Homework 7 (due 2021-6-6)

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The goal of homework 7 is to practice the analysis procedures for multiple time series data. We will use an economic data set. The data consist of3 economic variables (CPI, GDP, Unemployment rate) for G7 countries (Canada, France, Germany, Italy, Japan, UK, USA) in 1991-2019 (seasonal data with 4 observations in a year).

Part1 : The data for CPI (7 series)—李嘉蓉

Read the data

```
library(autoTS)
library(vars)
```

```
## Loading required package: MASS
```

```
## Loading required package: strucchange
```

```
## Loading required package: zoo
```

```
##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
## Loading required package: sandwich
```

```
## Loading required package: urca
```

```
## Loading required package: lmtest
```

```
library(forecast)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```r
library(aTSA)
```

```
##
## Attaching package: 'aTSA'
```

```
## The following object is masked from 'package:forecast':
##
##     forecast
```

```
## The following object is masked from 'package:vars':
##
##     arch.test
```

```
## The following object is masked from 'package:graphics':
##
##     identify
```

```r
library(astsa)
```

```
##
## Attaching package: 'astsa'
```

```
## The following object is masked from 'package:forecast':
##
##     gas
```

```r
dat1 = read.csv("C:/Users/cindy/time_series/econ_data.csv") #put the data under a correct direct
ory
dim(dat1)
```

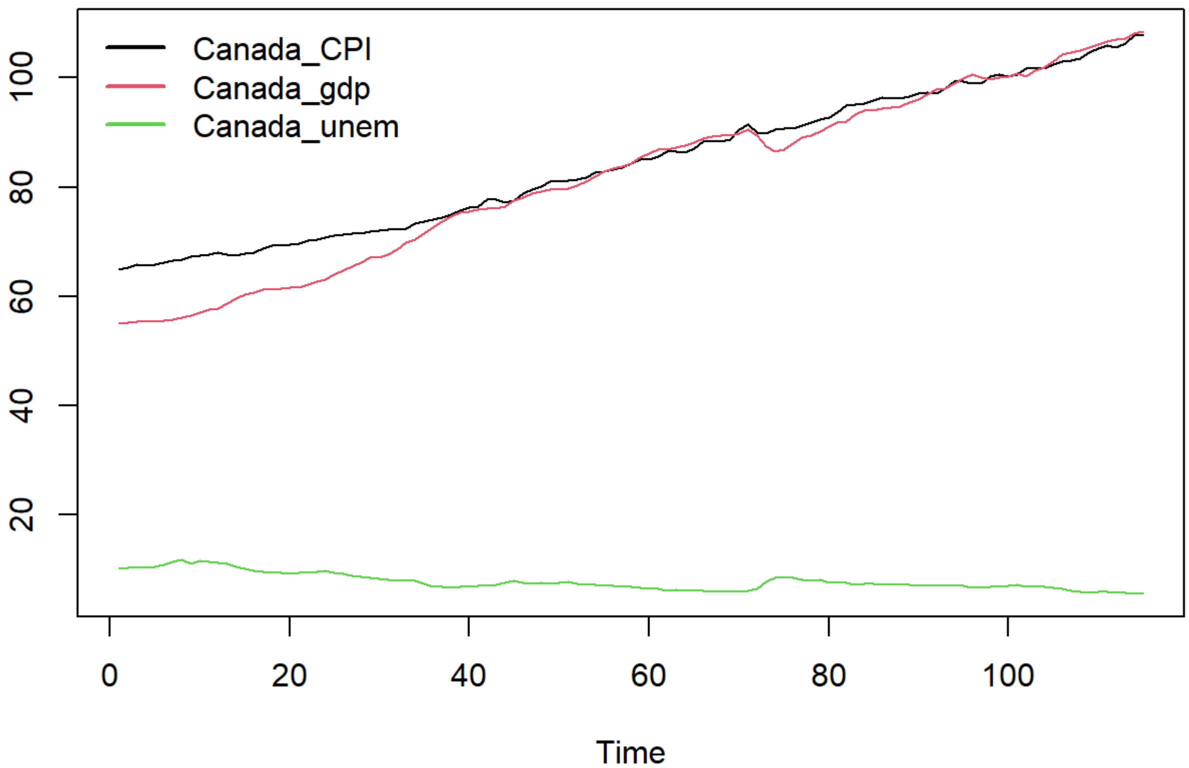
```
## [1] 115  22
```

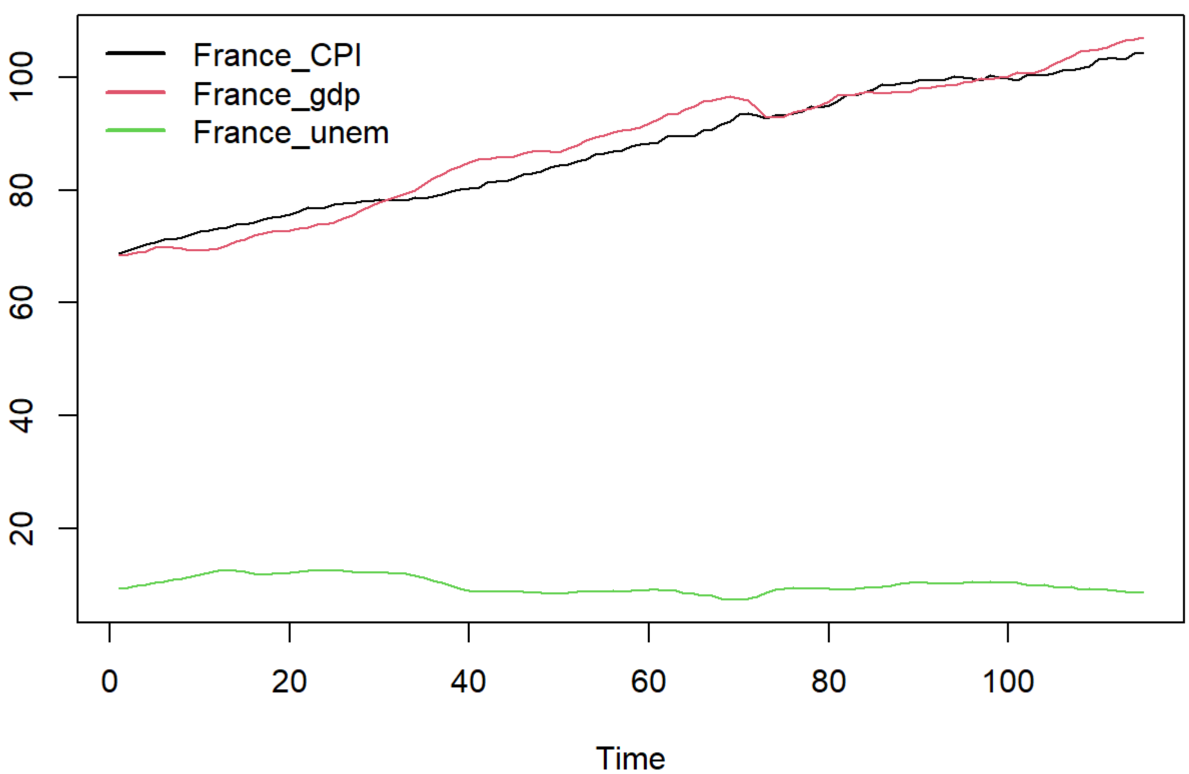
```r
head(dat1)
```

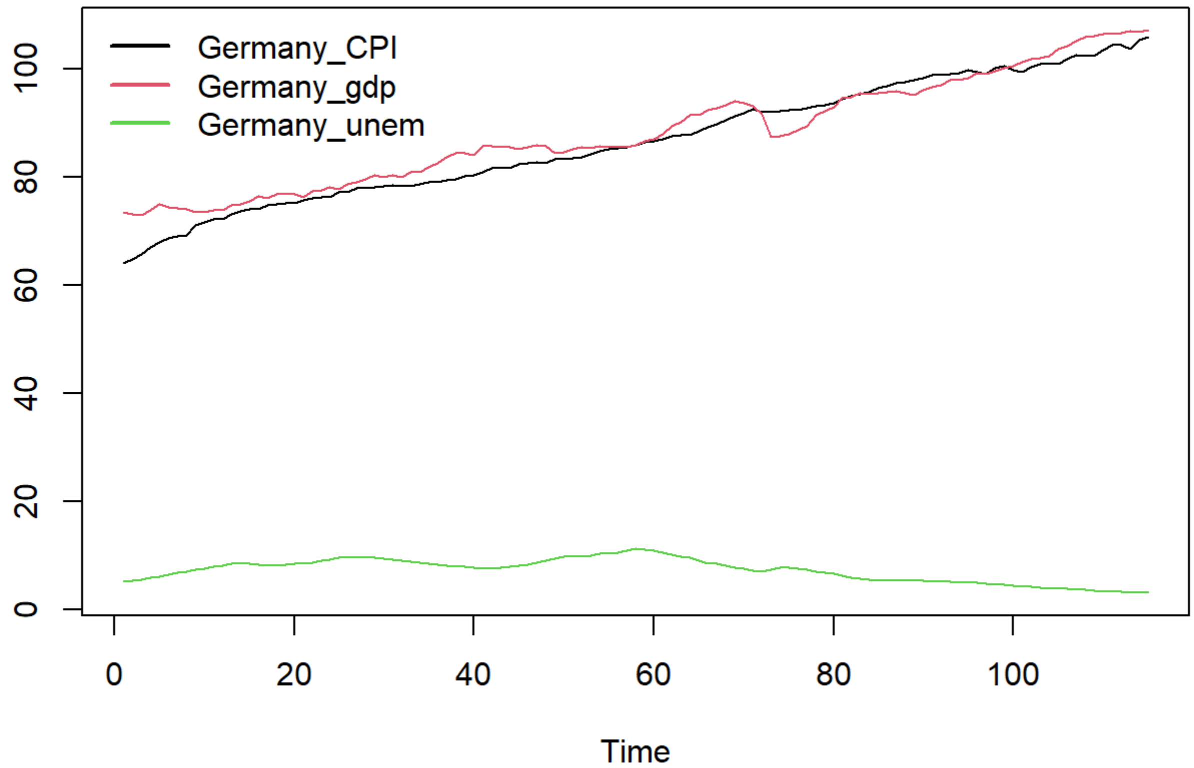
```
##        DATE Canada_CPI Canada_gdp Canada_unem France_CPI France_gdp France_unem
## 1  1991/1/1   64.86700   54.97783    10.16667   68.80333   68.47505    9.266667
## 2  1991/4/1   65.34106   55.23417    10.33333   69.36333   68.70599    9.433333
## 3  1991/7/1   65.73611   55.30814    10.43333   69.82000   68.92958    9.700000
## 4 1991/10/1   65.63076   55.41020    10.33333   70.38333   69.21646   10.000000
## 5  1992/1/1   65.89413   55.45059    10.60000   70.71667   69.92935   10.266667
## 6  1992/4/1   66.23650   55.51830    11.00000   71.26000   69.87414   10.500000
##   Germany_CPI Germany_gdp Germany_unem Italy_CPI Italy_gdp Italy_unem Japan_CPI
## 1    64.18494    73.48000     5.233333  54.07411  86.44591   8.633333  93.20000
## 2    64.80689    73.11000     5.333333  54.85303  85.97284   8.466667  94.16667
## 3    65.86420    72.95000     5.600000  55.49347  86.00364   8.433333  94.16667
## 4    67.10809    73.88000     5.900000  56.27239  87.01302   8.600000  95.30000
## 5    67.97882    74.99748     6.100000  57.15516  87.24703   8.666667  95.00000
## 6    68.75625    74.45750     6.400000  57.86484  87.19709   8.666667  96.33333
##   Japan_gdp Japan_unem United.Kingdom_CPI United.Kingdom_gdp
## 1  79.99816   2.100000               57.2           59.47495
## 2  80.94727   2.100000               59.1           59.39998
## 3  80.76807   2.100000               59.8           59.27025
## 4  81.33533   2.066667               60.5           59.37321
## 5  81.36063   2.066667               60.8           59.37750
## 6  81.64551   2.100000               62.1           59.30743
##   United.Kingdom_unem United.States_CPI United.States_gdp United.States_unem
## 1            7.766667          56.87356          53.26046           6.600000
## 2            8.466667          57.21109          53.67574           6.833333
## 3            8.933333          57.66112          53.94696           6.866667
## 4            9.100000          58.09710          54.13497           7.100000
## 5            9.300000          58.50495          54.78299           7.366667
## 6            9.666667          58.98311          55.37703           7.600000
```

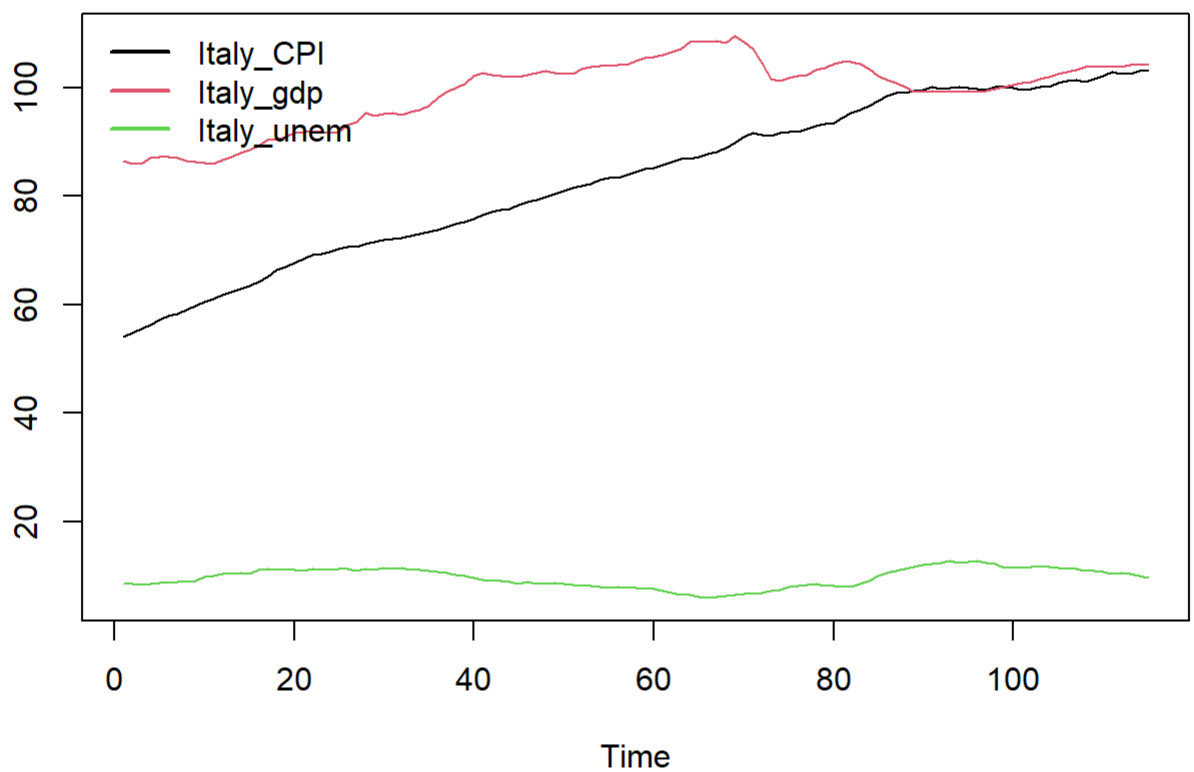
繪製每個國家之下，三個變數對應的時間序列圖形

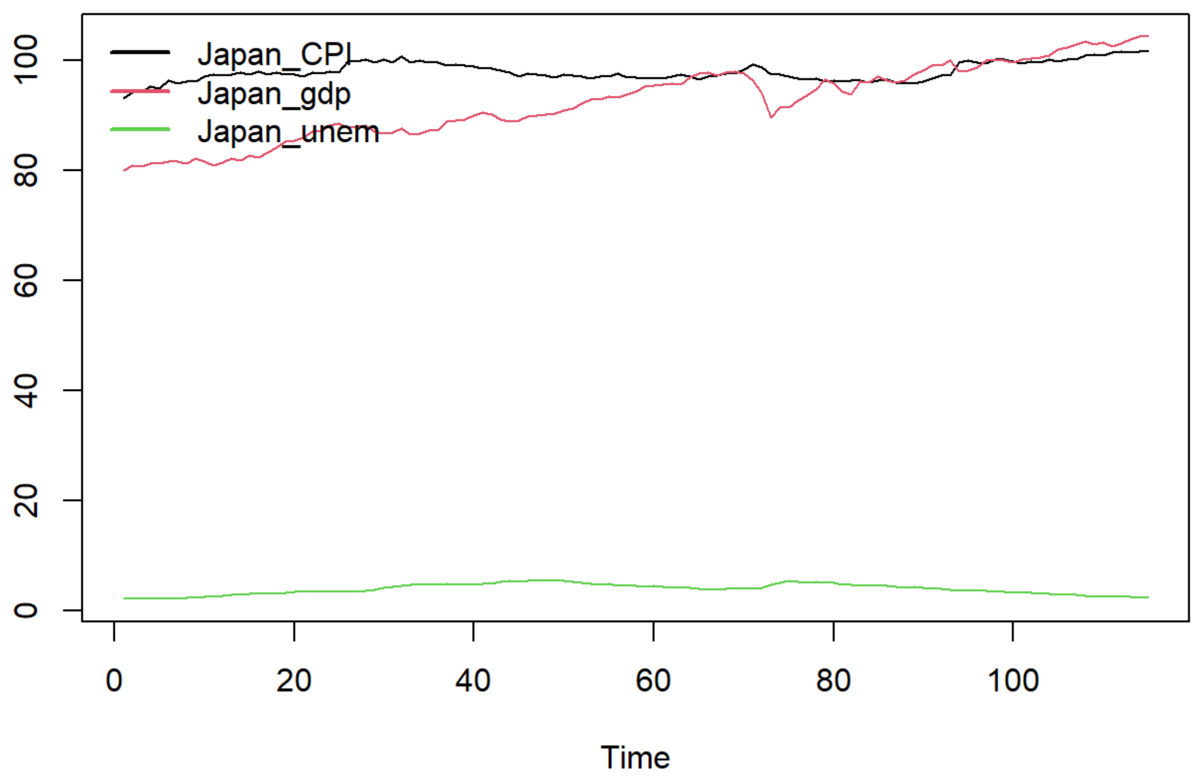
```
series_name = colnames(dat1)
for (i in 1:7){
    y = dat1[,1+(i-1)*3+(1:3)]
    ts.plot(y, col=1:3)
    legend("topleft", legend=series_name[1+(i-1)*3+(1:3)], col=1:3, lty=1, lwd=2, bty="n")
}
```

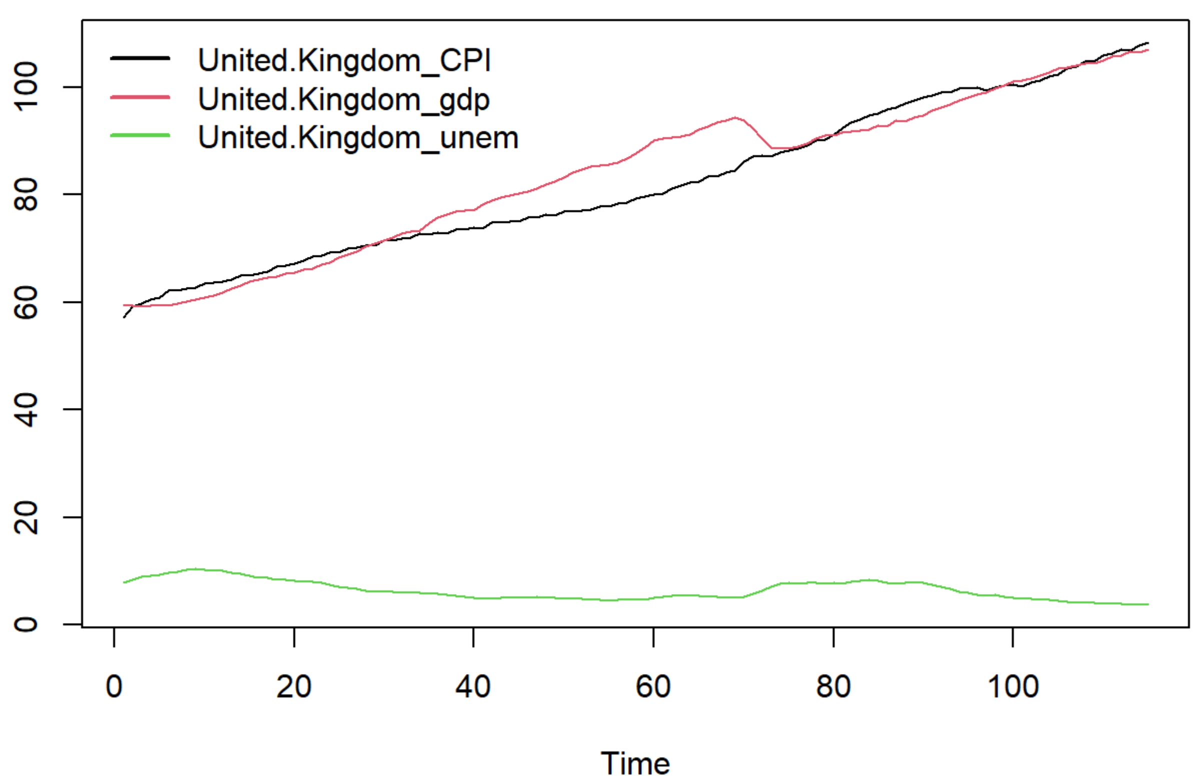

Canada_CPI
Canada_gdp
Canada_unem
20
40
60
80
100
0
20
40
60
80
100
Time

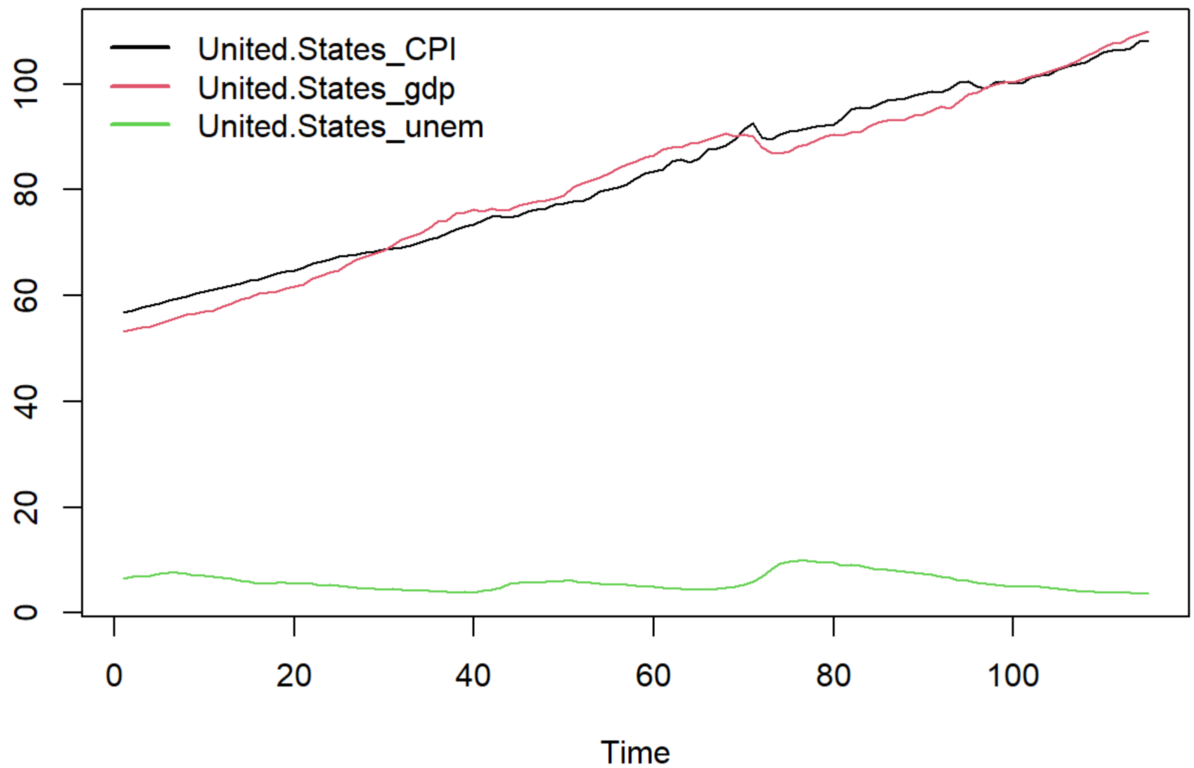

France_CPI
France_gdp
France_unem
20
40
60
80
100
0
20
40
60
80
100
Time


Germany_CPI
Germany_gdp
Germany_unem
0
20
40
60
80
100
Time


Italy_CPI
Italy_gdp
Italy_unem
20
40
60
80
100
0
Time

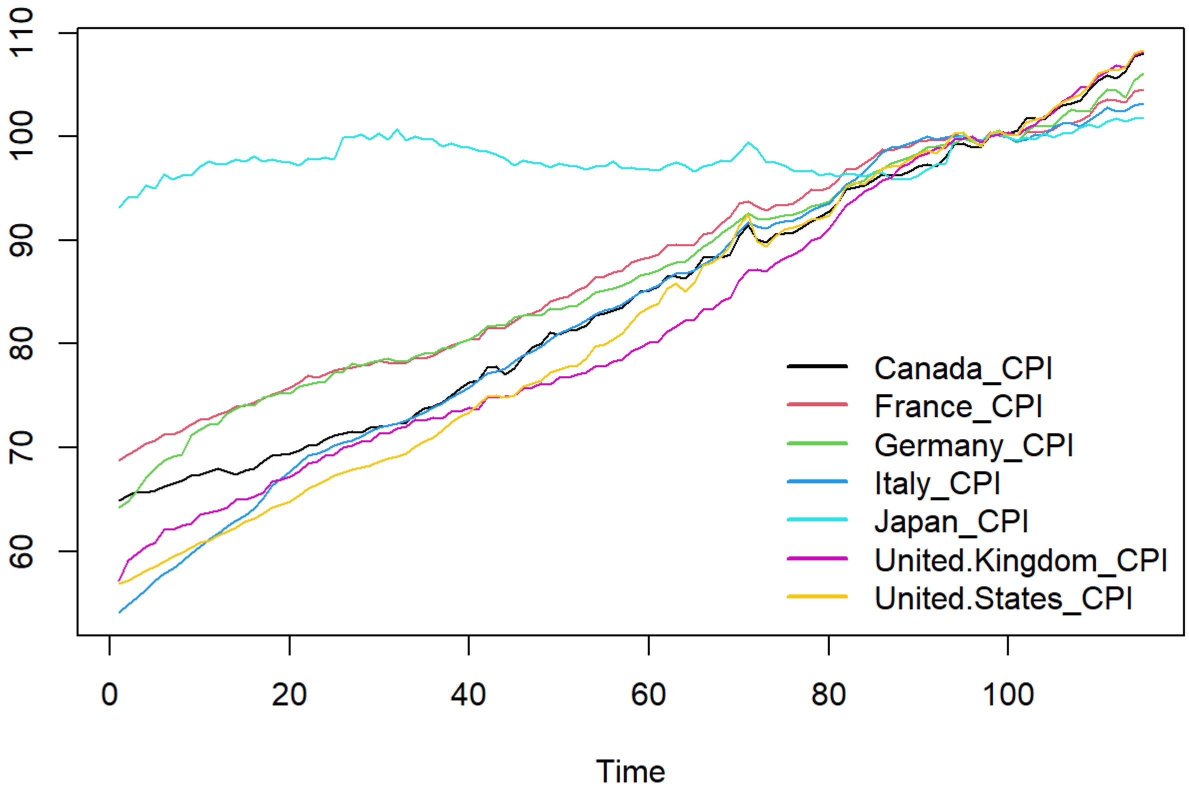

Japan_CPI
Japan_gdp
Japan_unem
0
20
40
60
80
100
Time

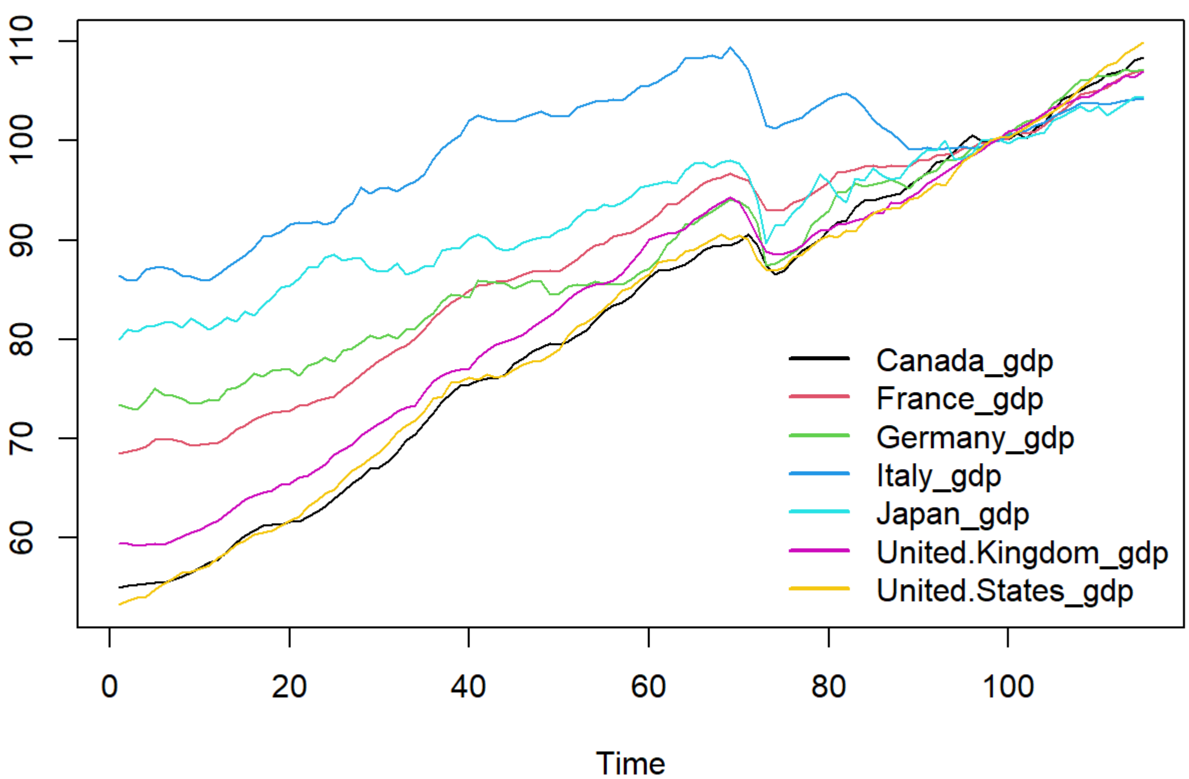

United.Kingdom_CPI
United.Kingdom_gdp
United.Kingdom_unem
0
20
40
60
80
100
Time

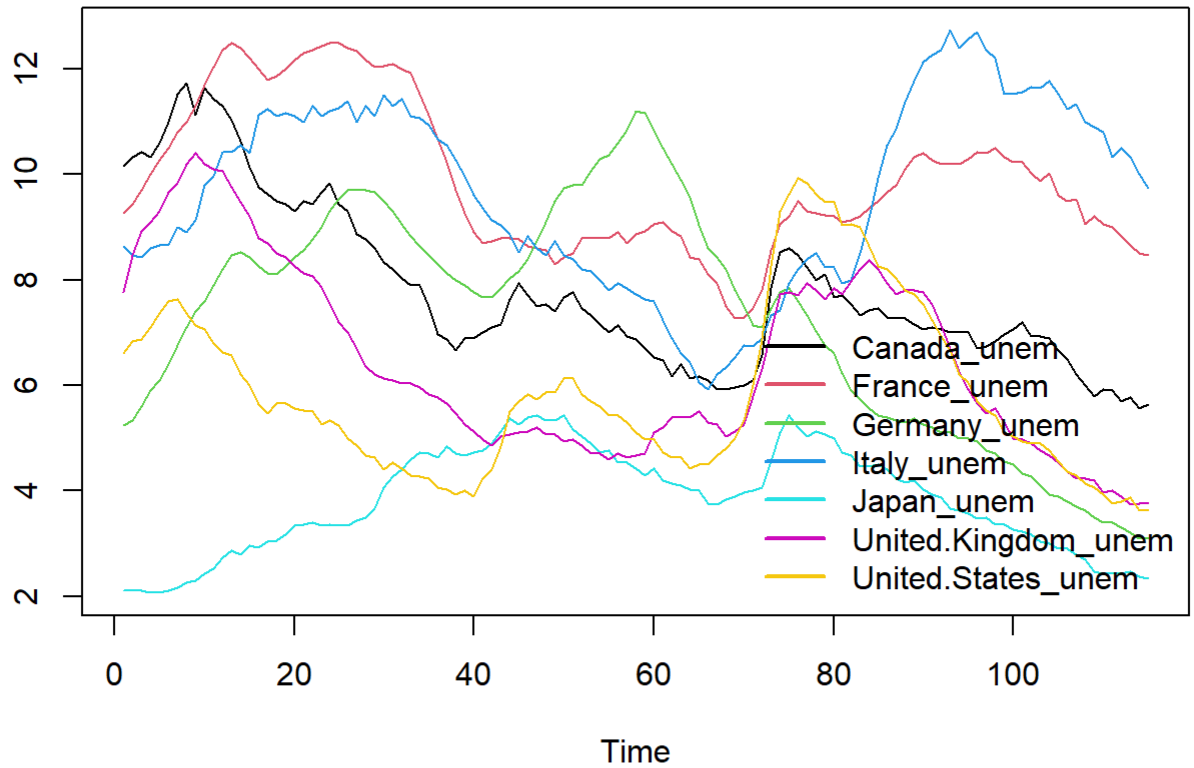


繪製每個變數之下，七個國家所對應的時間序列圖

```
#series_name = colnames(dat1)
for (i in 1:3){
    y = dat1[,i+seq(1,21,by=3)]
    ts.plot(y, col=1:7)
    legend("bottomright", legend=series_name[i+seq(1,21,by=3)], col=1:7, lty=1, lwd=2, bty="n")
}
```


110
100
90
80
70
60
0
20
40
60
80
100
Time
Canada_CPI
France_CPI
Germany_CPI
Italy_CPI
Japan_CPI
United.Kingdom_CPI
United.States_CPI


110
100
90
80
70
60
0
20
40
60
80
100
Time
Canada_gdp
France_gdp
Germany_gdp
Italy_gdp
Japan_gdp
United.Kingdom_gdp
United.States_gdp

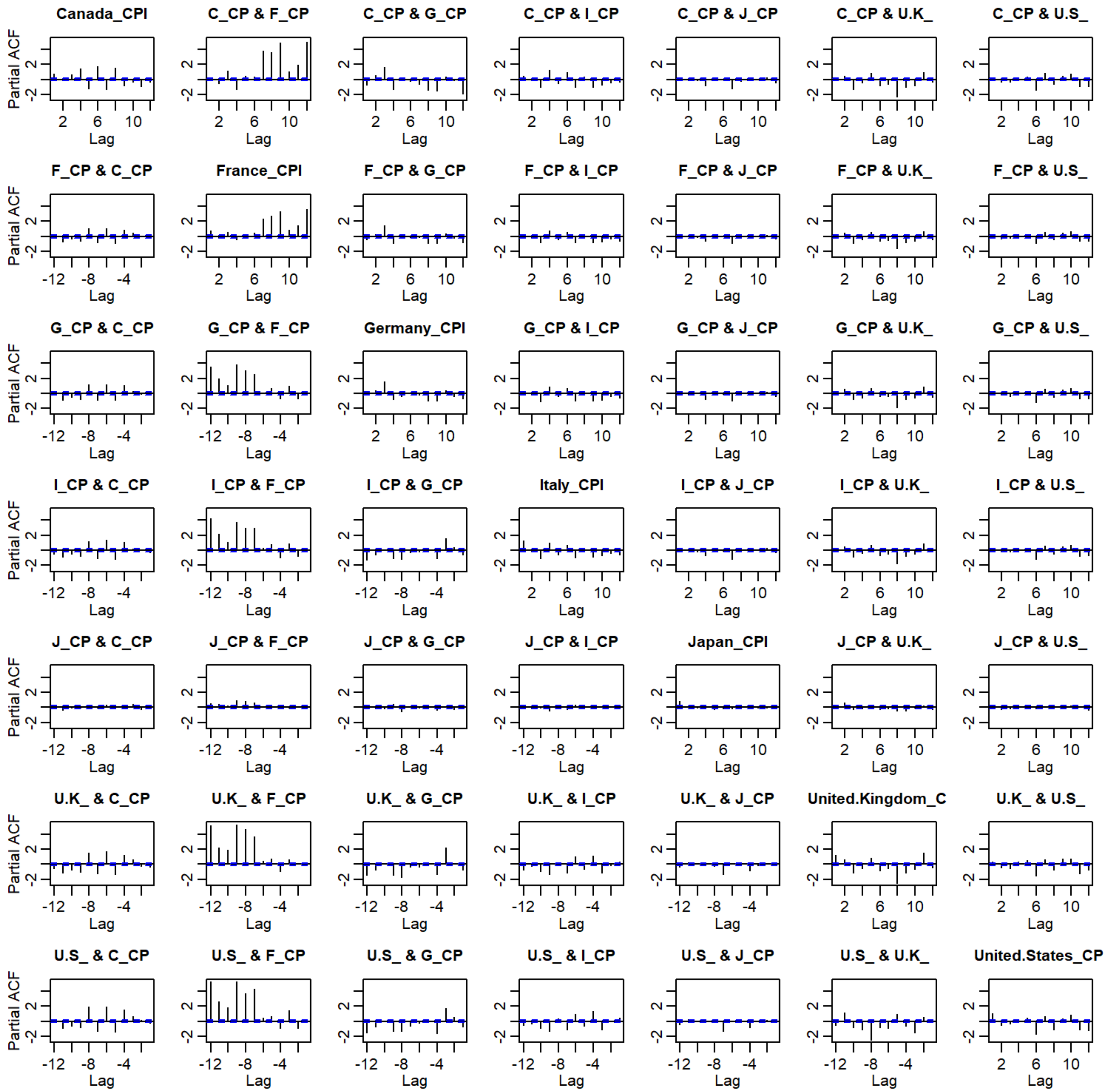


繪製七組時間序列之ACF圖

```
#ACF for CPI
acf(dat1[,1+seq(1,21,by=3)], max.mfrow = 7, mar = c(3, 2.25, 2, 0.55))
```

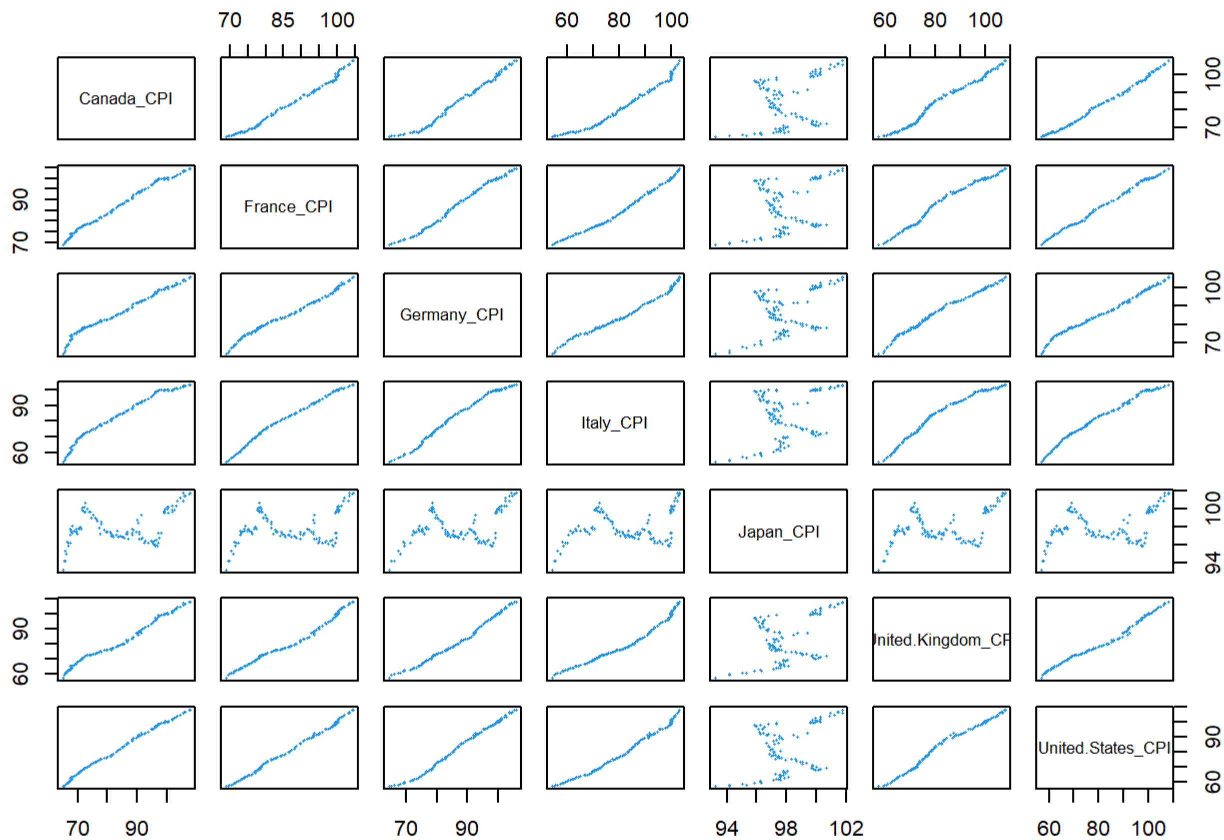
繪製七組時間序列之PACF圖

```
pacf(dat1[,1+seq(1,21,by=3)], max.mfrow = 7, mar = c(3, 2.25, 2, 0.55))
```



繪製七組時間序列之散布圖，可發現除了日本以外，其餘國家的CPI皆呈現高度正相關

```
pairs(dat1[,1+seq(1,21,by=3)], pch=16, cex=0.3, col=4)
```



以summary函數呈現基本統計量

```
summary(dat1[,1+seq(1,21,by=3)])
```

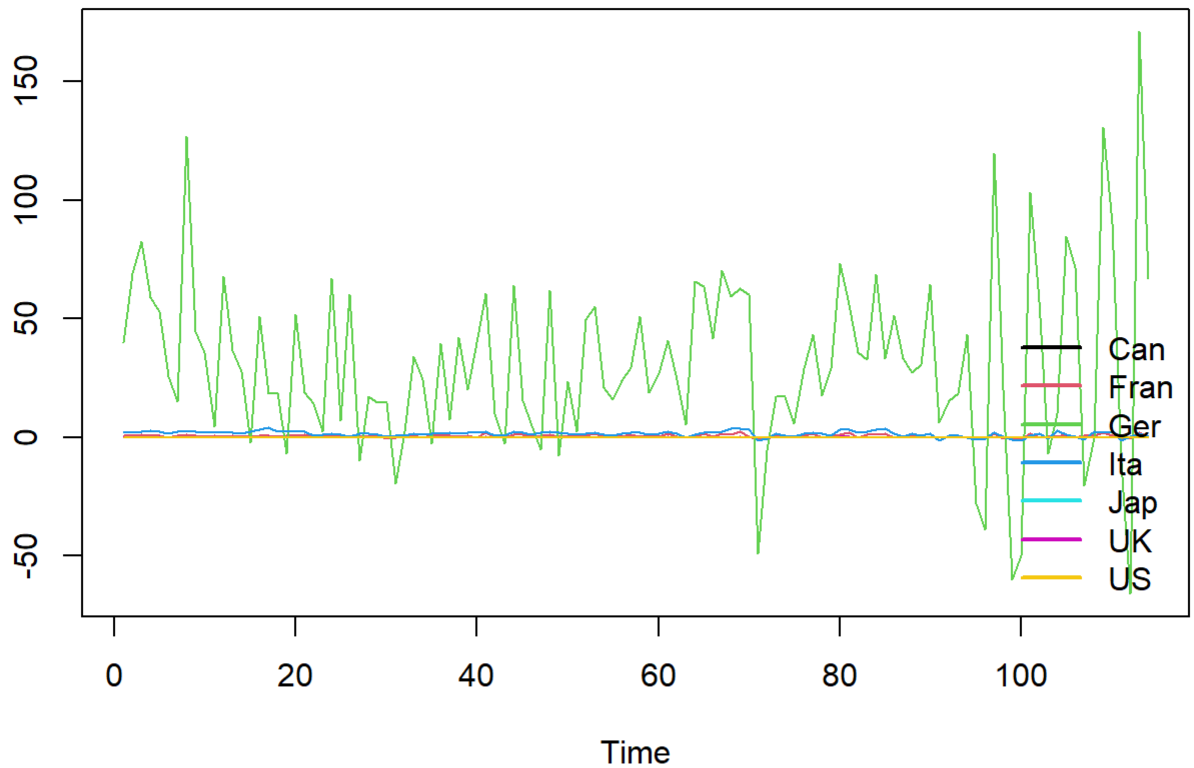
```
##    Canada_CPI        France_CPI       Germany_CPI        Italy_CPI
##  Min.   : 64.87   Min.   : 68.80   Min.   : 64.18   Min.   : 54.07
##  1st Qu.: 71.99   1st Qu.: 78.08   1st Qu.: 78.24   1st Qu.: 71.79
##  Median : 84.33   Median : 87.84   Median : 85.98   Median : 84.42
##  Mean   : 84.43   Mean   : 87.49   Mean   : 86.79   Mean   : 83.22
##  3rd Qu.: 96.21   3rd Qu.: 98.74   3rd Qu.: 97.13   3rd Qu.: 98.71
##  Max.   :107.98   Max.   :104.58   Max.   :106.08   Max.   :103.17
##    Japan_CPI      United.Kingdom_CPI United.States_CPI
##  Min.   : 93.20   Min.   : 57.20     Min.   : 56.87
##  1st Qu.: 96.87   1st Qu.: 71.00     1st Qu.: 68.49
##  Median : 97.57   Median : 79.30     Median : 82.06
##  Mean   : 98.03   Mean   : 82.20     Mean   : 82.35
##  3rd Qu.: 99.70   3rd Qu.: 95.95     3rd Qu.: 97.06
##  Max.   :101.77   Max.   :108.20     Max.   :108.27
```

將七組時間序列各別先取log轉換，使得變異數齊一，再取一階差分使得序列變平穩。

```
lambda1 <- BoxCox.lambda(dat1[,2])
a=BoxCox(dat1[,2], lambda = lambda1)

lambda2 <- BoxCox.lambda(dat1[,5])
b=BoxCox(dat1[,5], lambda = lambda2)

lambda3 <- BoxCox.lambda(dat1[,8])
c=BoxCox(dat1[,8], lambda = lambda3)

lambda4 <- BoxCox.lambda(dat1[,11])
d=BoxCox(dat1[,11], lambda = lambda4)

lambda5 <- BoxCox.lambda(dat1[,14])
e=BoxCox(dat1[,14], lambda = lambda5)

lambda6 <- BoxCox.lambda(dat1[,17])
f=BoxCox(dat1[,17], lambda = lambda6)

lambda7 <- BoxCox.lambda(dat1[,20])
g=BoxCox(dat1[,20], lambda = lambda7)

Can=diff(a,1)
Fran=diff(b,1)
Ger=diff(c,1)
Ita=diff(d,1)
Jap=diff(e,1)
UK=diff(f,1)
US=diff(g,1)

df=data.frame("Can"=Can,"Fran"=Fran,"Ger"=Ger,"Ita"=Ita,"Jap"=Jap,"UK"=UK,"US"=US)
```

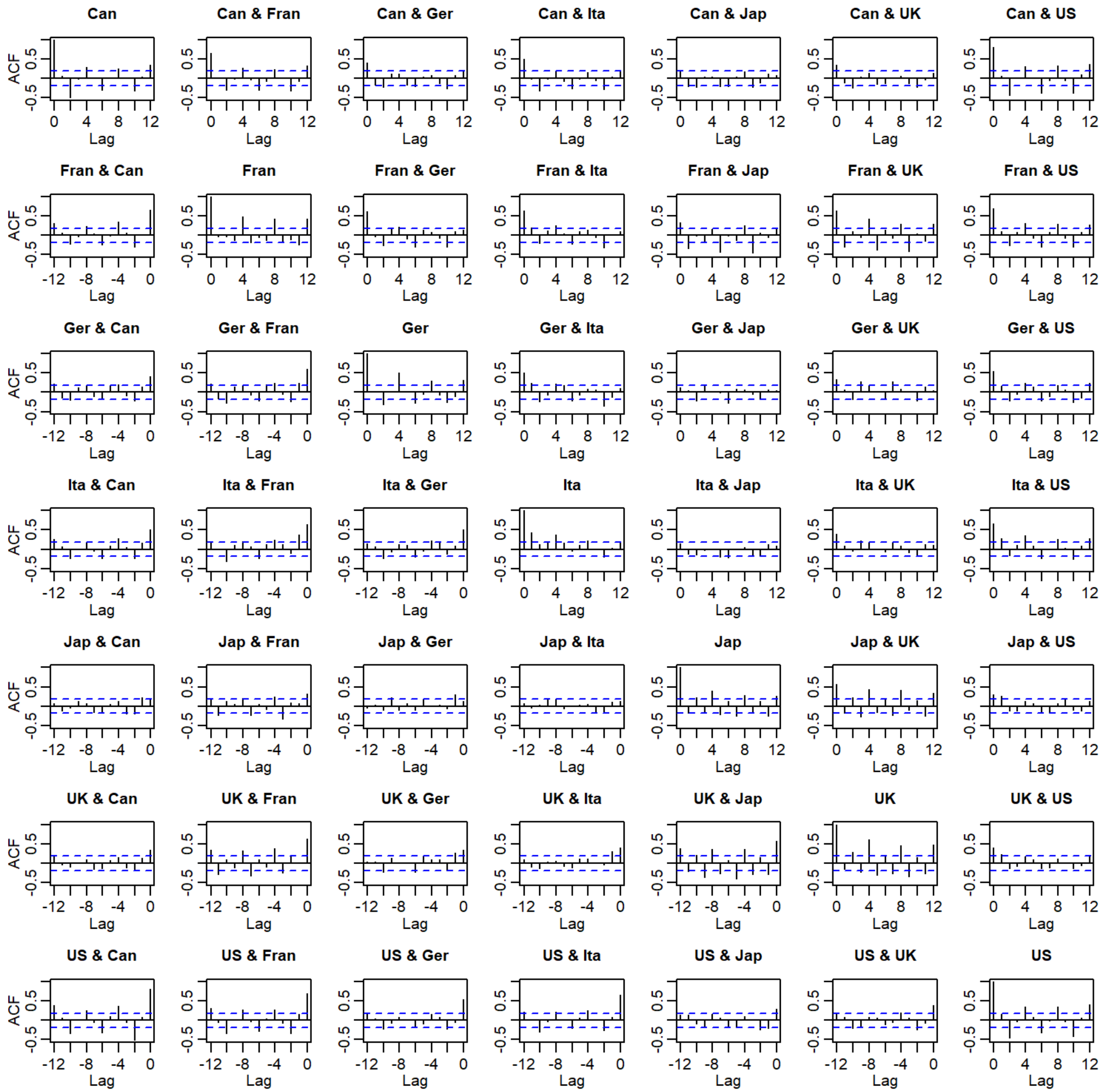
將轉換、差分過後的序列繪製時間序列圖。可見原本趨勢已去除，變異數亦相對齊一

```
df_name=colnames(df)

y = df[,1:7]
ts.plot(y, col=1:7)
legend("bottomright", legend=df_name[1:7], col=1:7, lty=1, lwd=2, bty="n")
```



繪製ACF、PACF圖，可見單變量時間序列之下仍存在週期性的相關性。其中，週期可約莫看出為4。

```
acf(df[,1:7], max.mfrow = 7, mar = c(3, 2.25, 2, 0.55))
```



```
pacf(df[,1:7],max.mfrow = 7, mar = c(3, 2.25, 2, 0.55))
```

以summary函數呈現處理過後數據之基本統計量，可見平均皆近乎等於0

```
summary(df[,1:7])
```

```
##       Can                  Fran              Ger                Ita
##  Min.   :-0.007430   Min.   :-0.8529   Min.   :-65.939   Min.   :-1.507
##  1st Qu.: 0.000728   1st Qu.: 0.1236   1st Qu.:  8.093   1st Qu.: 0.895
##  Median : 0.002230   Median : 0.5321   Median : 27.486   Median : 1.629
##  Mean   : 0.002229   Mean   : 0.5476   Mean   : 31.280   Mean   : 1.493
##  3rd Qu.: 0.003767   3rd Qu.: 0.9201   3rd Qu.: 55.802   3rd Qu.: 2.242
##  Max.   : 0.010008   Max.   : 2.2604   Max.   :170.871   Max.   : 4.005
##       Jap                  UK                 US
##  Min.   :-1.282e-04   Min.   :-0.06152   Min.   :-0.043601
##  1st Qu.:-2.718e-05   1st Qu.: 0.02045   1st Qu.: 0.005190
##  Median :-1.670e-06   Median : 0.04582   Median : 0.008524
##  Mean   : 7.926e-06   Mean   : 0.05093   Mean   : 0.008465
##  3rd Qu.: 3.894e-05   3rd Qu.: 0.08336   3rd Qu.: 0.012919
##  Max.   : 2.508e-04   Max.   : 0.25441   Max.   : 0.032974
```

基於information criteria 選取最佳VAR order。可得結果為:AIC準則選出的模型為VAR(10)，BIC(SC)準則選出的模型為VAR(1)，FPE準則選出的模型為VAR(2)

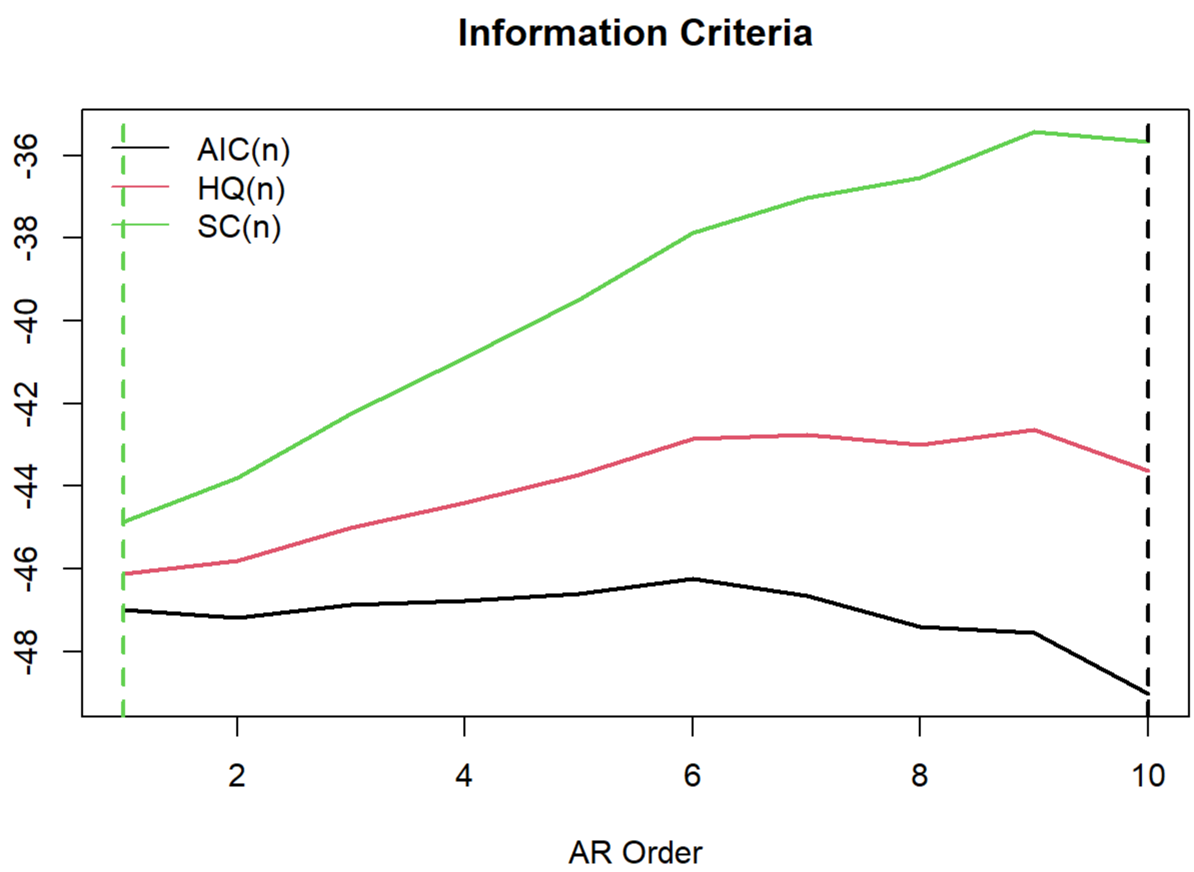
```
fit.aic=VARselect(y,lag.max = 10, type = "both",season = 4)
fit.aic
```

```
## $selection
## AIC(n)  HQ(n)  SC(n) FPE(n)
##     10      1      1      2
##
## $criteria
##                    1             2             3             4             5
## AIC(n) -4.699337e+01 -4.718322e+01 -4.687496e+01 -4.677920e+01 -4.660452e+01
## HQ(n)  -4.612807e+01 -4.581316e+01 -4.500015e+01 -4.439963e+01 -4.372020e+01
## SC(n)  -4.485752e+01 -4.380145e+01 -4.224727e+01 -4.090560e+01 -3.948501e+01
## FPE(n)  3.928049e-21  3.320481e-21  4.735600e-21  5.662615e-21  7.703415e-21
##                    6             7             8             9            10
## AIC(n) -4.623872e+01 -4.664574e+01 -4.739722e+01 -4.754427e+01 -4.902599e+01
## HQ(n)  -4.284964e+01 -4.275190e+01 -4.299862e+01 -4.264092e+01 -4.361788e+01
## SC(n)  -3.787330e+01 -3.703439e+01 -3.653996e+01 -3.544110e+01 -3.567690e+01
## FPE(n)  1.357513e-20  1.210145e-20  8.660294e-21  1.351767e-20  7.190616e-21
```

```
#AIC:10, HQ:1, BIC(SC):1, FPE:1
```

繪製出AIC、BIC、HQ curve

```
#plot information criteria(with seasonal mean)
par(mfcol=c(1,1))
ts.plot(t(fit.aic$crit[1:3,]), col=1:3, lwd=2, xlab="AR Order")
abline(v=fit.aic$sel[1:3],lty=2,col=1:3,lwd=2)
legend("topleft",legend=rownames(fit.aic$crit[1:3,]),col=1:3,lty=1, bty="n")
title("Information Criteria")
```



根據FPE的order選取結果建模，即VAR(2)且season=4。再繪製model fitted line及對模型殘差做ACF、PACF圖。可看出模型無法捕捉一些偏離較大的值。另外，由ACF、PACF圖可觀察出單變量之下，殘差似white noise process的結構。

```
fit1 = VAR(y, p=2, type="both",season=4)# "both" means fitting (constant + linear trend)
summary(fit1)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: Can, Fran, Ger, Ita, Jap, UK, US
## Deterministic variables: both
## Sample size: 112
## Log Likelihood: 1665.258
## Roots of the characteristic polynomial:
## 0.7424 0.7424 0.7048 0.5879 0.5879  0.58 0.571 0.5294 0.5294 0.4761 0.4761 0.3708 0.3708 0.32
26
## Call:
## VAR(y = y, p = 2, type = "both", season = 4L)
##
##
## Estimation results for equation Can:
## ====================================
## Can = Can.l1 + Fran.l1 + Ger.l1 + Ita.l1 + Jap.l1 + UK.l1 + US.l1 + Can.l2 + Fran.l2 + Ger.l2
+ Ita.l2 + Jap.l2 + UK.l2 + US.l2 + const + trend + sd1 + sd2 + sd3
##
##           Estimate Std. Error t value Pr(>|t|)
## Can.l1   1.039e-01  1.535e-01   0.676 0.500451
## Fran.l1 -2.605e-04  9.343e-04  -0.279 0.781034
## Ger.l1  -1.127e-05  9.555e-06  -1.180 0.241097
## Ita.l1  -2.740e-04  3.606e-04  -0.760 0.449162
## Jap.l1  -6.176e+00  5.135e+00  -1.203 0.232134
## UK.l1    2.490e-03  1.022e-02   0.244 0.808062
## US.l1    6.899e-02  5.002e-02   1.379 0.171152
## Can.l2  -3.245e-01  1.528e-01  -2.124 0.036363 *
## Fran.l2  1.140e-03  9.988e-04   1.141 0.256683
## Ger.l2  -1.013e-06  1.026e-05  -0.099 0.921616
## Ita.l2  -1.809e-06  3.527e-04  -0.005 0.995919
## Jap.l2  -1.999e+00  5.074e+00  -0.394 0.694484
## UK.l2   -5.338e-03  8.187e-03  -0.652 0.516056
## US.l2   -1.360e-02  4.899e-02  -0.278 0.781951
## const    2.786e-03  8.056e-04   3.458 0.000823 ***
## trend   -7.052e-07  8.452e-06  -0.083 0.933691
## sd1      6.487e-04  1.244e-03   0.521 0.603342
## sd2     -1.201e-03  7.934e-04  -1.514 0.133332
## sd3     -2.187e-03  1.499e-03  -1.459 0.147862
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.002143 on 93 degrees of freedom
## Multiple R-Squared: 0.4357,  Adjusted R-squared: 0.3264
## F-statistic: 3.989 on 18 and 93 DF,  p-value: 5.171e-06
##
##
## Estimation results for equation Fran:
## =====================================
## Fran = Can.l1 + Fran.l1 + Ger.l1 + Ita.l1 + Jap.l1 + UK.l1 + US.l1 + Can.l2 + Fran.l2 + Ger.l
2 + Ita.l2 + Jap.l2 + UK.l2 + US.l2 + const + trend + sd1 + sd2 + sd3
##
```

```
##           Estimate Std. Error t value Pr(>|t|)
## Can.l1  -6.404e+01  3.232e+01  -1.981  0.05054 .
## Fran.l1  3.422e-01  1.967e-01   1.740  0.08525 .
## Ger.l1  -4.070e-03  2.012e-03  -2.023  0.04593 *
## Ita.l1   1.094e-01  7.591e-02   1.442  0.15277
## Jap.l1  -6.878e+01  1.081e+03  -0.064  0.94941
## UK.l1   -1.681e+00  2.152e+00  -0.781  0.43664
## US.l1    1.934e+01  1.053e+01   1.836  0.06955 .
## Can.l2  -7.354e+00  3.217e+01  -0.229  0.81966
## Fran.l2  9.488e-03  2.103e-01   0.045  0.96411
## Ger.l2  -1.485e-03  2.161e-03  -0.687  0.49365
## Ita.l2  -2.792e-02  7.426e-02  -0.376  0.70777
## Jap.l2  -1.557e+03  1.068e+03  -1.458  0.14826
## UK.l2    1.572e+00  1.724e+00   0.912  0.36414
## US.l2   -4.173e+00  1.031e+01  -0.405  0.68671
## const    3.701e-01  1.696e-01   2.182  0.03163 *
## trend    1.259e-03  1.779e-03   0.707  0.48116
## sd1      5.982e-01  2.619e-01   2.284  0.02467 *
## sd2     -5.160e-01  1.670e-01  -3.089  0.00265 **
## sd3      1.235e-01  3.156e-01   0.391  0.69648
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.4512 on 93 degrees of freedom
## Multiple R-Squared: 0.5653,  Adjusted R-squared: 0.4812
## F-statistic: 6.719 on 18 and 93 DF,  p-value: 2.017e-10
##
##
## Estimation results for equation Ger:
## ====================================
## Ger = Can.l1 + Fran.l1 + Ger.l1 + Ita.l1 + Jap.l1 + UK.l1 + US.l1 + Can.l2 + Fran.l2 + Ger.l2
+ Ita.l2 + Jap.l2 + UK.l2 + US.l2 + const + trend + sd1 + sd2 + sd3
##
##           Estimate Std. Error t value Pr(>|t|)
## Can.l1  -1.565e+03  2.239e+03  -0.699  0.48615
## Fran.l1 -4.935e+00  1.362e+01  -0.362  0.71800
## Ger.l1   7.079e-03  1.393e-01   0.051  0.95959
## Ita.l1   1.675e+01  5.258e+00   3.185  0.00197 **
## Jap.l1   8.614e+04  7.488e+04   1.150  0.25292
## UK.l1   -2.302e+02  1.491e+02  -1.545  0.12583
## US.l1    4.760e+01  7.295e+02   0.065  0.94812
## Can.l2  -2.693e+03  2.228e+03  -1.209  0.22992
## Fran.l2  1.269e+01  1.456e+01   0.871  0.38592
## Ger.l2  -5.030e-01  1.497e-01  -3.361  0.00113 **
## Ita.l2  -1.052e+01  5.144e+00  -2.046  0.04359 *
## Jap.l2  -5.123e+04  7.399e+04  -0.692  0.49036
## UK.l2    3.427e+02  1.194e+02   2.870  0.00508 **
## US.l2    8.458e+02  7.144e+02   1.184  0.23947
## const    1.923e+01  1.175e+01   1.637  0.10494
## trend    1.487e-01  1.233e-01   1.206  0.23071
## sd1     -3.412e+01  1.814e+01  -1.880  0.06318 .
## sd2     -1.191e+00  1.157e+01  -0.103  0.91821
## sd3     -6.848e+01  2.186e+01  -3.133  0.00231 **
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 31.25 on 93 degrees of freedom
## Multiple R-Squared: 0.4239,  Adjusted R-squared: 0.3124
## F-statistic: 3.801 on 18 and 93 DF,  p-value: 1.101e-05
##
##
## Estimation results for equation Ita:
## ====================================
## Ita = Can.l1 + Fran.l1 + Ger.l1 + Ita.l1 + Jap.l1 + UK.l1 + US.l1 + Can.l2 + Fran.l2 + Ger.l2
+ Ita.l2 + Jap.l2 + UK.l2 + US.l2 + const + trend + sd1 + sd2 + sd3
##
##           Estimate Std. Error t value Pr(>|t|)
## Can.l1  -9.222e+01  5.787e+01  -1.594 0.114399
## Fran.l1  1.156e+00  3.521e-01   3.283 0.001448 **
## Ger.l1  -9.210e-03  3.601e-03  -2.557 0.012162 *
## Ita.l1   1.685e-01  1.359e-01   1.240 0.218067
## Jap.l1  -1.480e+03  1.935e+03  -0.765 0.446440
## UK.l1   -2.298e+00  3.853e+00  -0.596 0.552373
## US.l1    3.987e+01  1.885e+01   2.115 0.037142 *
## Can.l2   2.244e+01  5.759e+01   0.390 0.697639
## Fran.l2 -1.943e-01  3.764e-01  -0.516 0.606953
## Ger.l2  -1.643e-03  3.869e-03  -0.425 0.672081
## Ita.l2   2.635e-01  1.329e-01   1.982 0.050430 .
## Jap.l2   2.239e+02  1.912e+03   0.117 0.907058
## UK.l2    3.667e+00  3.086e+00   1.188 0.237776
## US.l2   -3.762e+01  1.846e+01  -2.037 0.044443 *
## const    1.057e+00  3.036e-01   3.480 0.000765 ***
## trend   -5.927e-03  3.186e-03  -1.860 0.065979 .
## sd1      4.560e-02  4.689e-01   0.097 0.922745
## sd2     -1.047e+00  2.990e-01  -3.502 0.000711 ***
## sd3     -5.595e-01  5.649e-01  -0.990 0.324588
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.8077 on 93 degrees of freedom
## Multiple R-Squared: 0.5824,  Adjusted R-squared: 0.5016
## F-statistic: 7.206 on 18 and 93 DF,  p-value: 3.922e-11
##
##
## Estimation results for equation Jap:
## ====================================
## Jap = Can.l1 + Fran.l1 + Ger.l1 + Ita.l1 + Jap.l1 + UK.l1 + US.l1 + Can.l2 + Fran.l2 + Ger.l2
+ Ita.l2 + Jap.l2 + UK.l2 + US.l2 + const + trend + sd1 + sd2 + sd3
##
##           Estimate Std. Error t value Pr(>|t|)
## Can.l1   2.582e-03  3.051e-03   0.846  0.39949
## Fran.l1  2.111e-05  1.856e-05   1.137  0.25849
## Ger.l1   3.352e-07  1.899e-07   1.765  0.08076 .
## Ita.l1  -1.425e-05  7.165e-06  -1.989  0.04963 *
## Jap.l1  -3.281e-02  1.020e-01  -0.322  0.74852
```

```
## UK.l1   -1.930e-04  2.031e-04  -0.950  0.34460
## US.l1    8.085e-04  9.940e-04   0.813  0.41808
## Can.l2  -3.383e-03  3.036e-03  -1.114  0.26808
## Fran.l2 -7.270e-06  1.985e-05  -0.366  0.71495
## Ger.l2   5.247e-07  2.040e-07   2.573  0.01168 *
## Ita.l2  -7.299e-06  7.009e-06  -1.041  0.30038
## Jap.l2   8.996e-02  1.008e-01   0.892  0.37453
## UK.l2    2.140e-04  1.627e-04   1.315  0.19163
## US.l2    3.635e-04  9.735e-04   0.373  0.70967
## const    6.959e-06  1.601e-05   0.435  0.66479
## trend   -1.917e-07  1.680e-07  -1.141  0.25675
## sd1      7.258e-05  2.472e-05   2.936  0.00419 **
## sd2      1.346e-05  1.577e-05   0.854  0.39552
## sd3      2.549e-05  2.978e-05   0.856  0.39435
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 4.258e-05 on 93 degrees of freedom
## Multiple R-Squared: 0.5031,  Adjusted R-squared: 0.4069
## F-statistic: 5.231 on 18 and 93 DF,  p-value: 4.147e-08
##
##
## Estimation results for equation UK:
## ===================================
## UK = Can.l1 + Fran.l1 + Ger.l1 + Ita.l1 + Jap.l1 + UK.l1 + US.l1 + Can.l2 + Fran.l2 + Ger.l2
+ Ita.l2 + Jap.l2 + UK.l2 + US.l2 + const + trend + sd1 + sd2 + sd3
##
##           Estimate Std. Error t value Pr(>|t|)
## Can.l1  -2.030e+00  1.809e+00  -1.122   0.2647
## Fran.l1 -4.263e-03  1.101e-02  -0.387   0.6995
## Ger.l1   1.110e-04  1.126e-04   0.986   0.3269
## Ita.l1   8.748e-03  4.249e-03   2.059   0.0423 *
## Jap.l1  -2.457e+01  6.051e+01  -0.406   0.6856
## UK.l1    7.761e-02  1.204e-01   0.644   0.5210
## US.l1    1.063e+00  5.894e-01   1.803   0.0746 .
## Can.l2  -3.281e+00  1.800e+00  -1.822   0.0716 .
## Fran.l2 -3.461e-03  1.177e-02  -0.294   0.7693
## Ger.l2  -8.051e-05  1.209e-04  -0.666   0.5072
## Ita.l2   4.297e-03  4.156e-03   1.034   0.3039
## Jap.l2  -1.287e+01  5.978e+01  -0.215   0.8300
## UK.l2    1.218e-01  9.647e-02   1.262   0.2099
## US.l2    3.640e-01  5.773e-01   0.631   0.5299
## const    8.264e-03  9.492e-03   0.871   0.3862
## trend    2.403e-04  9.960e-05   2.413   0.0178 *
## sd1      6.324e-02  1.466e-02   4.314    4e-05 ***
## sd2     -1.250e-02  9.349e-03  -1.337   0.1844
## sd3      2.099e-02  1.766e-02   1.188   0.2377
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.02525 on 93 degrees of freedom
## Multiple R-Squared: 0.7062,  Adjusted R-squared: 0.6493
```

```
## F-statistic: 12.42 on 18 and 93 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation US:
## ===================================
## US = Can.l1 + Fran.l1 + Ger.l1 + Ita.l1 + Jap.l1 + UK.l1 + US.l1 + Can.l2 + Fran.l2 + Ger.l2
+ Ita.l2 + Jap.l2 + UK.l2 + US.l2 + const + trend + sd1 + sd2 + sd3
##
##            Estimate Std. Error t value Pr(>|t|)
## Can.l1  -5.782e-01  5.024e-01  -1.151   0.2527
## Fran.l1  5.532e-03  3.057e-03   1.810   0.0736 .
## Ger.l1  -4.363e-05  3.126e-05  -1.395   0.1662
## Ita.l1  -1.077e-03  1.180e-03  -0.913   0.3636
## Jap.l1  -1.987e+01  1.680e+01  -1.183   0.2399
## UK.l1   -1.601e-02  3.345e-02  -0.479   0.6334
## US.l1    3.593e-01  1.637e-01   2.195   0.0307 *
## Can.l2  -4.897e-01  4.999e-01  -0.979   0.3299
## Fran.l2 -1.021e-03  3.268e-03  -0.312   0.7554
## Ger.l2   3.904e-05  3.358e-05   1.163   0.2480
## Ita.l2   2.698e-04  1.154e-03   0.234   0.8157
## Jap.l2  -1.660e+01  1.660e+01  -1.000   0.3199
## UK.l2    2.946e-03  2.679e-02   0.110   0.9127
## US.l2   -2.134e-01  1.603e-01  -1.332   0.1863
## const    1.171e-02  2.636e-03   4.441 2.46e-05 ***
## trend   -3.989e-05  2.766e-05  -1.442   0.1525
## sd1      5.325e-03  4.071e-03   1.308   0.1941
## sd2     -3.732e-03  2.596e-03  -1.438   0.1539
## sd3     -1.781e-03  4.904e-03  -0.363   0.7174
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.007012 on 93 degrees of freedom
## Multiple R-Squared: 0.4925,  Adjusted R-squared: 0.3942
## F-statistic: 5.013 on 18 and 93 DF,  p-value: 9.445e-08
##
##
##
## Covariance matrix of residuals:
##             Can      Fran       Ger       Ita       Jap        UK        US
## Can  4.592e-06 6.129e-04 2.290e-02 8.818e-04 7.475e-09 1.552e-05 1.070e-05
## Fran 6.129e-04 2.036e-01 1.029e+01 2.486e-01 1.340e-06 4.810e-03 2.185e-03
## Ger  2.290e-02 1.029e+01 9.766e+02 1.305e+01 2.641e-04 3.706e-01 1.259e-01
## Ita  8.818e-04 2.486e-01 1.305e+01 6.524e-01 1.434e-06 8.168e-03 3.232e-03
## Jap  7.475e-09 1.340e-06 2.641e-04 1.434e-06 1.813e-09 2.974e-07 5.631e-08
## UK   1.552e-05 4.810e-03 3.706e-01 8.168e-03 2.974e-07 6.377e-04 7.833e-05
## US   1.070e-05 2.185e-03 1.259e-01 3.232e-03 5.631e-08 7.833e-05 4.917e-05
##
## Correlation matrix of residuals:
##           Can    Fran    Ger     Ita     Jap     UK     US
## Can  1.00000 0.63390 0.3419 0.50946 0.08191 0.2869 0.7123
## Fran 0.63390 1.00000 0.7296 0.68207 0.06973 0.4222 0.6906
## Ger  0.34189 0.72958 1.0000 0.51695 0.19843 0.4696 0.5744
## Ita  0.50946 0.68207 0.5170 1.00000 0.04169 0.4005 0.5707
```

```
## Jap  0.08191 0.06973 0.1984 0.04169 1.00000 0.2766 0.1886
## UK   0.28687 0.42216 0.4696 0.40047 0.27659 1.0000 0.4424
## US   0.71232 0.69063 0.5744 0.57071 0.18856 0.4424 1.0000
```

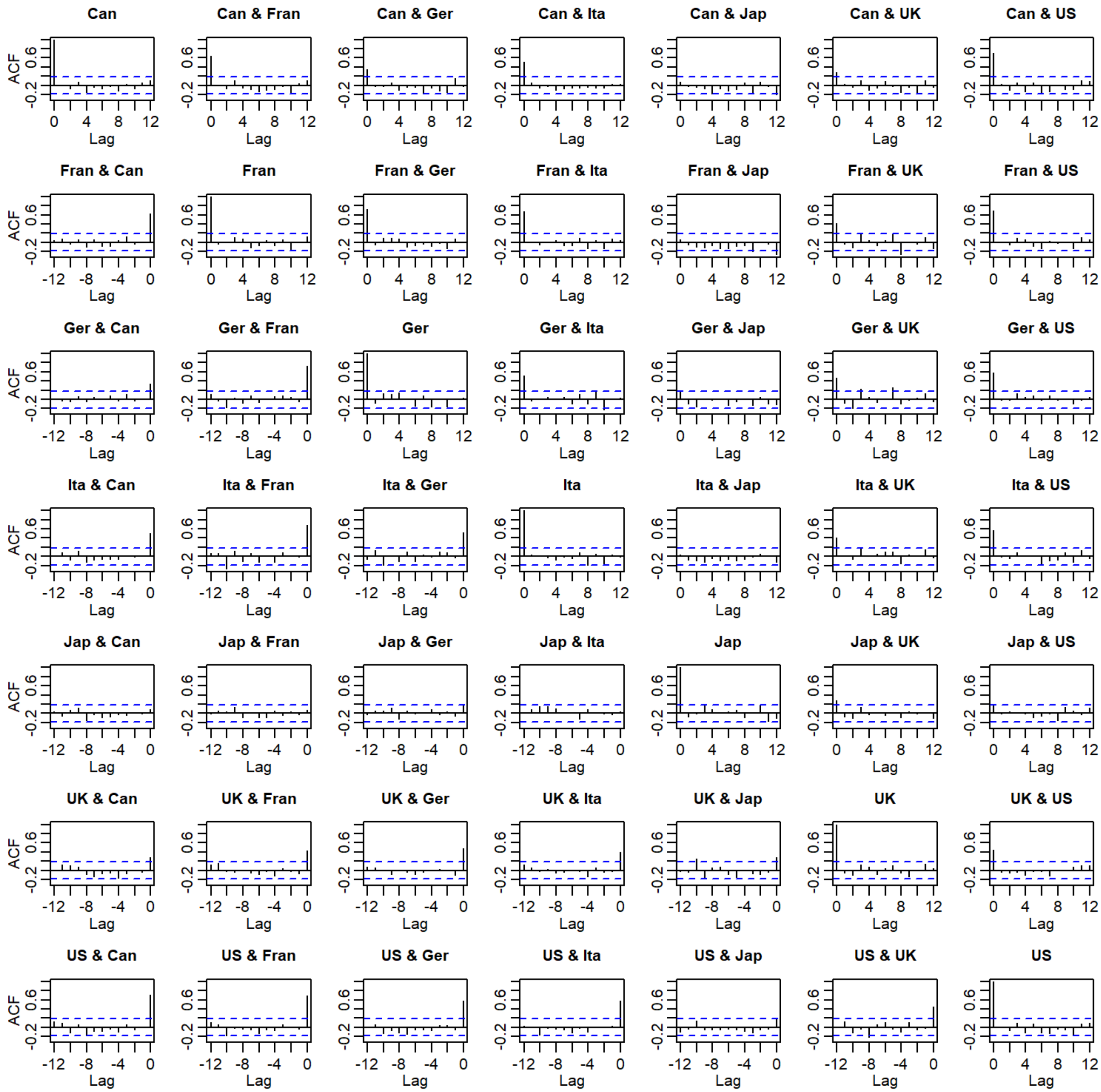
```
coef(fit1)   #including s.d(estimate)
```

```
## $Can
##              Estimate   Std. Error      t value     Pr(>|t|)
## Can.l1   1.038532e-01 1.535328e-01  0.676423292 0.5004507920
## Fran.l1 -2.604562e-04 9.342789e-04 -0.278777765 0.7810343312
## Ger.l1  -1.127228e-05 9.554560e-06 -1.179780579 0.2410965941
## Ita.l1  -2.740400e-04 3.605669e-04 -0.760025501 0.4491617775
## Jap.l1  -6.175910e+00 5.134907e+00 -1.202730582 0.2321337240
## UK.l1    2.490272e-03 1.022198e-02  0.243619334 0.8080622430
## US.l1    6.899062e-02 5.002352e-02  1.379163771 0.1711520105
## Can.l2  -3.244504e-01 1.527875e-01 -2.123541042 0.0363631692
## Fran.l2  1.139870e-03 9.987631e-04  1.141281311 0.2566831236
## Ger.l2  -1.012697e-06 1.026397e-05 -0.098665287 0.9216163849
## Ita.l2  -1.809146e-06 3.527268e-04 -0.005129029 0.9959186310
## Jap.l2  -1.998979e+00 5.073571e+00 -0.393998357 0.6944842031
## UK.l2   -5.337571e-03 8.187363e-03 -0.651927927 0.5160556375
## US.l2   -1.359844e-02 4.898923e-02 -0.277580142 0.7819507954
## const    2.785666e-03 8.055776e-04  3.457973979 0.0008227134
## trend   -7.051539e-07 8.452282e-06 -0.083427636 0.9336908281
## sd1      6.486927e-04 1.244187e-03  0.521378802 0.6033423098
## sd2     -1.201444e-03 7.933797e-04 -1.514336862 0.1333317653
## sd3     -2.187228e-03 1.498856e-03 -1.459265615 0.1478615182
##
## $Fran
##              Estimate   Std. Error     t value    Pr(>|t|)
## Can.l1  -6.403511e+01 3.232372e+01 -1.98105642 0.050538726
## Fran.l1  3.421625e-01 1.966965e-01  1.73954535 0.085247364
## Ger.l1  -4.069635e-03 2.011550e-03 -2.02313400 0.045930030
## Ita.l1   1.094359e-01 7.591121e-02  1.44163037 0.152765618
## Jap.l1  -6.877880e+01 1.081067e+03 -0.06362121 0.949408389
## UK.l1   -1.681313e+00 2.152064e+00 -0.78125595 0.436636269
## US.l1    1.933649e+01 1.053160e+01  1.83604544 0.069546103
## Can.l2  -7.354089e+00 3.216679e+01 -0.22862363 0.819663431
## Fran.l2  9.487855e-03 2.102725e-01  0.04512170 0.964107048
## Ger.l2  -1.485027e-03 2.160904e-03 -0.68722483 0.493651286
## Ita.l2  -2.792208e-02 7.426062e-02 -0.37600124 0.707772327
## Jap.l2  -1.557176e+03 1.068154e+03 -1.45781953 0.148258996
## UK.l2    1.571982e+00 1.723710e+00  0.91197616 0.364139238
## US.l2   -4.172819e+00 1.031385e+01 -0.40458410 0.686712264
## const    3.700641e-01 1.696006e-01  2.18197357 0.031631543
## trend    1.258600e-03 1.779484e-03  0.70728395 0.481158637
## sd1      5.981643e-01 2.619424e-01  2.28357224 0.024673352
## sd2     -5.159577e-01 1.670326e-01 -3.08896469 0.002648574
## sd3      1.234754e-01 3.155585e-01  0.39129153 0.696476818
##
## $Ger
##              Estimate   Std. Error     t value    Pr(>|t|)
## Can.l1  -1.565492e+03 2.238884e+03 -0.69922885 0.486154163
## Fran.l1 -4.935135e+00 1.362407e+01 -0.36223642 0.717997199
## Ger.l1   7.078993e-03 1.393289e-01  0.05080780 0.959587638
## Ita.l1   1.674817e+01 5.257947e+00  3.18530626 0.001968428
## Jap.l1   8.614279e+04 7.487950e+04  1.15041883 0.252920917
## UK.l1   -2.302463e+02 1.490615e+02 -1.54464004 0.125829506
## US.l1    4.759633e+01 7.294652e+02  0.06524825 0.948116397
```

```
## Can.l2  -2.692567e+03 2.228015e+03 -1.20850490 0.229916956
## Fran.l2  1.268758e+01 1.456441e+01  0.87113625 0.385922993
## Ger.l2  -5.029857e-01 1.496738e-01 -3.36054636 0.001129726
## Ita.l2  -1.052330e+01 5.143619e+00 -2.04589458 0.043590493
## Jap.l2  -5.123314e+04 7.398508e+04 -0.69247936 0.490361846
## UK.l2    3.426840e+02 1.193918e+02  2.87024810 0.005077520
## US.l2    8.457540e+02 7.143828e+02  1.18389465 0.239471929
## const    1.923434e+01 1.174729e+01  1.63734237 0.104938440
## trend    1.486978e-01 1.232549e-01  1.20642502 0.230713652
## sd1     -3.411733e+01 1.814329e+01 -1.88043798 0.063177958
## sd2     -1.191212e+00 1.156942e+01 -0.10296213 0.918214719
## sd3     -6.848079e+01 2.185698e+01 -3.13313156 0.002313402
##
## $Ita
##              Estimate   Std. Error     t value     Pr(>|t|)
## Can.l1  -9.222313e+01 5.786768e+01 -1.59368980 0.1143986988
## Fran.l1  1.156040e+00 3.521367e-01  3.28292886 0.0014482241
## Ger.l1  -9.209801e-03 3.601186e-03 -2.55743551 0.0121624388
## Ita.l1   1.685268e-01 1.359004e-01  1.24007625 0.2180669343
## Jap.l1  -1.479815e+03 1.935385e+03 -0.76460993 0.4464396548
## UK.l1   -2.297682e+00 3.852741e+00 -0.59637582 0.5523730851
## US.l1    3.986853e+01 1.885424e+01  2.11456590 0.0371417663
## Can.l2   2.244239e+01 5.758674e+01  0.38971446 0.6976387507
## Fran.l2 -1.943119e-01 3.764413e-01 -0.51618123 0.6069529850
## Ger.l2  -1.642740e-03 3.868567e-03 -0.42463783 0.6720814978
## Ita.l2   2.634994e-01 1.329454e-01  1.98201207 0.0504298389
## Jap.l2   2.238669e+02 1.912267e+03  0.11706884 0.9070579550
## UK.l2    3.666676e+00 3.085879e+00  1.18821139 0.2377756750
## US.l2   -3.762083e+01 1.846441e+01 -2.03747820 0.0444434073
## const    1.056679e+00 3.036283e-01  3.48017467 0.0007647280
## trend   -5.926951e-03 3.185729e-03 -1.86046944 0.0659791833
## sd1      4.560002e-02 4.689434e-01  0.09723992 0.9227451269
## sd2     -1.047235e+00 2.990308e-01 -3.50209775 0.0007112673
## sd3     -5.594604e-01 5.649299e-01 -0.99031811 0.3245882887
##
## $Jap
##              Estimate   Std. Error    t value    Pr(>|t|)
## Can.l1   2.582299e-03 3.050845e-03  0.8464209 0.399490722
## Fran.l1  2.110698e-05 1.856502e-05  1.1369221 0.258491775
## Ger.l1   3.351941e-07 1.898583e-07  1.7654962 0.080762614
## Ita.l1  -1.425147e-05 7.164810e-06 -1.9890926 0.049629284
## Jap.l1  -3.280914e-02 1.020355e-01 -0.3215462 0.748518043
## UK.l1   -1.929553e-04 2.031206e-04 -0.9499547 0.344597427
## US.l1    8.085119e-04 9.940153e-04  0.8133798 0.418077911
## Can.l2  -3.382630e-03 3.036033e-03 -1.1141611 0.268081226
## Fran.l2 -7.270282e-06 1.984638e-05 -0.3663278 0.714952520
## Ger.l2   5.247048e-07 2.039549e-07  2.5726505 0.011675147
## Ita.l2  -7.299330e-06 7.009020e-06 -1.0414194 0.300380698
## Jap.l2   8.996033e-02 1.008167e-01  0.8923155 0.374526905
## UK.l2    2.139922e-04 1.626908e-04  1.3153311 0.191632386
## US.l2    3.635343e-04 9.734631e-04  0.3734444 0.709667631
## const    6.958517e-06 1.600760e-05  0.4347008 0.664786619
## trend   -1.916554e-07 1.679550e-07 -1.1411118 0.256753284
## sd1      7.258044e-05 2.472319e-05  2.9357232 0.004192883
```

```
## sd2      1.345717e-05 1.576522e-05  0.8535985 0.395520770
## sd3      2.548650e-05 2.978370e-05  0.8557197 0.394352151
##
## $UK
##               Estimate   Std. Error    t value     Pr(>|t|)
## Can.l1  -2.030052e+00 1.809130e+00 -1.1221146 2.647025e-01
## Fran.l1 -4.263120e-03 1.100893e-02 -0.3872419 6.994619e-01
## Ger.l1   1.109595e-04 1.125847e-04  0.9855647 3.269039e-01
## Ita.l1   8.747678e-03 4.248684e-03  2.0589144 4.229875e-02
## Jap.l1  -2.457082e+01 6.050639e+01 -0.4060864 6.856120e-01
## UK.l1    7.760565e-02 1.204491e-01  0.6443023 5.209654e-01
## US.l1    1.063000e+00 5.894444e-01  1.8033924 7.456481e-02
## Can.l2  -3.280820e+00 1.800347e+00 -1.8223260 7.161933e-02
## Fran.l2 -3.461367e-03 1.176877e-02 -0.2941146 7.693258e-01
## Ger.l2  -8.051365e-05 1.209439e-04 -0.6657106 5.072439e-01
## Ita.l2   4.296692e-03 4.156302e-03  1.0337777 3.039209e-01
## Jap.l2  -1.286848e+01 5.978365e+01 -0.2152509 8.300431e-01
## UK.l2    1.217939e-01 9.647453e-02  1.2624460 2.099449e-01
## US.l2    3.639917e-01 5.772571e-01  0.6305539 5.298787e-01
## const    8.264059e-03 9.492400e-03  0.8705974 3.862157e-01
## trend    2.402896e-04 9.959617e-05  2.4126393 1.779916e-02
## sd1      6.323915e-02 1.466069e-02  4.3135198 3.999294e-05
## sd2     -1.250186e-02 9.348668e-03 -1.3372877 1.843904e-01
## sd3      2.098741e-02 1.766153e-02  1.1883121 2.377362e-01
##
## $US
##               Estimate   Std. Error    t value     Pr(>|t|)
## Can.l1  -5.781895e-01 5.023692e-01 -1.1509253 2.527135e-01
## Fran.l1  5.532326e-03 3.057020e-03  1.8097121 7.357067e-02
## Ger.l1  -4.362596e-05 3.126313e-05 -1.3954443 1.662050e-01
## Ita.l1  -1.077232e-03 1.179798e-03 -0.9130648 3.635695e-01
## Jap.l1  -1.987322e+01 1.680174e+01 -1.1828068 2.399008e-01
## UK.l1   -1.600673e-02 3.344697e-02 -0.4785705 6.333674e-01
## US.l1    3.592707e-01 1.636801e-01  2.1949562 3.065625e-02
## Can.l2  -4.896747e-01 4.999303e-01 -0.9794859 3.298811e-01
## Fran.l2 -1.021078e-03 3.268017e-03 -0.3124458 7.554012e-01
## Ger.l2   3.904325e-05 3.358436e-05  1.1625424 2.479898e-01
## Ita.l2   2.698065e-04 1.154145e-03  0.2337718 8.156760e-01
## Jap.l2  -1.660082e+01 1.660105e+01 -0.9999863 3.199119e-01
## UK.l2    2.946343e-03 2.678957e-02  0.1099810 9.126615e-01
## US.l2   -2.134417e-01 1.602959e-01 -1.3315483 1.862632e-01
## const    1.170640e-02 2.635901e-03  4.4411354 2.462546e-05
## trend   -3.989240e-05 2.765641e-05 -1.4424287 1.525409e-01
## sd1      5.325247e-03 4.071059e-03  1.3080740 1.940722e-01
## sd2     -3.731908e-03 2.595989e-03 -1.4375669 1.539132e-01
## sd3     -1.780692e-03 4.904351e-03 -0.3630841 7.173660e-01
```

```
fit1.pred = fitted(fit1) #output fitted value
fit1$resid = resid(fit1) #output resid
acf(fit1$resid,max.mfrow = 7, mar = c(3, 2.25, 2, 0.55))          #check WN for resid
```


Can
Can & Fran
Can & Ger
Can & Ita
Can & Jap
Can & UK
Can & US
Fran & Can
Fran
Fran & Ger
Fran & Ita
Fran & Jap
Fran & UK
Fran & US
Ger & Can
Ger & Fran
Ger
Ger & Ita
Ger & Jap
Ger & UK
Ger & US
Ita & Can
Ita & Fran
Ita & Ger
Ita
Ita & Jap
Ita & UK
Ita & US
Jap & Can
Jap & Fran
Jap & Ger
Jap & Ita
Jap
Jap & UK
Jap & US
UK & Can
UK & Fran
UK & Ger
UK & Ita
UK & Jap
UK
UK & US
US & Can
US & Fran
US & Ger
US & Ita
US & Jap
US & UK
US
ACF
Lag

```
plot(fit1)
```

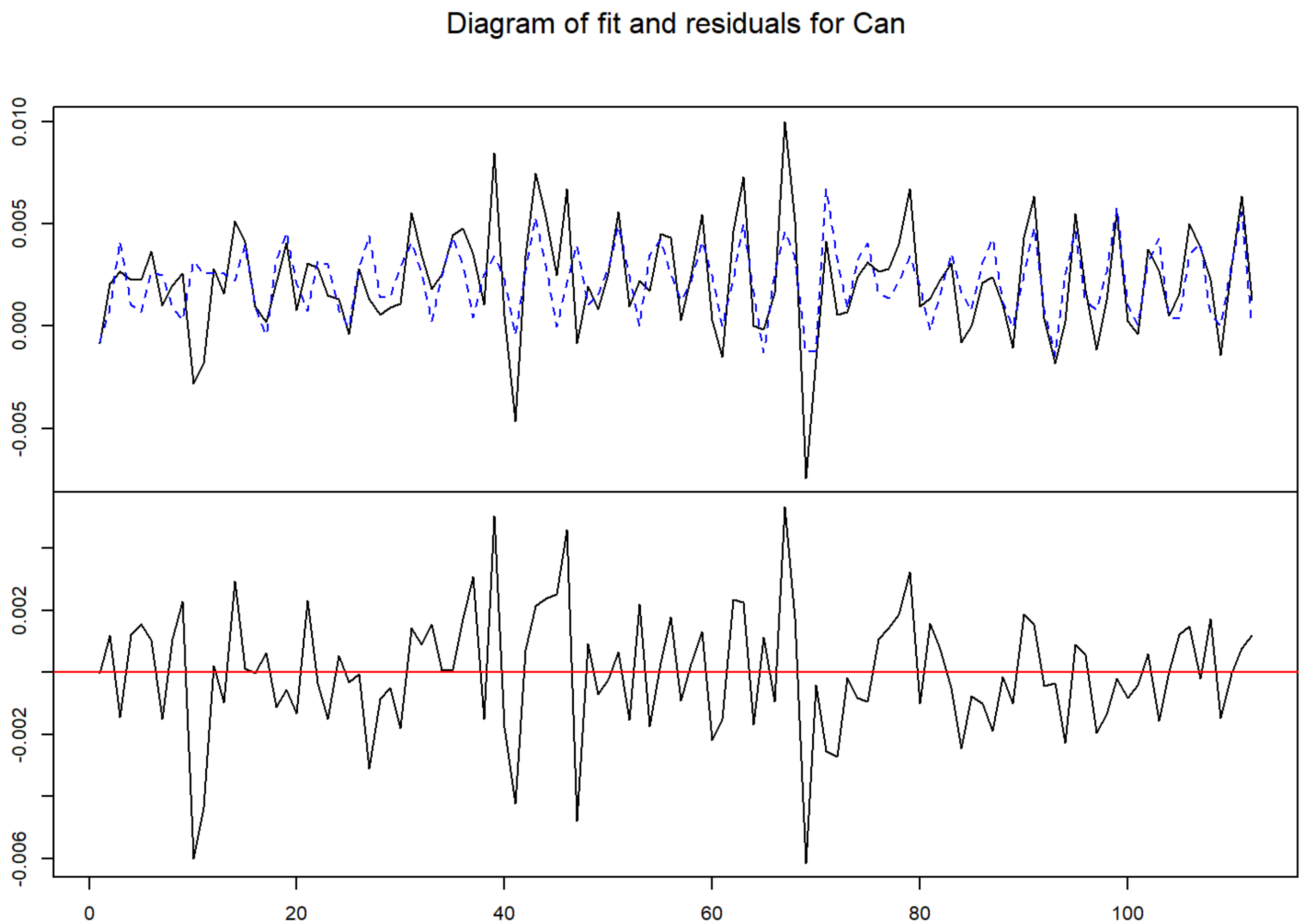
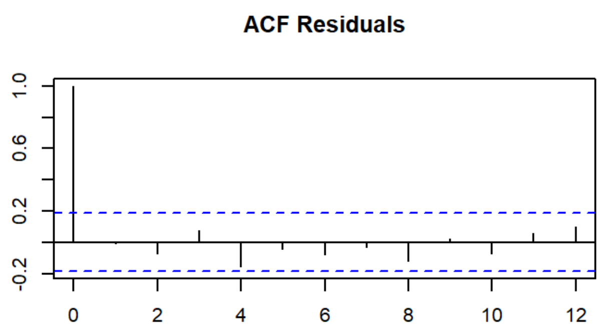
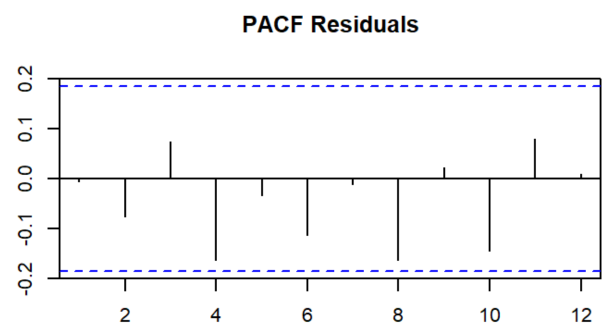


Diagram of fit and residuals for Can
0.010
0.005
0.000
-0.005
0.002
-0.002
-0.006
0
20
40
60
80
100


ACF Residuals
1.0
0.6
0.2
-0.2
0
2
4
6
8
10
12


PACF Residuals
0.2
0.1
0.0
-0.1
-0.2
2
4
6
8
10
12

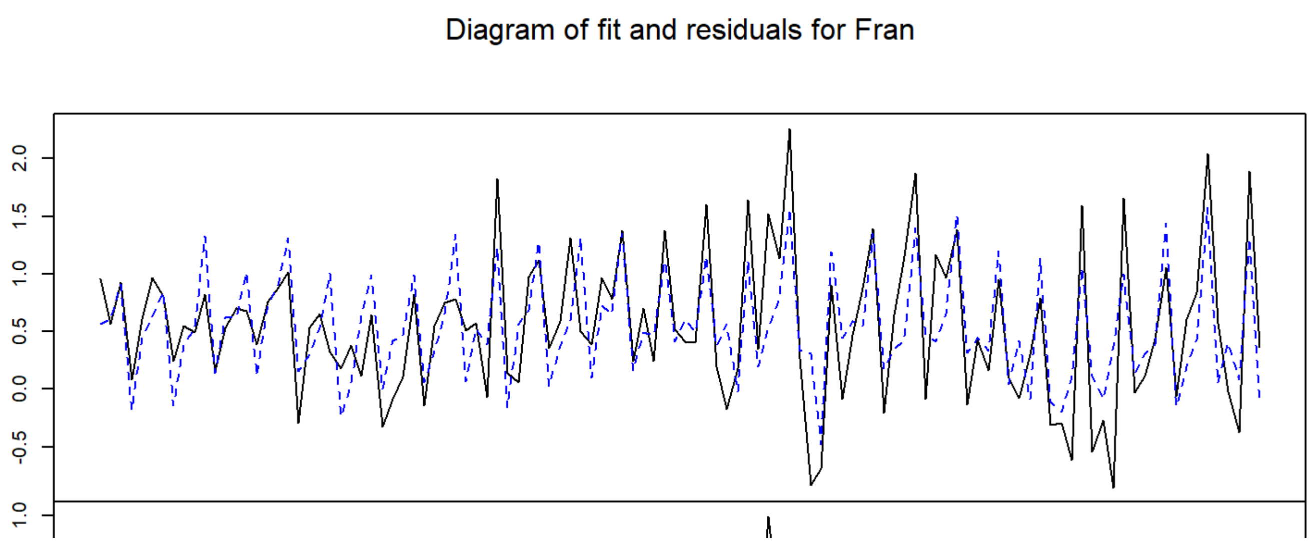
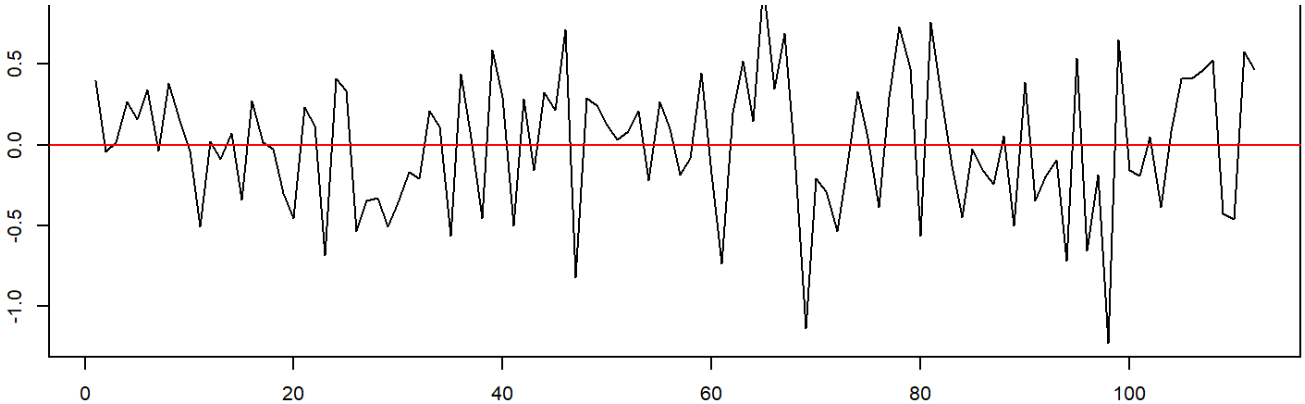
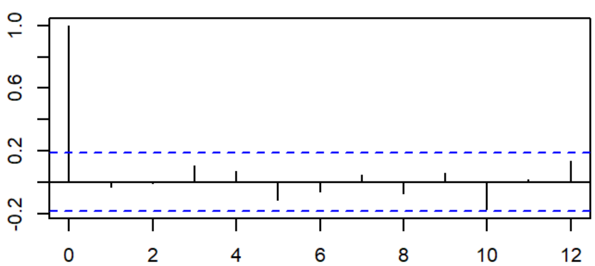


Diagram of fit and residuals for Fran
2.0
1.5
1.0
0.5
0.0
-0.5
1.0



ACF Residuals



PACF Residuals

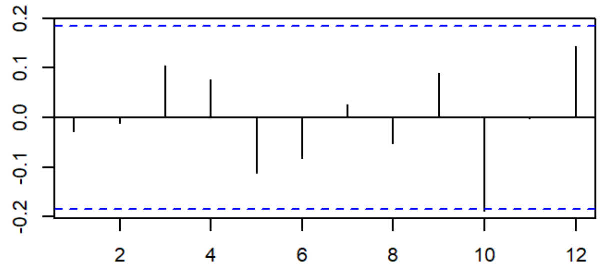
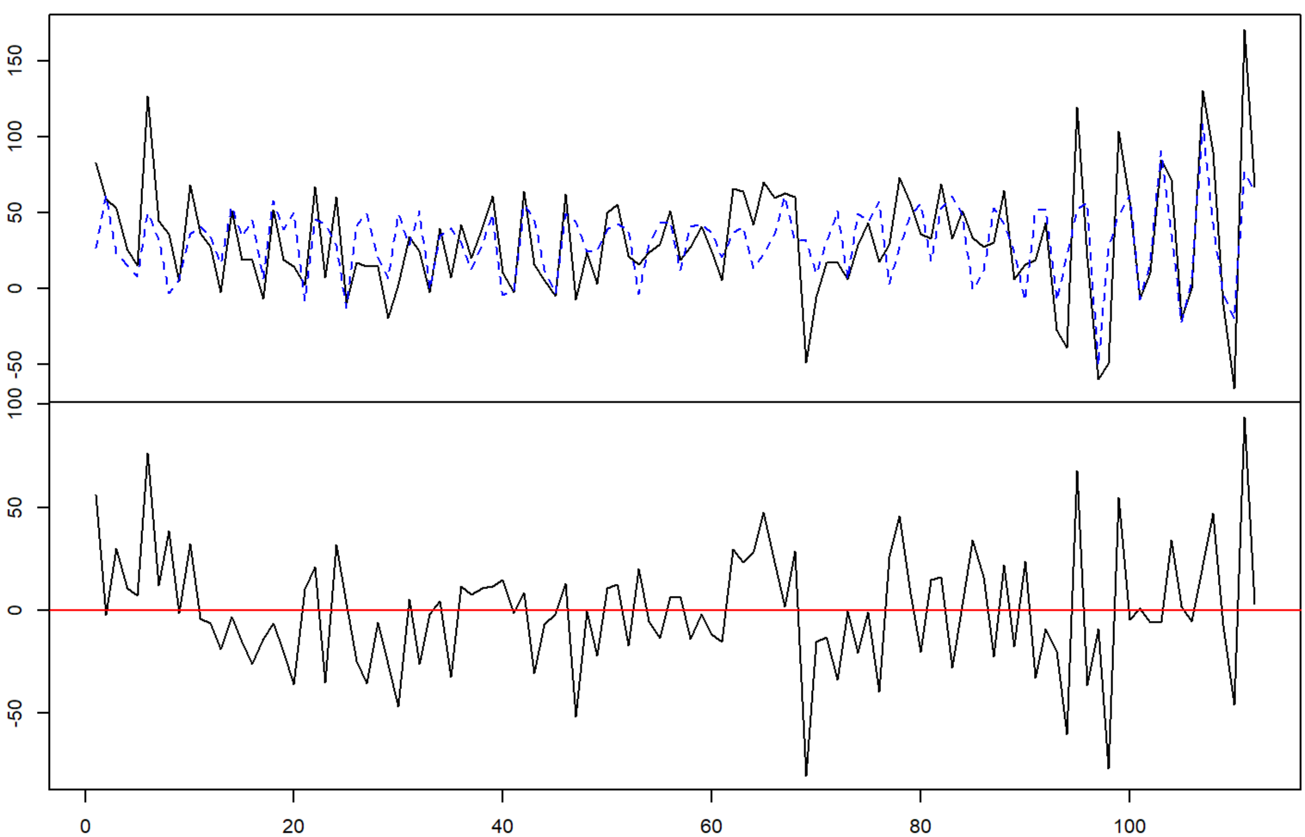
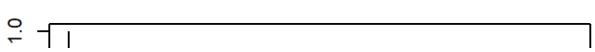


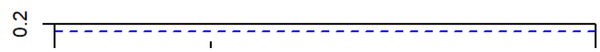
Diagram of fit and residuals for Ger

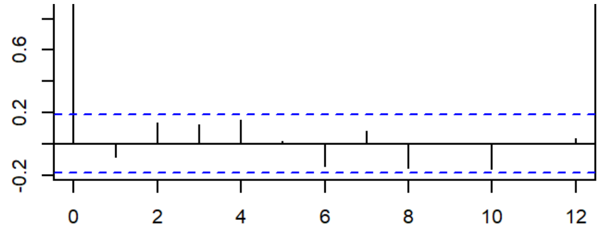


ACF Residuals



PACF Residuals





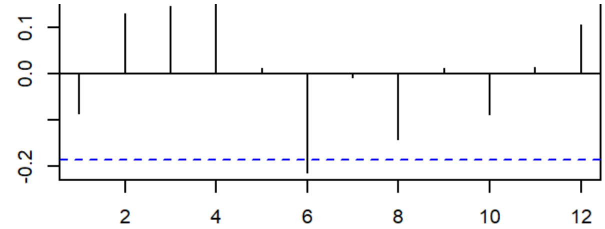
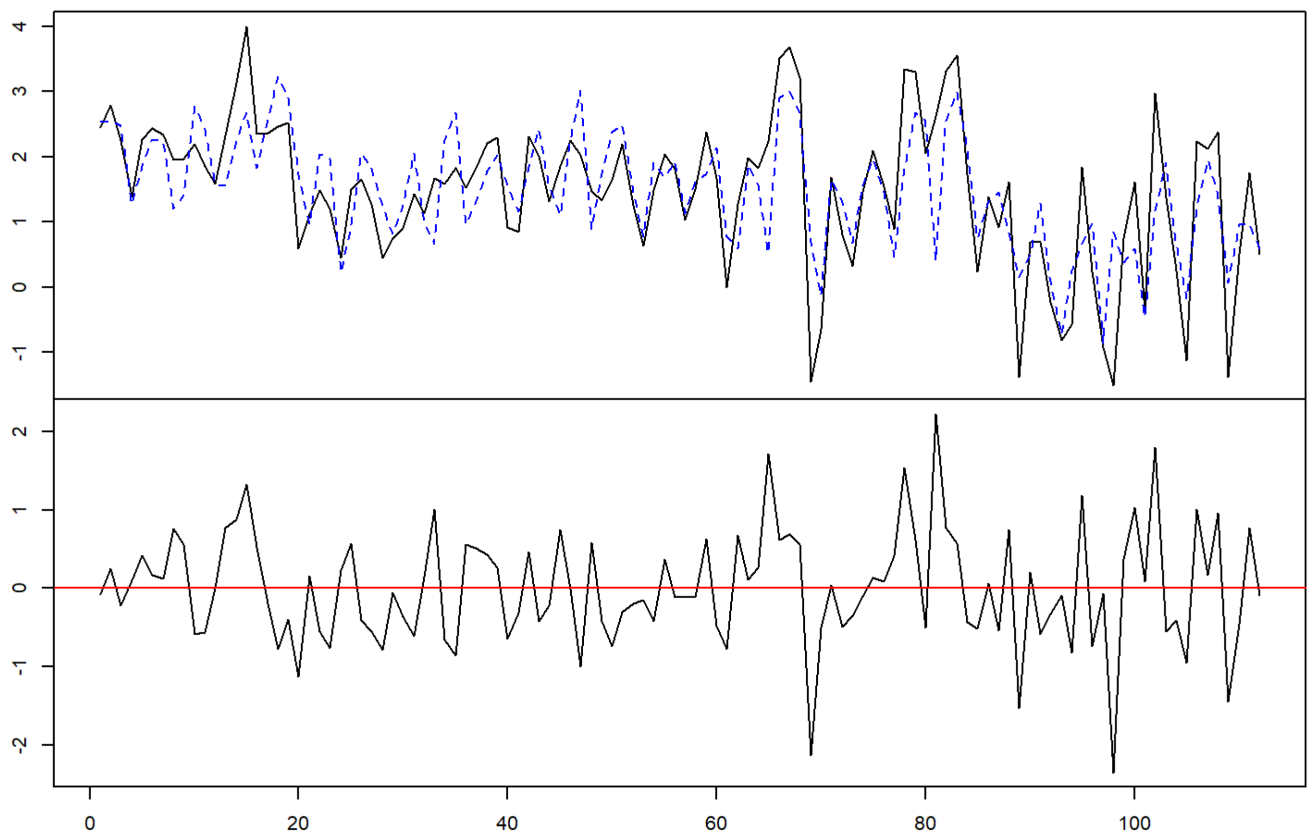
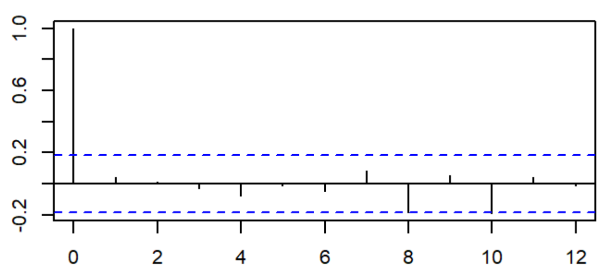


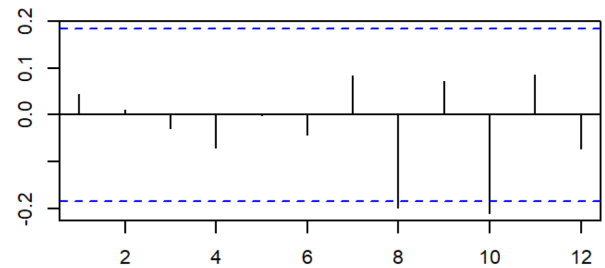
Diagram of fit and residuals for Ita



ACF Residuals



PACF Residuals



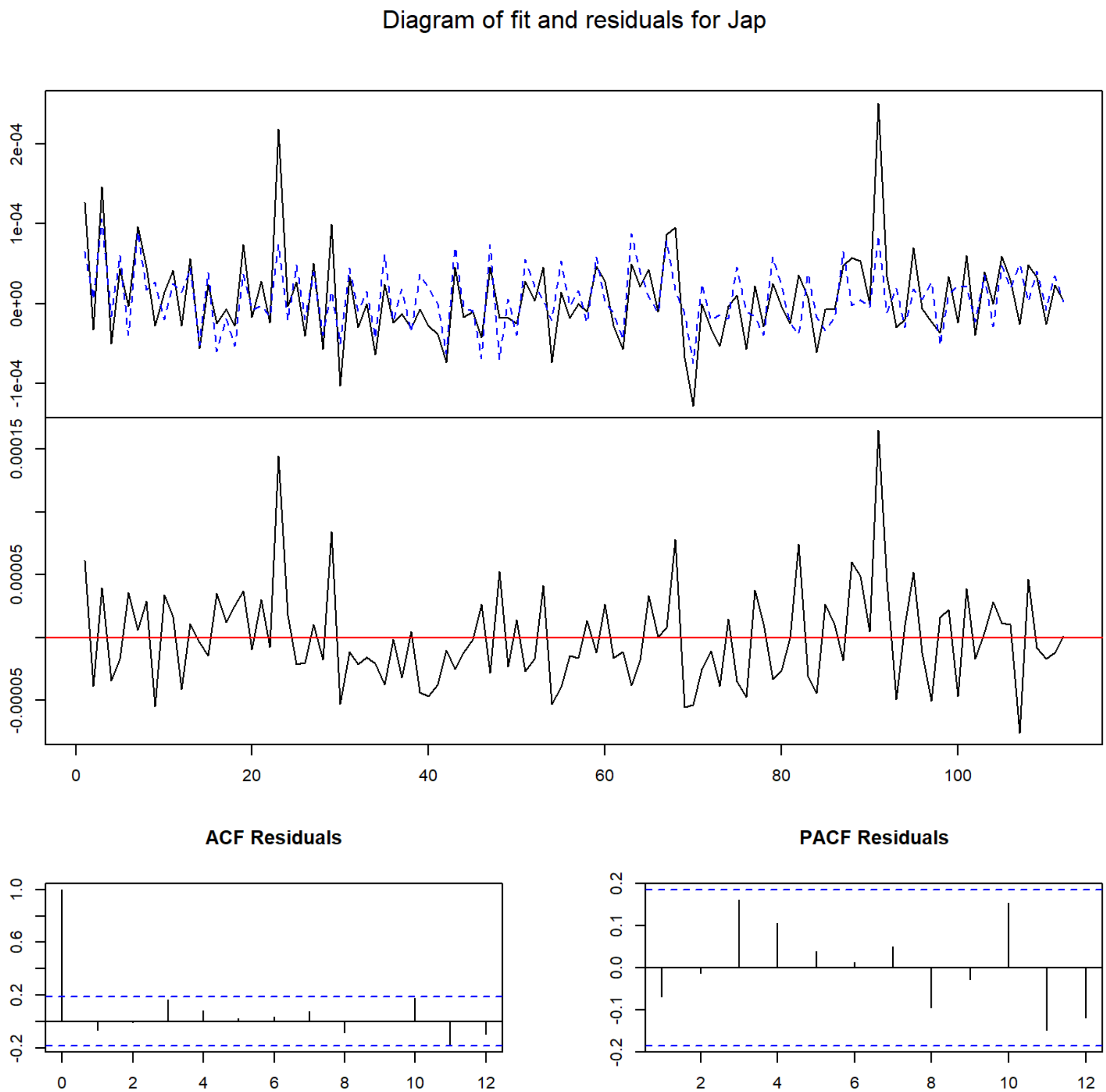


Diagram of fit and residuals for Jap
2e-04
1e-04
0e+00
-1e-04
0.00015
0.00005
-0.00005
0
20
40
60
80
100
ACF Residuals
1.0
0.6
0.2
-0.2
0
2
4
6
8
10
12
PACF Residuals
0.2
0.1
0.0
-0.1
-0.2
2
4
6
8
10
12

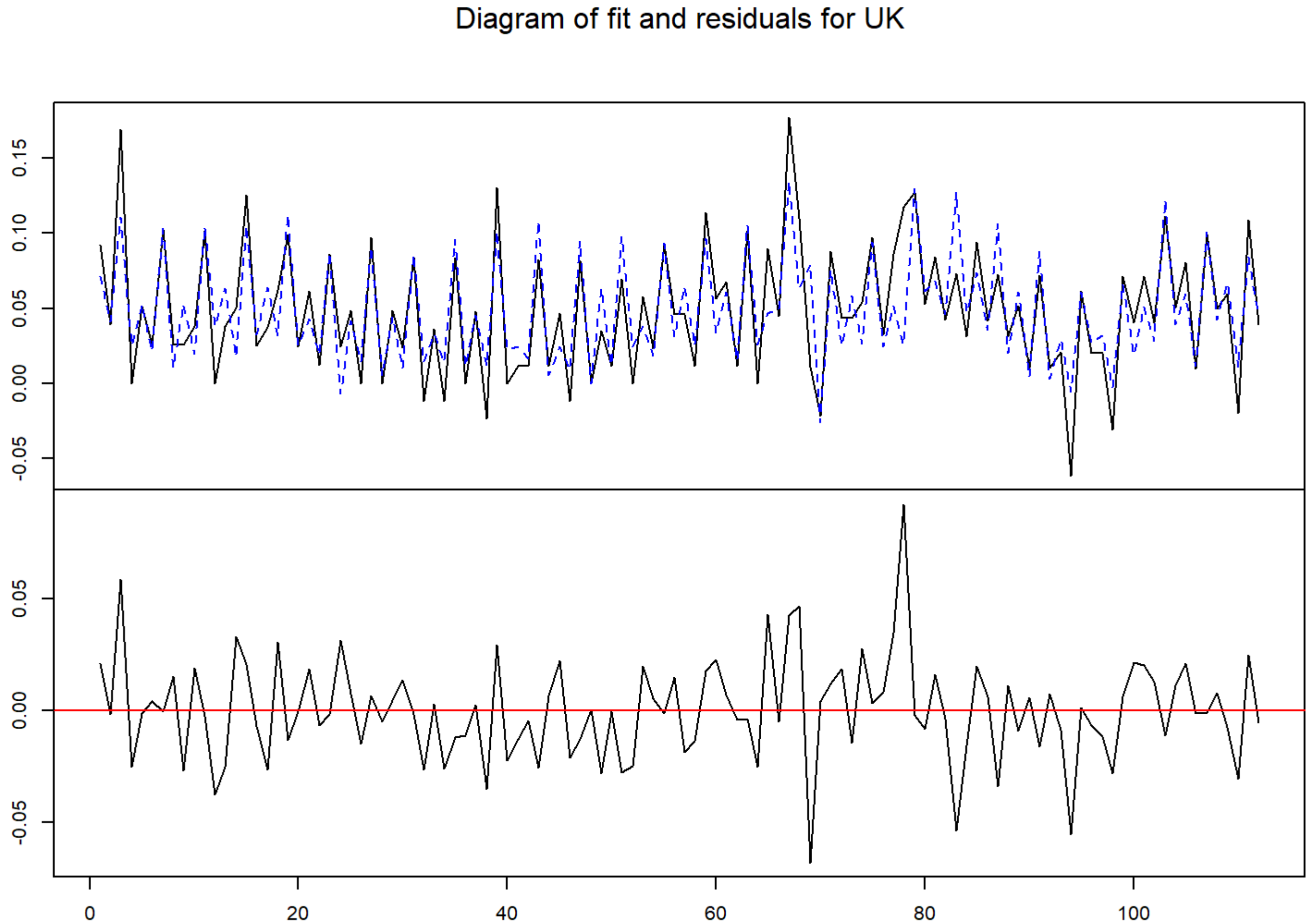
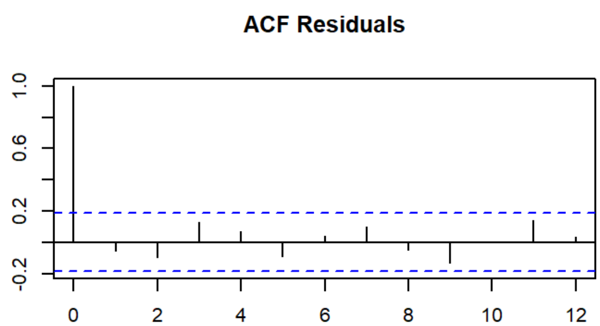
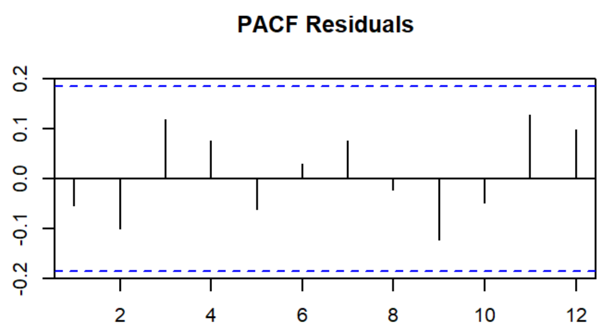
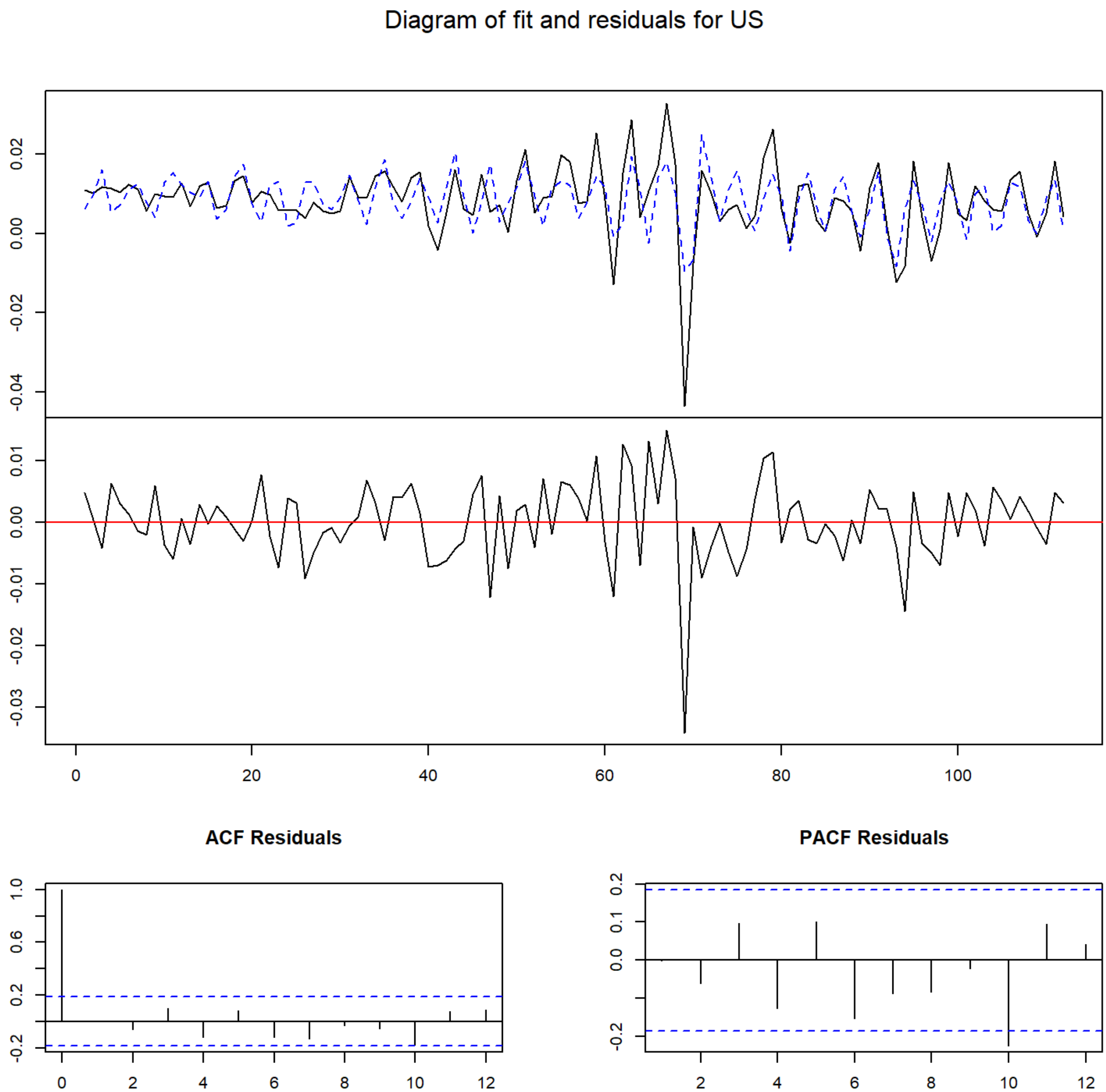


Diagram of fit and residuals for UK
0.15
0.10
0.05
0.00
-0.05
0.05
0.00
-0.05
0
20
40
60
80
100

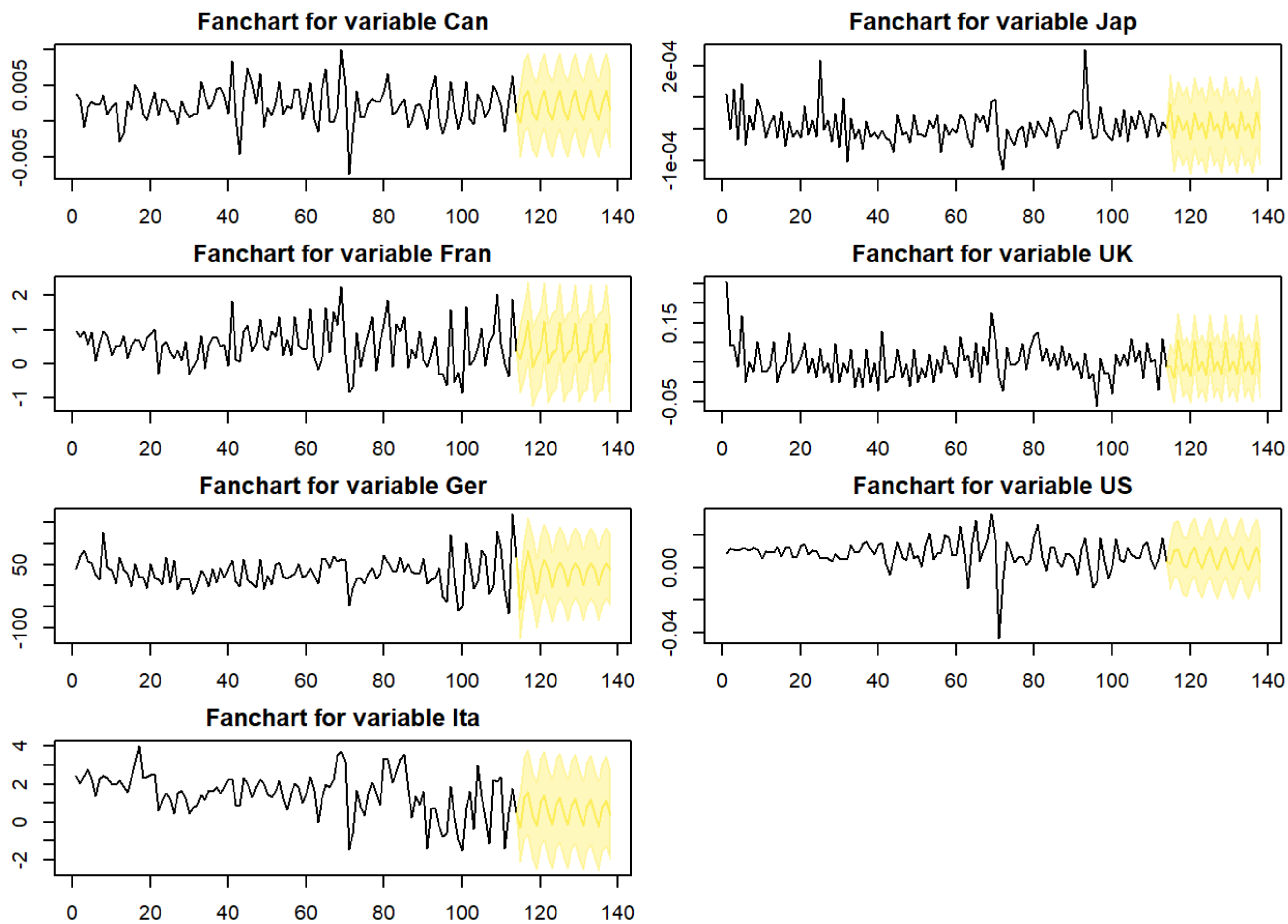

ACF Residuals
1.0
0.6
0.2
-0.2
0
2
4
6
8
10
12


PACF Residuals
0.2
0.1
0.0
-0.1
-0.2
2
4
6
8
10
12



繪製擬合模型的預測結果

```
fit1$pred = predict(fit1, n.ahead = 24, ci = 0.95)
fanchart(fit1$pred, ci=c(0.025,0.975), colors=c("#F7E6204D", "#FDE7254D"),mar = c(2,2,2,1))
```



Some diagnostic tests: 對配適好模型之殘差做Portmanteau test。可看出兩模型殘差的Portmanteau test p-value值皆不顯著，即殘差無法拒絕服從white noise process的假設。可知配適結果並非不好的模型。

```
#fit1.new1
serial.test(fit1, lags.pt=8, type="PT.adjusted")  #make finite sample adjustment
```

```
##
##  Portmanteau Test (adjusted)
##
## data:  Residuals of VAR object fit1
## Chi-squared = 324.81, df = 294, p-value = 0.1045
```

檢查序列間是否存在granger causality 和 instantaneous causality。舉例而言，觀察日本、美國、英國三個序列分別對其餘序列的因果關係。就結果可知，日本及英國對其餘時間序列均無granger causality，顯示兩序列的過去值對其餘序列無顯著解釋能力。然而，美國的過去值則對其餘序列有顯著解釋能力，即美國這筆序列對其餘序列具有預測能力。

```
causality(fit1,cause= c("Jap"))
```

```
## $Granger
##
##  Granger causality H0: Jap do not Granger-cause Can Fran Ger Ita UK US
##
## data:  VAR object fit1
## F-Test = 1.0787, df1 = 12, df2 = 651, p-value = 0.3752
##
##
## $Instant
##
##  H0: No instantaneous causality between: Jap and Can Fran Ger Ita UK US
##
## data:  VAR object fit1
## Chi-squared = 12.233, df = 6, p-value = 0.05696
```

```
causality(fit1, cause= "US")
```

```
## $Granger
##
##  Granger causality H0: US do not Granger-cause Can Fran Ger Ita Jap UK
##
## data:  VAR object fit1
## F-Test = 1.8753, df1 = 12, df2 = 651, p-value = 0.03437
##
##
## $Instant
##
##  H0: No instantaneous causality between: US and Can Fran Ger Ita Jap UK
##
## data:  VAR object fit1
## Chi-squared = 44.45, df = 6, p-value = 6.017e-08
```

```
causality(fit1, cause= "UK")
```

```
## $Granger
##
##  Granger causality H0: UK do not Granger-cause Can Fran Ger Ita Jap US
##
## data:  VAR object fit1
## F-Test = 1.2572, df1 = 12, df2 = 651, p-value = 0.2399
##
##
## $Instant
##
##  H0: No instantaneous causality between: UK and Can Fran Ger Ita Jap US
##
## data:  VAR object fit1
## Chi-squared = 26.597, df = 6, p-value = 0.0001722
```

Part2 : The data for UK (3 series)—何羿樺

Library

```
library(aTSA)
library(vars)
library(forecast)
library(tseries)
```

Data cleaning

```
econ<-read.csv("C:/Users/cindy/time_series/econ_data.csv", header = T)
dim(econ)
```

```
## [1] 115  22
```

```
head(econ)
```

```
##        DATE Canada_CPI Canada_gdp Canada_unem France_CPI France_gdp France_unem
## 1  1991/1/1   64.86700   54.97783    10.16667   68.80333   68.47505    9.266667
## 2  1991/4/1   65.34106   55.23417    10.33333   69.36333   68.70599    9.433333
## 3  1991/7/1   65.73611   55.30814    10.43333   69.82000   68.92958    9.700000
## 4 1991/10/1   65.63076   55.41020    10.33333   70.38333   69.21646   10.000000
## 5  1992/1/1   65.89413   55.45059    10.60000   70.71667   69.92935   10.266667
## 6  1992/4/1   66.23650   55.51830    11.00000   71.26000   69.87414   10.500000
##   Germany_CPI Germany_gdp Germany_unem Italy_CPI Italy_gdp Italy_unem Japan_CPI
## 1    64.18494    73.48000     5.233333  54.07411  86.44591   8.633333  93.20000
## 2    64.80689    73.11000     5.333333  54.85303  85.97284   8.466667  94.16667
## 3    65.86420    72.95000     5.600000  55.49347  86.00364   8.433333  94.16667
## 4    67.10809    73.88000     5.900000  56.27239  87.01302   8.600000  95.30000
## 5    67.97882    74.99748     6.100000  57.15516  87.24703   8.666667  95.00000
## 6    68.75625    74.45750     6.400000  57.86484  87.19709   8.666667  96.33333
##   Japan_gdp Japan_unem United.Kingdom_CPI United.Kingdom_gdp
## 1  79.99816   2.100000               57.2           59.47495
## 2  80.94727   2.100000               59.1           59.39998
## 3  80.76807   2.100000               59.8           59.27025
## 4  81.33533   2.066667               60.5           59.37321
## 5  81.36063   2.066667               60.8           59.37750
## 6  81.64551   2.100000               62.1           59.30743
##   United.Kingdom_unem United.States_CPI United.States_gdp United.States_unem
## 1            7.766667          56.87356          53.26046           6.600000
## 2            8.466667          57.21109          53.67574           6.833333
## 3            8.933333          57.66112          53.94696           6.866667
## 4            9.100000          58.09710          54.13497           7.100000
## 5            9.300000          58.50495          54.78299           7.366667
## 6            9.666667          58.98311          55.37703           7.600000
```

```
econ[is.na(econ)] #no NAs
```

```
## character(0)
```

```
econ$DATE<-as.Date(econ$DATE, format =  "%Y/%m/%d" )
summary(econ)
```

```
##       DATE              Canada_CPI       Canada_gdp       Canada_unem    
##  Min.   :1991-01-01   Min.   : 64.87   Min.   : 54.98   Min.   : 5.567  
##  1st Qu.:1998-02-15   1st Qu.: 71.99   1st Qu.: 67.05   1st Qu.: 6.850  
##  Median :2005-04-01   Median : 84.33   Median : 84.34   Median : 7.333  
##  Mean   :2005-04-01   Mean   : 84.43   Mean   : 81.75   Mean   : 7.810  
##  3rd Qu.:2012-05-16   3rd Qu.: 96.21   3rd Qu.: 94.40   3rd Qu.: 8.600  
##  Max.   :2019-07-01   Max.   :107.98   Max.   :108.39   Max.   :11.733  
##    France_CPI       France_gdp       France_unem      Germany_CPI    
##  Min.   : 68.80   Min.   : 68.48   Min.   : 7.267   Min.   : 64.18  
##  1st Qu.: 78.08   1st Qu.: 77.49   1st Qu.: 8.817   1st Qu.: 78.24  
##  Median : 87.84   Median : 90.77   Median : 9.500   Median : 85.98  
##  Mean   : 87.49   Mean   : 88.30   Mean   : 9.913   Mean   : 86.79  
##  3rd Qu.: 98.74   3rd Qu.: 97.39   3rd Qu.:10.733   3rd Qu.: 97.13  
##  Max.   :104.58   Max.   :107.11   Max.   :12.500   Max.   :106.08  
##   Germany_gdp      Germany_unem      Italy_CPI        Italy_gdp     
##  Min.   : 72.95   Min.   : 3.100   Min.   : 54.07   Min.   : 85.94  
##  1st Qu.: 80.10   1st Qu.: 5.333   1st Qu.: 71.79   1st Qu.: 95.04  
##  Median : 86.04   Median : 7.767   Median : 84.42   Median :101.57  
##  Mean   : 88.32   Mean   : 7.263   Mean   : 83.22   Mean   : 99.13  
##  3rd Qu.: 95.67   3rd Qu.: 8.800   3rd Qu.: 98.71   3rd Qu.:103.79  
##  Max.   :107.23   Max.   :11.200   Max.   :103.17   Max.   :109.42  
##    Italy_unem       Japan_CPI        Japan_gdp        Japan_unem   
##  Min.   : 5.933   Min.   : 93.20   Min.   : 80.00   Min.   :2.067  
##  1st Qu.: 8.317   1st Qu.: 96.87   1st Qu.: 87.32   1st Qu.:3.033  
##  Median : 9.967   Median : 97.57   Median : 92.97   Median :3.933  
##  Mean   : 9.673   Mean   : 98.03   Mean   : 92.47   Mean   :3.825  
##  3rd Qu.:11.183   3rd Qu.: 99.70   3rd Qu.: 97.76   3rd Qu.:4.667  
##  Max.   :12.733   Max.   :101.77   Max.   :104.35   Max.   :5.433  
##  United.Kingdom_CPI United.Kingdom_gdp United.Kingdom_unem United.States_CPI
##  Min.   : 57.20     Min.   : 59.27     Min.   : 3.733      Min.   : 56.87   
##  1st Qu.: 71.00     1st Qu.: 71.19     1st Qu.: 5.033      1st Qu.: 68.49   
##  Median : 79.30     Median : 87.71     Median : 5.933      Median : 82.06   
##  Mean   : 82.20     Mean   : 83.80     Mean   : 6.454      Mean   : 82.35   
##  3rd Qu.: 95.95     3rd Qu.: 93.79     3rd Qu.: 7.817      3rd Qu.: 97.06   
##  Max.   :108.20     Max.   :106.95     Max.   :10.400      Max.   :108.27   
##  United.States_gdp United.States_unem
##  Min.   : 53.26    Min.   :3.633     
##  1st Qu.: 68.30    1st Qu.:4.633     
##  Median : 85.27    Median :5.500     
##  Mean   : 81.78    Mean   :5.869     
##  3rd Qu.: 93.14    3rd Qu.:6.850     
##  Max.   :109.87    Max.   :9.933     
```

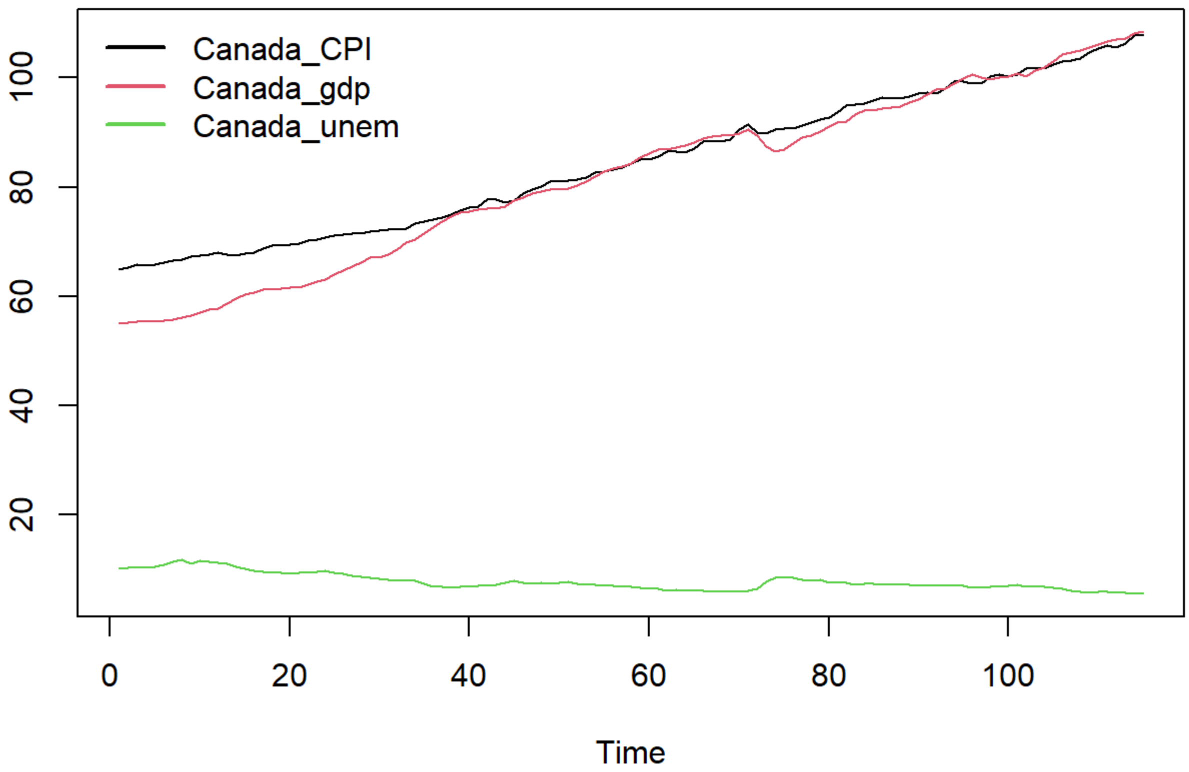
```
str(econ)
```

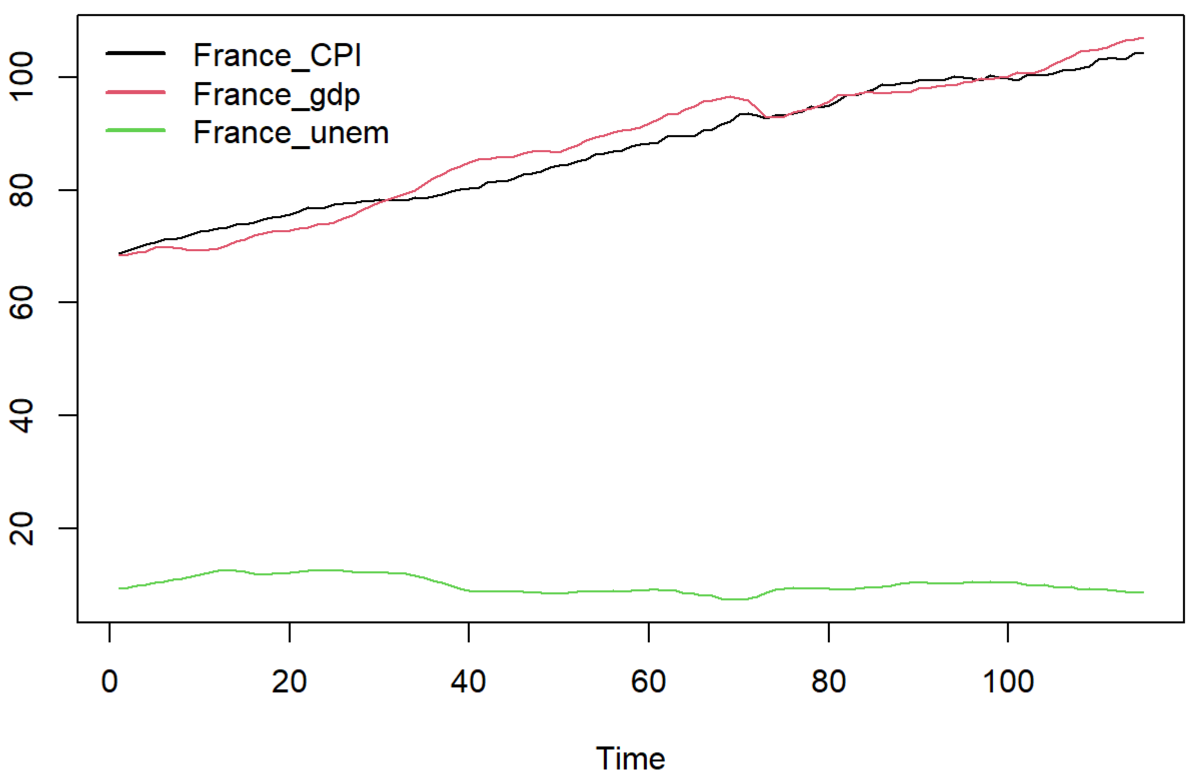
```
## 'data.frame':    115 obs. of  22 variables:
##  $ DATE               : Date, format: "1991-01-01" "1991-04-01" ...
##  $ Canada_CPI         : num  64.9 65.3 65.7 65.6 65.9 ...
##  $ Canada_gdp         : num  55 55.2 55.3 55.4 55.5 ...
##  $ Canada_unem        : num  10.2 10.3 10.4 10.3 10.6 ...
##  $ France_CPI         : num  68.8 69.4 69.8 70.4 70.7 ...
##  $ France_gdp         : num  68.5 68.7 68.9 69.2 69.9 ...
##  $ France_unem        : num  9.27 9.43 9.7 10 10.27 ...
##  $ Germany_CPI        : num  64.2 64.8 65.9 67.1 68 ...
##  $ Germany_gdp        : num  73.5 73.1 73 73.9 75 ...
##  $ Germany_unem       : num  5.23 5.33 5.6 5.9 6.1 ...
##  $ Italy_CPI          : num  54.1 54.9 55.5 56.3 57.2 ...
##  $ Italy_gdp          : num  86.4 86 86 87 87.2 ...
##  $ Italy_unem         : num  8.63 8.47 8.43 8.6 8.67 ...
##  $ Japan_CPI          : num  93.2 94.2 94.2 95.3 95 ...
##  $ Japan_gdp          : num  80 80.9 80.8 81.3 81.4 ...
##  $ Japan_unem         : num  2.1 2.1 2.1 2.07 2.07 ...
##  $ United.Kingdom_CPI : num  57.2 59.1 59.8 60.5 60.8 62.1 62.1 62.5 62.7 63.5 ...
##  $ United.Kingdom_gdp : num  59.5 59.4 59.3 59.4 59.4 ...
##  $ United.Kingdom_unem: num  7.77 8.47 8.93 9.1 9.3 ...
##  $ United.States_CPI  : num  56.9 57.2 57.7 58.1 58.5 ...
##  $ United.States_gdp  : num  53.3 53.7 53.9 54.1 54.8 ...
##  $ United.States_unem : num  6.6 6.83 6.87 7.1 7.37 ...
```

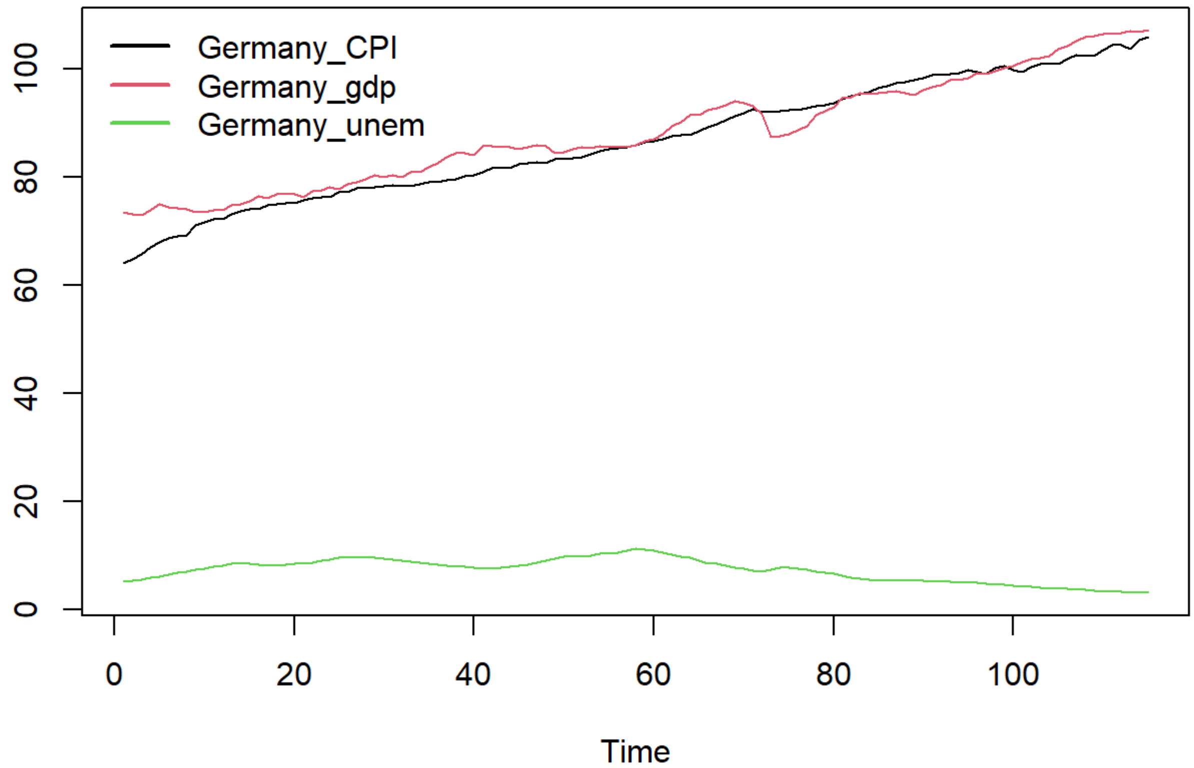
> 確保資料型態以及缺失值等等。

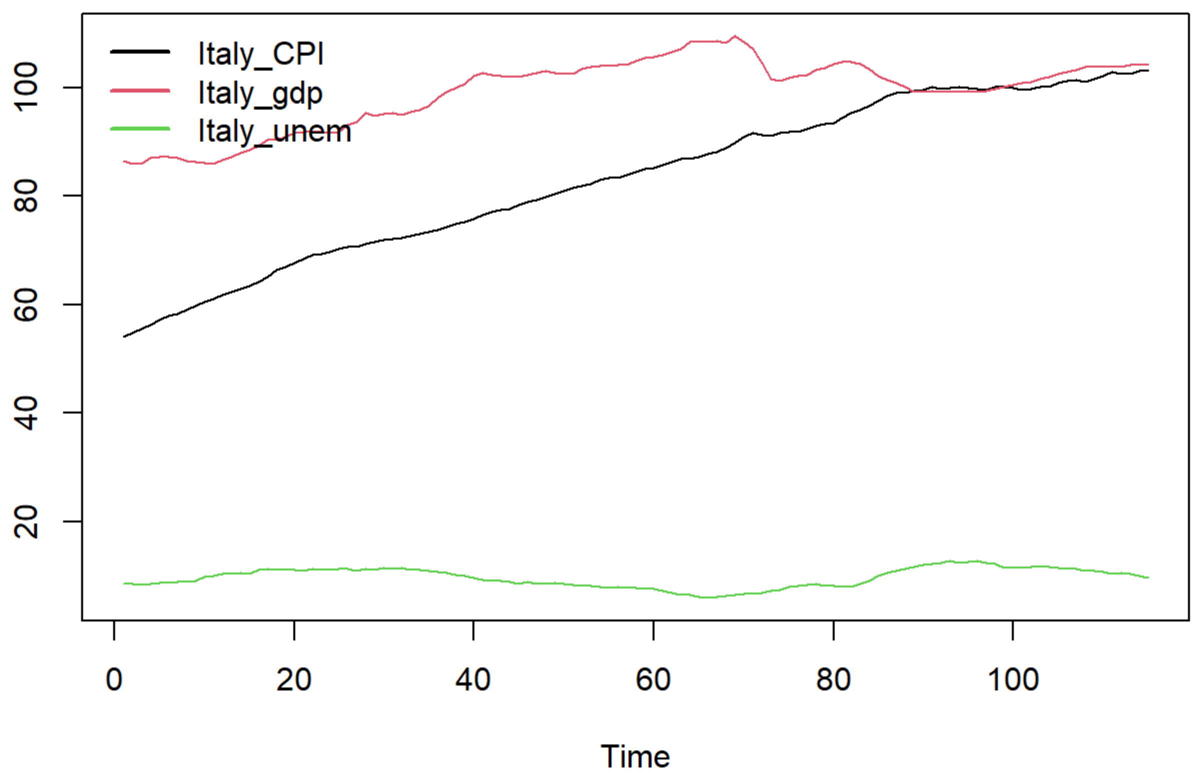
EDA

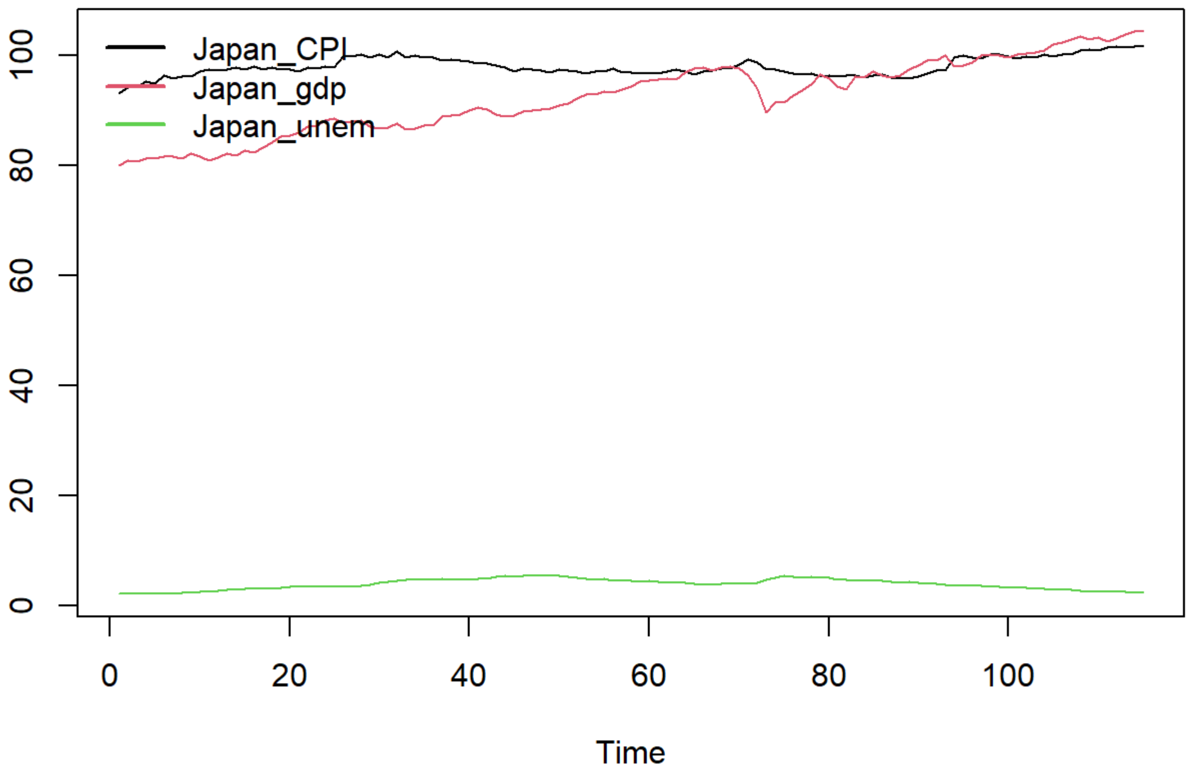
```
par(mfrow=c(1,1))
series_name = colnames(econ)
for (i in 1:7){
  y = econ[,1+(i-1)*3+(1:3)]
  ts.plot(y, col=1:3)
  legend("topleft", legend=series_name[1+(i-1)*3+(1:3)], col=1:3, lty=1, lwd=2, bty="n")
}
```

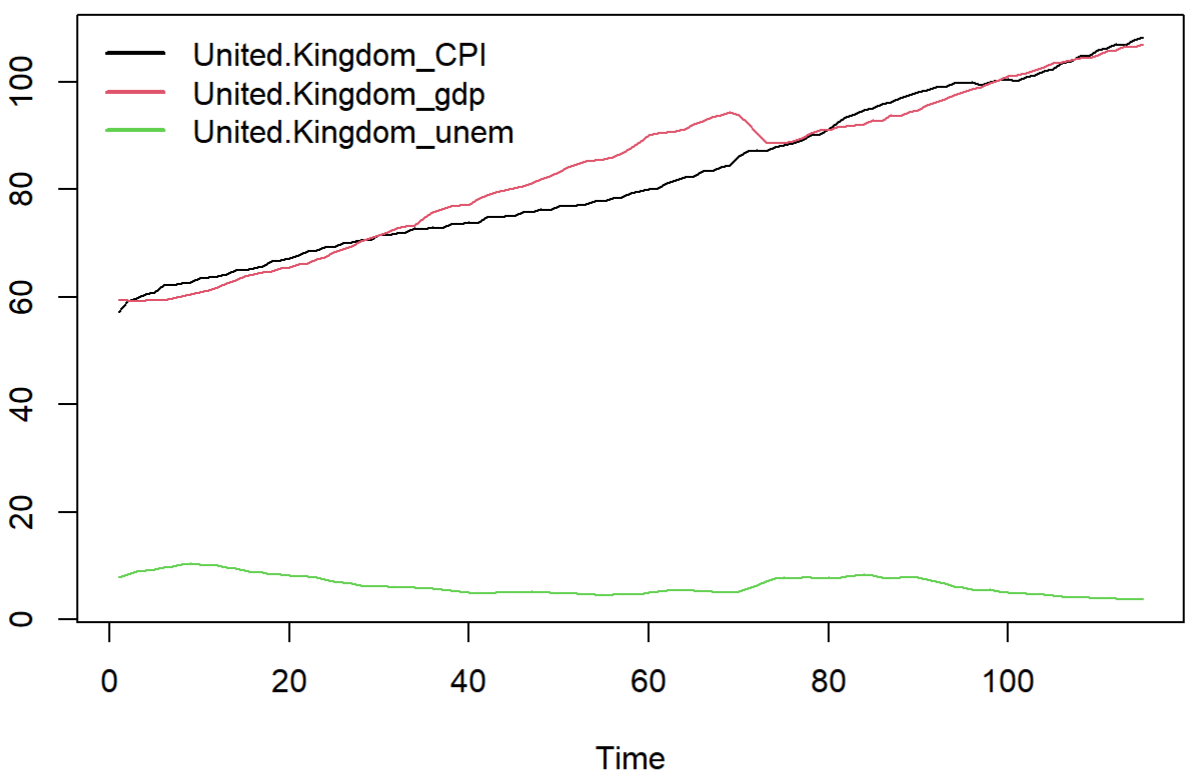

Canada_CPI
Canada_gdp
Canada_unem
Time

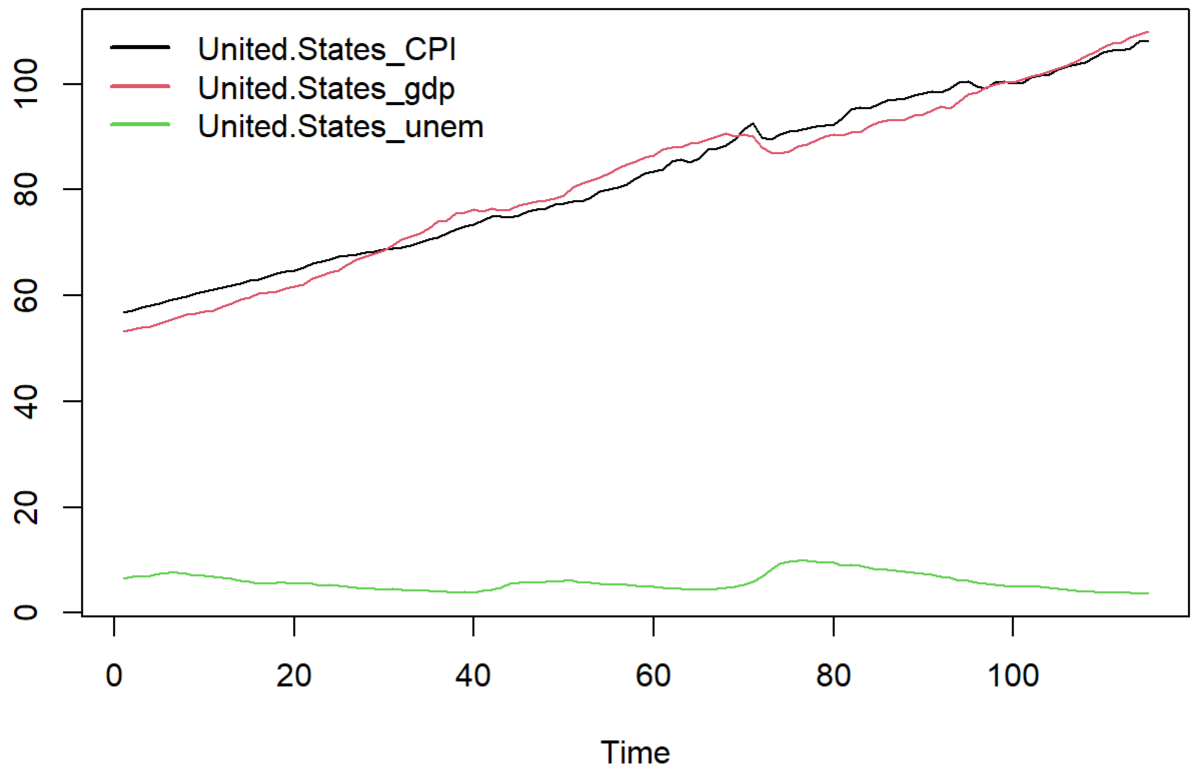

France_CPI
France_gdp
France_unem
Time


Germany_CPI
Germany_gdp
Germany_unem
0
20
40
60
80
100
Time


Italy_CPI
Italy_gdp
Italy_unem
20
40
60
80
100
0
Time


Japan_CPI
Japan_gdp
Japan_unem
0
20
40
60
80
100
Time


United.Kingdom_CPI
United.Kingdom_gdp
United.Kingdom_unem
0
20
40
60
80
100
Time



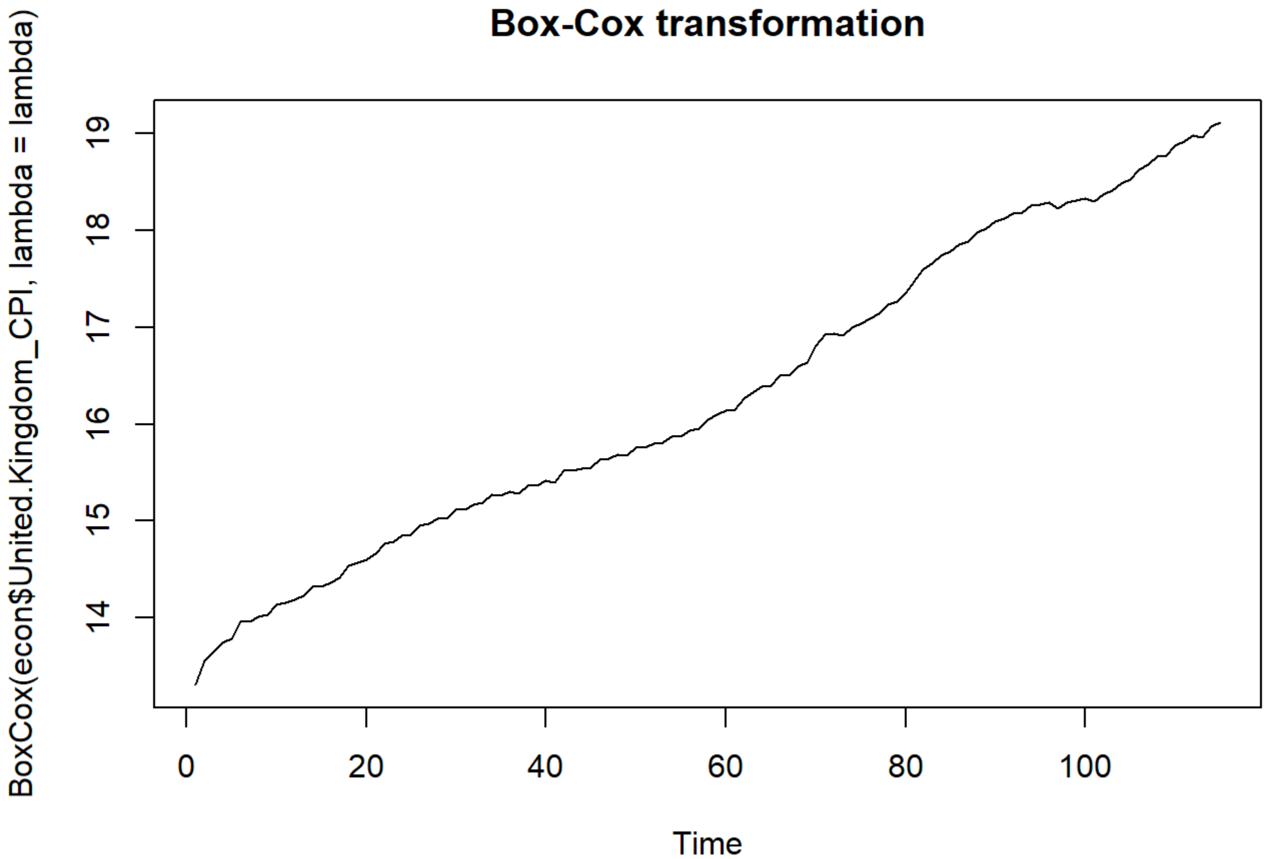
Data Preprocessing

box-cox

```
# box-cox
par(mfrow=c(1,1))
lambda <- BoxCox.lambda(econ$United.Kingdom_CPI)
print(lambda)
```

```
## [1] 0.5051194
```

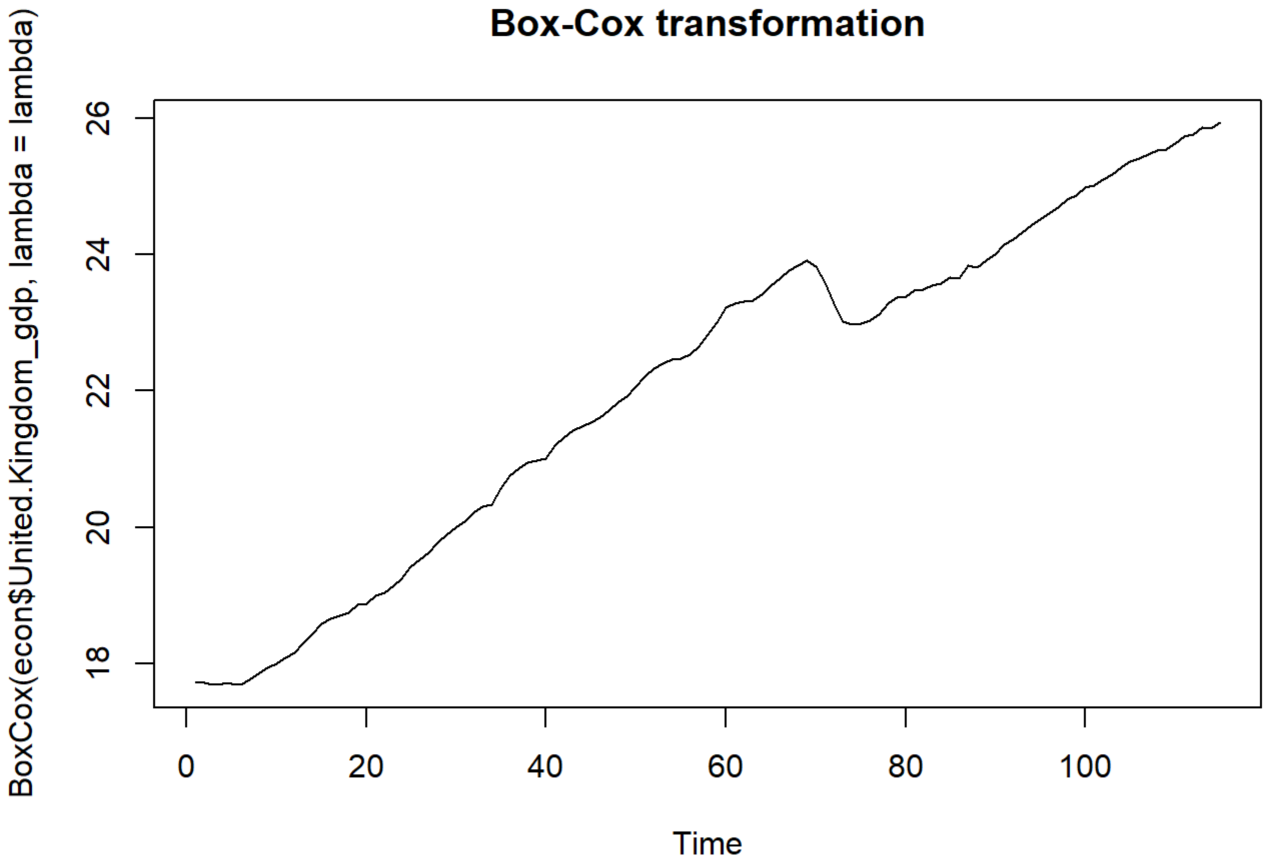
```
plot.ts(BoxCox(econ$United.Kingdom_CPI, lambda = lambda), main='Box-Cox transformation')
```




```
kc1<-BoxCox(econ$United.Kingdom_CPI, lambda = lambda)

par(mfrow=c(1,1))
lambda <- BoxCox.lambda(econ$United.Kingdom_gdp)
print(lambda)
```

```
## [1] 0.6011321
```

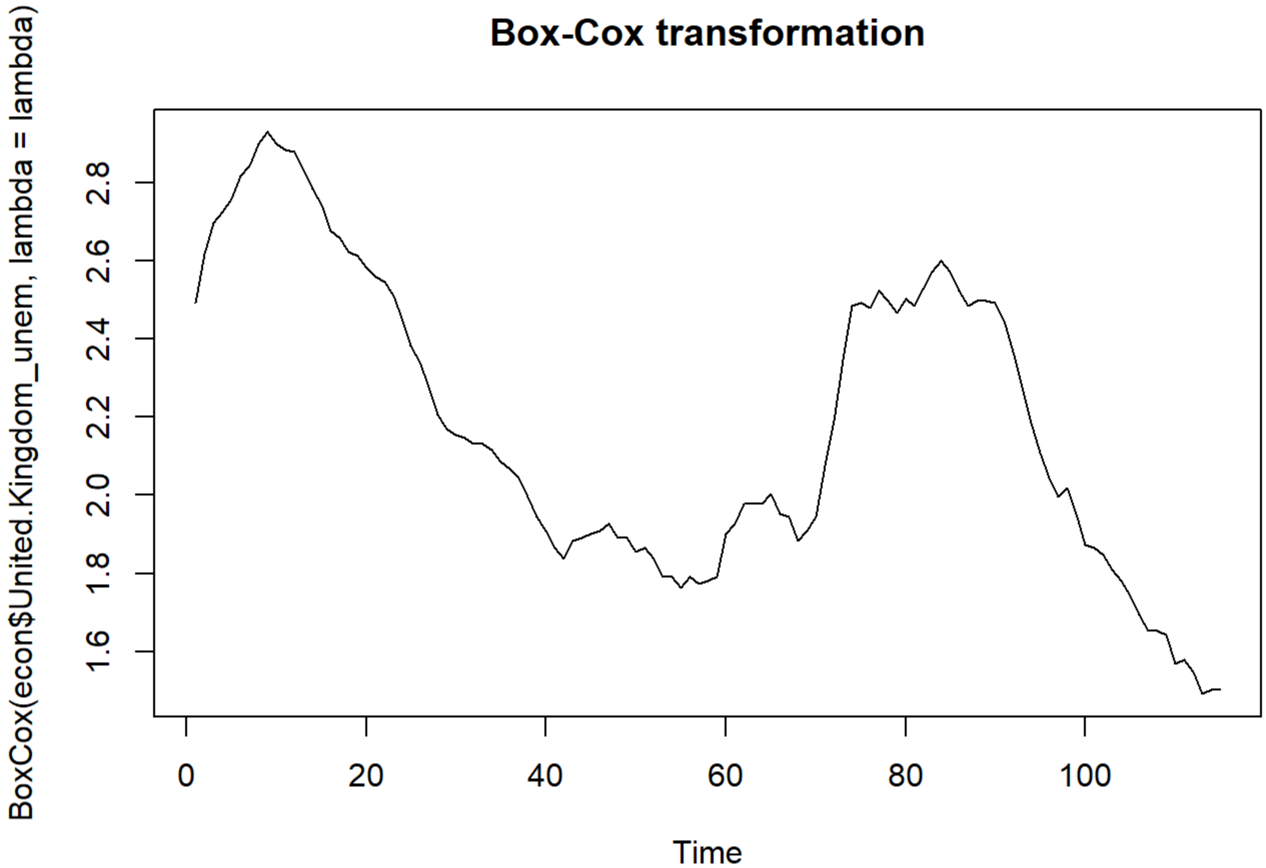
```
plot.ts(BoxCox(econ$United.Kingdom_gdp, lambda = lambda), main='Box-Cox transformation')
```




```
kg1<-BoxCox(econ$United.Kingdom_gdp, lambda = lambda)

par(mfrow=c(1,1))
lambda <- BoxCox.lambda(econ$United.Kingdom_unem)
print(lambda)
```

```
## [1] 0.1844793
```

```
plot.ts(BoxCox(econ$United.Kingdom_unem, lambda = lambda), main='Box-Cox transformation')
```

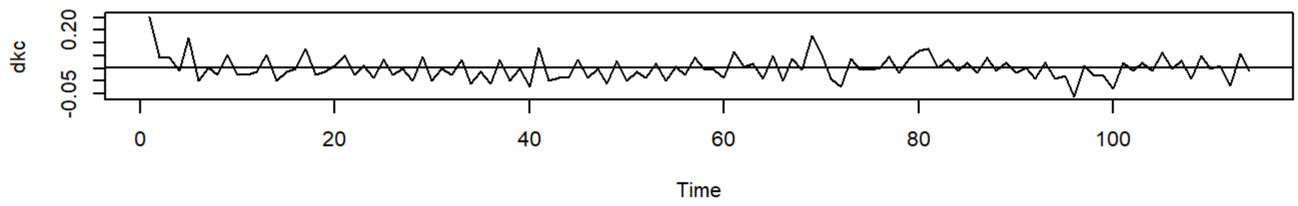


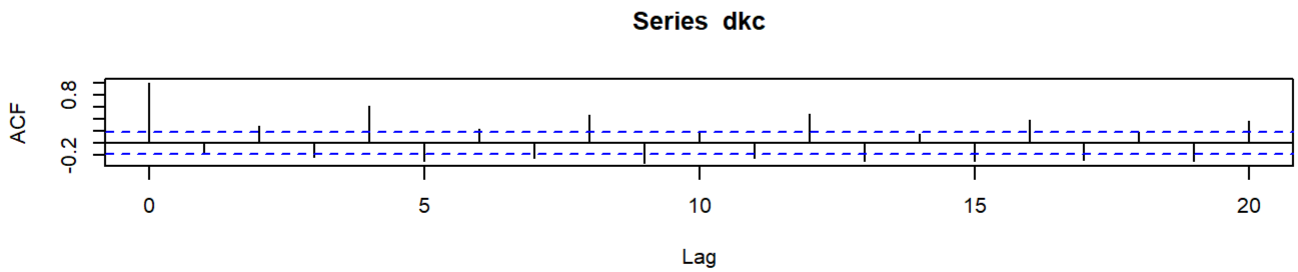

```
ku1<-BoxCox(econ$United.Kingdom_unem, lambda = lambda)
```

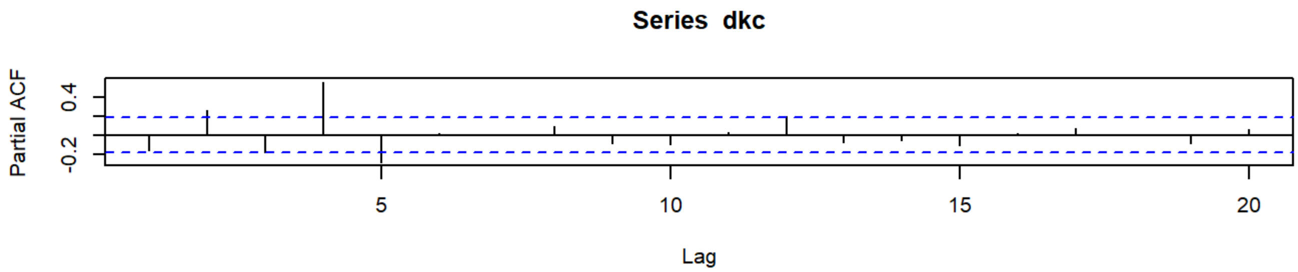
> 利用Box-Cox transformation，使轉換後的資料變異數齊一，更似常態分佈。 其中，計算出的lambda值分別為-0.5051194，0.6011321，0.1844793，並將轉換後的資料繪製成圖，並存為新的變數。

Differencing

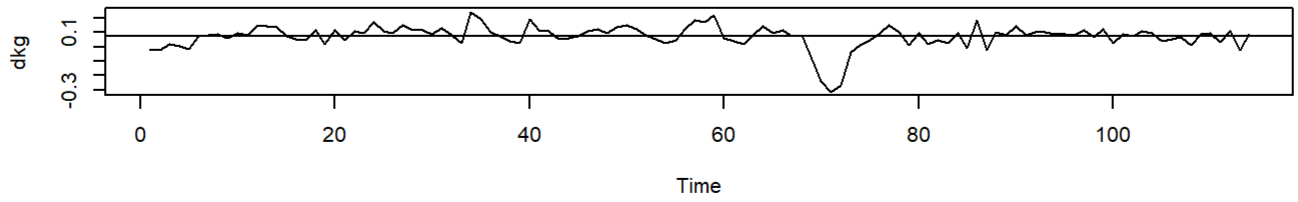
```
dkc<-diff(kc1)
{par(mfrow=c(3,1))
  {ts.plot(dkc)
    abline(h=mean(dkc))
}
  acf(dkc)
  pacf(dkc)}
```

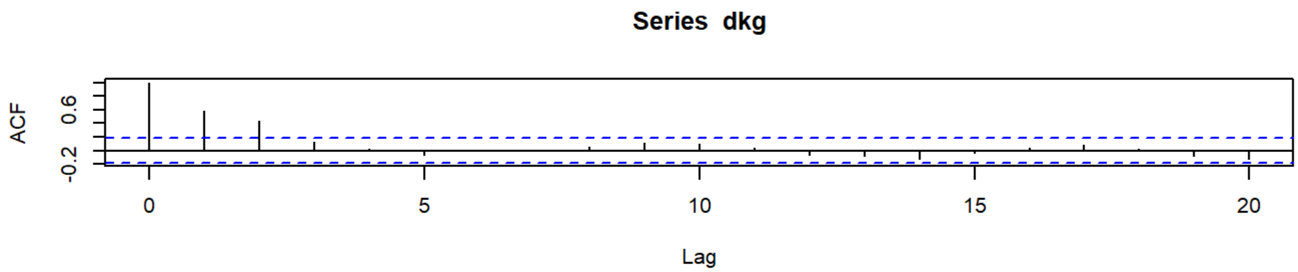


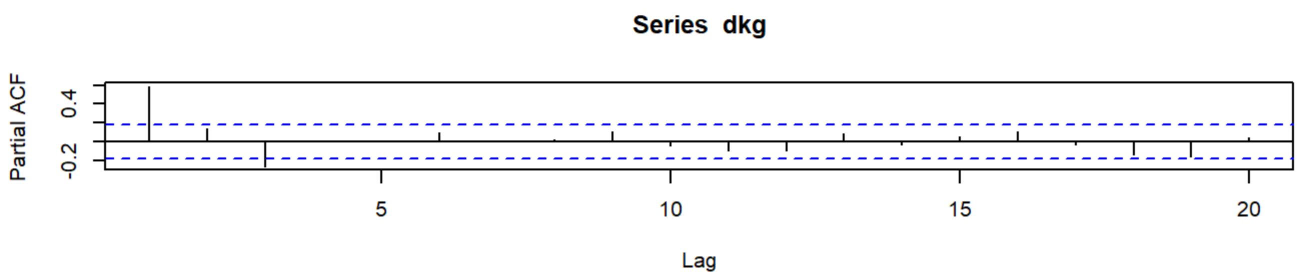




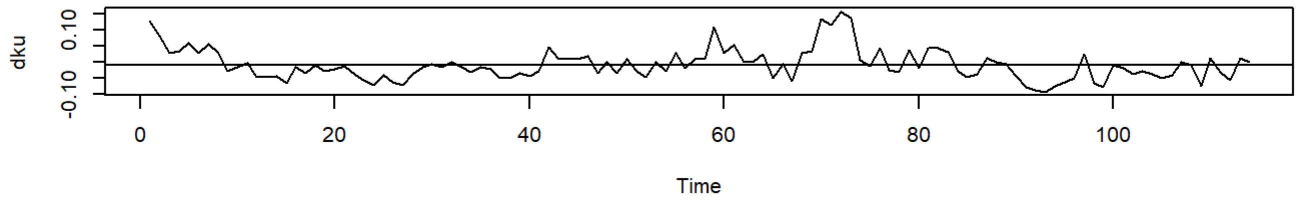

```
dkg<-diff(kg1)
{par(mfrow=c(3,1))
  {ts.plot(dkg)
    abline(h=mean(dkg))
}
  acf(dkg)
  pacf(dkg)}
```

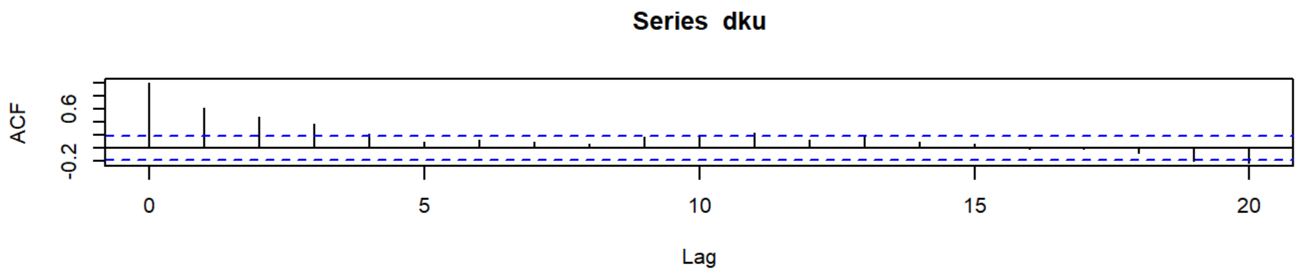


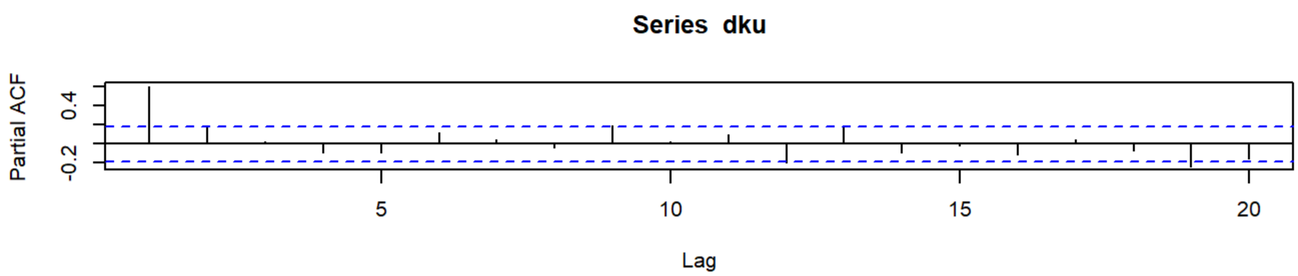





```
dku<-diff(ku1)
{par(mfrow=c(3,1))
  {ts.plot(dku)
    abline(h=mean(dku))
}
  acf(dku)
  pacf(dku)}
```







ADF Test

```
adf.test(dkc)
```

```
## Warning in adf.test(dkc): p-value smaller than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  dkc
## Dickey-Fuller = -4.1775, Lag order = 4, p-value = 0.01
## alternative hypothesis: stationary
```

```
adf.test(dkg)
```

```
## Warning in adf.test(dkg): p-value smaller than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  dkg
## Dickey-Fuller = -4.4138, Lag order = 4, p-value = 0.01
## alternative hypothesis: stationary
```

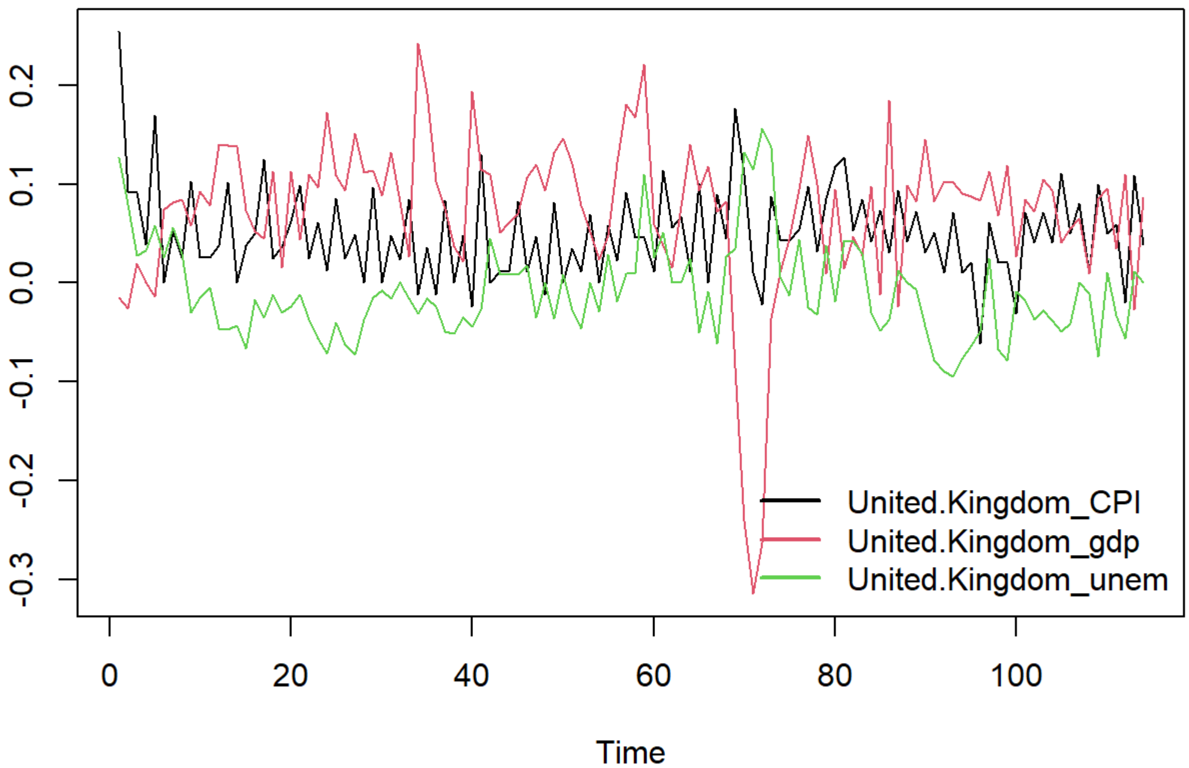
```
adf.test(dku)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  dku
## Dickey-Fuller = -3.9256, Lag order = 4, p-value = 0.01534
## alternative hypothesis: stationary
```

> 經由一次差分消除local trend，並利用ADF Test測試stationality. ADF Test的p-value為0.01，0.01，0.015，說明這三個序列皆為stationary.

Plots

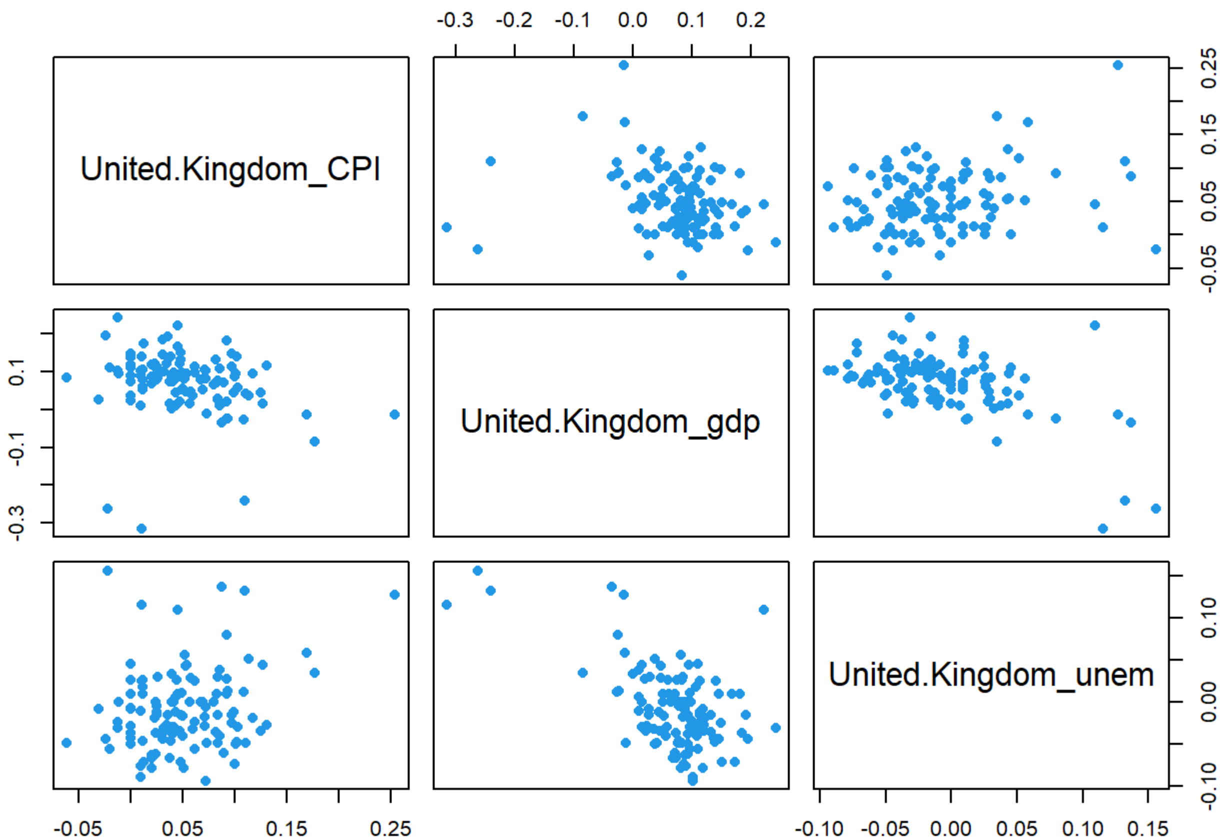
```
uk=cbind(dkc,dkg,dku)
colnames(uk)=c('United.Kingdom_CPI','United.Kingdom_gdp','United.Kingdom_unem')
ts.plot(uk, col=1:3)
legend("bottomright", legend=colnames(uk), col=1:3, lty=1, lwd=2, bty="n")
```



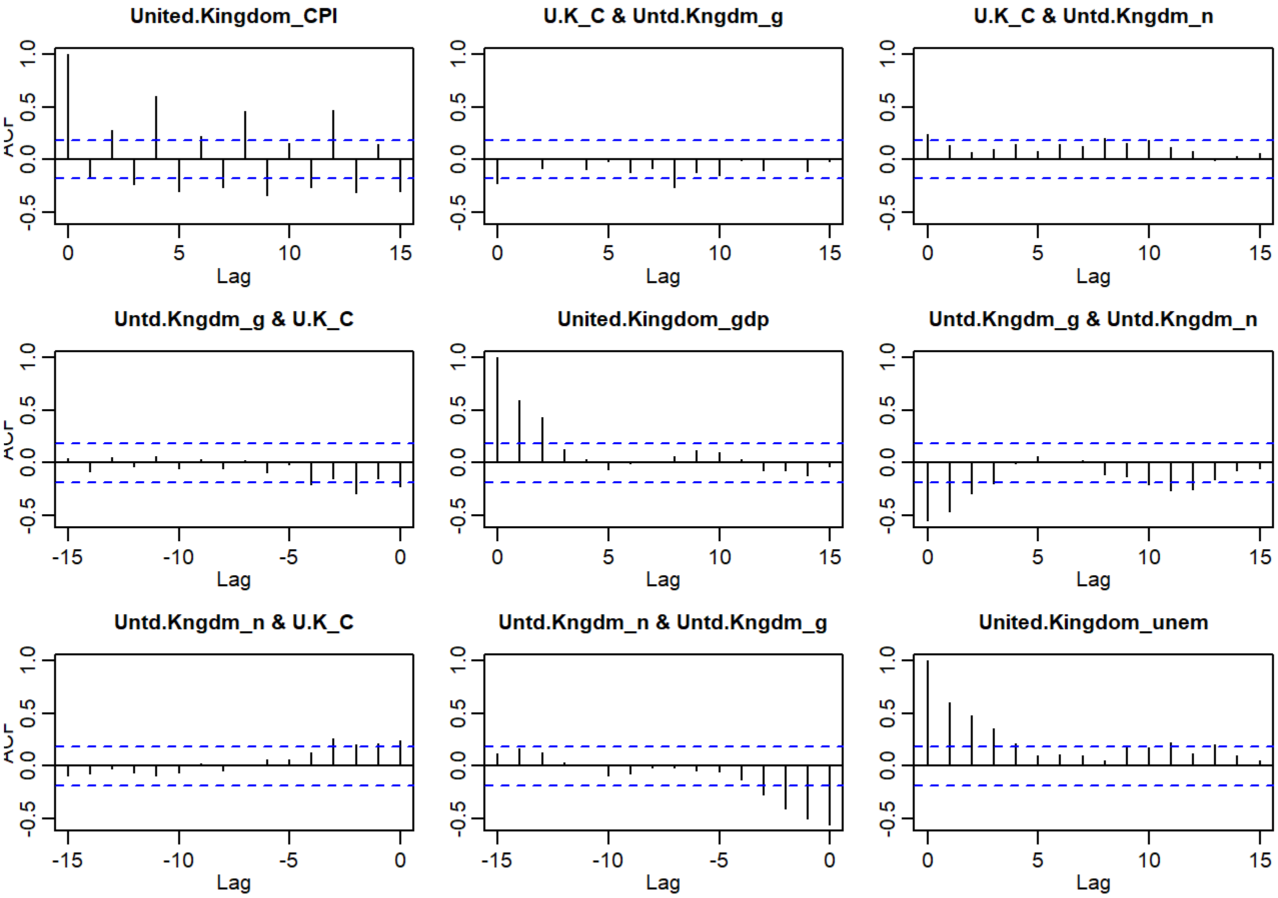
> 此圖為經過差分的序列圖。

ACF plots

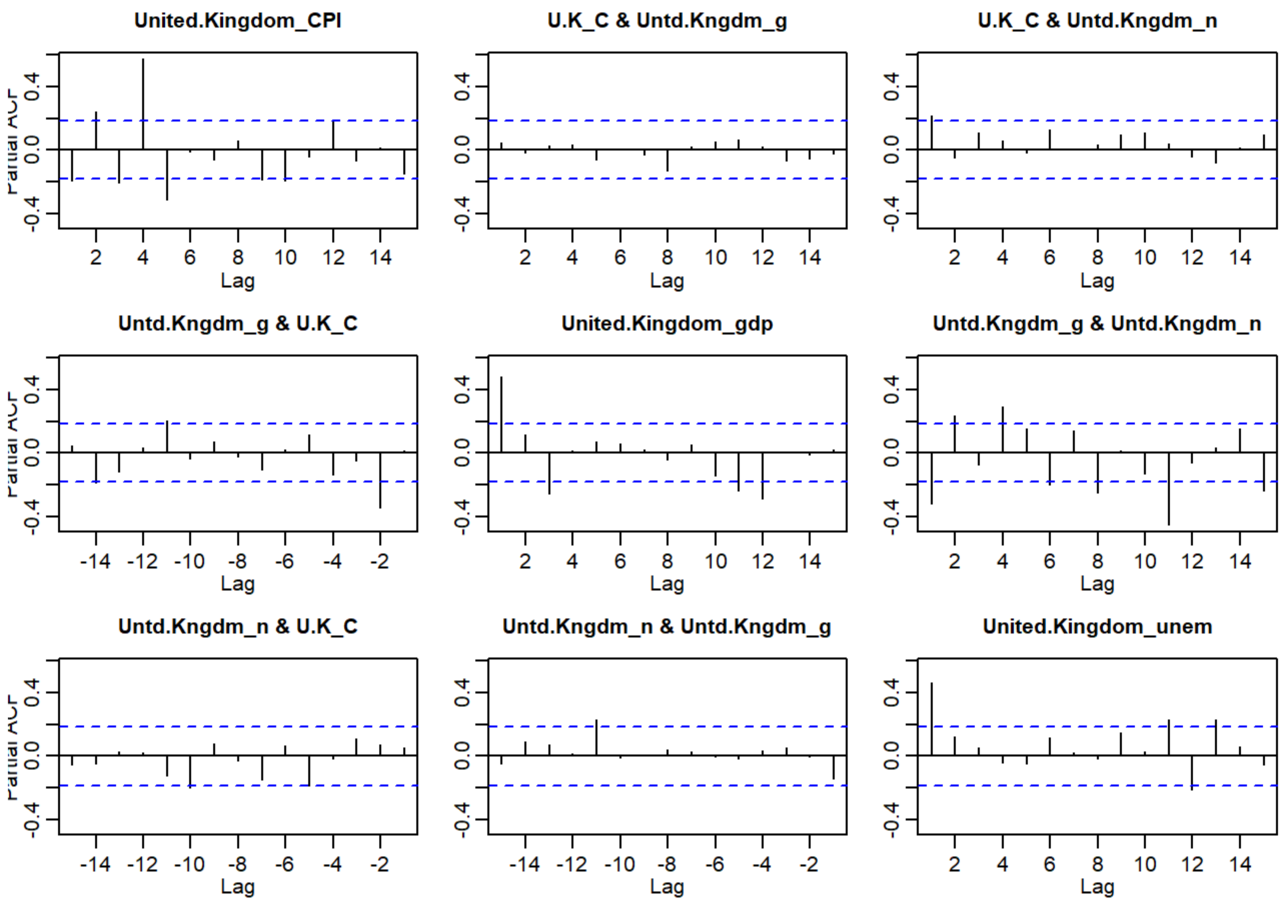
```
pairs(uk, col=4, pch=16)
```




```
par(mfrow=c(1,1))
acf(uk) #sample ccf
```


United.Kingdom_CPI
U.K_C & Untd.Kngdm_g
U.K_C & Untd.Kngdm_n
Untd.Kngdm_g & U.K_C
United.Kingdom_gdp
Untd.Kngdm_g & Untd.Kngdm_n
Untd.Kngdm_n & U.K_C
Untd.Kngdm_n & Untd.Kngdm_g
United.Kingdom_unem
Lag
ACF

```
pacf(uk)
```



> 繪製出這三筆data的scatter plot 和 ccf。

Model fitting

order selection

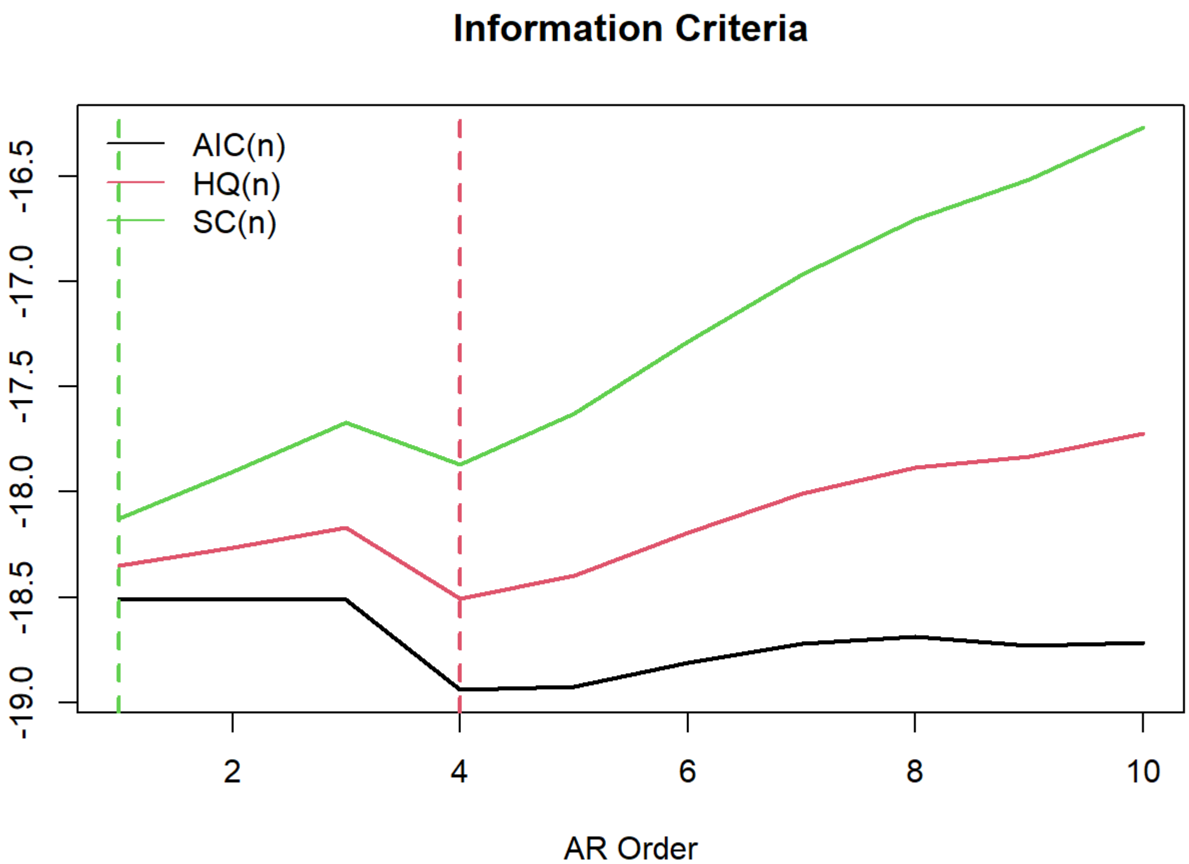
```
fit = VARselect(uk, lag.max=10, type="both")
fit
```

```
## $selection
## AIC(n)  HQ(n)  SC(n) FPE(n)
##      4      4      1      4
##
## $criteria
##                     1             2             3             4             5
## AIC(n) -1.850576e+01 -1.851196e+01 -1.851010e+01 -1.893852e+01 -1.892371e+01
## HQ(n)  -1.835124e+01 -1.826473e+01 -1.817016e+01 -1.850587e+01 -1.839835e+01
## SC(n)  -1.812435e+01 -1.790171e+01 -1.767101e+01 -1.787059e+01 -1.762694e+01
## FPE(n)  9.186478e-09  9.135994e-09  9.166462e-09  5.987548e-09  6.100908e-09
##                     6             7             8             9            10
## AIC(n) -1.881057e+01 -1.871960e+01 -1.868730e+01 -1.872649e+01 -1.871254e+01
## HQ(n)  -1.819250e+01 -1.800882e+01 -1.788381e+01 -1.783029e+01 -1.772363e+01
## SC(n)  -1.728496e+01 -1.696515e+01 -1.670401e+01 -1.651436e+01 -1.627156e+01
## FPE(n)  6.870573e-09  7.583329e-09  7.912502e-09  7.708181e-09  7.944977e-09
```

```
names(fit)
```

```
## [1] "selection" "criteria"
```

```
par(mfcol=c(1,1))
ts.plot(t(fit$crit[1:3,]), col=1:3, lwd=2, xlab="AR Order")
abline(v=fit$sel[1:3],lty=2,col=1:3,lwd=2)
legend("topleft",legend=rownames(fit$crit[1:3,]),col=1:3,lty=1, bty="n")
title("Information Criteria")
```



利用VARselect選擇出最適合的order。根據BIC，應該選擇order=1，根據AIC、FPC、HQ，應選擇order=4。因此以下為兩個模型的fitting。

Order=1

```
fit1 = VAR(uk, p=1, type="both",season=4)
summary(fit1)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: United.Kingdom_CPI, United.Kingdom_gdp, United.Kingdom_unem
## Deterministic variables: both
## Sample size: 113
## Log Likelihood: 638.761
## Roots of the characteristic polynomial:
## 0.6822 0.3002 0.3002
## Call:
## VAR(y = uk, p = 1, type = "both", season = 4L)
##
##
## Estimation results for equation United.Kingdom_CPI:
## ===================================================
## United.Kingdom_CPI = United.Kingdom_CPI.l1 + United.Kingdom_gdp.l1 + United.Kingdom_unem.l1 +
const + trend + sd1 + sd2 + sd3
##
##                         Estimate Std. Error t value Pr(>|t|)
## United.Kingdom_CPI.l1   3.616e-01  8.270e-02   4.373 2.90e-05 ***
## United.Kingdom_gdp.l1   3.311e-02  3.909e-02   0.847 0.398846
## United.Kingdom_unem.l1  9.361e-02  6.547e-02   1.430 0.155767
## const                   2.681e-02  7.821e-03   3.428 0.000869 ***
## trend                   4.605e-05  7.904e-05   0.583 0.561460
## sd1                     9.327e-02  7.649e-03  12.193  < 2e-16 ***
## sd2                    -9.341e-03  8.224e-03  -1.136 0.258608
## sd3                     4.273e-02  7.410e-03   5.766 8.24e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.02654 on 105 degrees of freedom
## Multiple R-Squared: 0.637,   Adjusted R-squared: 0.6128
## F-statistic: 26.32 on 7 and 105 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation United.Kingdom_gdp:
## ===================================================
## United.Kingdom_gdp = United.Kingdom_CPI.l1 + United.Kingdom_gdp.l1 + United.Kingdom_unem.l1 +
const + trend + sd1 + sd2 + sd3
##
##                         Estimate Std. Error t value Pr(>|t|)
## United.Kingdom_CPI.l1  -0.4063769  0.1915779  -2.121 0.036258 *
## United.Kingdom_gdp.l1   0.4676006  0.0905510   5.164 1.15e-06 ***
## United.Kingdom_unem.l1 -0.2722675  0.1516749  -1.795 0.075519 .
## const                   0.0717401  0.0181165   3.960 0.000137 ***
## trend                  -0.0002512  0.0001831  -1.372 0.173026
## sd1                    -0.0367210  0.0177199  -2.072 0.040687 *
## sd2                     0.0256654  0.0190515   1.347 0.180829
## sd3                    -0.0331039  0.0171664  -1.928 0.056504 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
##
## Residual standard error: 0.06148 on 105 degrees of freedom
## Multiple R-Squared: 0.4357,  Adjusted R-squared: 0.3981
## F-statistic: 11.58 on 7 and 105 DF,  p-value: 7.791e-11
##
##
## Estimation results for equation United.Kingdom_unem:
## ====================================================
## United.Kingdom_unem = United.Kingdom_CPI.l1 + United.Kingdom_gdp.l1 + United.Kingdom_unem.l1
+ const + trend + sd1 + sd2 + sd3
##
##                          Estimate Std. Error t value Pr(>|t|)
## United.Kingdom_CPI.l1   1.577e-01  1.111e-01   1.419  0.15900
## United.Kingdom_gdp.l1  -1.502e-01  5.253e-02  -2.860  0.00512 **
## United.Kingdom_unem.l1  4.321e-01  8.799e-02   4.911 3.35e-06 ***
## const                   1.005e-04  1.051e-02   0.010  0.99239
## trend                  -5.792e-05  1.062e-04  -0.545  0.58672
## sd1                     7.261e-03  1.028e-02   0.706  0.48154
## sd2                    -9.314e-03  1.105e-02  -0.843  0.40134
## sd3                     5.568e-03  9.959e-03   0.559  0.57732
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.03567 on 105 degrees of freedom
## Multiple R-Squared: 0.4526,  Adjusted R-squared: 0.4161
## F-statistic:  12.4 on 7 and 105 DF,  p-value: 1.732e-11
##
##
##
## Covariance matrix of residuals:
##                     United.Kingdom_CPI United.Kingdom_gdp United.Kingdom_unem
## United.Kingdom_CPI           7.044e-04         -0.0001358           8.282e-05
## United.Kingdom_gdp          -1.358e-04          0.0037801          -6.237e-04
## United.Kingdom_unem          8.282e-05         -0.0006237           1.272e-03
##
## Correlation matrix of residuals:
##                     United.Kingdom_CPI United.Kingdom_gdp United.Kingdom_unem
## United.Kingdom_CPI             1.00000            -0.0832             0.08749
## United.Kingdom_gdp            -0.08320             1.0000            -0.28439
## United.Kingdom_unem            0.08749            -0.2844             1.00000
```

Serial Test

```
serial.test(fit1, lags.pt = 8, type = "PT.asymptotic")
```

```
##
##  Portmanteau Test (asymptotic)
##
## data:  Residuals of VAR object fit1
## Chi-squared = 69.418, df = 63, p-value = 0.2702
```

> 由於serial test的結果不顯著，可知無法拒絕服從white noise process的假設。

Coefficients

```
Acoef(fit1)  #estimated AR coeff matrix
```

```
## [[1]]
##                      United.Kingdom_CPI.l1 United.Kingdom_gdp.l1
## United.Kingdom_CPI              0.3616111            0.03311371
## United.Kingdom_gdp             -0.4063769            0.46760055
## United.Kingdom_unem             0.1576601           -0.15022029
##                      United.Kingdom_unem.l1
## United.Kingdom_CPI              0.09361068
## United.Kingdom_gdp             -0.27226749
## United.Kingdom_unem             0.43211423
```

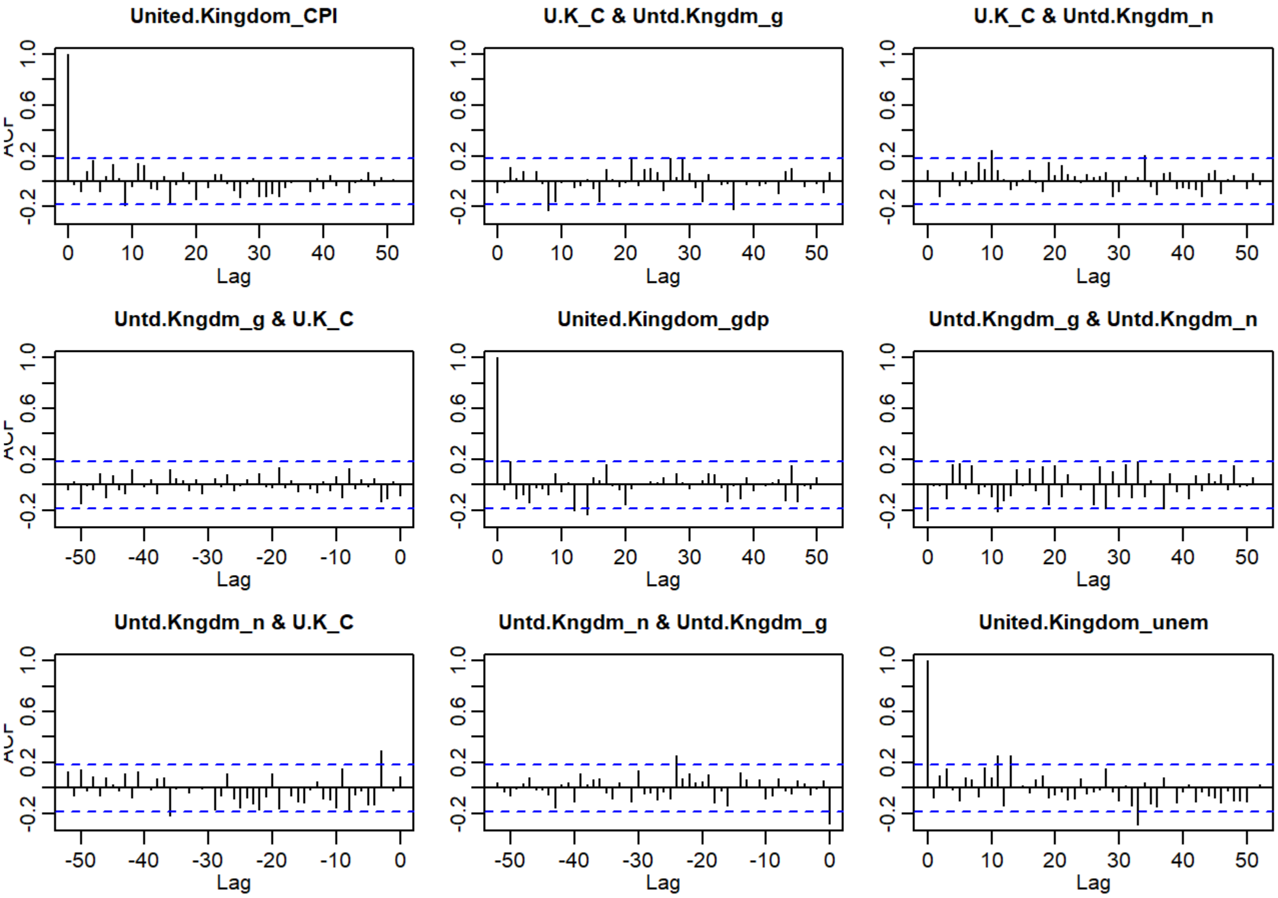
```
round(Bcoef(fit1),5)  #all estimated coeff
```

```
##                      United.Kingdom_CPI.l1 United.Kingdom_gdp.l1
## United.Kingdom_CPI                0.36161               0.03311
## United.Kingdom_gdp               -0.40638               0.46760
## United.Kingdom_unem               0.15766              -0.15022
##                      United.Kingdom_unem.l1   const    trend      sd1      sd2
## United.Kingdom_CPI                  0.09361 0.02681  0.00005  0.09327 -0.00934
## United.Kingdom_gdp                 -0.27227 0.07174 -0.00025 -0.03672  0.02567
## United.Kingdom_unem                 0.43211 0.00010 -0.00006  0.00726 -0.00931
##                           sd3
## United.Kingdom_CPI    0.04273
## United.Kingdom_gdp   -0.03310
## United.Kingdom_unem   0.00557
```

> 上述結果即為參數估計的結果。

Residuals

```
fit1$resid = resid(fit1)
acf(fit1$resid,52)
```

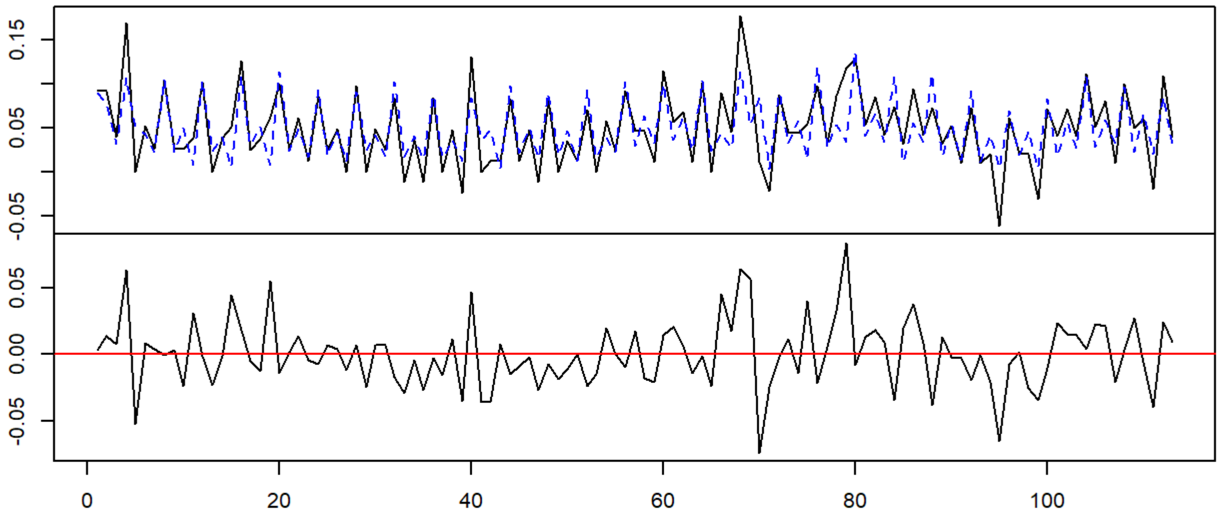


> 由上圖可知，residuals 是 white noise process.

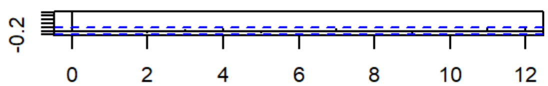
Model plots

```
plot(fit1) #plot fitted values and residuals w/ ACF and PACF
```

Diagram of fit and residuals for United.Kingdom_CPI



ACF Residuals



PACF Residuals

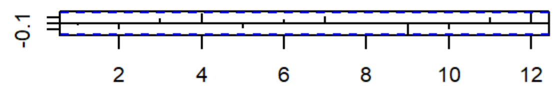
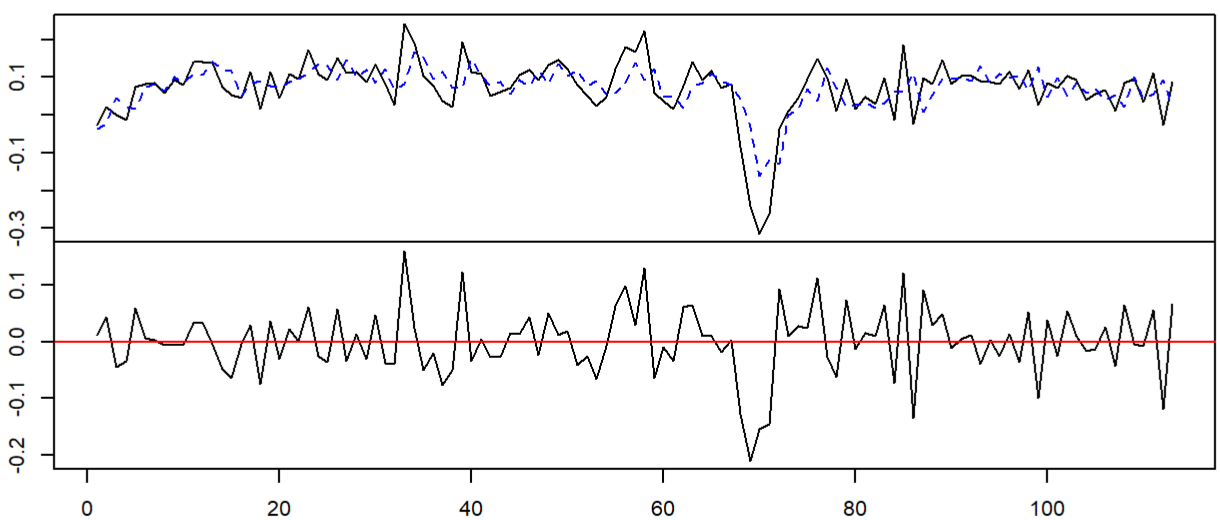
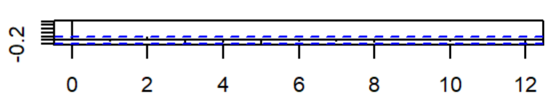


Diagram of fit and residuals for United.Kingdom_gdp



ACF Residuals



PACF Residuals

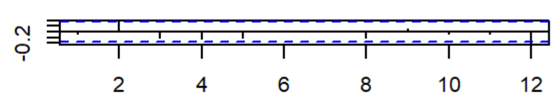
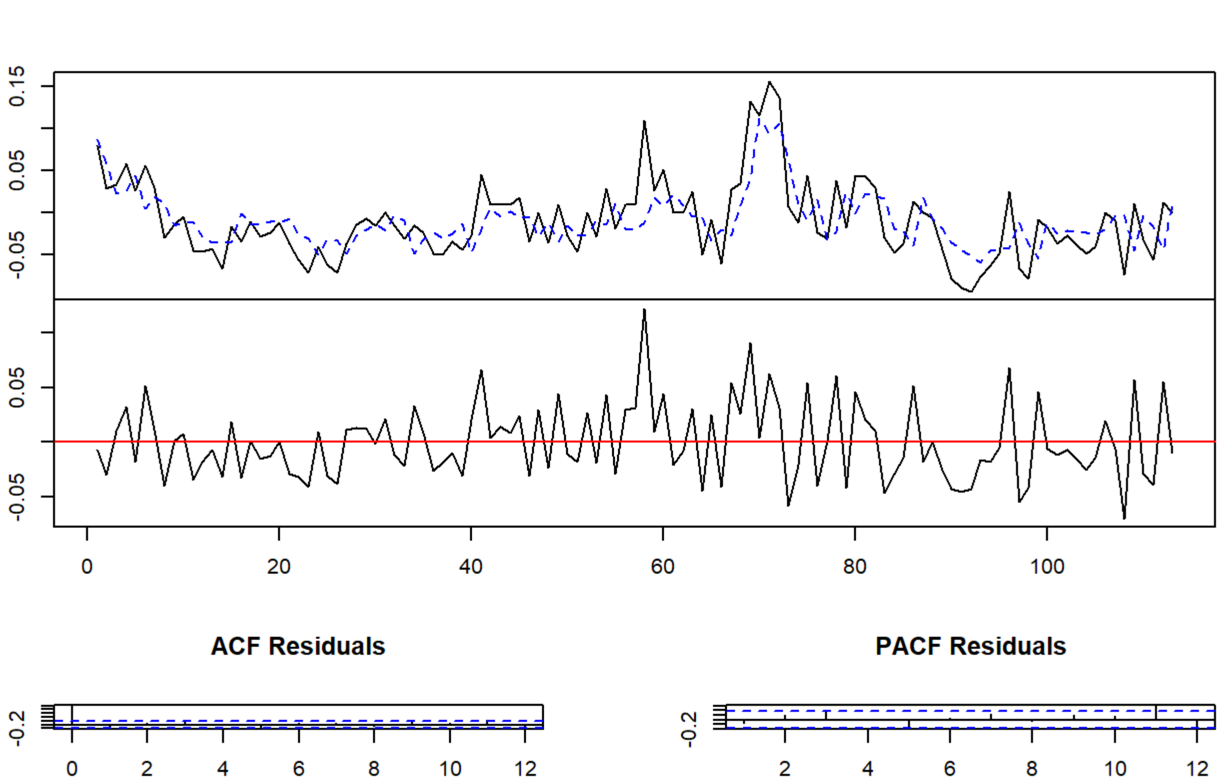


Diagram of fit and residuals for United.Kingdom_unem



> 結果為正相關。

Causality

```
causality(fit1, cause= "United.Kingdom_CPI")
```

```
## $Granger
##
##  Granger causality H0: United.Kingdom_CPI do not Granger-cause
##  United.Kingdom_gdp United.Kingdom_unem
##
## data:  VAR object fit1
## F-Test = 2.6113, df1 = 2, df2 = 315, p-value = 0.07502
##
##
## $Instant
##
##  H0: No instantaneous causality between: United.Kingdom_CPI and
##  United.Kingdom_gdp United.Kingdom_unem
##
## data:  VAR object fit1
## Chi-squared = 1.2687, df = 2, p-value = 0.5303
```

```
causality(fit1, cause= "United.Kingdom_gdp")
```

```
## $Granger
##
##  Granger causality H0: United.Kingdom_gdp do not Granger-cause
##  United.Kingdom_CPI United.Kingdom_unem
##
## data:  VAR object fit1
## F-Test = 4.6951, df1 = 2, df2 = 315, p-value = 0.009789
##
##
## $Instant
##
##  H0: No instantaneous causality between: United.Kingdom_gdp and
##  United.Kingdom_CPI United.Kingdom_unem
##
## data:  VAR object fit1
## Chi-squared = 8.7856, df = 2, p-value = 0.01237
```

```
causality(fit1, cause= "United.Kingdom_unem")
```

```
## $Granger
##
##  Granger causality H0: United.Kingdom_unem do not Granger-cause
##  United.Kingdom_CPI United.Kingdom_gdp
##
## data:  VAR object fit1
## F-Test = 2.4365, df1 = 2, df2 = 315, p-value = 0.08911
##
##
## $Instant
##
##  H0: No instantaneous causality between: United.Kingdom_unem and
##  United.Kingdom_CPI United.Kingdom_gdp
##
## data:  VAR object fit1
## Chi-squared = 8.8503, df = 2, p-value = 0.01197
```

> 檢查序列間是否存在granger causality 和 instantaneous causality。舉例而言，觀察United.Kingdom_CPI、United.Kingdom_gdp、United.Kingdom_unem分別對其他兩者的因果關係。

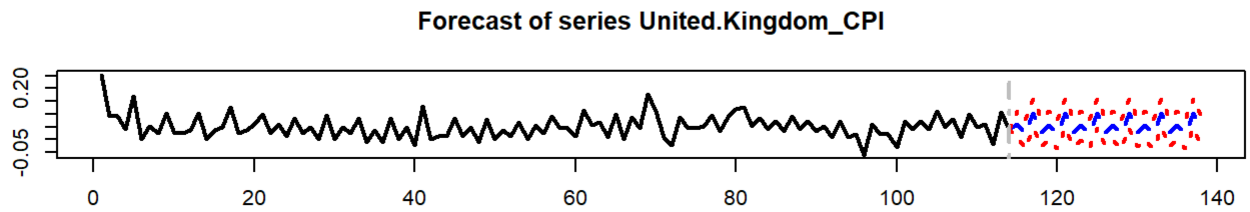
- 由第一個causality的結果可知不能拒絕H0，United.Kingdom_CPI 對於United.Kingdom_gdp和United.Kingdom_unem無顯著相關性，然而他們有當期的相關性。
- 由第二個causality的結果可知可以拒絕H0，United.Kingdom_gdp 對於United.Kingdom_CPI和United.Kingdom_unem有相關性，並且同時有當期的相關性。

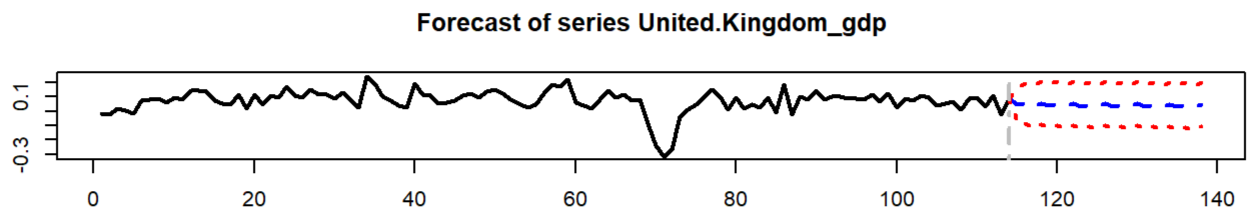
- 由第一個causality的結果可知不能拒絕H0，United.Kingdom_unem 對於United.Kingdom_gdp和United.Kingdom_CPI無顯著相關性，然而他們有當期的相關性。

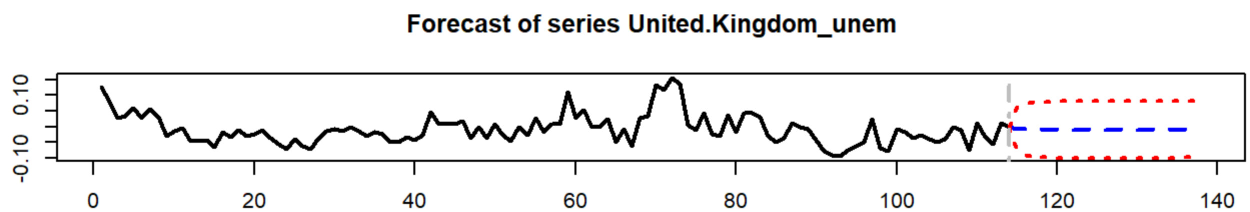
> 就結果可知，United.Kingdom_CPI及United.Kingdom_unem對其餘時間序列均無granger causality，顯示兩序列的過去值對其餘序列無顯著解釋能力。然而，United.Kingdom_gdp則對其餘序列有顯著解釋能力，即United.Kingdom_gdp這筆序列對其餘序列具有預測能力。

Prediction

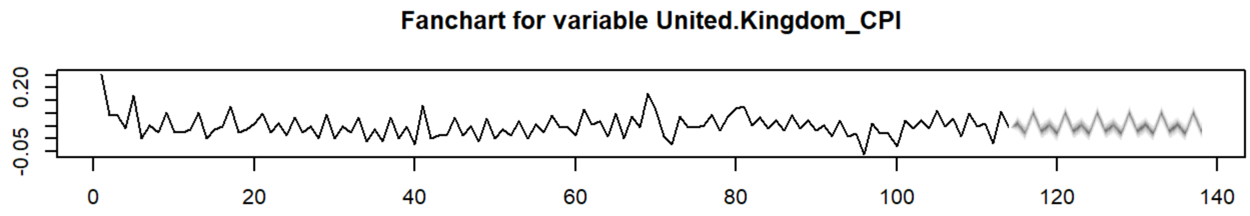
```
fit1$pred = predict(fit1, n.ahead = 24, ci = 0.95)
plot(fit1$pred, lwd=2)
```

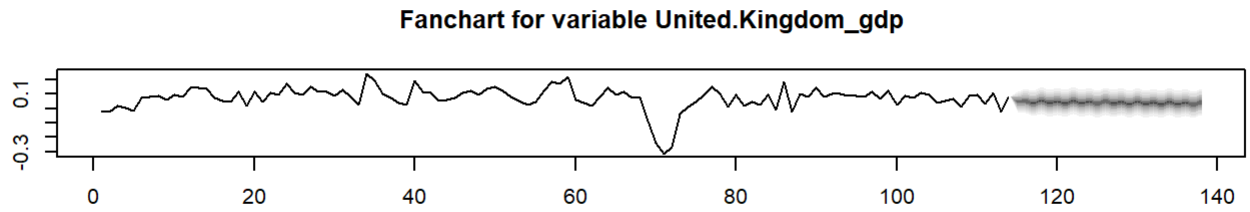


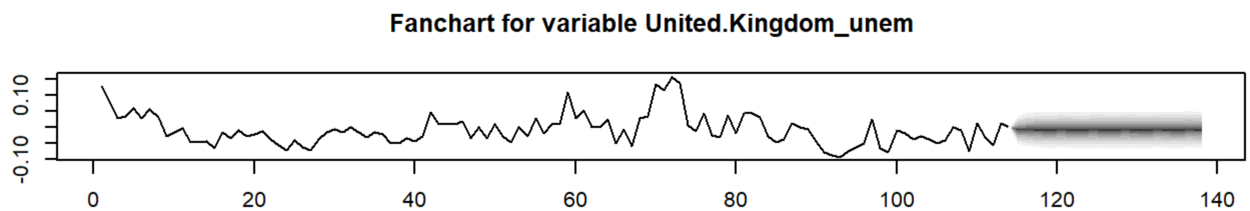





```
fanchart(fit1$pred)
```







Order=4

```
fit4 = VAR(uk, p=4, type="both",season=4)
summary(fit4)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: United.Kingdom_CPI, United.Kingdom_gdp, United.Kingdom_unem
## Deterministic variables: both
## Sample size: 110
## Log Likelihood: 642.203
## Roots of the characteristic polynomial:
## 0.8258 0.7649 0.7649 0.6679 0.6679 0.6566 0.6566 0.6336 0.5721 0.5721 0.5224 0.5224
## Call:
## VAR(y = uk, p = 4, type = "both", season = 4L)
##
##
## Estimation results for equation United.Kingdom_CPI:
## ===================================================
## United.Kingdom_CPI = United.Kingdom_CPI.l1 + United.Kingdom_gdp.l1 + United.Kingdom_unem.l1 +
United.Kingdom_CPI.l2 + United.Kingdom_gdp.l2 + United.Kingdom_unem.l2 + United.Kingdom_CPI.l3 +
United.Kingdom_gdp.l3 + United.Kingdom_unem.l3 + United.Kingdom_CPI.l4 + United.Kingdom_gdp.l4 +
United.Kingdom_unem.l4 + const + trend + sd1 + sd2 + sd3
##
##                          Estimate Std. Error t value Pr(>|t|)
## United.Kingdom_CPI.l1   2.961e-01  1.014e-01   2.921  0.00438 **
## United.Kingdom_gdp.l1   4.044e-02  4.683e-02   0.864  0.38999
## United.Kingdom_unem.l1  4.570e-02  7.854e-02   0.582  0.56204
## United.Kingdom_CPI.l2  -4.422e-02  1.049e-01  -0.421  0.67440
## United.Kingdom_gdp.l2   2.866e-02  5.020e-02   0.571  0.56941
## United.Kingdom_unem.l2 -6.384e-02  8.250e-02  -0.774  0.44099
## United.Kingdom_CPI.l3   1.344e-01  1.072e-01   1.253  0.21323
## United.Kingdom_gdp.l3  -1.021e-02  5.042e-02  -0.203  0.83992
## United.Kingdom_unem.l3  6.108e-02  8.347e-02   0.732  0.46619
## United.Kingdom_CPI.l4   1.660e-01  9.628e-02   1.724  0.08797 .
## United.Kingdom_gdp.l4   4.175e-02  4.683e-02   0.892  0.37493
## United.Kingdom_unem.l4  9.138e-02  7.552e-02   1.210  0.22933
## const                   8.287e-03  1.120e-02   0.740  0.46105
## trend                   1.077e-04  8.511e-05   1.266  0.20871
## sd1                     8.924e-02  1.657e-02   5.384 5.43e-07 ***
## sd2                    -1.165e-03  1.002e-02  -0.116  0.90762
## sd3                     5.040e-02  1.675e-02   3.009  0.00337 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.02651 on 93 degrees of freedom
## Multiple R-Squared: 0.6729,  Adjusted R-squared: 0.6166
## F-statistic: 11.96 on 16 and 93 DF,  p-value: 3.004e-16
##
##
## Estimation results for equation United.Kingdom_gdp:
## ===================================================
## United.Kingdom_gdp = United.Kingdom_CPI.l1 + United.Kingdom_gdp.l1 + United.Kingdom_unem.l1 +
United.Kingdom_CPI.l2 + United.Kingdom_gdp.l2 + United.Kingdom_unem.l2 + United.Kingdom_CPI.l3 +
United.Kingdom_gdp.l3 + United.Kingdom_unem.l3 + United.Kingdom_CPI.l4 + United.Kingdom_gdp.l4 +
United.Kingdom_unem.l4 + const + trend + sd1 + sd2 + sd3
```

```
##
##                         Estimate Std. Error t value Pr(>|t|)
## United.Kingdom_CPI.l1  -0.4245142  0.2303546  -1.843 0.068533 .
## United.Kingdom_gdp.l1   0.4148563  0.1064111   3.899 0.000182 ***
## United.Kingdom_unem.l1 -0.3455959  0.1784714  -1.936 0.055852 .
## United.Kingdom_CPI.l2  -0.2325828  0.2384102  -0.976 0.331815
## United.Kingdom_gdp.l2   0.1857235  0.1140757   1.628 0.106893
## United.Kingdom_unem.l2  0.0390147  0.1874675   0.208 0.835595
## United.Kingdom_CPI.l3  -0.1747921  0.2437072  -0.717 0.475035
## United.Kingdom_gdp.l3  -0.2135097  0.1145791  -1.863 0.065558 .
## United.Kingdom_unem.l3 -0.1701933  0.1896794  -0.897 0.371893
## United.Kingdom_CPI.l4   0.0932001  0.2187936   0.426 0.671112
## United.Kingdom_gdp.l4  -0.0138669  0.1064215  -0.130 0.896609
## United.Kingdom_unem.l4  0.2927588  0.1716066   1.706 0.091348 .
## const                   0.0936216  0.0254403   3.680 0.000391 ***
## trend                  -0.0002309  0.0001934  -1.194 0.235639
## sd1                    -0.0458946  0.0376604  -1.219 0.226061
## sd2                     0.0145639  0.0227604   0.640 0.523824
## sd3                    -0.0292472  0.0380614  -0.768 0.444183
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.06024 on 93 degrees of freedom
## Multiple R-Squared: 0.5076,  Adjusted R-squared: 0.4228
## F-statistic: 5.991 on 16 and 93 DF,  p-value: 8.25e-09
##
##
## Estimation results for equation United.Kingdom_unem:
## =====================================================
## United.Kingdom_unem = United.Kingdom_CPI.l1 + United.Kingdom_gdp.l1 + United.Kingdom_unem.l1
+ United.Kingdom_CPI.l2 + United.Kingdom_gdp.l2 + United.Kingdom_unem.l2 + United.Kingdom_CPI.l3
+ United.Kingdom_gdp.l3 + United.Kingdom_unem.l3 + United.Kingdom_CPI.l4 + United.Kingdom_gdp.l4
+ United.Kingdom_unem.l4 + const + trend + sd1 + sd2 + sd3
##
##                         Estimate Std. Error t value Pr(>|t|)
## United.Kingdom_CPI.l1   2.235e-01  1.332e-01   1.678 0.096651 .
## United.Kingdom_gdp.l1  -9.973e-02  6.152e-02  -1.621 0.108348
## United.Kingdom_unem.l1  3.864e-01  1.032e-01   3.745 0.000313 ***
## United.Kingdom_CPI.l2  -2.449e-03  1.378e-01  -0.018 0.985859
## United.Kingdom_gdp.l2  -1.902e-02  6.595e-02  -0.288 0.773622
## United.Kingdom_unem.l2  1.454e-01  1.084e-01   1.341 0.183057
## United.Kingdom_CPI.l3   3.996e-01  1.409e-01   2.837 0.005596 **
## United.Kingdom_gdp.l3  -2.699e-03  6.624e-02  -0.041 0.967589
## United.Kingdom_unem.l3  1.079e-01  1.097e-01   0.984 0.327706
## United.Kingdom_CPI.l4  -2.966e-01  1.265e-01  -2.345 0.021159 *
## United.Kingdom_gdp.l4   4.451e-02  6.152e-02   0.723 0.471237
## United.Kingdom_unem.l4 -8.516e-02  9.921e-02  -0.858 0.392851
## const                  -1.088e-02  1.471e-02  -0.740 0.461457
## trend                  -6.583e-05  1.118e-04  -0.589 0.557470
## sd1                     6.304e-02  2.177e-02   2.896 0.004716 **
## sd2                     8.642e-03  1.316e-02   0.657 0.512918
## sd3                     4.920e-02  2.200e-02   2.236 0.027738 *
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.03483 on 93 degrees of freedom
## Multiple R-Squared: 0.5148,  Adjusted R-squared: 0.4313
## F-statistic: 6.167 on 16 and 93 DF,  p-value: 4.547e-09
##
##
##
## Covariance matrix of residuals:
##                     United.Kingdom_CPI United.Kingdom_gdp United.Kingdom_unem
## United.Kingdom_CPI           0.0007028         -0.0002160           0.0001043
## United.Kingdom_gdp          -0.0002160          0.0036293          -0.0005354
## United.Kingdom_unem          0.0001043         -0.0005354           0.0012129
##
## Correlation matrix of residuals:
##                     United.Kingdom_CPI United.Kingdom_gdp United.Kingdom_unem
## United.Kingdom_CPI              1.0000            -0.1353              0.1130
## United.Kingdom_gdp             -0.1353             1.0000             -0.2552
## United.Kingdom_unem             0.1130            -0.2552              1.0000
```

Serial Test

```
serial.test(fit4, lags.pt = 8, type = "PT.asymptotic")
```

```
##
##  Portmanteau Test (asymptotic)
##
## data:  Residuals of VAR object fit4
## Chi-squared = 30.983, df = 36, p-value = 0.7059
```

> 由於serial test的結果不顯著，可知無法拒絕服從white noise process的假設。

Coefficients

```
Acoef(fit4)  #estimated AR coeff matrix
```

```
## [[1]]
##                      United.Kingdom_CPI.l1 United.Kingdom_gdp.l1
## United.Kingdom_CPI               0.2961087            0.04044376
## United.Kingdom_gdp              -0.4245142            0.41485629
## United.Kingdom_unem              0.2234930           -0.09973400
##                      United.Kingdom_unem.l1
## United.Kingdom_CPI               0.04570134
## United.Kingdom_gdp              -0.34559590
## United.Kingdom_unem              0.38636363
##
## [[2]]
##                      United.Kingdom_CPI.l2 United.Kingdom_gdp.l2
## United.Kingdom_CPI            -0.044216670            0.02866171
## United.Kingdom_gdp            -0.232582756            0.18572352
## United.Kingdom_unem           -0.002449422           -0.01902442
##                      United.Kingdom_unem.l2
## United.Kingdom_CPI              -0.06383873
## United.Kingdom_gdp               0.03901469
## United.Kingdom_unem              0.14537445
##
## [[3]]
##                      United.Kingdom_CPI.l3 United.Kingdom_gdp.l3
## United.Kingdom_CPI               0.1344138           -0.01021385
## United.Kingdom_gdp              -0.1747921           -0.21350973
## United.Kingdom_unem              0.3996444           -0.00269866
##                      United.Kingdom_unem.l3
## United.Kingdom_CPI               0.06107519
## United.Kingdom_gdp              -0.17019326
## United.Kingdom_unem              0.10789095
##
## [[4]]
##                      United.Kingdom_CPI.l4 United.Kingdom_gdp.l4
## United.Kingdom_CPI              0.16602575            0.04175325
## United.Kingdom_gdp              0.09320006           -0.01386687
## United.Kingdom_unem            -0.29659539            0.04450655
##                      United.Kingdom_unem.l4
## United.Kingdom_CPI               0.09137970
## United.Kingdom_gdp               0.29275881
## United.Kingdom_unem             -0.08516346
```

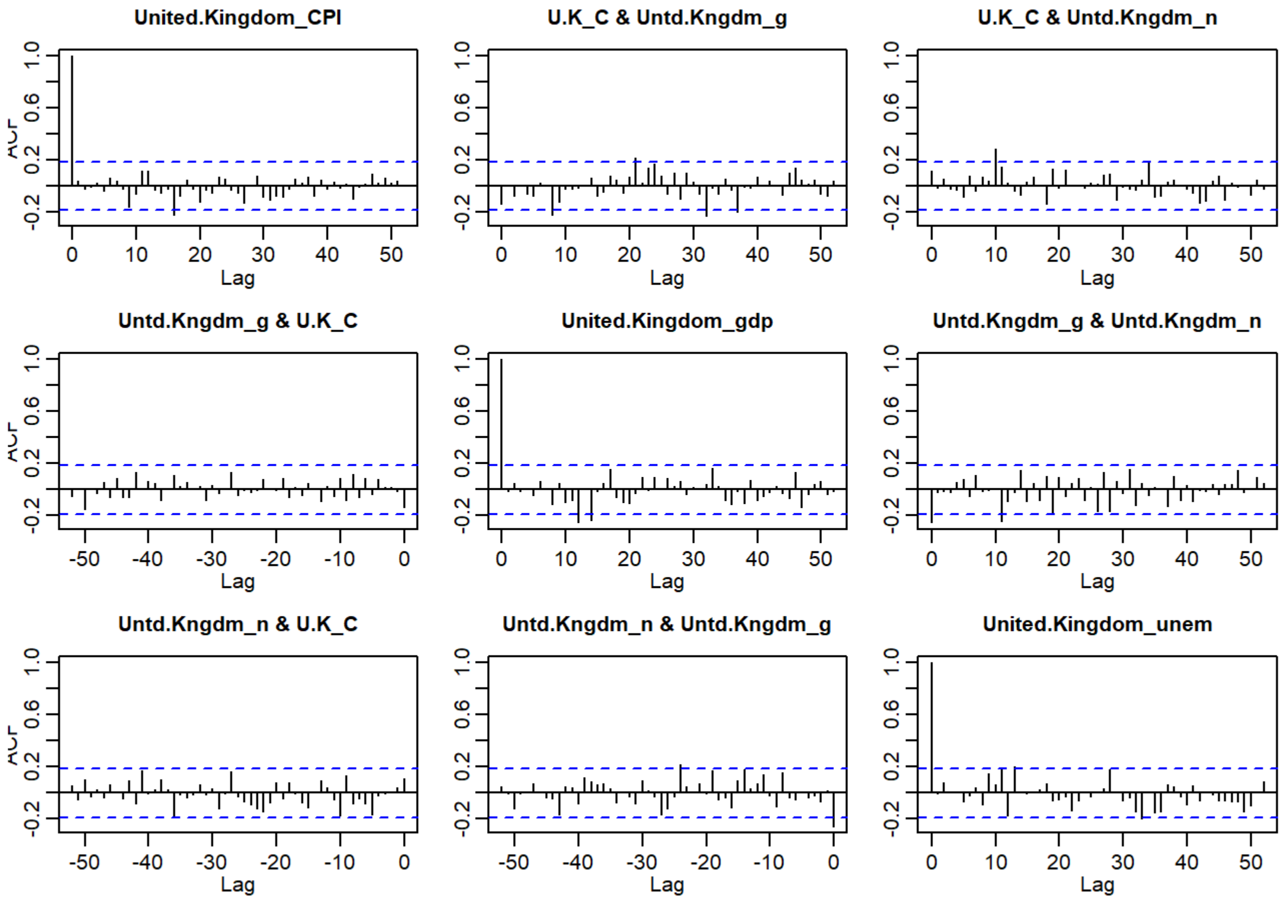
```
round(Bcoef(fit4),5)  #all estimated coeff
```

```
##                     United.Kingdom_CPI.l1 United.Kingdom_gdp.l1
## United.Kingdom_CPI                0.29611               0.04044
## United.Kingdom_gdp               -0.42451               0.41486
## United.Kingdom_unem               0.22349              -0.09973
##                     United.Kingdom_unem.l1 United.Kingdom_CPI.l2
## United.Kingdom_CPI                 0.04570              -0.04422
## United.Kingdom_gdp                -0.34560              -0.23258
## United.Kingdom_unem                0.38636              -0.00245
##                     United.Kingdom_gdp.l2 United.Kingdom_unem.l2
## United.Kingdom_CPI                0.02866               -0.06384
## United.Kingdom_gdp                0.18572                0.03901
## United.Kingdom_unem              -0.01902                0.14537
##                     United.Kingdom_CPI.l3 United.Kingdom_gdp.l3
## United.Kingdom_CPI                0.13441              -0.01021
## United.Kingdom_gdp               -0.17479              -0.21351
## United.Kingdom_unem               0.39964              -0.00270
##                     United.Kingdom_unem.l3 United.Kingdom_CPI.l4
## United.Kingdom_CPI                 0.06108               0.16603
## United.Kingdom_gdp                -0.17019               0.09320
## United.Kingdom_unem                0.10789              -0.29660
##                     United.Kingdom_gdp.l4 United.Kingdom_unem.l4    const
## United.Kingdom_CPI                0.04175                0.09138  0.00829
## United.Kingdom_gdp               -0.01387                0.29276  0.09362
## United.Kingdom_unem               0.04451               -0.08516 -0.01088
##                        trend      sd1      sd2      sd3
## United.Kingdom_CPI   0.00011  0.08924 -0.00117  0.05040
## United.Kingdom_gdp  -0.00023 -0.04589  0.01456 -0.02925
## United.Kingdom_unem -0.00007  0.06304  0.00864  0.04920
```

> 上述結果即為參數估計的結果。

Residuals

```
fit4$resid = resid(fit4)
acf(fit4$resid,52)
```

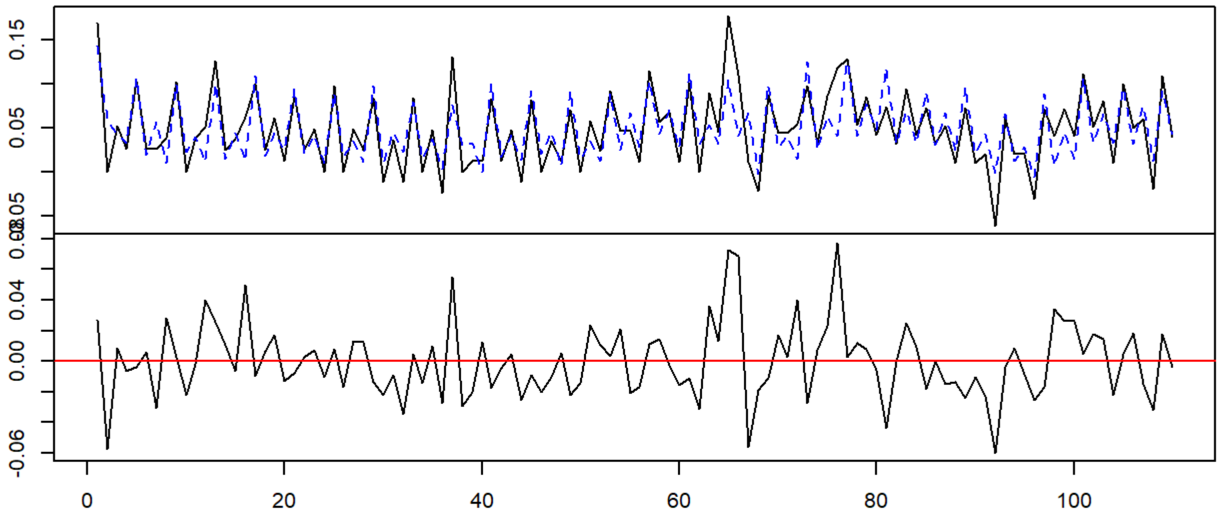


由上圖可知，residuals 是 white noise process.

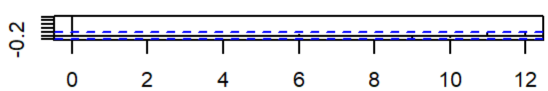
Model plots

```
plot(fit4) #plot fitted values and residuals w/ ACF and PACF
```

Diagram of fit and residuals for United.Kingdom_CPI



ACF Residuals



PACF Residuals

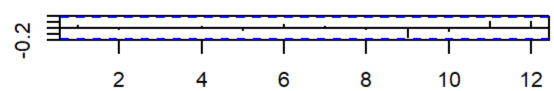
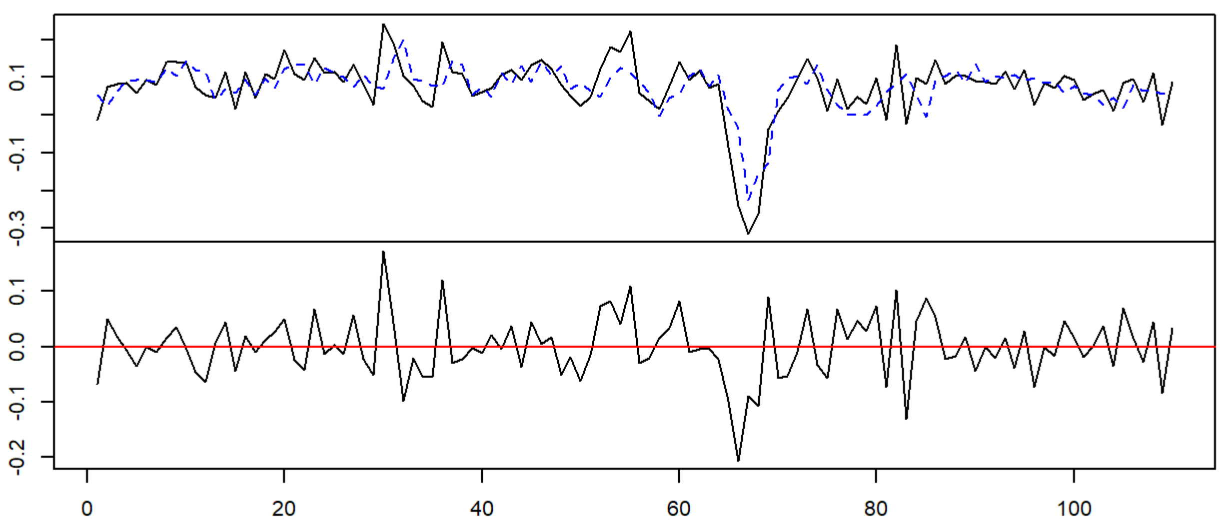
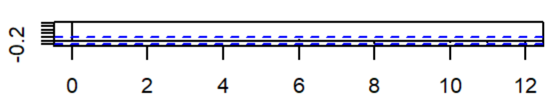


Diagram of fit and residuals for United.Kingdom_gdp



ACF Residuals



PACF Residuals

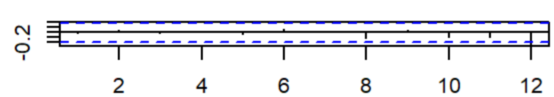
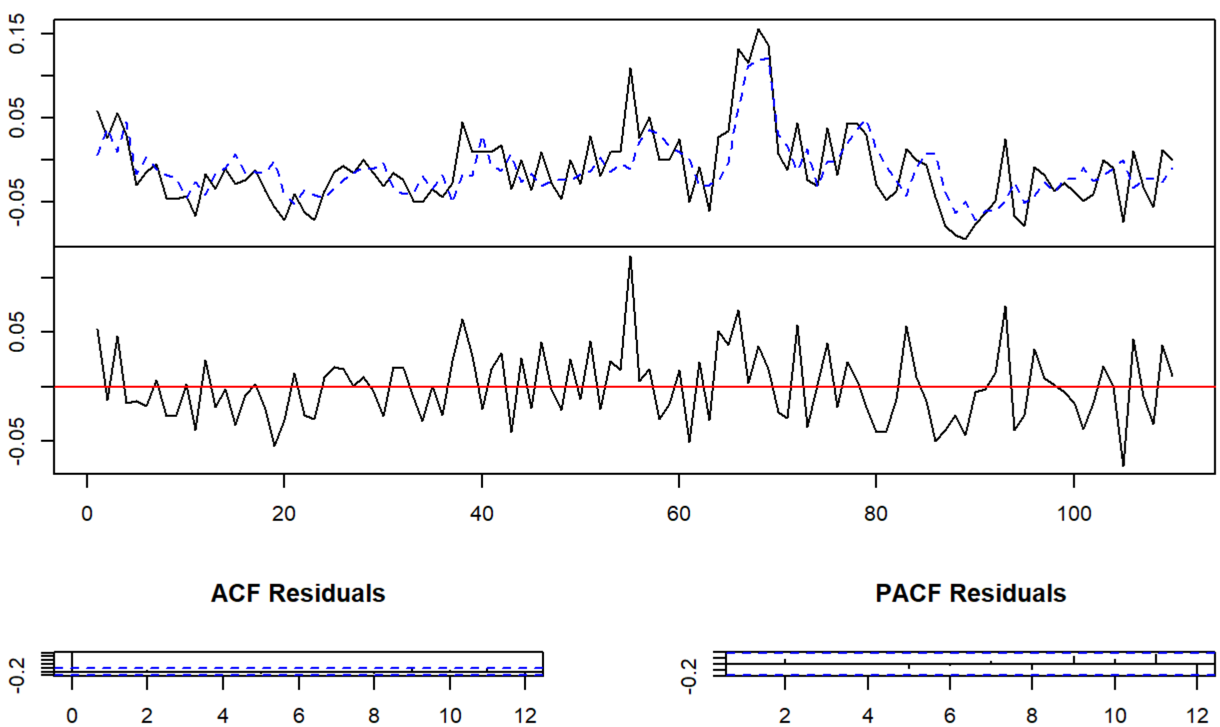


Diagram of fit and residuals for United.Kingdom_unem



結果為正相關。

Causality

```
causality(fit4, cause= "United.Kingdom_CPI")
```

```
## $Granger
##
##  Granger causality H0: United.Kingdom_CPI do not Granger-cause
##  United.Kingdom_gdp United.Kingdom_unem
##
## data:  VAR object fit4
## F-Test = 2.3, df1 = 8, df2 = 279, p-value = 0.02116
##
##
## $Instant
##
##  H0: No instantaneous causality between: United.Kingdom_CPI and
##  United.Kingdom_gdp United.Kingdom_unem
##
## data:  VAR object fit4
## Chi-squared = 2.6701, df = 2, p-value = 0.2631
```

```
causality(fit4, cause= "United.Kingdom_gdp")
```

```
## $Granger
##
##  Granger causality H0: United.Kingdom_gdp do not Granger-cause
##  United.Kingdom_CPI United.Kingdom_unem
##
## data:  VAR object fit4
## F-Test = 1.0008, df1 = 8, df2 = 279, p-value = 0.4356
##
##
## $Instant
##
##  H0: No instantaneous causality between: United.Kingdom_gdp and
##  United.Kingdom_CPI United.Kingdom_unem
##
## data:  VAR object fit4
## Chi-squared = 7.8267, df = 2, p-value = 0.01997
```

```
causality(fit4, cause= "United.Kingdom_unem")
```

```
## $Granger
##
##  Granger causality H0: United.Kingdom_unem do not Granger-cause
##  United.Kingdom_CPI United.Kingdom_gdp
##
## data:  VAR object fit4
## F-Test = 1.3763, df1 = 8, df2 = 279, p-value = 0.2066
##
##
## $Instant
##
##  H0: No instantaneous causality between: United.Kingdom_unem and
##  United.Kingdom_CPI United.Kingdom_gdp
##
## data:  VAR object fit4
## Chi-squared = 7.33, df = 2, p-value = 0.0256
```

> 檢查序列間是否存在granger causality 和 instantaneous causality並觀察United.Kingdom_CPI、United.Kingdom_gdp、United.Kingdom_unem分別對其他兩者的因果關係。

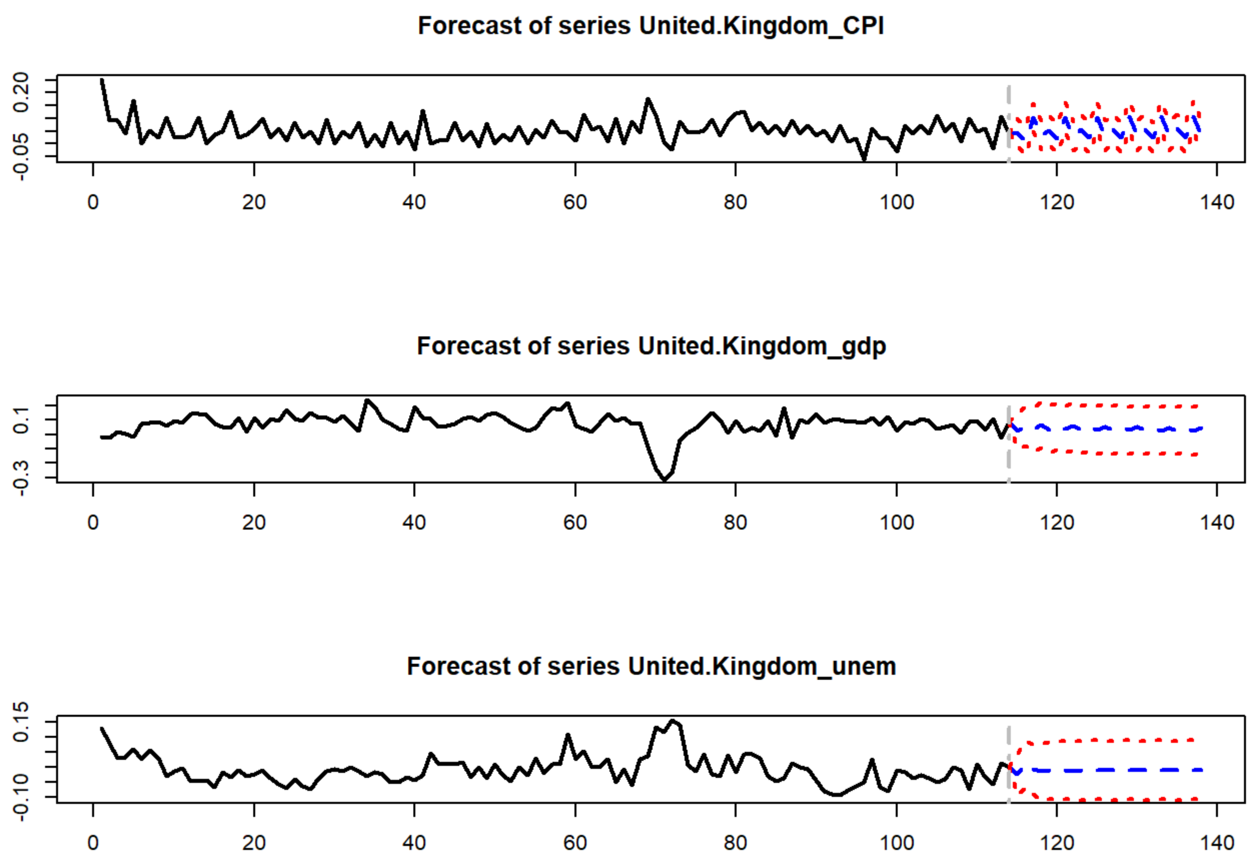
- 由第一個causality的結果可知可以拒絕H0，United.Kingdom_CPI 對於United.Kingdom_gdp和United.Kingdom_unem有相關性，然而，並沒有當期的相關性。
- 由第二個causality的結果可知不能拒絕H0，United.Kingdom_gdp 對於United.Kingdom_CPI和United.Kingdom_unem無顯著相關性，然而他們有當期的相關性。

- 由第一個causality的結果可知不能拒絕H0，United.Kingdom_unem 對於United.Kingdom_gdp和United.Kingdom_CPI無顯著相關性，然而他們有當期的相關性。

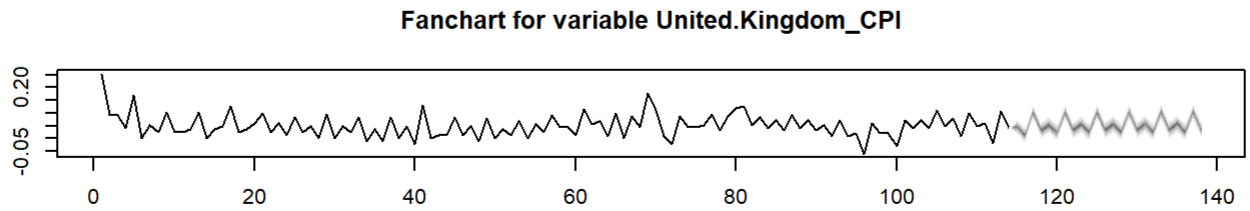
> 就結果可知，United.Kingdom_gdp及United.Kingdom_unem對其餘時間序列均無granger causality，顯示兩序列的過去值對其餘序列無顯著解釋能力。然而，United.Kingdom_CPI則對其餘序列有顯著解釋能力，即United.Kingdom_gdp這筆序列對其餘序列具有預測能力。

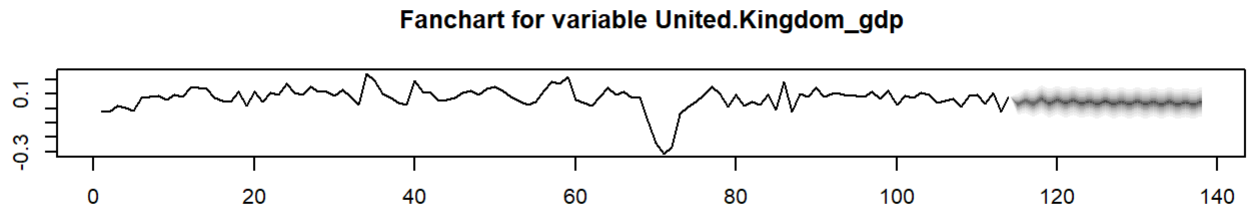
Prediction

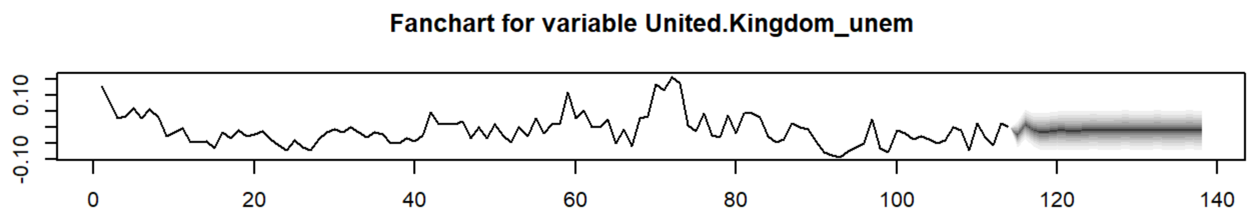
```
fit4$pred = predict(fit4, n.ahead = 24, ci = 0.95)
plot(fit4$pred, lwd=2)
```




```
fanchart(fit4$pred)
```







AIC

```
AIC1<-2*9-2*638.761
AIC1
```

```
## [1] -1259.522
```

```
AIC4<-2*17-2*642.203
AIC4
```

```
## [1] -1250.406
```

> 由於AIC1(-1259.522) < AIC4(-1250.406)，可得知fit1模型的預測能力較好。