Time Series HW 3

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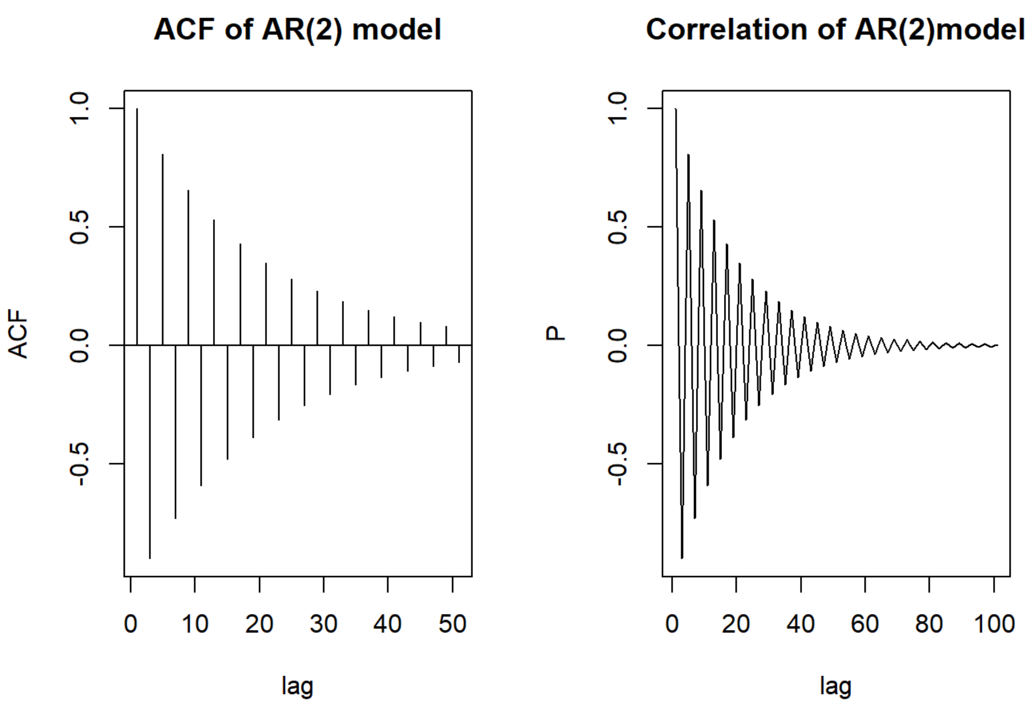
Question 3.6

```
#Question 3.6

z<-c(1,0,0.9) # Find the roots of the autoregressive polynomial
polyroot(z)
```

```
## [1] 0+1.054093i 0-1.054093i
```

```
par(mfrow=c(1,2))
# ACF plot
ACF=ARMAacf(ar=c(0,-0.9), ma=0,50)
plot(ACF, type='h', xlab="lag", main="ACF of AR(2) model")
abline(h=0)
#correlation plot
P=ARMAacf(ar=c(0,-0.9),lag.max=100)
plot(P, type='l', xlab="lag", main="Correlation of AR(2)model")
```



Qusetion 3.7

(a)

```
#(a) x_t + 1.6x_t-1 + .64x_t-2 = w_t
z1<-c(1,1.6,0.64)
polyroot(z1)
```

```
## [1] -1.25-0i -1.25+0i
```

```
# In z1, the roots are real and equal(Z1=Z2=Z0), then rho(h)=Z0^-h*(c1+c2h)
```

Solve for constant

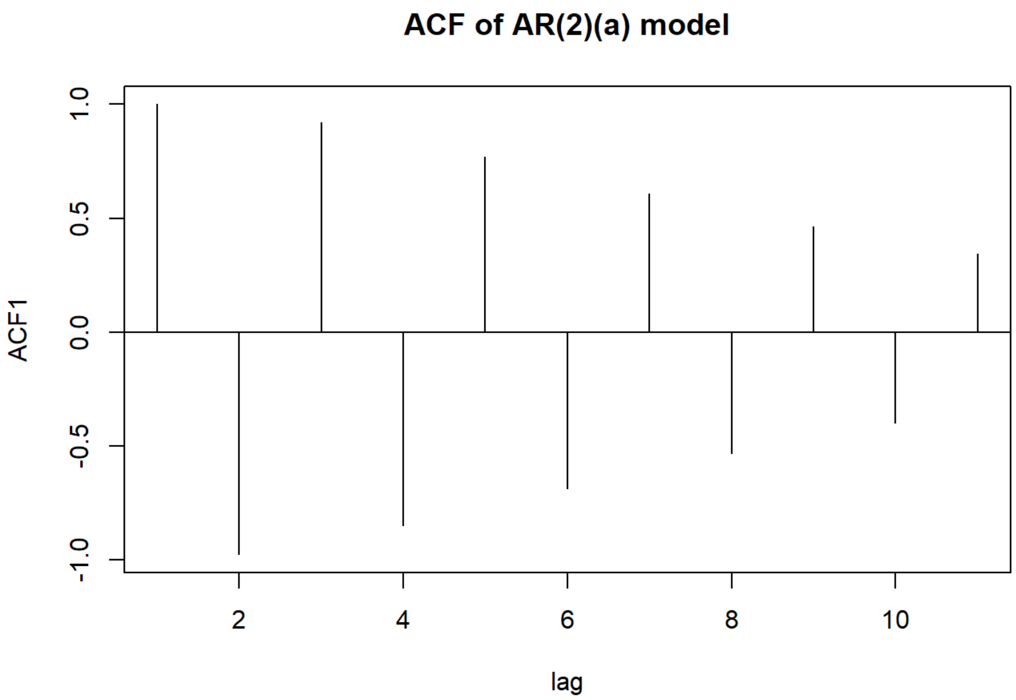
```
#solve for constant
A<-matrix(c(1,0,-1.25,-1.25),2,2,T)
B<-matrix(c(1,-1.6/1.64),2,1)
solve(A,B)
```

```
##            [,1]
## [1,]  1.0000000
## [2,] -0.2195122
```

```
# The answer is c1 and c2(constant)
```

plot ACF

```
#plot ACF
ACF1=ARMAacf(ar=c(-1.6,-.64), lag.max=10)
plot(ACF1, type='h', xlab="lag", main="ACF of AR(2)(a) model")
abline(h=0)
```



(b)

```
#(b) x_t - .40x_t-1 - .45x_t-2 = w_t
z2<-c(1,-.4,-.45)
polyroot(z2)
```

```
## [1]  1.111111-0i -2.000000+0i
```

```
# In z2, the roots (Z1 and Z2) are real and distinct, so the rho(h)=c1*Z1^-h+c2*Z2^-h
```

solve for constant

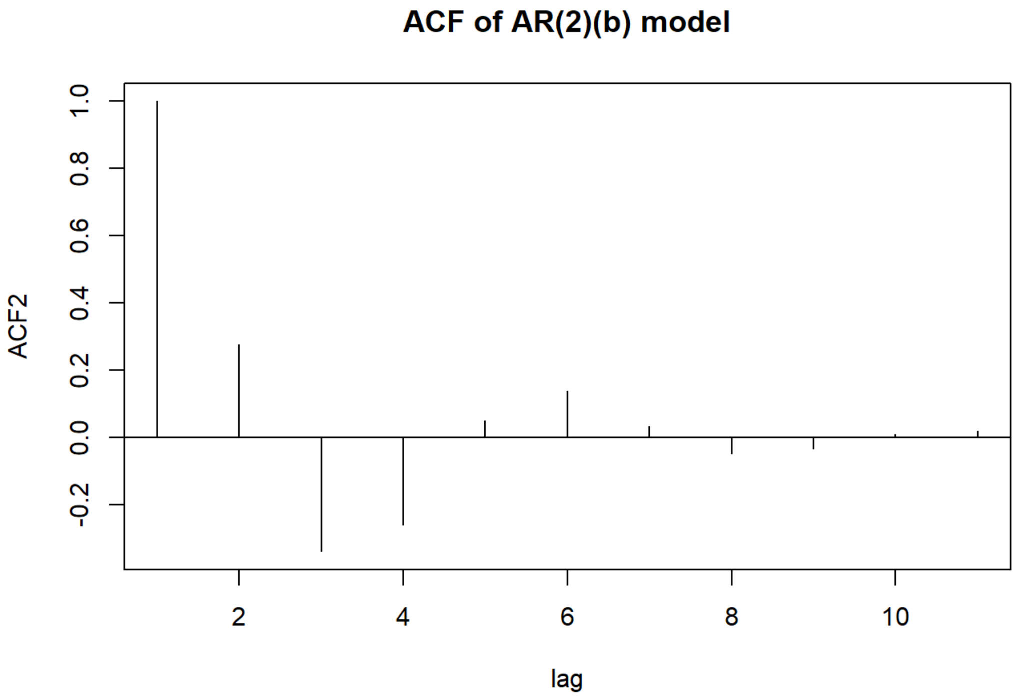
```
A<-matrix(c(1.11,-2,1,1),2,2,T)
B<-matrix(c(0.4/0.55,1),2,1)
solve(A,B)
```

```
##           [,1]
## [1,] 0.8769366
## [2,] 0.1230634
```

```
# The answer is c1 and c2(constant)
```

plot ACF

```
ACF2=ARMAacf(ar=c(.4,-.45), lag.max=10)
plot(ACF2, type='h', xlab="lag", main="ACF of AR(2)(b) model")
abline(h=0)
```



(c)

```
#(c) x_t - 1.2x_t-1 + .85x_t-2 = w_t
z3<-c(1,-1.2,.85)
polyroot(z3)
```

```
## [1] 0.7058824+0.8235294i 0.7058824-0.8235294i
```

```
# In z3, the two roots (Z1 and Z2) are a complex and conjugate pair, Z1=Z2_bar, then c2=c1_bar(because the rho(h) is r
eal), and rou(h)=c1*Z1^-h + c1_bar*Z1_bar^-h.
```

solve for constant

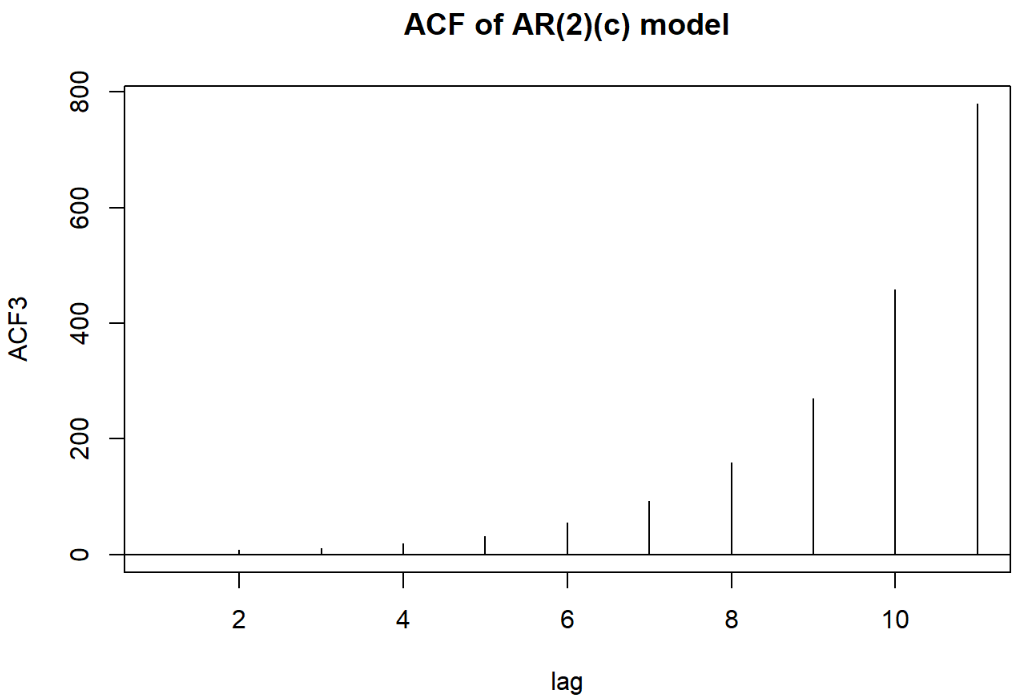
```
Z1<-polyroot(z3)[1]
Z2<-polyroot(z3)[2]
A<-matrix(c(1,1,Z1,Z2),2,2,T)
B<-matrix(c(1,1.2/1.85),2,1)
solve(A,B)
```

```
##                [,1]
## [1,] 0.5+0.034749i
## [2,] 0.5-0.034749i
```

```
# The answer is c1 and c2(constant)
```

plot ACF

```
ACF3=ARMAacf(ar=c(1.2,.85), lag.max=10)
plot(ACF3, type='h', xlab="lag", main="ACF of AR(2)(c) model")
abline(h=0)
```



Qusetion 3.9

```
# Generate 100 observation for three models: ARMA(1,1), ARMA(0,1),ARMA(1,0); theta = .9, phi = .6

ARMA11 <- arima.sim(model=list(ar=.6,ma=.9),n=100)
ARMA01 <- arima.sim(model=list(ma=.9),n=100)
ARMA10 <- arima.sim(model=list(ar=.6),n=100)
```

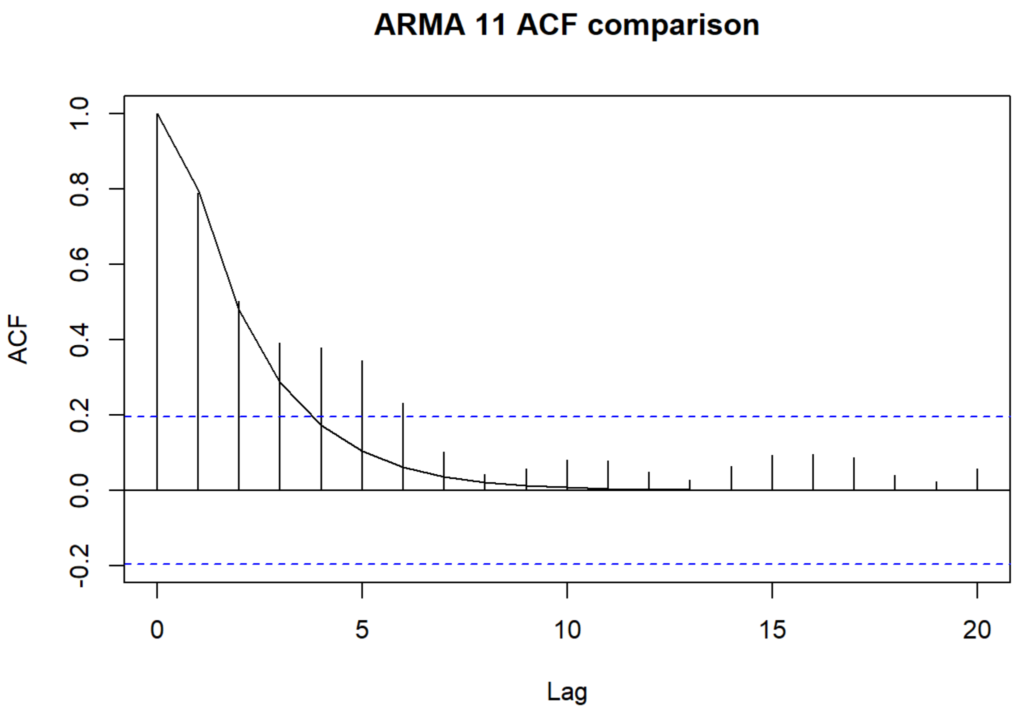
```
# compute sample ACF for all three simulations

ARMA11acf <- acf(ARMA11,plot=F)
ARMA01acf <- acf(ARMA01,plot=F)
ARMA10acf <- acf(ARMA10,plot=F)
```

```
# compute theoretical ACFs for all models
#
acf11 <- ARMAacf(ar=.6,ma=.9,lag.max=20)
acf01 <- ARMAacf(ma=.9,lag.max=20)
acf10 <- ARMAacf(ar=.6,lag.max=20)
```

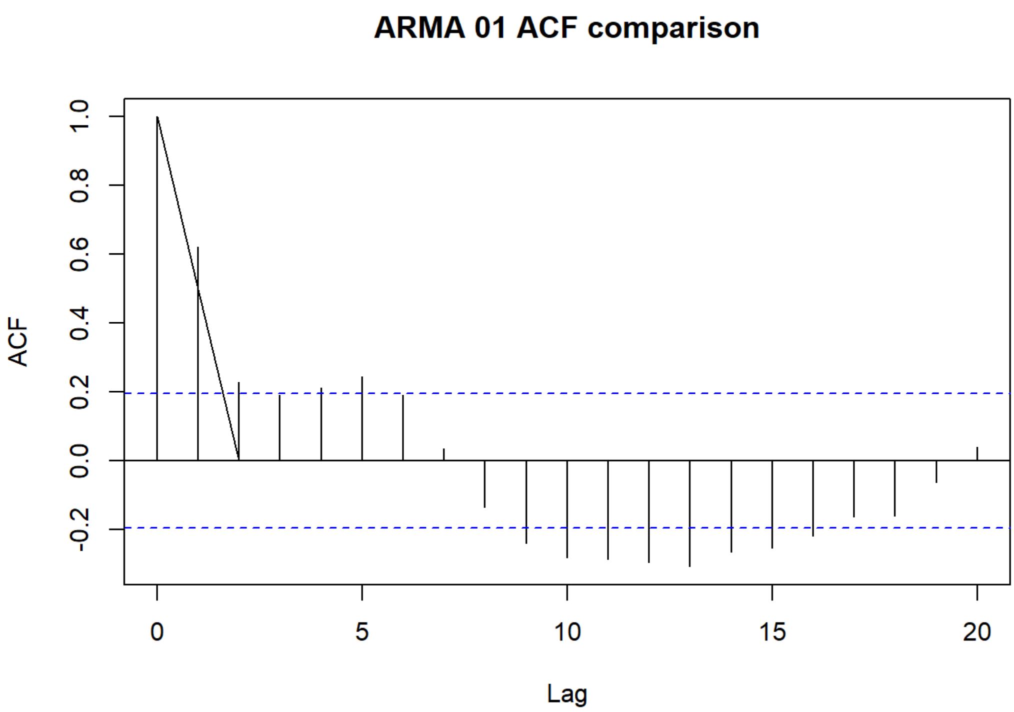
```
# plot and compare each other

plot(ARMA11acf, main="ARMA 11 ACF comparison")
lines(ARMA11acf$lag,acf11)
```



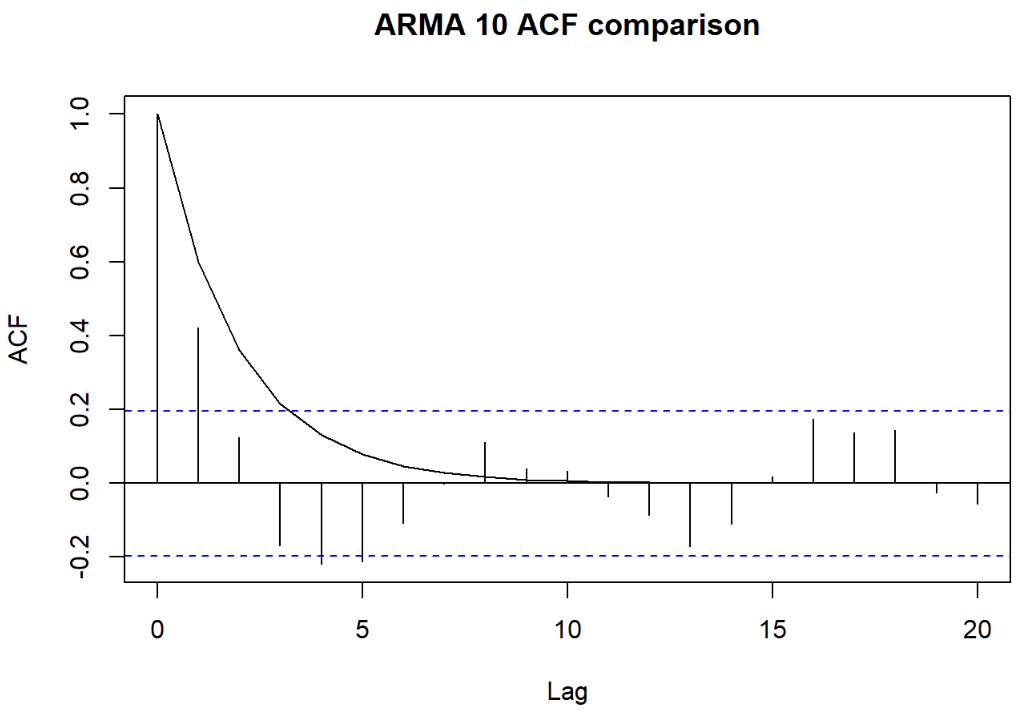

```
# Comparison: In the graph1, the theoretical ACF gradually converge to 0, while the sample ARMA 11 ACF still larger th
an 0 after the theoretical ACF become near to 0.
```

```
plot(ARMA01acf, main="ARMA 01 ACF comparison")
lines(ARMA01acf$lag,acf01)
```




```
# Comparison: In the graph2, the theoretical ACF quickly converge to 0, while the sample ARMA 01 ACF still larger than
0 after the theoretical ACF become near to 0.
```

```
plot(ARMA10acf, main="ARMA 10 ACF comparison")
lines(ARMA10acf$lag,acf10)
```

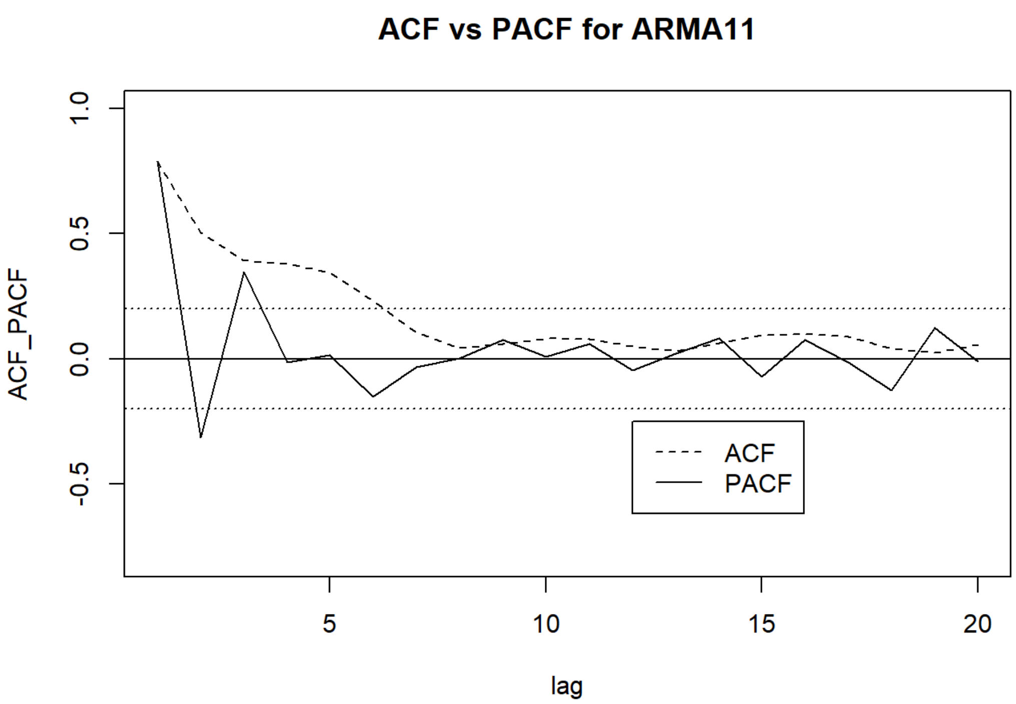



```
# Comparison: In the graph3, the theoretical ACF gradually converge to 0, while the sample ARMA 10 ACF still larger th
an 0 after the theoretical ACF become near to 0.
```

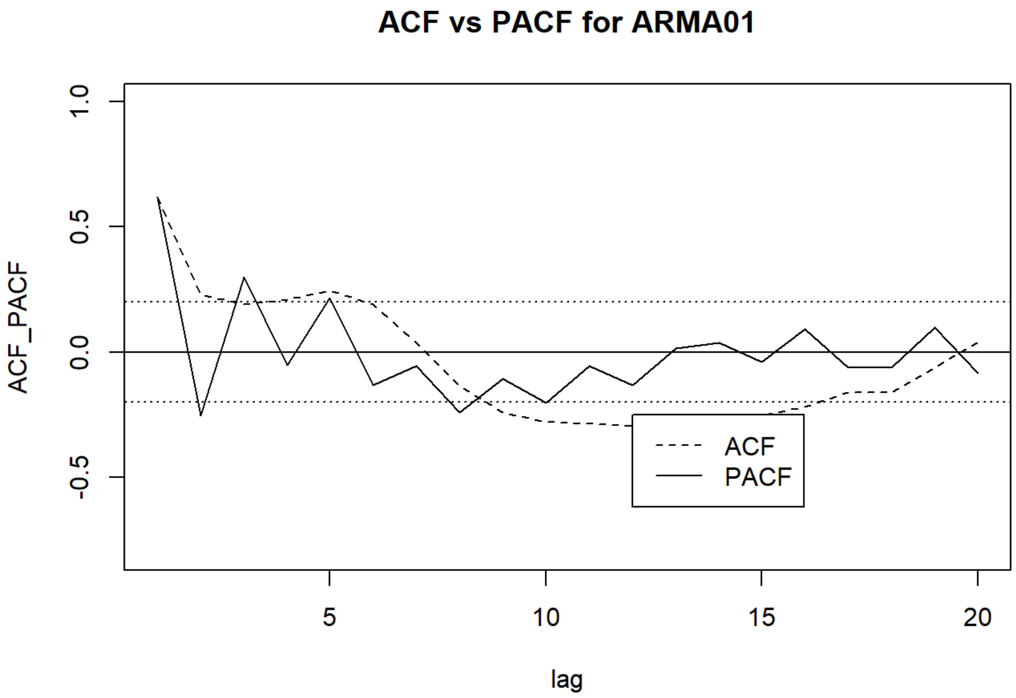
```
# compute sample PACF and plot against sample ACF

ARMA11pacf <- pacf(ARMA11,plot=F)
ARMA01pacf <- pacf(ARMA01,plot=F)
ARMA10pacf <- pacf(ARMA10,plot=F)
```

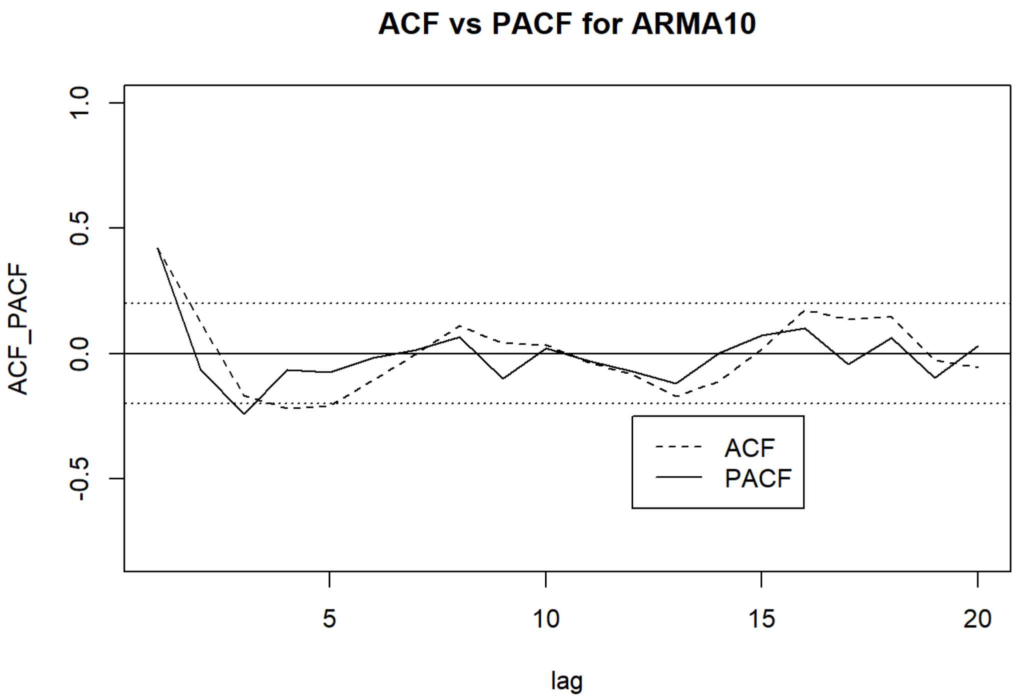
```
plot(ARMA11acf$acf[-1],type='l',lty=2,ylim=c(-.8,1),ylab='ACF_PACF',
     xlab='lag',main="ACF vs PACF for ARMA11")
abline(h=0)
abline(h=c(.2,-.2),lty=3)
lines(ARMA11acf$lag[-1],ARMA11pacf$acf)
legend(12,-.25,c("ACF","PACF"),lty=c(2,1))
```



```
plot(ARMA01acf$acf[-1],type='l',lty=2,ylim=c(-.8,1),ylab='ACF_PACF',
     xlab='lag',main="ACF vs PACF for ARMA01")
abline(h=0)
abline(h=c(.2,-.2),lty=3)
lines(ARMA01acf$lag[-1],ARMA01pacf$acf)
legend(12,-.25,c("ACF","PACF"),lty=c(2,1))
```




```
plot(ARMA10acf$acf[-1],type='l',lty=2,ylim=c(-.8,1),ylab='ACF_PACF',
     xlab='lag',main="ACF vs PACF for ARMA10")
abline(h=0)
abline(h=c(.2,-.2),lty=3)
lines(ARMA10acf$lag[-1],ARMA10pacf$acf)
legend(12,-.25,c("ACF","PACF"),lty=c(2,1))
```



```
# We can see that the sample PACF for the ARMA11 and MA1 'tail off' around lag 2, where the sample PACF for AR1 cuts o
ffafter lag p=1. Similarly, the sample ACF for the ARMA11 andappear to tail off slightly while the MA1 cutts off after
lag1. Note however, for AR1/MA1, the tailing off happens early (lag1-2), which makes sense since we had set p=1; q=1.
```