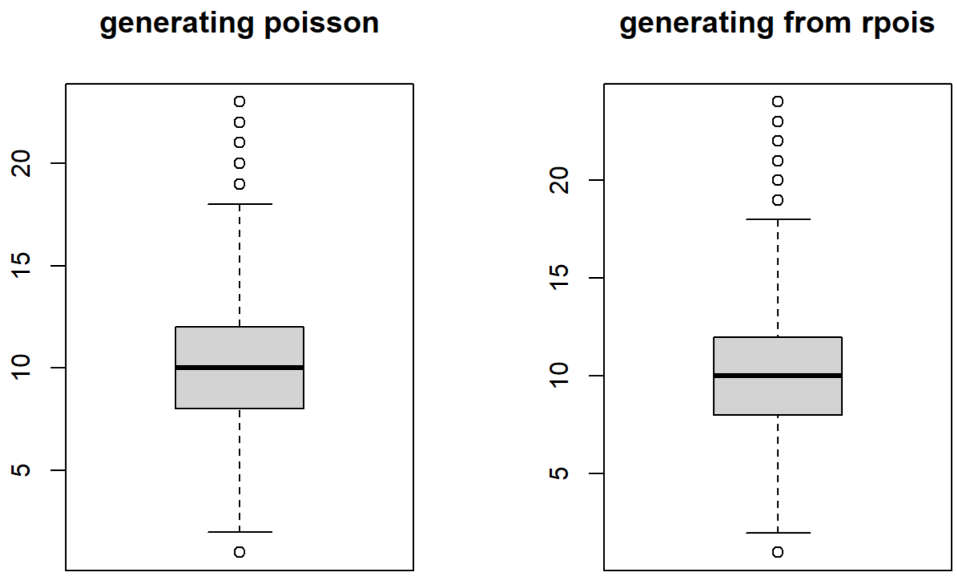
Problem 2

(a)

```
#Q2(a)
set.seed(1986)
#Lambda = 10
p_x <- rep(0,10000)

pois<-function(lambda){
  t <- 0
  X <- -1
  while(t < 1){
    t<- t-(1/lambda*log(runif(1)))
    X<-X+1
    }
    return(X)
}
t0 <- Sys.time()
for(i in 1:length(p_x)){
  p_x[i]<-pois(10)
}
t1 <- Sys.time()
poi_generate_time<-t1-t0
t2<-Sys.time()
b<-rpois(10000,10)
t3<-Sys.time()
rpois_time<-t3-t2
par(mfrow=c(1,2))
hist(p_x,prob=T,breaks = 100, main="generating poisson")
hist(rpois(10000,10),prob=T,breaks = 100, main="generating from rpois")
```




```
par(mfrow=c(1,2))
boxplot(p_x,prob=T,breaks = 100, main="generating poisson")
boxplot(rpois(10000,10),prob=T,breaks = 100, main="generating from rpois")
```




```
poi_generate_time
```

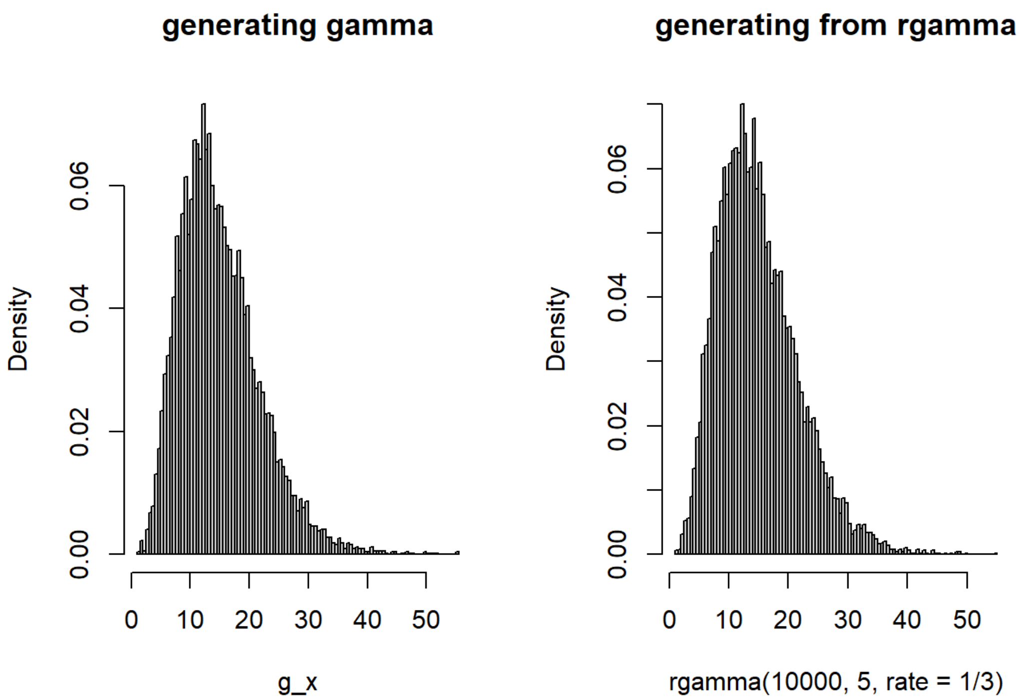
```
## Time difference of 0.8085401 secs
```

```
rpois_time
```

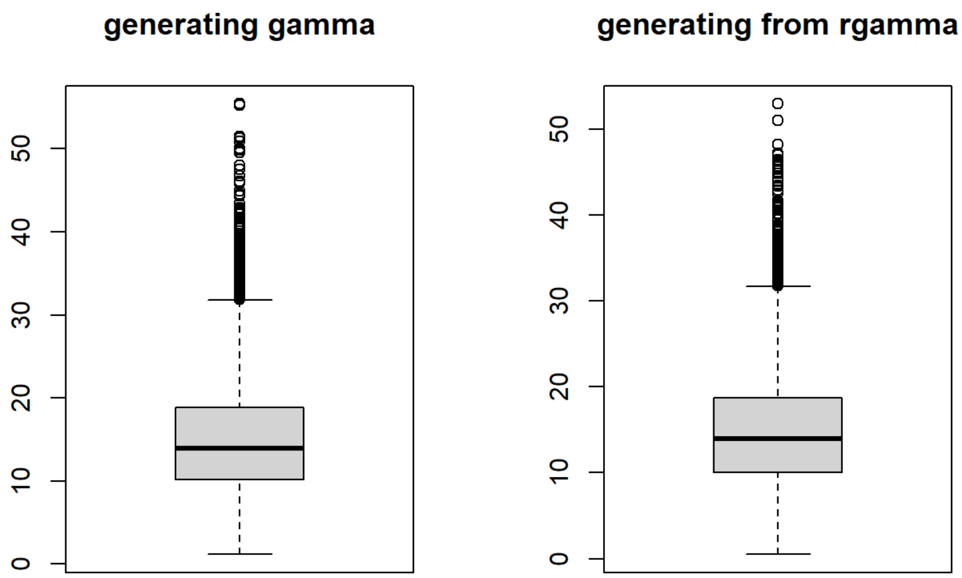
```
## Time difference of 0.01000094 secs
```

(b)

```r
#(b)
set.seed(1130)
g_x <- rep(0,10000)
gam<-function(a,b){
  U<-runif(a,0,1)
  X1<- -b*log(U)
  X<-sum(X1)
  return(X)
}
t0<-Sys.time()
for(i in 1:length(g_x)){
  g_x[i]<-gam(5,3)
}
t1<-Sys.time()
generate_gamma_time<- t1-t0
t2<-Sys.time()
a<-rgamma(10000,5,rate = 1/3)
t3<-Sys.time()
rgamma_time<- t3-t2
par(mfrow=c(1,2))
hist(g_x,prob=T,breaks = 100, main="generating gamma")
hist(rgamma(10000,5,rate = 1/3),prob=T,breaks = 100, main="generating from rgamma")
```




```r
par(mfrow=c(1,2))
boxplot(g_x,prob=T,breaks = 100, main="generating gamma")
boxplot(rgamma(10000,5,rate=1/3),prob=T,breaks = 100, main="generating from rgamma")
```




```
generate_gamma_time
```

```
## Time difference of 0.08393383 secs
```

```
rgamma_time
```

```
## Time difference of 0.004998922 secs
```

Problem 3

(a)

$$\text{Derive the marginal distribution of X}$$

$$\text{Suppose } \; X|\mu \sim Poi(\mu), \mu \sim Gamma(\alpha, \beta)$$

$$f(x;\mu) = \frac{\mu^x}{x!}e^{-\mu}\frac{1}{\Gamma(\alpha)\beta^\alpha}\mu^{\alpha-1}e^{\frac{-\mu}{\beta}}$$

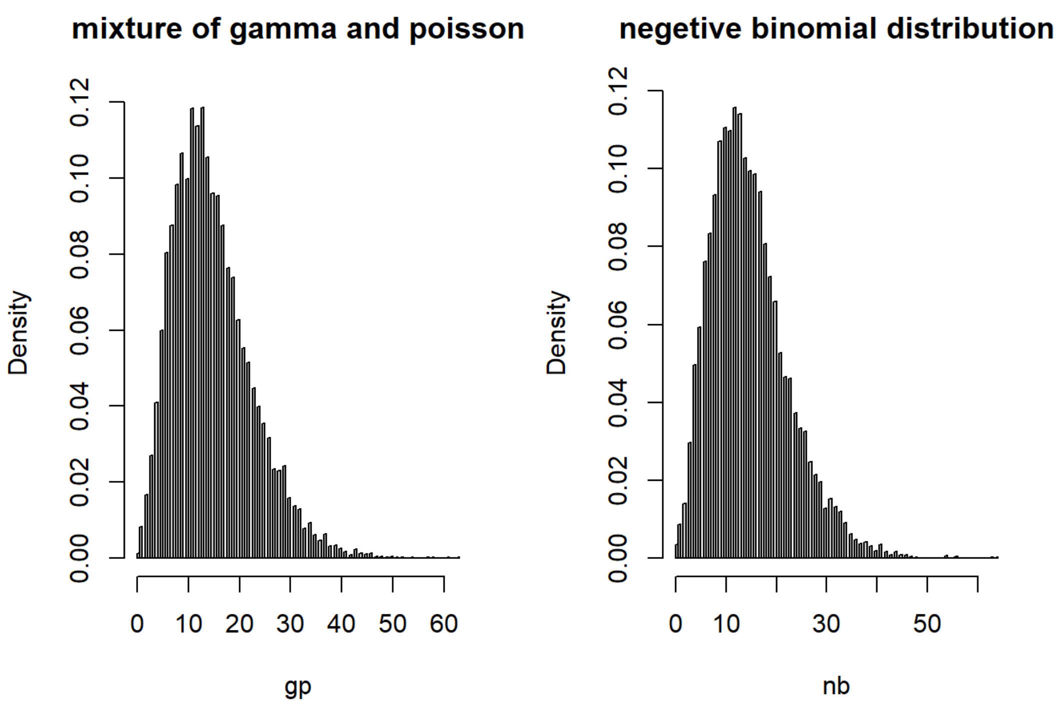
$$\begin{aligned} f(x) = & \frac{1}{\Gamma(\alpha)\beta^\alpha x!}\int_0^\infty \mu^{x+\alpha-1}e^{-(1+1/\beta)\mu}d\mu \\ = & \frac{1}{\Gamma(x+1)\Gamma(\alpha)\beta^\alpha}\Gamma(\alpha+x)(\frac{\beta}{1+\beta}) \\ = & \binom{\alpha-1+x}{x}(\frac{1}{1+\beta})^\alpha(1-\frac{1}{1+\beta})^x \end{aligned}$$

$$\text{which } \; p = \frac{1}{1+\beta}$$

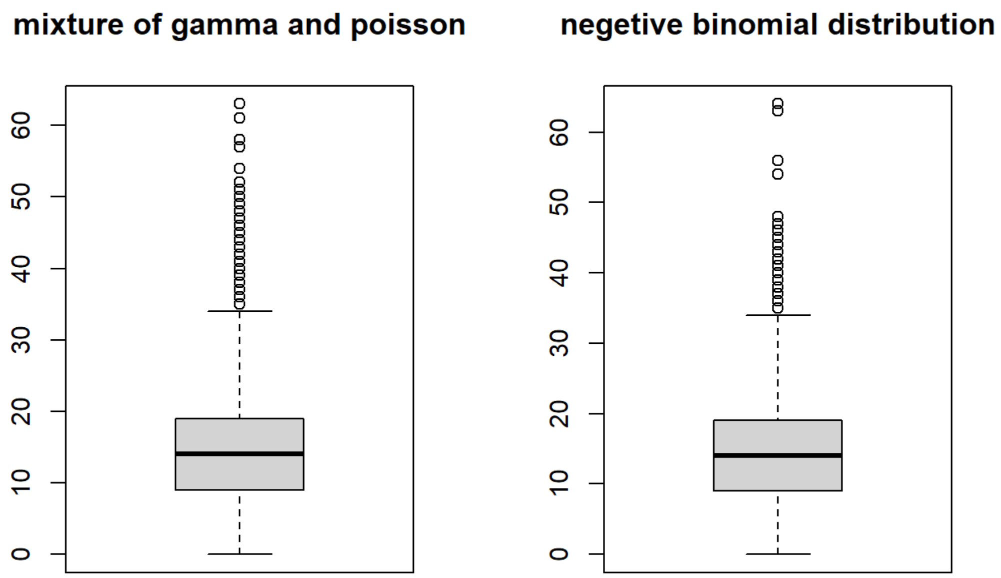
$$\text{Thus, } \; X \sim NB(\alpha, \frac{1}{1+\beta})$$

(b)

```r
#(b)
set.seed(1209)
lambda <- rep(0,10000)
gam<-function(a,b){
  U<-runif(a,0,1)
  X1<- -b*log(U)
  X<-sum(X1)
  return(X)
}
pois<-function(lambda){
  t <- 0
  X <- -1
  while(t < 1){
    t<- t-(1/lambda*log(runif(1)))
    X<-X+1
  }
  return(X)
}
t0<- Sys.time()
for(i in 1:length(lambda)){
  lambda[i]<-gam(5,3)
}
gp=rep(0,10000)
for(i in 1:length(lambda)){
  gp[i]<-pois(lambda[i])
}
t1<-Sys.time()
time_mix<-t1-t0
t2<-Sys.time()
nb<-rnbinom(10000, 5, 1/4)
t3<-Sys.time()
nbtime<-t3-t2
par(mfrow=c(1,2))
hist(gp,prob=T,breaks = 100, main="mixture of gamma and poisson")
hist(nb,prob=T,breaks = 100,main="negetive binomial distribution")
```




```r
par(mfrow=c(1,2))
boxplot(gp,prob=T,breaks = 100, main="mixture of gamma and poisson")
boxplot(nb,prob=T,breaks = 100, main="negetive binomial distribution")
```




```
time_mix
```

```
## Time difference of 0.499716 secs
```

```
nbtime
```

```
## Time difference of 0.007993937 secs
```

(c)

```
#(c)
paste("The mean of X is",mean(gp), "The mean of NB(5,1/4) is", mean(nb))
```

```
## [1] "The mean of X is 14.9614 The mean of NB(5,1/4) is 14.8885"
```

```
paste("The variance of X is",var(gp), "The varance of NB(5,1/4) is", var(nb))
```

```
## [1] "The variance of X is 60.2589359335934 The varance of NB(5,1/4) is 59.5582235723572"
```

Problem 4

(a)

$n = 2$, $(p_1, p_2) = (p_1, 1 - p_1)$, $\mu = (0, 3)$, and $\sigma = (1, 1)$

$$\text{The finite mixture distribution is}$$

$$p_1 N(0,1) + (1 - p_1) N(3,1)$$

$$f(x) = \frac{p_1}{\sqrt{2\pi}} exp\{-\frac{1}{2}(x)^2\} + \frac{1 - p_1}{\sqrt{2\pi}} exp\{-\frac{1}{2}(x-3)^2\}$$

(b)

```r
#Q4(b)
set.seed(0525)
U <-runif(10000,0,1)
rand<-rep(0, 10000)
for(i in 1:length(rand)){
  if(U[i]<.75){
    rand[i]<-rnorm(1,0,1)
  }else{
    rand[i]<-rnorm(1,3,1)
  }
}
hist(rand,prob=T,breaks = 500, main="The distribution of X")
```