Time Series HW7

106070020

2021年6月13日

Library

```
library(aTSA)
library(vars)
library(forecast)
library(tseries)
```

Data cleaning

```
econ<-read.csv("C:/Users/eva/Desktop/作業 上課資料(清大)/大四下/時序/hw7/econ_data.csv", header =
T)
dim(econ)
```

```
## [1] 115  22
```

```
head(econ)
```

```
##        DATE Canada_CPI Canada_gdp Canada_unem France_CPI France_gdp France_unem
## 1  1991/1/1   64.86700   54.97783    10.16667   68.80333   68.47505    9.266667
## 2  1991/4/1   65.34106   55.23417    10.33333   69.36333   68.70599    9.433333
## 3  1991/7/1   65.73611   55.30814    10.43333   69.82000   68.92958    9.700000
## 4 1991/10/1   65.63076   55.41020    10.33333   70.38333   69.21646   10.000000
## 5  1992/1/1   65.89413   55.45059    10.60000   70.71667   69.92935   10.266667
## 6  1992/4/1   66.23650   55.51830    11.00000   71.26000   69.87414   10.500000
##   Germany_CPI Germany_gdp Germany_unem Italy_CPI Italy_gdp Italy_unem Japan_CPI
## 1    64.18494    73.48000     5.233333  54.07411  86.44591   8.633333  93.20000
## 2    64.80689    73.11000     5.333333  54.85303  85.97284   8.466667  94.16667
## 3    65.86420    72.95000     5.600000  55.49347  86.00364   8.433333  94.16667
## 4    67.10809    73.88000     5.900000  56.27239  87.01302   8.600000  95.30000
## 5    67.97882    74.99748     6.100000  57.15516  87.24703   8.666667  95.00000
## 6    68.75625    74.45750     6.400000  57.86484  87.19709   8.666667  96.33333
##   Japan_gdp Japan_unem United.Kingdom_CPI United.Kingdom_gdp
## 1  79.99816   2.100000               57.2           59.47495
## 2  80.94727   2.100000               59.1           59.39998
## 3  80.76807   2.100000               59.8           59.27025
## 4  81.33533   2.066667               60.5           59.37321
## 5  81.36063   2.066667               60.8           59.37750
## 6  81.64551   2.100000               62.1           59.30743
##   United.Kingdom_unem United.States_CPI United.States_gdp United.States_unem
## 1            7.766667          56.87356          53.26046           6.600000
## 2            8.466667          57.21109          53.67574           6.833333
## 3            8.933333          57.66112          53.94696           6.866667
## 4            9.100000          58.09710          54.13497           7.100000
## 5            9.300000          58.50495          54.78299           7.366667
## 6            9.666667          58.98311          55.37703           7.600000
```

```
econ[is.na(econ)] #no NAs
```

```
## character(0)
```

```
econ$DATE<-as.Date(econ$DATE, format =  "%Y/%m/%d" )
summary(econ)
```

```
##       DATE              Canada_CPI       Canada_gdp      Canada_unem
##  Min.   :1991-01-01   Min.   : 64.87   Min.   : 54.98   Min.   : 5.567
##  1st Qu.:1998-02-15   1st Qu.: 71.99   1st Qu.: 67.05   1st Qu.: 6.850
##  Median :2005-04-01   Median : 84.33   Median : 84.34   Median : 7.333
##  Mean   :2005-04-01   Mean   : 84.43   Mean   : 81.75   Mean   : 7.810
##  3rd Qu.:2012-05-16   3rd Qu.: 96.21   3rd Qu.: 94.40   3rd Qu.: 8.600
##  Max.   :2019-07-01   Max.   :107.98   Max.   :108.39   Max.   :11.733
##    France_CPI       France_gdp      France_unem      Germany_CPI
##  Min.   : 68.80   Min.   : 68.48   Min.   : 7.267   Min.   : 64.18
##  1st Qu.: 78.08   1st Qu.: 77.49   1st Qu.: 8.817   1st Qu.: 78.24
##  Median : 87.84   Median : 90.77   Median : 9.500   Median : 85.98
##  Mean   : 87.49   Mean   : 88.30   Mean   : 9.913   Mean   : 86.79
##  3rd Qu.: 98.74   3rd Qu.: 97.39   3rd Qu.:10.733   3rd Qu.: 97.13
##  Max.   :104.58   Max.   :107.11   Max.   :12.500   Max.   :106.08
##   Germany_gdp      Germany_unem      Italy_CPI        Italy_gdp
##  Min.   : 72.95   Min.   : 3.100   Min.   : 54.07   Min.   : 85.94
##  1st Qu.: 80.10   1st Qu.: 5.333   1st Qu.: 71.79   1st Qu.: 95.04
##  Median : 86.04   Median : 7.767   Median : 84.42   Median :101.57
##  Mean   : 88.32   Mean   : 7.263   Mean   : 83.22   Mean   : 99.13
##  3rd Qu.: 95.67   3rd Qu.: 8.800   3rd Qu.: 98.71   3rd Qu.:103.79
##  Max.   :107.23   Max.   :11.200   Max.   :103.17   Max.   :109.42
##    Italy_unem       Japan_CPI        Japan_gdp        Japan_unem
##  Min.   : 5.933   Min.   : 93.20   Min.   : 80.00   Min.   :2.067
##  1st Qu.: 8.317   1st Qu.: 96.87   1st Qu.: 87.32   1st Qu.:3.033
##  Median : 9.967   Median : 97.57   Median : 92.97   Median :3.933
##  Mean   : 9.673   Mean   : 98.03   Mean   : 92.47   Mean   :3.825
##  3rd Qu.:11.183   3rd Qu.: 99.70   3rd Qu.: 97.76   3rd Qu.:4.667
##  Max.   :12.733   Max.   :101.77   Max.   :104.35   Max.   :5.433
##  United.Kingdom_CPI United.Kingdom_gdp United.Kingdom_unem United.States_CPI
##  Min.   : 57.20     Min.   : 59.27     Min.   : 3.733      Min.   : 56.87
##  1st Qu.: 71.00     1st Qu.: 71.19     1st Qu.: 5.033      1st Qu.: 68.49
##  Median : 79.30     Median : 87.71     Median : 5.933      Median : 82.06
##  Mean   : 82.20     Mean   : 83.80     Mean   : 6.454      Mean   : 82.35
##  3rd Qu.: 95.95     3rd Qu.: 93.79     3rd Qu.: 7.817      3rd Qu.: 97.06
##  Max.   :108.20     Max.   :106.95     Max.   :10.400      Max.   :108.27
##  United.States_gdp United.States_unem
##  Min.   : 53.26    Min.   :3.633
##  1st Qu.: 68.30    1st Qu.:4.633
##  Median : 85.27    Median :5.500
##  Mean   : 81.78    Mean   :5.869
##  3rd Qu.: 93.14    3rd Qu.:6.850
##  Max.   :109.87    Max.   :9.933
```

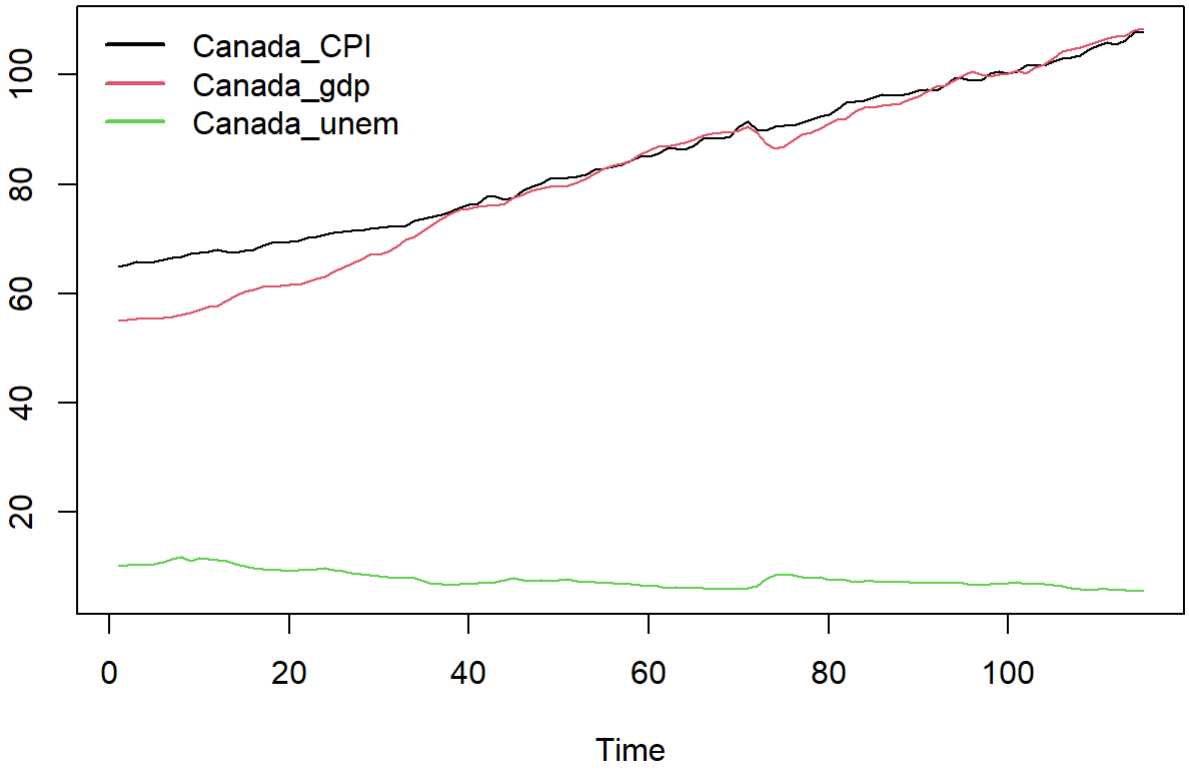
```
str(econ)
```

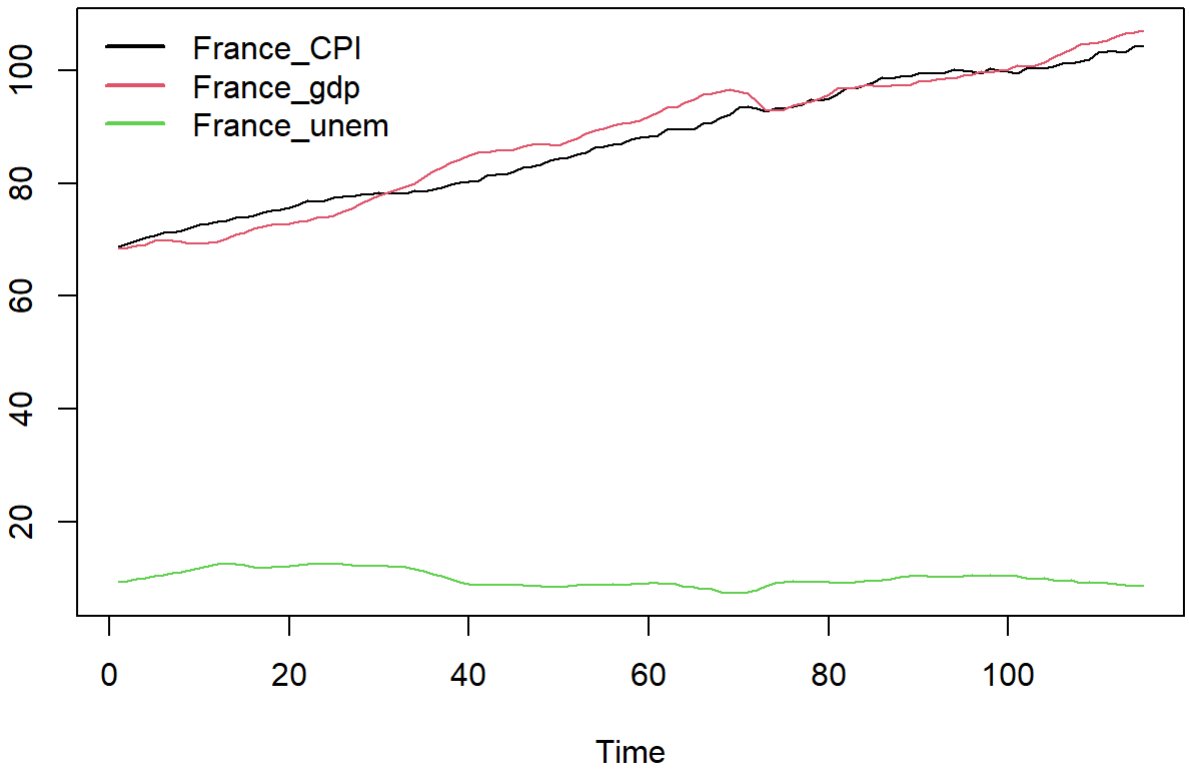
```
## 'data.frame':    115 obs. of  22 variables:
##  $ DATE               : Date, format: "1991-01-01" "1991-04-01" ...
##  $ Canada_CPI         : num  64.9 65.3 65.7 65.6 65.9 ...
##  $ Canada_gdp         : num  55 55.2 55.3 55.4 55.5 ...
##  $ Canada_unem        : num  10.2 10.3 10.4 10.3 10.6 ...
##  $ France_CPI         : num  68.8 69.4 69.8 70.4 70.7 ...
##  $ France_gdp         : num  68.5 68.7 68.9 69.2 69.9 ...
##  $ France_unem        : num  9.27 9.43 9.7 10 10.27 ...
##  $ Germany_CPI        : num  64.2 64.8 65.9 67.1 68 ...
##  $ Germany_gdp        : num  73.5 73.1 73 73.9 75 ...
##  $ Germany_unem       : num  5.23 5.33 5.6 5.9 6.1 ...
##  $ Italy_CPI          : num  54.1 54.9 55.5 56.3 57.2 ...
##  $ Italy_gdp          : num  86.4 86 86 87 87.2 ...
##  $ Italy_unem         : num  8.63 8.47 8.43 8.6 8.67 ...
##  $ Japan_CPI          : num  93.2 94.2 94.2 95.3 95 ...
##  $ Japan_gdp          : num  80 80.9 80.8 81.3 81.4 ...
##  $ Japan_unem         : num  2.1 2.1 2.1 2.07 2.07 ...
##  $ United.Kingdom_CPI : num  57.2 59.1 59.8 60.5 60.8 62.1 62.1 62.5 62.7 63.5 ...
##  $ United.Kingdom_gdp : num  59.5 59.4 59.3 59.4 59.4 ...
##  $ United.Kingdom_unem: num  7.77 8.47 8.93 9.1 9.3 ...
##  $ United.States_CPI  : num  56.9 57.2 57.7 58.1 58.5 ...
##  $ United.States_gdp  : num  53.3 53.7 53.9 54.1 54.8 ...
##  $ United.States_unem : num  6.6 6.83 6.87 7.1 7.37 ...
```

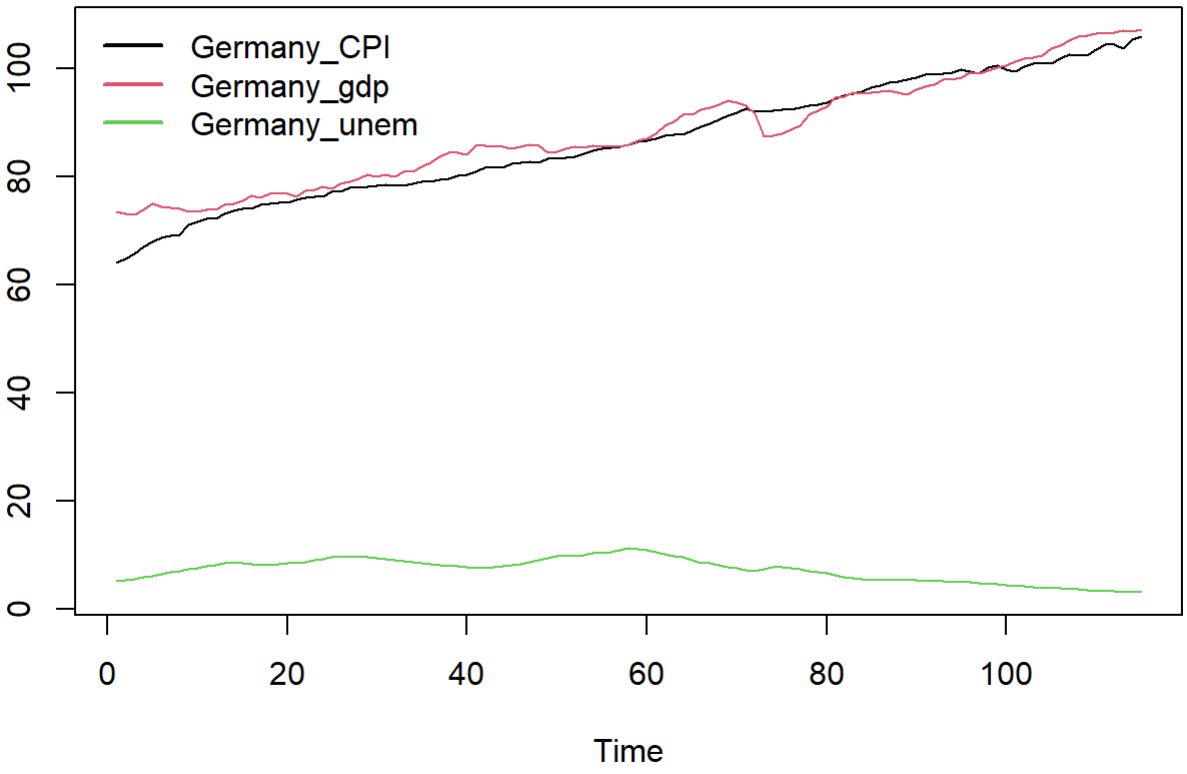
> 確保資料型態以及缺失值等等。

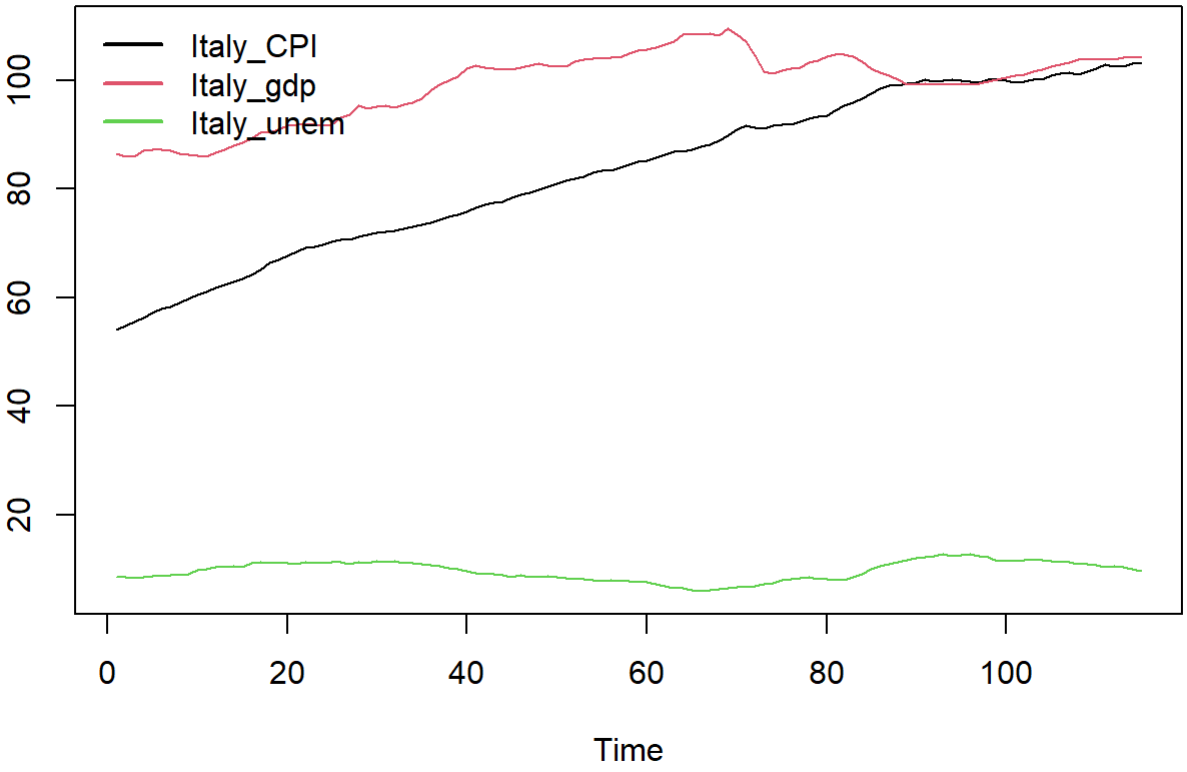
EDA

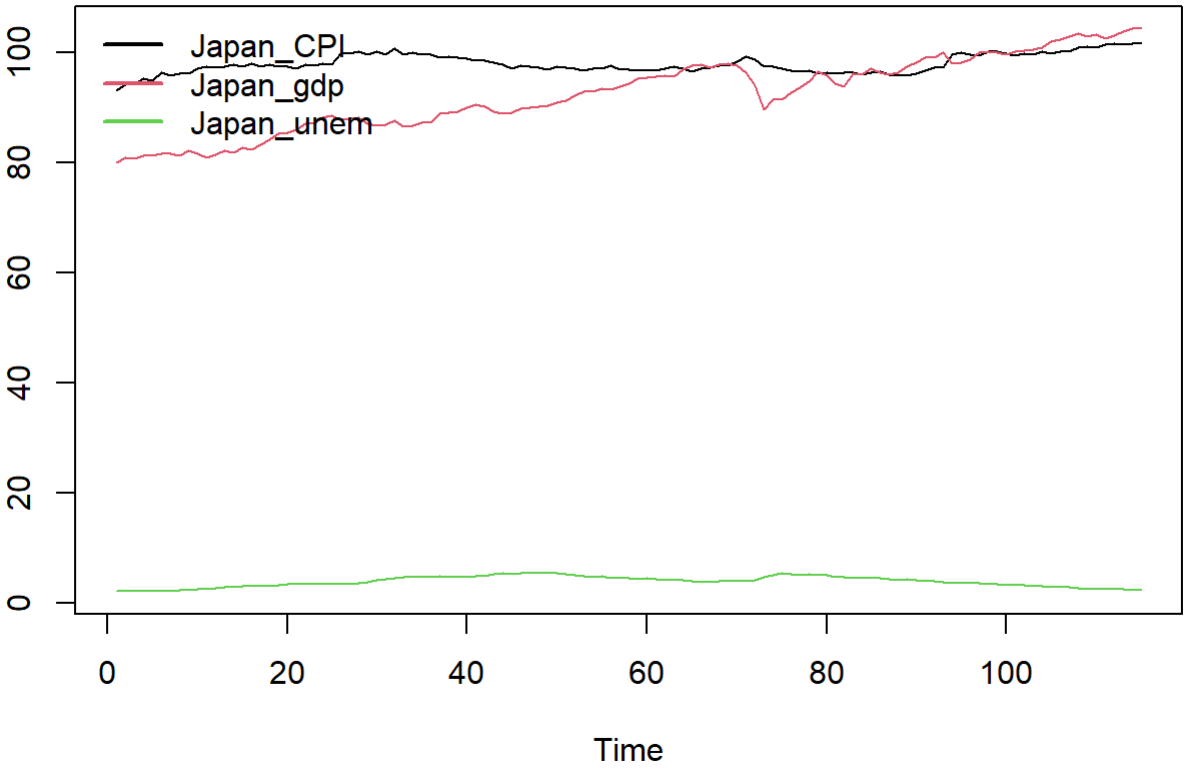
```
par(mfrow=c(1,1))
series_name = colnames(econ)
for (i in 1:7){
  y = econ[,1+(i-1)*3+(1:3)]
  ts.plot(y, col=1:3)
  legend("topleft", legend=series_name[1+(i-1)*3+(1:3)], col=1:3, lty=1, lwd=2, bty="n")
}
```

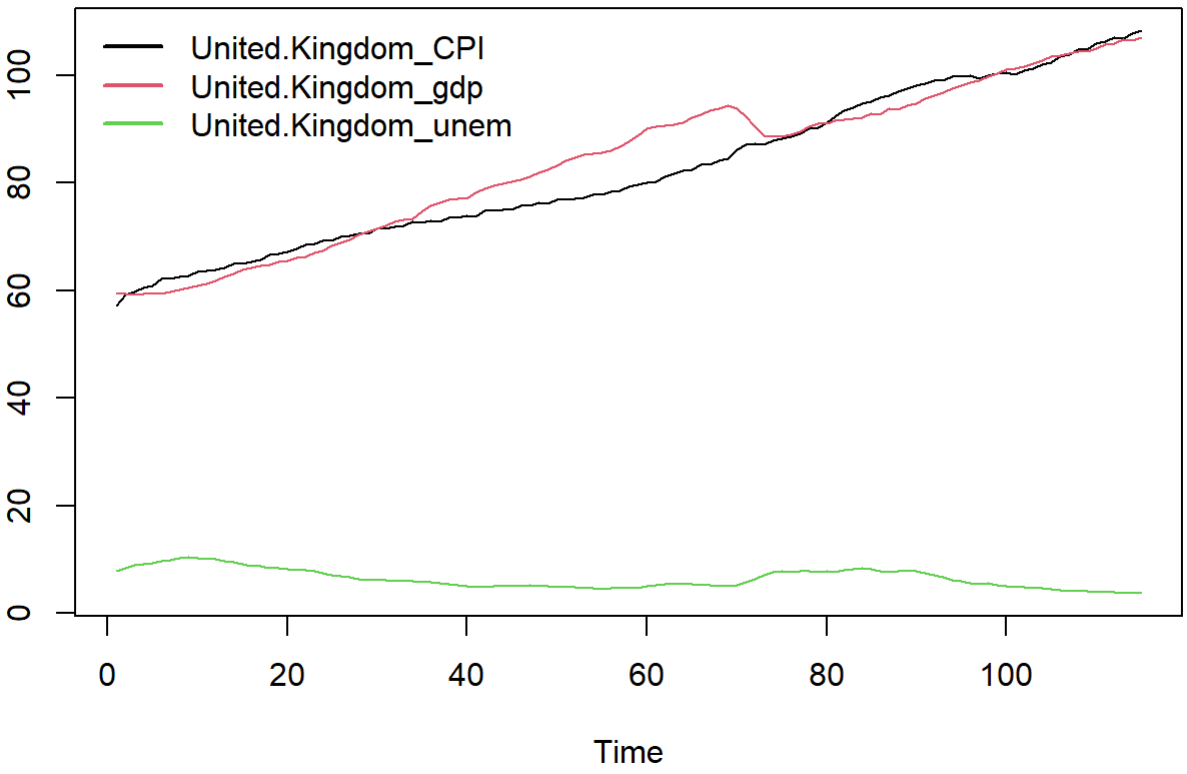

Canada_CPI
Canada_gdp
Canada_unem
20
40
60
80
100
0
20
40
60
80
100
Time

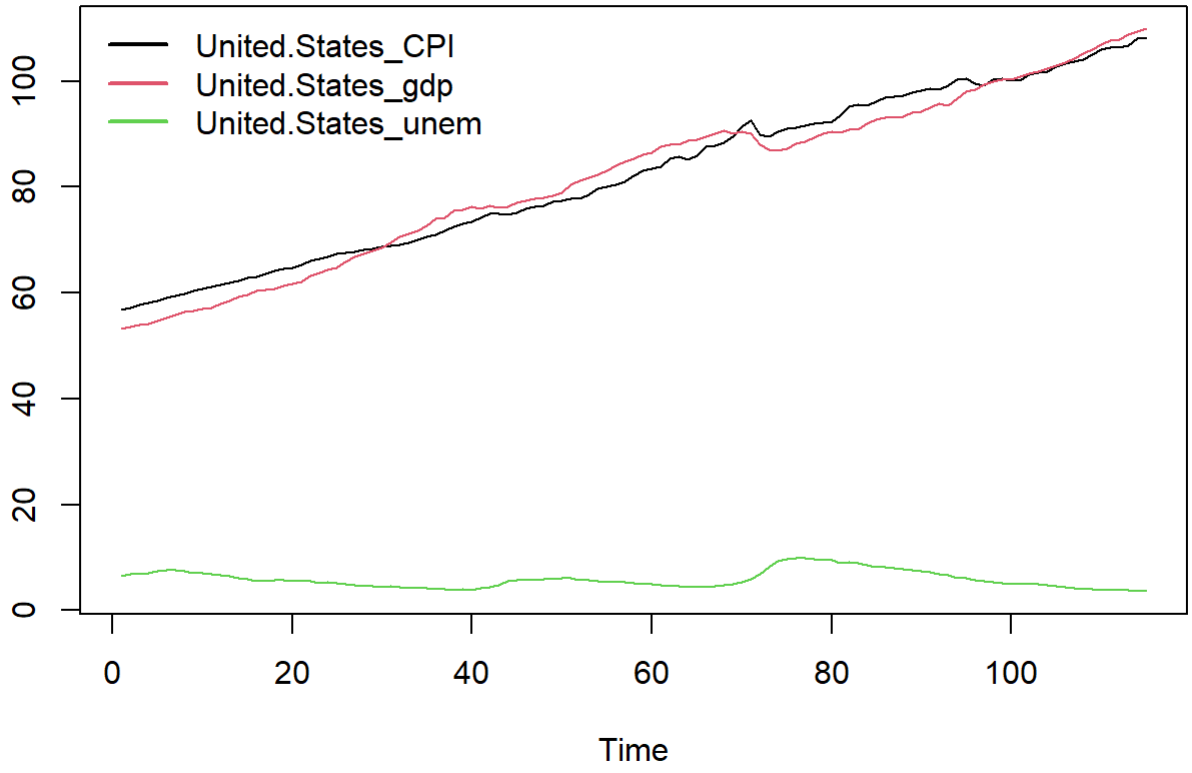

France_CPI
France_gdp
France_unem
20
40
60
80
100
0
20
40
60
80
100
Time


Germany_CPI
Germany_gdp
Germany_unem
0
20
40
60
80
100
Time


Italy_CPI
Italy_gdp
Italy_unem
20
40
60
80
100
0
20
40
60
80
100
Time


Japan_CPI
Japan_gdp
Japan_unem
0
20
40
60
80
100
Time


United.Kingdom_CPI
United.Kingdom_gdp
United.Kingdom_unem
0
20
40
60
80
100
Time



Data Preprocessing

box-cox

```
# box-cox
par(mfrow=c(1,1))
lambda <- BoxCox.lambda(econ$United.Kingdom_CPI)
print(lambda)
```

```
## [1] 0.5051194
```

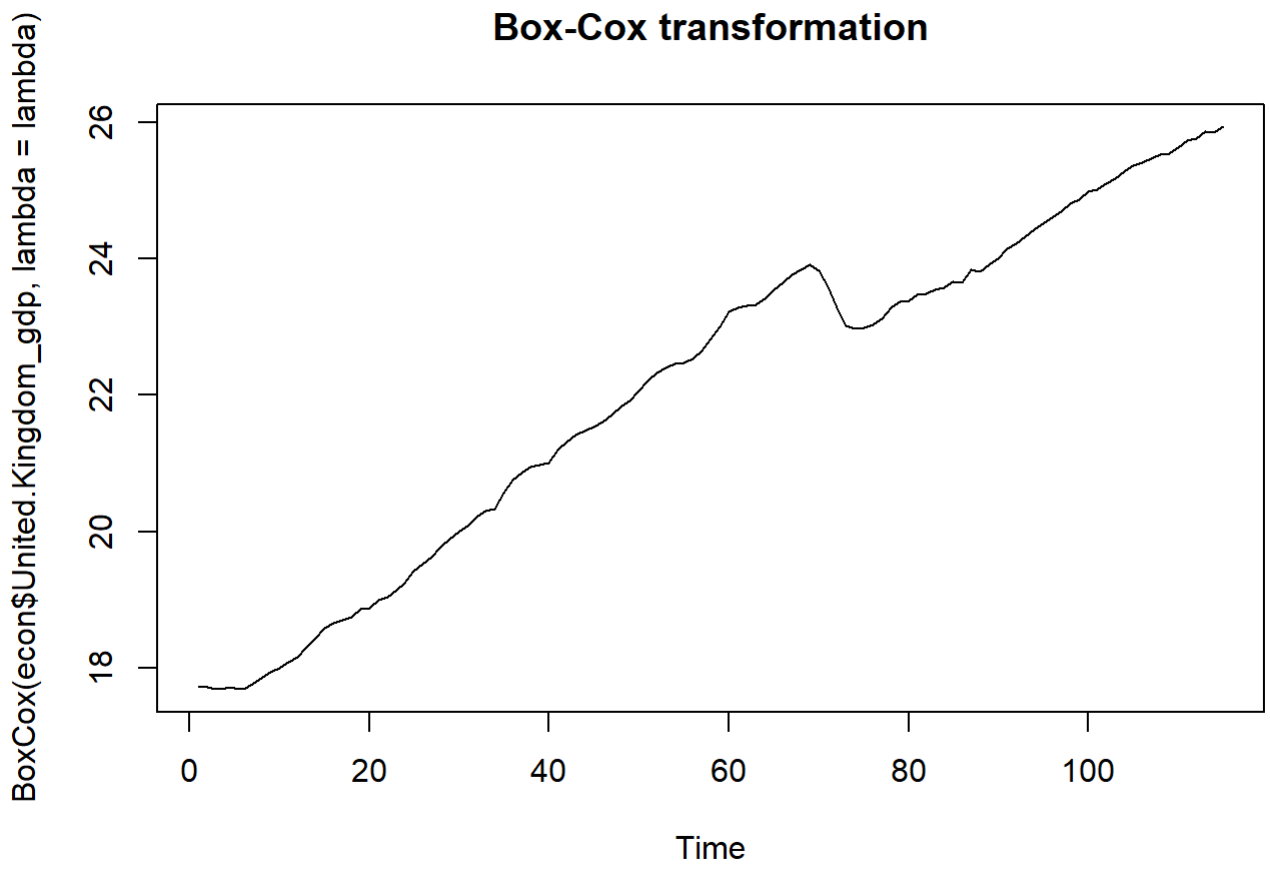
```
plot.ts(BoxCox(econ$United.Kingdom_CPI, lambda = lambda), main='Box-Cox transformation')
```


```
kc1<-BoxCox(econ$United.Kingdom_CPI, lambda = lambda)

par(mfrow=c(1,1))
lambda <- BoxCox.lambda(econ$United.Kingdom_gdp)
print(lambda)
```

```
## [1] 0.6011321
```

```
plot.ts(BoxCox(econ$United.Kingdom_gdp, lambda = lambda), main='Box-Cox transformation')
```

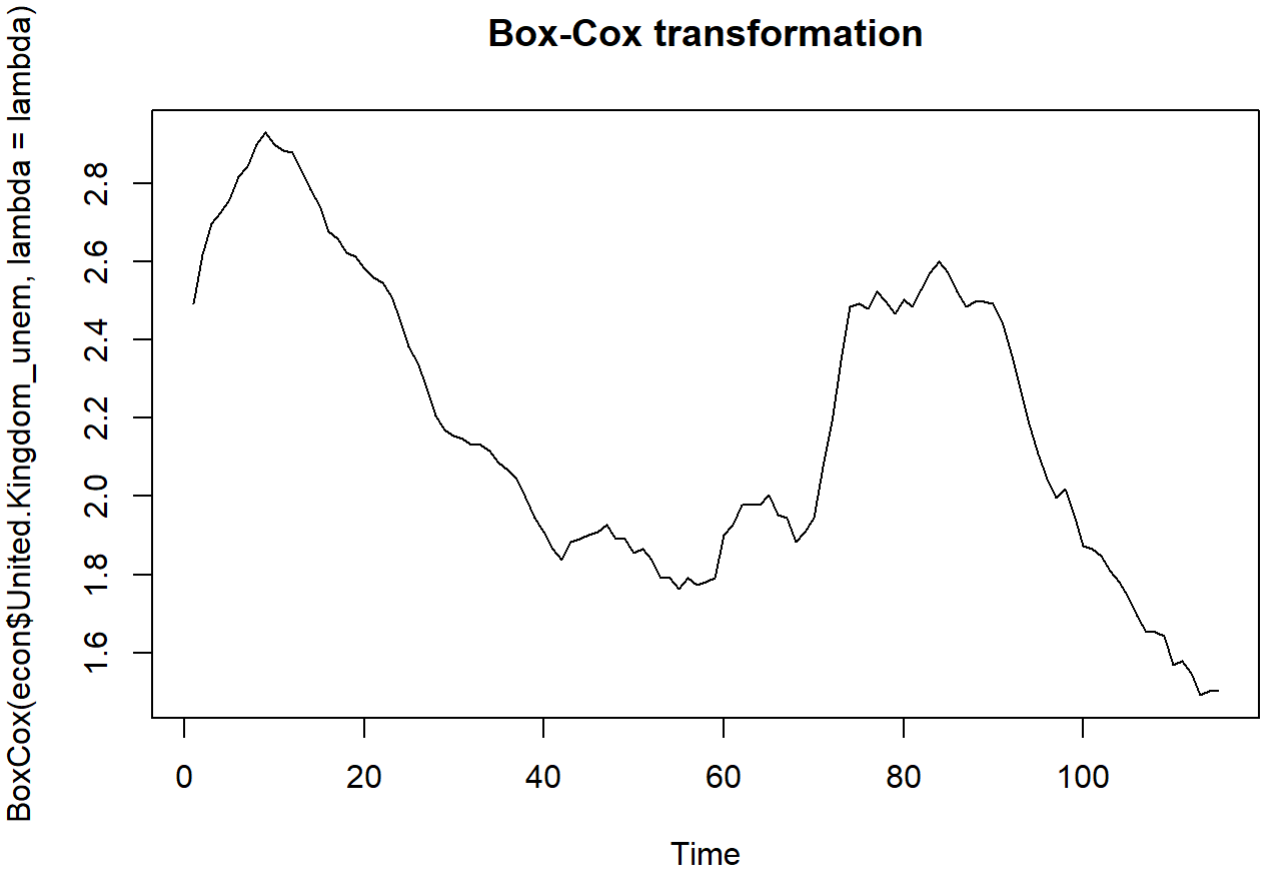



```
kg1<-BoxCox(econ$United.Kingdom_gdp, lambda = lambda)

par(mfrow=c(1,1))
lambda <- BoxCox.lambda(econ$United.Kingdom_unem)
print(lambda)
```

```
## [1] 0.1844793
```

```
plot.ts(BoxCox(econ$United.Kingdom_unem, lambda = lambda), main='Box-Cox transformation')
```

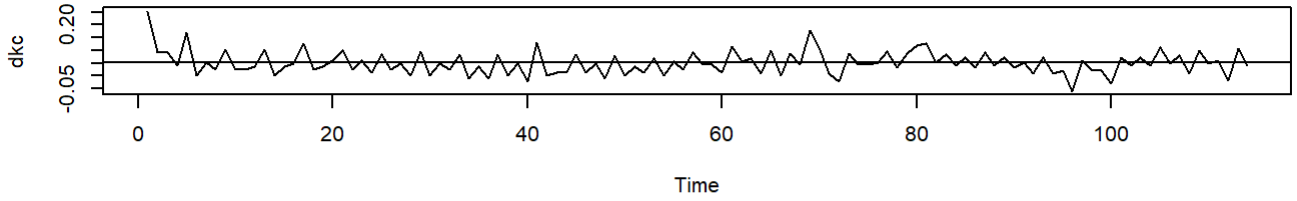


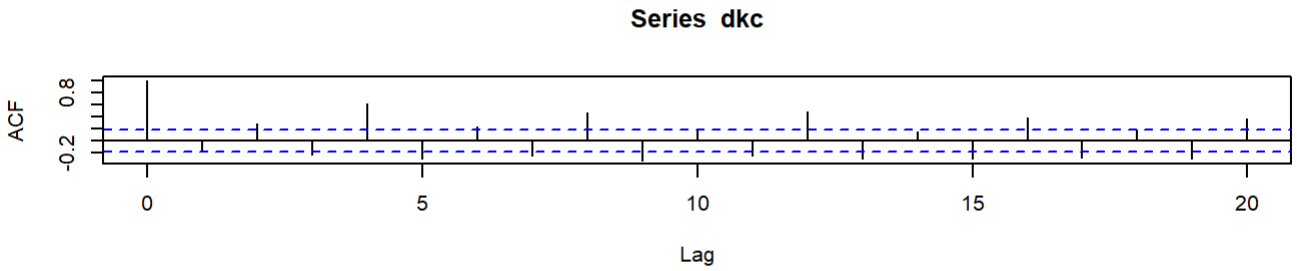

```
ku1<-BoxCox(econ$United.Kingdom_unem, lambda = lambda)
```

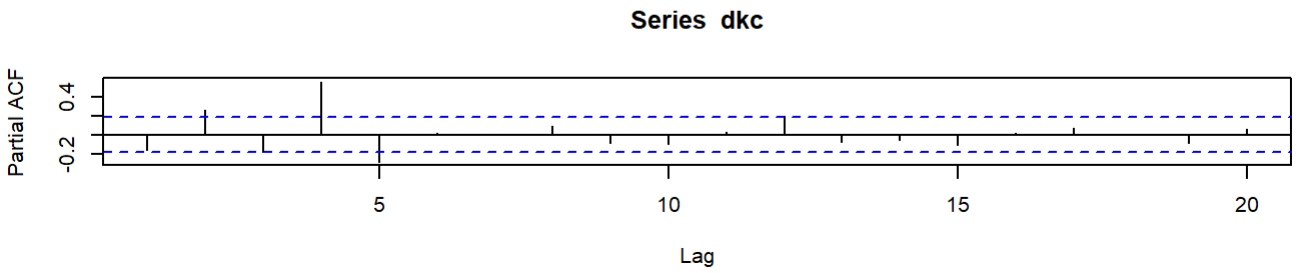
利用Box-Cox transformation，使轉換後的資料變異數齊一，更似常態分佈。 其中，計算出的lambda值分別為-0.5051194，0.6011321，0.1844793，並將轉換後的資料繪製成圖，並存為新的變數。

Differencing

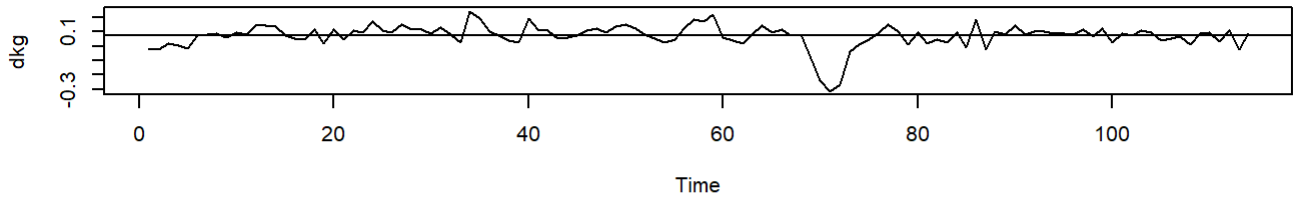
```
dkc<-diff(kc1)
{par(mfrow=c(3,1))
  {ts.plot(dkc)
    abline(h=mean(dkc))
}
  acf(dkc)
  pacf(dkc)}
```

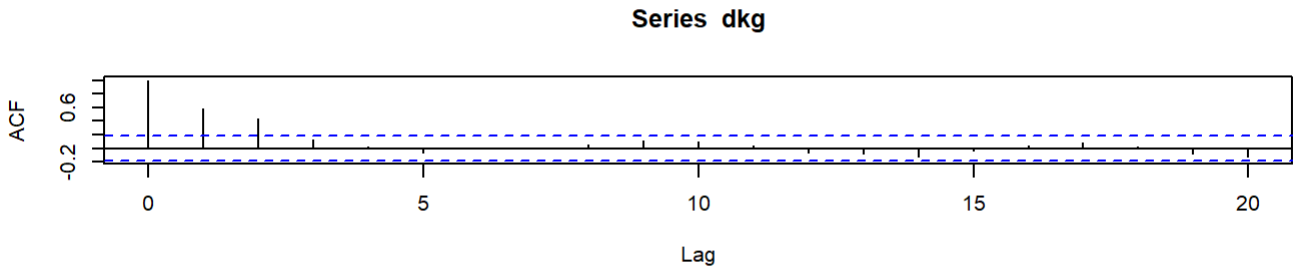


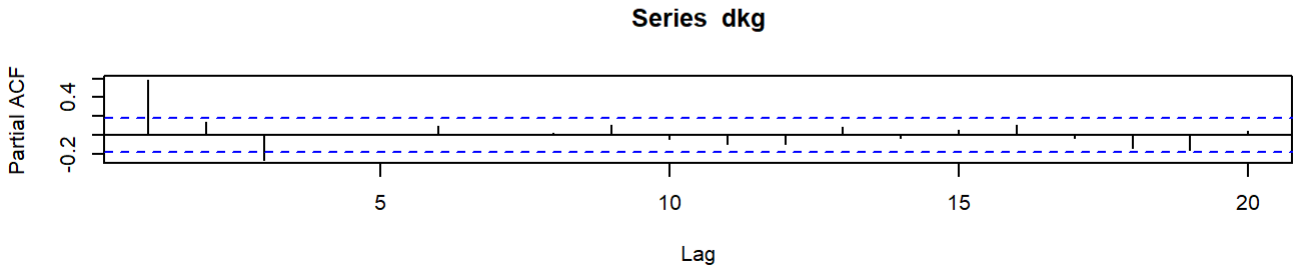




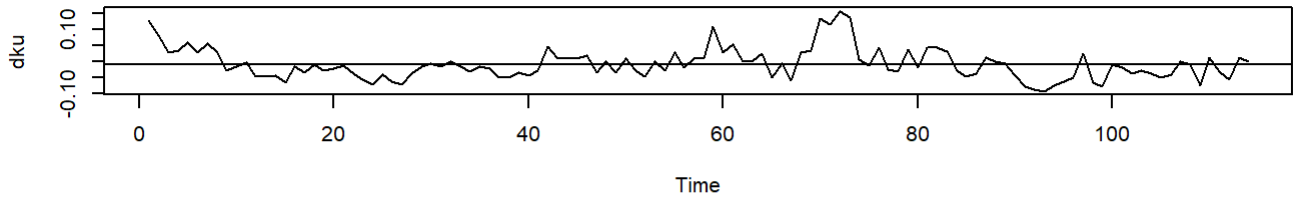

```
dkg<-diff(kg1)
{par(mfrow=c(3,1))
  {ts.plot(dkg)
    abline(h=mean(dkg))
}
  acf(dkg)
  pacf(dkg)}
```

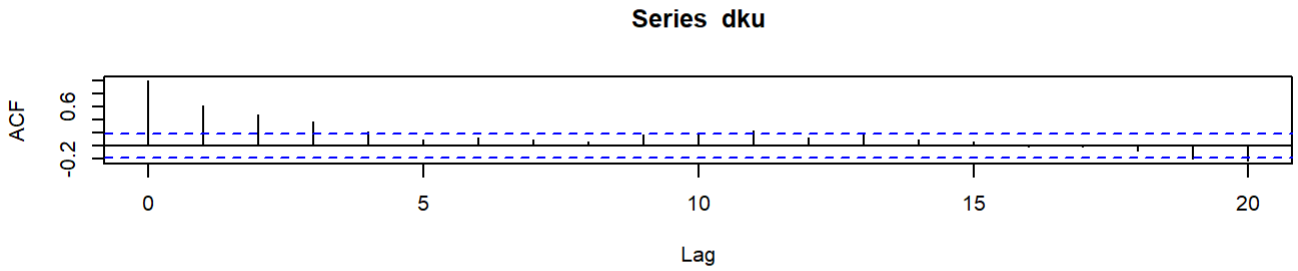


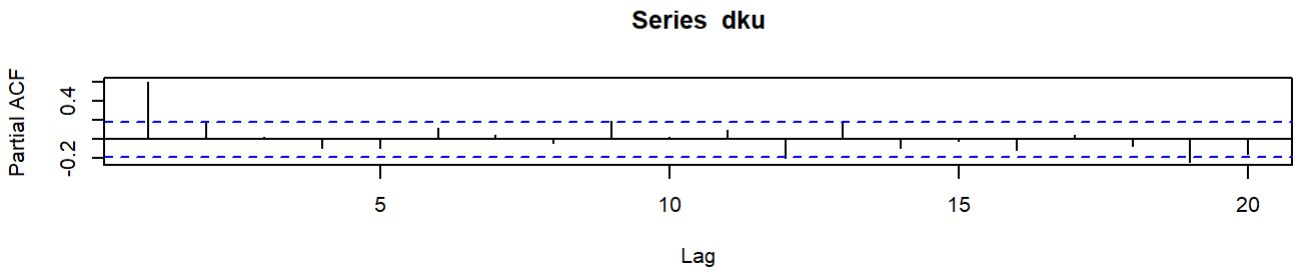





```
dku<-diff(ku1)
{par(mfrow=c(3,1))
  {ts.plot(dku)
    abline(h=mean(dku))
}
  acf(dku)
  pacf(dku)}
```







ADF Test

```
adf.test(dkc)
```

```
## Warning in adf.test(dkc): p-value smaller than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  dkc
## Dickey-Fuller = -4.1775, Lag order = 4, p-value = 0.01
## alternative hypothesis: stationary
```

```
adf.test(dkg)
```

```
## Warning in adf.test(dkg): p-value smaller than printed p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  dkg
## Dickey-Fuller = -4.4138, Lag order = 4, p-value = 0.01
## alternative hypothesis: stationary
```

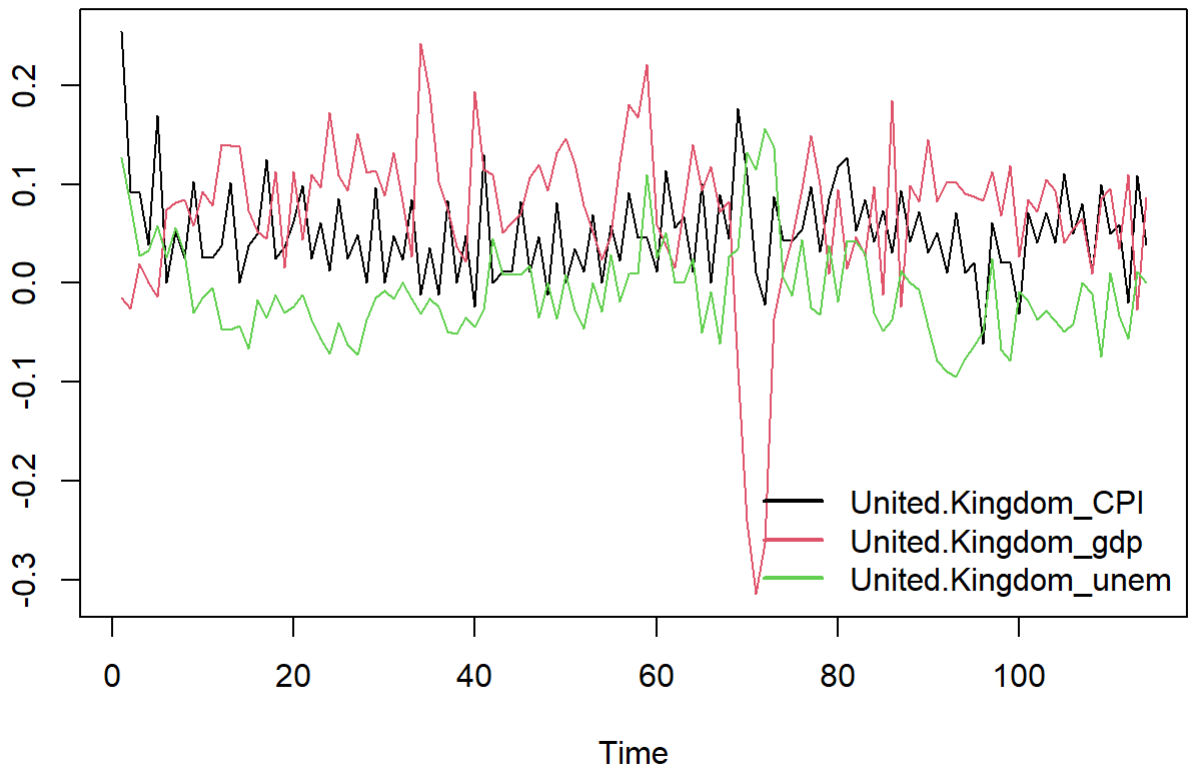
```
adf.test(dku)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  dku
## Dickey-Fuller = -3.9256, Lag order = 4, p-value = 0.01534
## alternative hypothesis: stationary
```

經由一次差分消除local trend，並利用ADF Test測試stationality. ADF Test的p-value為0.01，0.01，0.015，說明這三個序列皆為stationary.

Plots

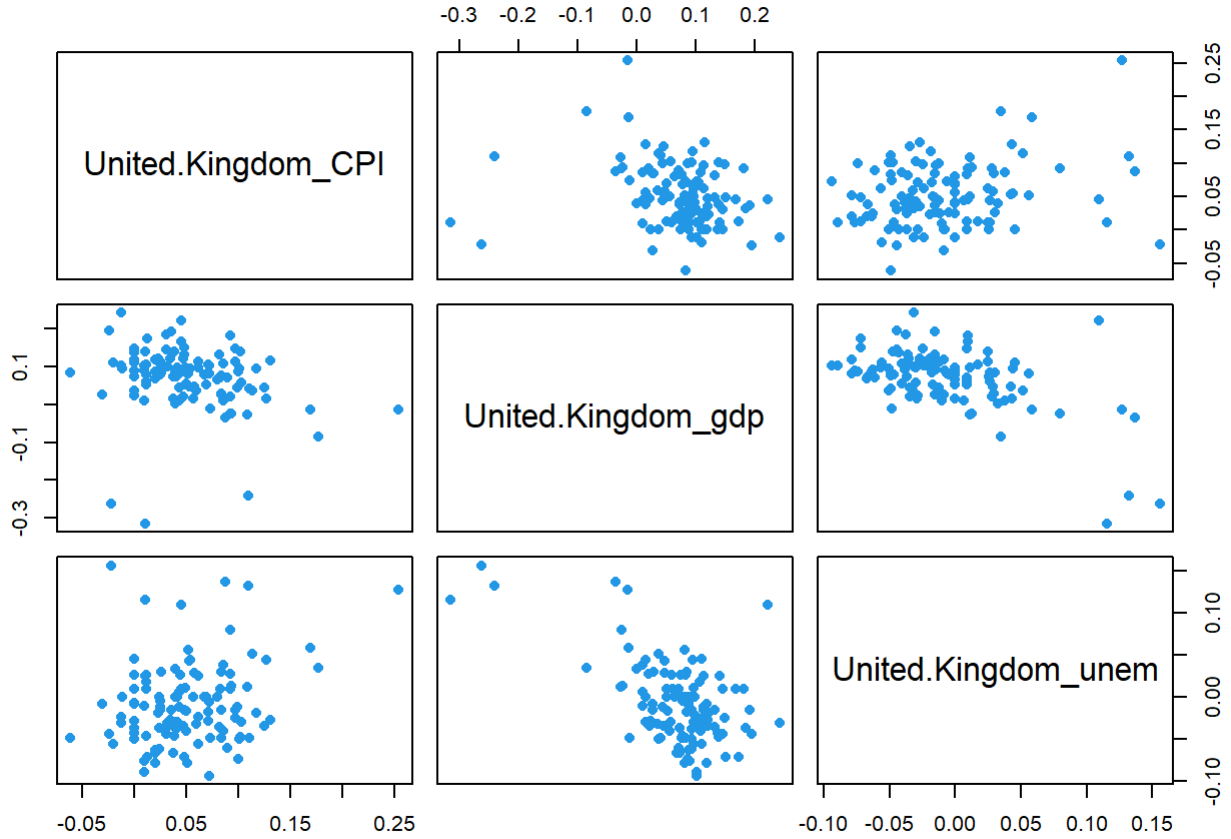
```
uk=cbind(dkc,dkg,dku)
colnames(uk)=c('United.Kingdom_CPI','United.Kingdom_gdp','United.Kingdom_unem')
ts.plot(uk, col=1:3)
legend("bottomright", legend=colnames(uk), col=1:3, lty=1, lwd=2, bty="n")
```



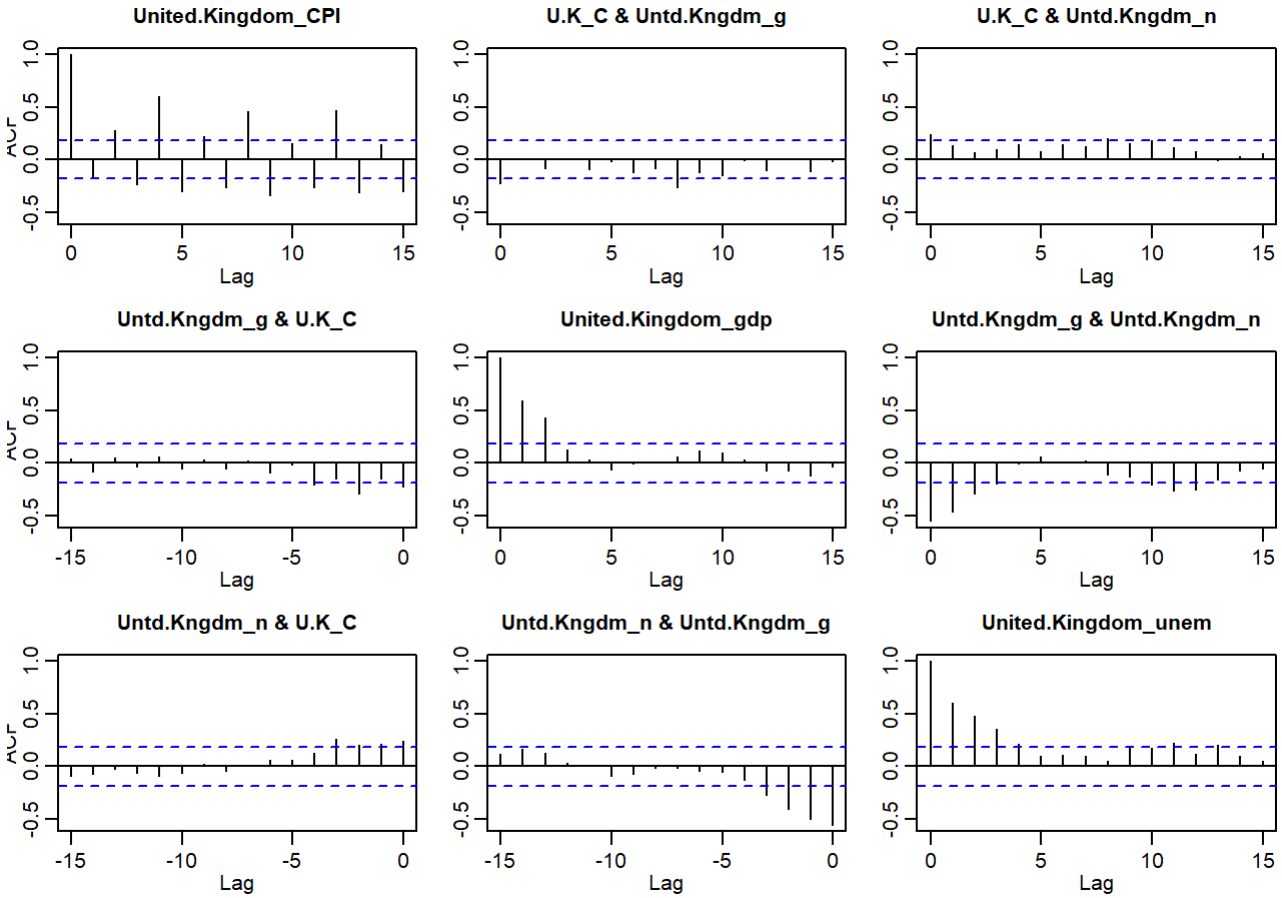
此圖為經過差分的序列圖。

ACF plots

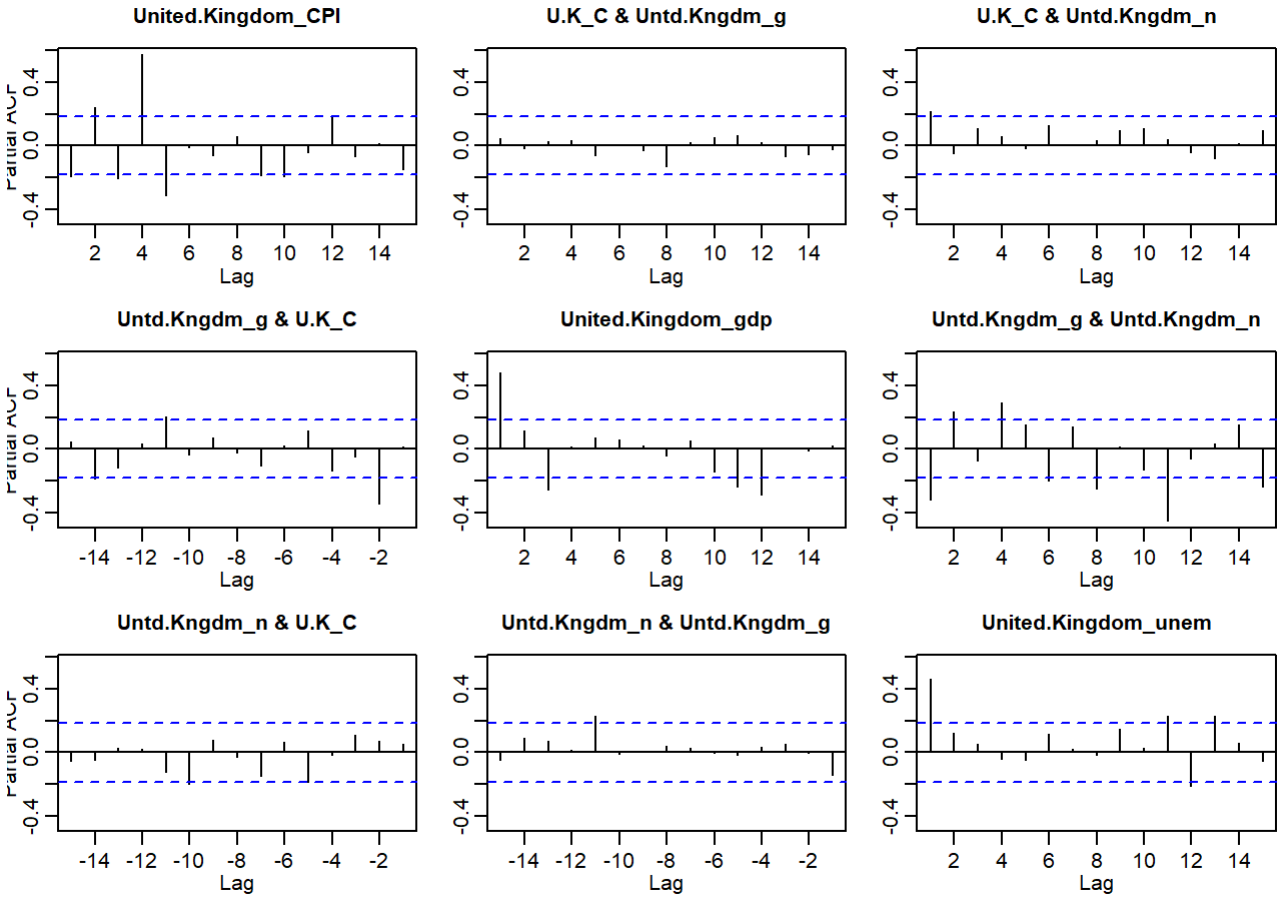
```
pairs(uk, col=4, pch=16)
```




```
par(mfrow=c(1,1))
acf(uk) #sample ccf
```


United.Kingdom_CPI
U.K_C & Untd.Kngdm_g
U.K_C & Untd.Kngdm_n
Untd.Kngdm_g & U.K_C
United.Kingdom_gdp
Untd.Kngdm_g & Untd.Kngdm_n
Untd.Kngdm_n & U.K_C
Untd.Kngdm_n & Untd.Kngdm_g
United.Kingdom_unem
ACF
Lag

```
pacf(uk)
```



繪製出這三筆data的scatter plot 和 ccf。

Model fitting

order selection

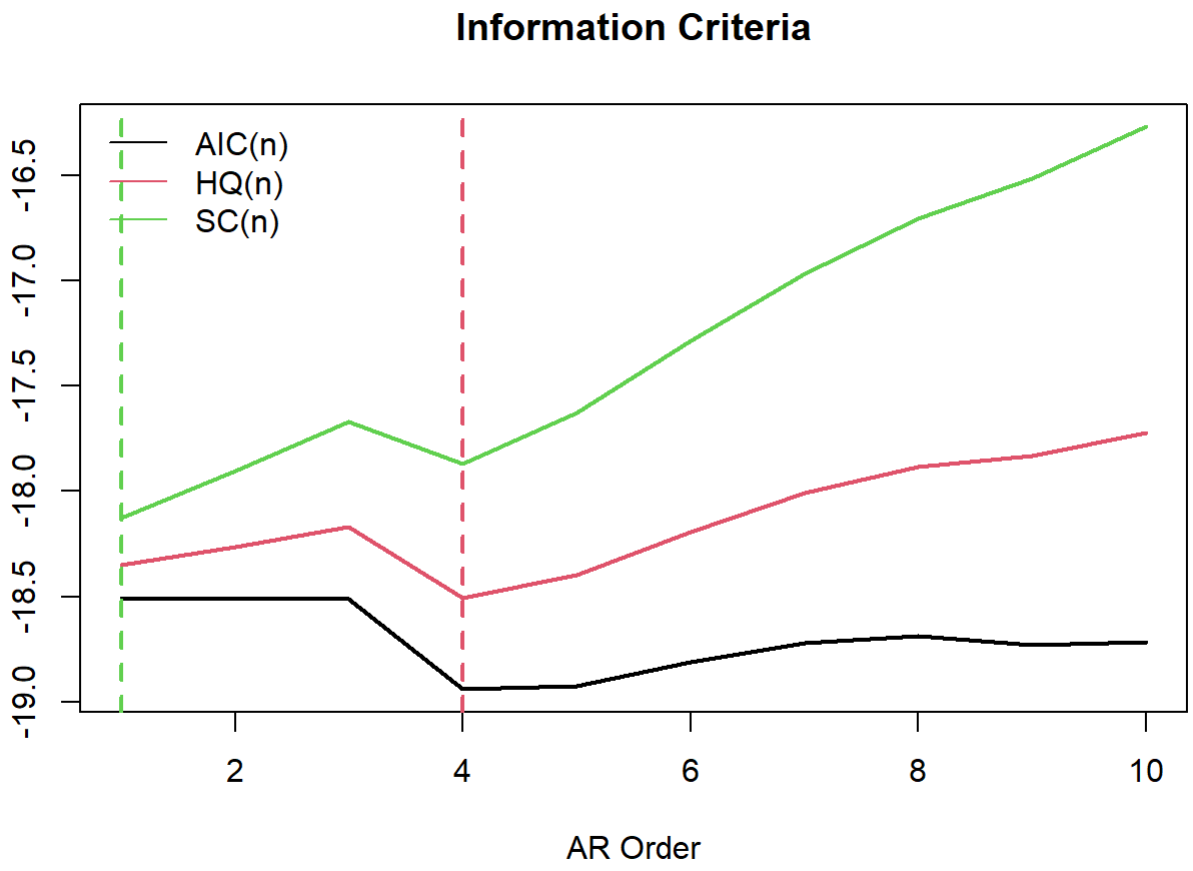
```
fit = VARselect(uk, lag.max=10, type="both")
fit
```

```
## $selection
## AIC(n)  HQ(n)  SC(n) FPE(n)
##      4      4      1      4
##
## $criteria
##                    1             2             3             4             5
## AIC(n) -1.850576e+01 -1.851196e+01 -1.851010e+01 -1.893852e+01 -1.892371e+01
## HQ(n)  -1.835124e+01 -1.826473e+01 -1.817016e+01 -1.850587e+01 -1.839835e+01
## SC(n)  -1.812435e+01 -1.790171e+01 -1.767101e+01 -1.787059e+01 -1.762694e+01
## FPE(n)  9.186478e-09  9.135994e-09  9.166462e-09  5.987548e-09  6.100908e-09
##                    6             7             8             9            10
## AIC(n) -1.881057e+01 -1.871960e+01 -1.868730e+01 -1.872649e+01 -1.871254e+01
## HQ(n)  -1.819250e+01 -1.800882e+01 -1.788381e+01 -1.783029e+01 -1.772363e+01
## SC(n)  -1.728496e+01 -1.696515e+01 -1.670401e+01 -1.651436e+01 -1.627156e+01
## FPE(n)  6.870573e-09  7.583329e-09  7.912502e-09  7.708181e-09  7.944977e-09
```

```
names(fit)
```

```
## [1] "selection" "criteria"
```

```
par(mfcol=c(1,1))
ts.plot(t(fit$crit[1:3,]), col=1:3, lwd=2, xlab="AR Order")
abline(v=fit$sel[1:3],lty=2,col=1:3,lwd=2)
legend("topleft",legend=rownames(fit$crit[1:3,]),col=1:3,lty=1, bty="n")
title("Information Criteria")
```



利用VARselect選擇出最適合的order。根據BIC，應該選擇order=1，根據AIC、FPC、HQ，應選擇order=4。因此以下為兩個模型的fitting。

Order=1

```
fit1 = VAR(uk, p=1, type="both",season=4)
summary(fit1)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: United.Kingdom_CPI, United.Kingdom_gdp, United.Kingdom_unem
## Deterministic variables: both
## Sample size: 113
## Log Likelihood: 638.761
## Roots of the characteristic polynomial:
## 0.6822 0.3002 0.3002
## Call:
## VAR(y = uk, p = 1, type = "both", season = 4L)
##
##
## Estimation results for equation United.Kingdom_CPI:
## ===================================================
## United.Kingdom_CPI = United.Kingdom_CPI.l1 + United.Kingdom_gdp.l1 + United.Kingdom_unem.l1 +
const + trend + sd1 + sd2 + sd3
##
##                          Estimate Std. Error t value Pr(>|t|)
## United.Kingdom_CPI.l1   3.616e-01  8.270e-02   4.373 2.90e-05 ***
## United.Kingdom_gdp.l1   3.311e-02  3.909e-02   0.847 0.398846
## United.Kingdom_unem.l1  9.361e-02  6.547e-02   1.430 0.155767
## const                   2.681e-02  7.821e-03   3.428 0.000869 ***
## trend                   4.605e-05  7.904e-05   0.583 0.561460
## sd1                     9.327e-02  7.649e-03  12.193  < 2e-16 ***
## sd2                    -9.341e-03  8.224e-03  -1.136 0.258608
## sd3                     4.273e-02  7.410e-03   5.766 8.24e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.02654 on 105 degrees of freedom
## Multiple R-Squared: 0.637,   Adjusted R-squared: 0.6128
## F-statistic: 26.32 on 7 and 105 DF,  p-value: < 2.2e-16
##
##
## Estimation results for equation United.Kingdom_gdp:
## ===================================================
## United.Kingdom_gdp = United.Kingdom_CPI.l1 + United.Kingdom_gdp.l1 + United.Kingdom_unem.l1 +
const + trend + sd1 + sd2 + sd3
##
##                          Estimate Std. Error t value Pr(>|t|)
## United.Kingdom_CPI.l1  -0.4063769  0.1915779  -2.121 0.036258 *
## United.Kingdom_gdp.l1   0.4676006  0.0905510   5.164 1.15e-06 ***
## United.Kingdom_unem.l1 -0.2722675  0.1516749  -1.795 0.075519 .
## const                   0.0717401  0.0181165   3.960 0.000137 ***
## trend                  -0.0002512  0.0001831  -1.372 0.173026
## sd1                    -0.0367210  0.0177199  -2.072 0.040687 *
## sd2                     0.0256654  0.0190515   1.347 0.180829
## sd3                    -0.0331039  0.0171664  -1.928 0.056504 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
##
## Residual standard error: 0.06148 on 105 degrees of freedom
## Multiple R-Squared: 0.4357,  Adjusted R-squared: 0.3981
## F-statistic: 11.58 on 7 and 105 DF,  p-value: 7.791e-11
##
##
## Estimation results for equation United.Kingdom_unem:
## ===================================================
## United.Kingdom_unem = United.Kingdom_CPI.l1 + United.Kingdom_gdp.l1 + United.Kingdom_unem.l1
+ const + trend + sd1 + sd2 + sd3
##
##                          Estimate Std. Error t value Pr(>|t|)
## United.Kingdom_CPI.l1   1.577e-01  1.111e-01   1.419  0.15900
## United.Kingdom_gdp.l1  -1.502e-01  5.253e-02  -2.860  0.00512 **
## United.Kingdom_unem.l1  4.321e-01  8.799e-02   4.911 3.35e-06 ***
## const                   1.005e-04  1.051e-02   0.010  0.99239
## trend                  -5.792e-05  1.062e-04  -0.545  0.58672
## sd1                     7.261e-03  1.028e-02   0.706  0.48154
## sd2                    -9.314e-03  1.105e-02  -0.843  0.40134
## sd3                     5.568e-03  9.959e-03   0.559  0.57732
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.03567 on 105 degrees of freedom
## Multiple R-Squared: 0.4526,  Adjusted R-squared: 0.4161
## F-statistic:  12.4 on 7 and 105 DF,  p-value: 1.732e-11
##
##
##
## Covariance matrix of residuals:
##                      United.Kingdom_CPI United.Kingdom_gdp United.Kingdom_unem
## United.Kingdom_CPI           7.044e-04         -0.0001358           8.282e-05
## United.Kingdom_gdp          -1.358e-04          0.0037801          -6.237e-04
## United.Kingdom_unem          8.282e-05         -0.0006237           1.272e-03
##
## Correlation matrix of residuals:
##                      United.Kingdom_CPI United.Kingdom_gdp United.Kingdom_unem
## United.Kingdom_CPI             1.00000            -0.0832             0.08749
## United.Kingdom_gdp            -0.08320             1.0000            -0.28439
## United.Kingdom_unem            0.08749            -0.2844             1.00000
```

Serial Test

```
serial.test(fit1, lags.pt = 8, type = "PT.asymptotic")
```

```
##
##  Portmanteau Test (asymptotic)
##
## data:  Residuals of VAR object fit1
## Chi-squared = 69.418, df = 63, p-value = 0.2702
```

由於serial test的結果不顯著，可知無法拒絕服從white noise process的假設。

Coefficients

```
Acoef(fit1)  #estimated AR coeff matrix
```

```
## [[1]]
##                      United.Kingdom_CPI.l1 United.Kingdom_gdp.l1
## United.Kingdom_CPI              0.3616111            0.03311371
## United.Kingdom_gdp             -0.4063769            0.46760055
## United.Kingdom_unem             0.1576601           -0.15022029
##                      United.Kingdom_unem.l1
## United.Kingdom_CPI              0.09361068
## United.Kingdom_gdp             -0.27226749
## United.Kingdom_unem             0.43211423
```

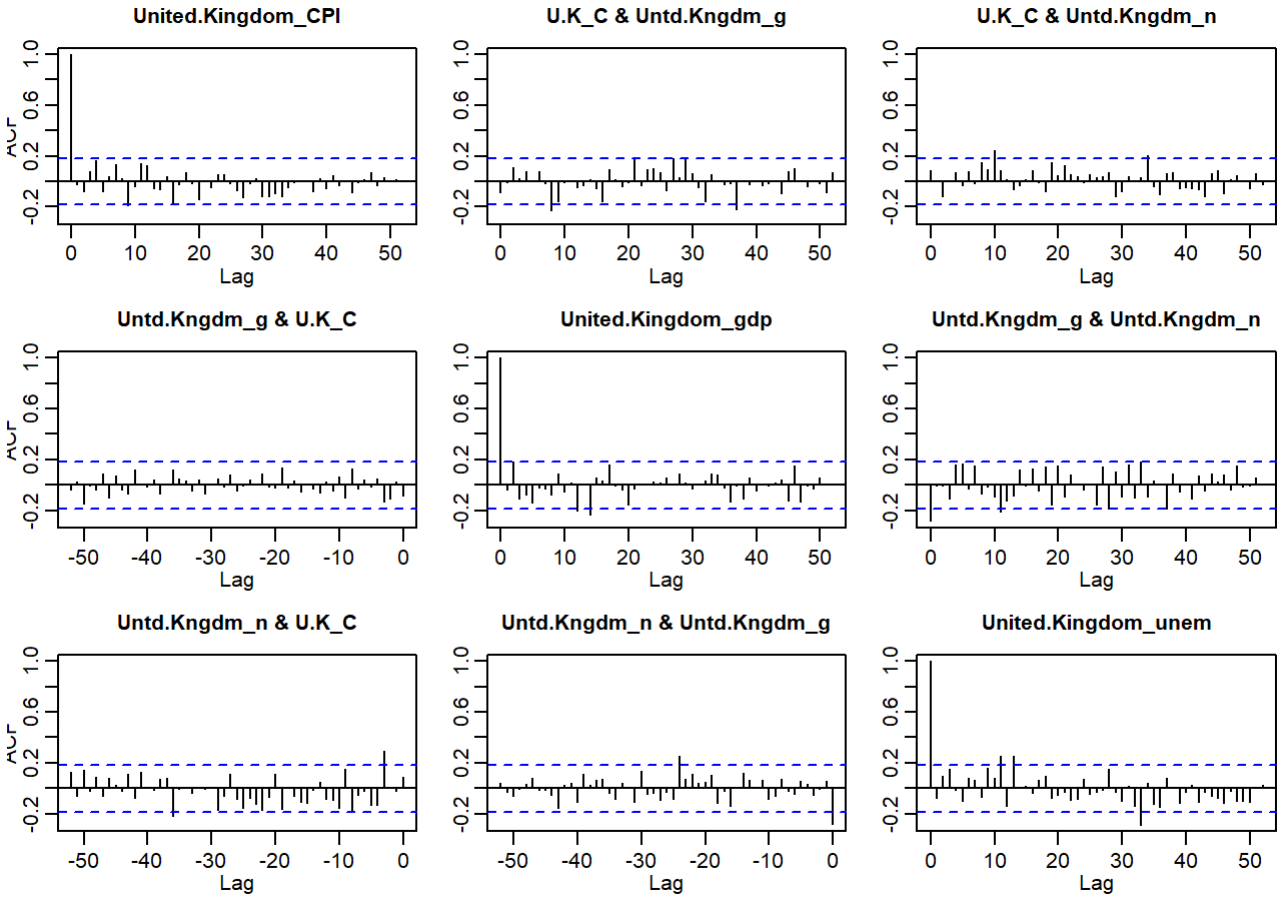
```
round(Bcoef(fit1),5)  #all estimated coeff
```

```
##                      United.Kingdom_CPI.l1 United.Kingdom_gdp.l1
## United.Kingdom_CPI                 0.36161               0.03311
## United.Kingdom_gdp                -0.40638               0.46760
## United.Kingdom_unem                0.15766              -0.15022
##                      United.Kingdom_unem.l1   const    trend      sd1      sd2
## United.Kingdom_CPI                  0.09361 0.02681  0.00005  0.09327 -0.00934
## United.Kingdom_gdp                 -0.27227 0.07174 -0.00025 -0.03672  0.02567
## United.Kingdom_unem                 0.43211 0.00010 -0.00006  0.00726 -0.00931
##                           sd3
## United.Kingdom_CPI    0.04273
## United.Kingdom_gdp   -0.03310
## United.Kingdom_unem   0.00557
```

上述結果即為參數估計的結果。

Residuals

```
fit1$resid = resid(fit1)
acf(fit1$resid,52)
```



由上圖可知，residuals 是 white noise process.

Model plots

```
plot(fit1) #plot fitted values and residuals w/ ACF and PACF
```

Diagram of fit and residuals for United.Kingdom_CPI

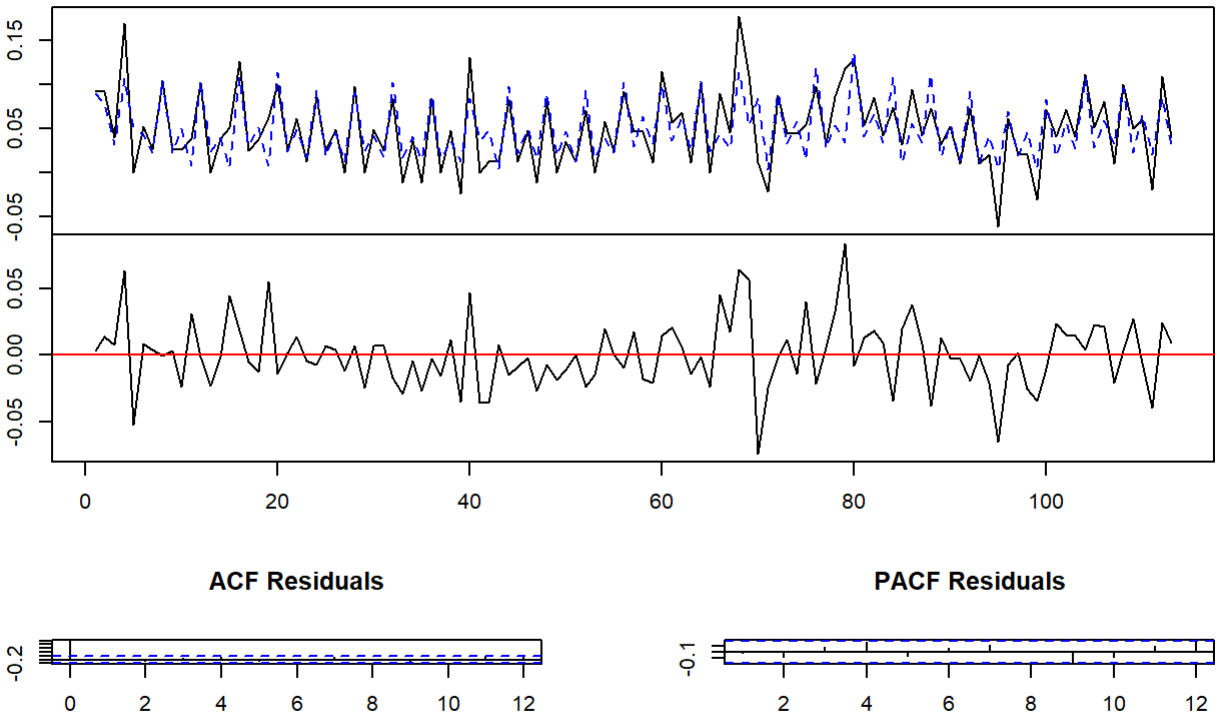


Diagram of fit and residuals for United.Kingdom_gdp

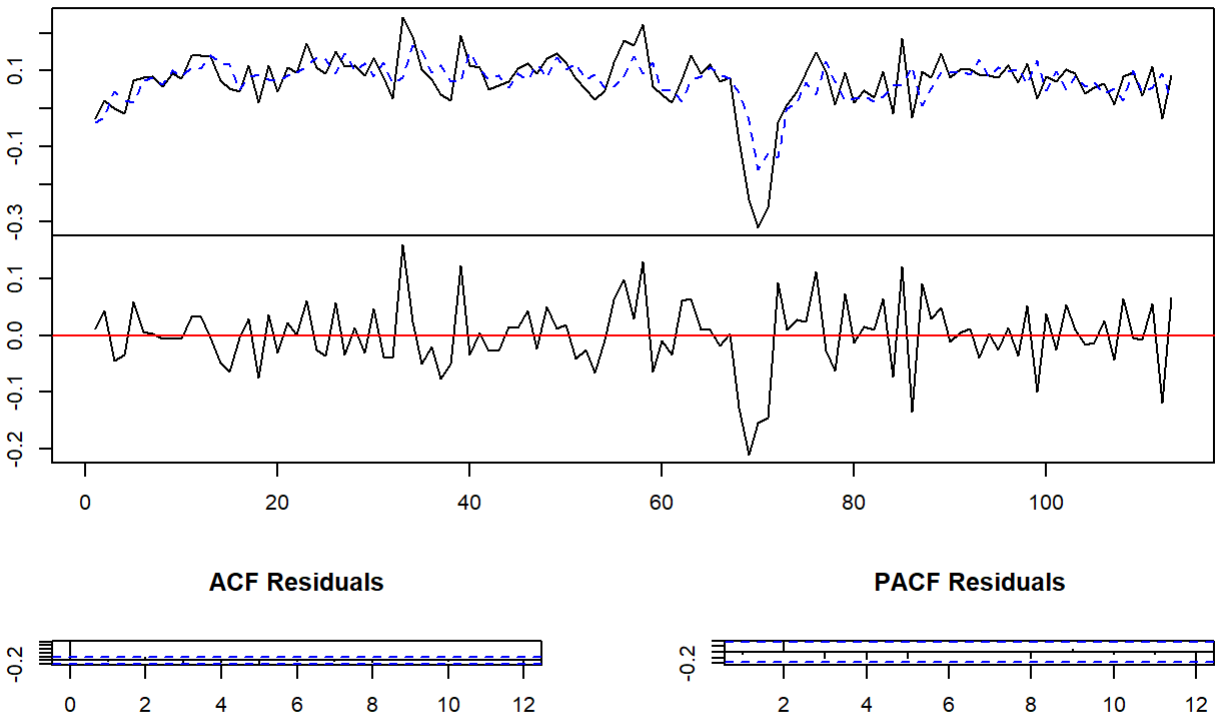
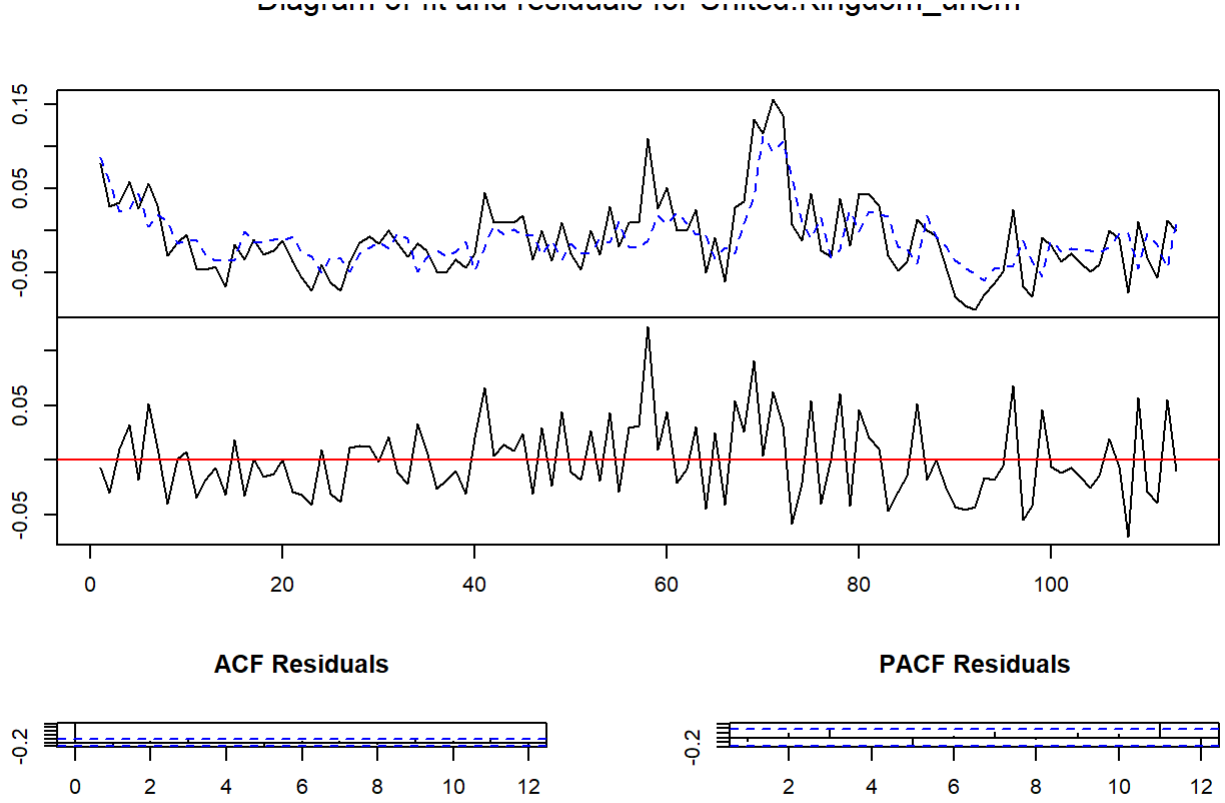


Diagram of fit and residuals for United.Kingdom_unem



結果為正相關。

Causality

```
causality(fit1, cause= "United.Kingdom_CPI")
```

```
## $Granger
##
##  Granger causality H0: United.Kingdom_CPI do not Granger-cause
##  United.Kingdom_gdp United.Kingdom_unem
##
## data:  VAR object fit1
## F-Test = 2.6113, df1 = 2, df2 = 315, p-value = 0.07502
##
##
## $Instant
##
##  H0: No instantaneous causality between: United.Kingdom_CPI and
##  United.Kingdom_gdp United.Kingdom_unem
##
## data:  VAR object fit1
## Chi-squared = 1.2687, df = 2, p-value = 0.5303
```

```
causality(fit1, cause= "United.Kingdom_gdp")
```

```
## $Granger
##
##  Granger causality H0: United.Kingdom_gdp do not Granger-cause
##  United.Kingdom_CPI United.Kingdom_unem
##
## data:  VAR object fit1
## F-Test = 4.6951, df1 = 2, df2 = 315, p-value = 0.009789
##
##
## $Instant
##
##  H0: No instantaneous causality between: United.Kingdom_gdp and
##  United.Kingdom_CPI United.Kingdom_unem
##
## data:  VAR object fit1
## Chi-squared = 8.7856, df = 2, p-value = 0.01237
```

```
causality(fit1, cause= "United.Kingdom_unem")
```

```
## $Granger
##
##  Granger causality H0: United.Kingdom_unem do not Granger-cause
##  United.Kingdom_CPI United.Kingdom_gdp
##
## data:  VAR object fit1
## F-Test = 2.4365, df1 = 2, df2 = 315, p-value = 0.08911
##
##
## $Instant
##
##  H0: No instantaneous causality between: United.Kingdom_unem and
##  United.Kingdom_CPI United.Kingdom_gdp
##
## data:  VAR object fit1
## Chi-squared = 8.8503, df = 2, p-value = 0.01197
```

檢查序列間是否存在granger causality 和 instantaneous causality。舉例而言，觀察United.Kingdom_CPI、United.Kingdom_gdp、United.Kingdom_unem分別對其他兩者的因果關係。

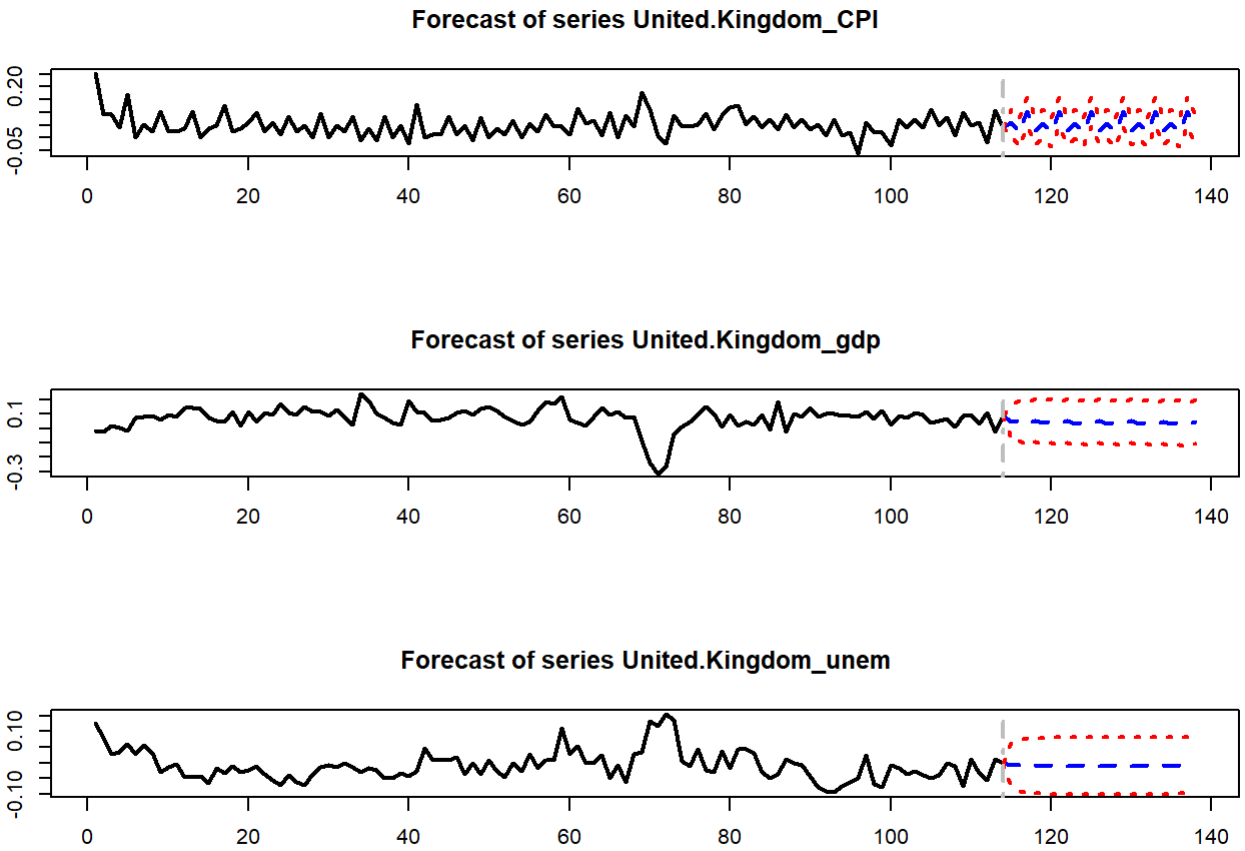
- 由第一個causality的結果可知不能拒絕H0，United.Kingdom_CPI 對於United.Kingdom_gdp和United.Kingdom_unem無顯著相關性，然而他們有當期的相關性。
- 由第二個causality的結果可知可以拒絕H0，United.Kingdom_gdp 對於United.Kingdom_CPI和United.Kingdom_unem有相關性，並且同時有當期的相關性。

- 由第一個causality的結果可知不能拒絕H0，United.Kingdom_unem 對於United.Kingdom_gdp和United.Kingdom_CPI無顯著相關性，然而他們有當期的相關性。

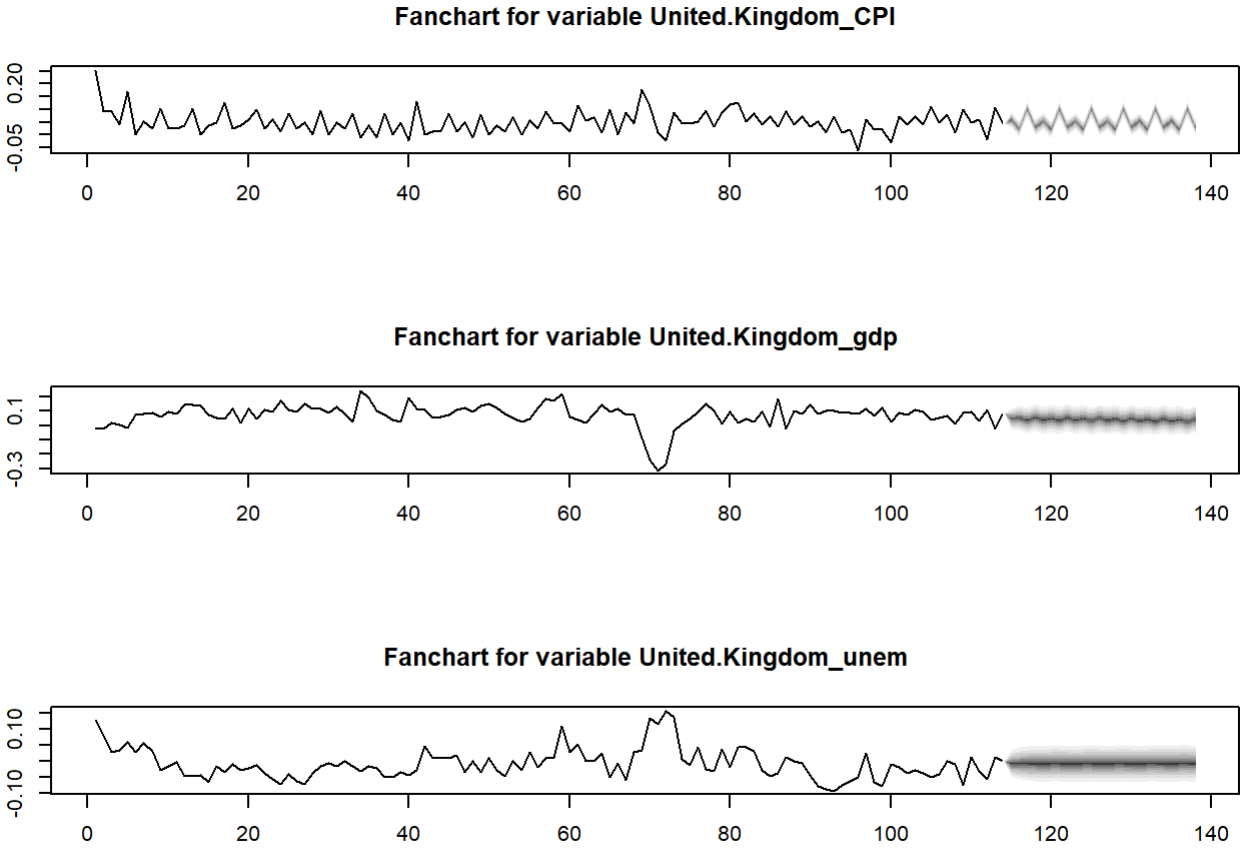
就結果可知，United.Kingdom_CPI及United.Kingdom_unem對其餘時間序列均無granger causality，顯示兩序列的過去值對其餘序列無顯著解釋能力。然而，United.Kingdom_gdp則對其餘序列有顯著解釋能力，即United.Kingdom_gdp這筆序列對其餘序列具有預測能力。

Prediction

```
fit1$pred = predict(fit1, n.ahead = 24, ci = 0.95)
plot(fit1$pred, lwd=2)
```




```
fanchart(fit1$pred)
```



Order=4

```
fit4 = VAR(uk, p=4, type="both",season=4)
summary(fit4)
```

```
##
## VAR Estimation Results:
## =========================
## Endogenous variables: United.Kingdom_CPI, United.Kingdom_gdp, United.Kingdom_unem
## Deterministic variables: both
## Sample size: 110
## Log Likelihood: 642.203
## Roots of the characteristic polynomial:
## 0.8258 0.7649 0.7649 0.6679 0.6679 0.6566 0.6566 0.6336 0.5721 0.5721 0.5224 0.5224
## Call:
## VAR(y = uk, p = 4, type = "both", season = 4L)
##
##
## Estimation results for equation United.Kingdom_CPI:
## ===================================================
## United.Kingdom_CPI = United.Kingdom_CPI.l1 + United.Kingdom_gdp.l1 + United.Kingdom_unem.l1 +
United.Kingdom_CPI.l2 + United.Kingdom_gdp.l2 + United.Kingdom_unem.l2 + United.Kingdom_CPI.l3 +
United.Kingdom_gdp.l3 + United.Kingdom_unem.l3 + United.Kingdom_CPI.l4 + United.Kingdom_gdp.l4 +
United.Kingdom_unem.l4 + const + trend + sd1 + sd2 + sd3
##
##                          Estimate Std. Error t value Pr(>|t|)
## United.Kingdom_CPI.l1   2.961e-01  1.014e-01   2.921  0.00438 **
## United.Kingdom_gdp.l1   4.044e-02  4.683e-02   0.864  0.38999
## United.Kingdom_unem.l1  4.570e-02  7.854e-02   0.582  0.56204
## United.Kingdom_CPI.l2  -4.422e-02  1.049e-01  -0.421  0.67440
## United.Kingdom_gdp.l2   2.866e-02  5.020e-02   0.571  0.56941
## United.Kingdom_unem.l2 -6.384e-02  8.250e-02  -0.774  0.44099
## United.Kingdom_CPI.l3   1.344e-01  1.072e-01   1.253  0.21323
## United.Kingdom_gdp.l3  -1.021e-02  5.042e-02  -0.203  0.83992
## United.Kingdom_unem.l3  6.108e-02  8.347e-02   0.732  0.46619
## United.Kingdom_CPI.l4   1.660e-01  9.628e-02   1.724  0.08797 .
## United.Kingdom_gdp.l4   4.175e-02  4.683e-02   0.892  0.37493
## United.Kingdom_unem.l4  9.138e-02  7.552e-02   1.210  0.22933
## const                   8.287e-03  1.120e-02   0.740  0.46105
## trend                   1.077e-04  8.511e-05   1.266  0.20871
## sd1                     8.924e-02  1.657e-02   5.384 5.43e-07 ***
## sd2                    -1.165e-03  1.002e-02  -0.116  0.90762
## sd3                     5.040e-02  1.675e-02   3.009  0.00337 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.02651 on 93 degrees of freedom
## Multiple R-Squared: 0.6729,  Adjusted R-squared: 0.6166
## F-statistic: 11.96 on 16 and 93 DF,  p-value: 3.004e-16
##
##
## Estimation results for equation United.Kingdom_gdp:
## ===================================================
## United.Kingdom_gdp = United.Kingdom_CPI.l1 + United.Kingdom_gdp.l1 + United.Kingdom_unem.l1 +
United.Kingdom_CPI.l2 + United.Kingdom_gdp.l2 + United.Kingdom_unem.l2 + United.Kingdom_CPI.l3 +
United.Kingdom_gdp.l3 + United.Kingdom_unem.l3 + United.Kingdom_CPI.l4 + United.Kingdom_gdp.l4 +
United.Kingdom_unem.l4 + const + trend + sd1 + sd2 + sd3
```

```
##
##                          Estimate Std. Error t value Pr(>|t|)
## United.Kingdom_CPI.l1  -0.4245142  0.2303546  -1.843 0.068533 .
## United.Kingdom_gdp.l1   0.4148563  0.1064111   3.899 0.000182 ***
## United.Kingdom_unem.l1 -0.3455959  0.1784714  -1.936 0.055852 .
## United.Kingdom_CPI.l2  -0.2325828  0.2384102  -0.976 0.331815
## United.Kingdom_gdp.l2   0.1857235  0.1140757   1.628 0.106893
## United.Kingdom_unem.l2  0.0390147  0.1874675   0.208 0.835595
## United.Kingdom_CPI.l3  -0.1747921  0.2437072  -0.717 0.475035
## United.Kingdom_gdp.l3  -0.2135097  0.1145791  -1.863 0.065558 .
## United.Kingdom_unem.l3 -0.1701933  0.1896794  -0.897 0.371893
## United.Kingdom_CPI.l4   0.0932001  0.2187936   0.426 0.671112
## United.Kingdom_gdp.l4  -0.0138669  0.1064215  -0.130 0.896609
## United.Kingdom_unem.l4  0.2927588  0.1716066   1.706 0.091348 .
## const                   0.0936216  0.0254403   3.680 0.000391 ***
## trend                  -0.0002309  0.0001934  -1.194 0.235639
## sd1                    -0.0458946  0.0376604  -1.219 0.226061
## sd2                     0.0145639  0.0227604   0.640 0.523824
## sd3                    -0.0292472  0.0380614  -0.768 0.444183
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.06024 on 93 degrees of freedom
## Multiple R-Squared: 0.5076,  Adjusted R-squared: 0.4228
## F-statistic: 5.991 on 16 and 93 DF,  p-value: 8.25e-09
##
##
## Estimation results for equation United.Kingdom_unem:
## ====================================================
## United.Kingdom_unem = United.Kingdom_CPI.l1 + United.Kingdom_gdp.l1 + United.Kingdom_unem.l1
+ United.Kingdom_CPI.l2 + United.Kingdom_gdp.l2 + United.Kingdom_unem.l2 + United.Kingdom_CPI.l3
+ United.Kingdom_gdp.l3 + United.Kingdom_unem.l3 + United.Kingdom_CPI.l4 + United.Kingdom_gdp.l4
+ United.Kingdom_unem.l4 + const + trend + sd1 + sd2 + sd3
##
##                          Estimate Std. Error t value Pr(>|t|)
## United.Kingdom_CPI.l1   2.235e-01  1.332e-01   1.678 0.096651 .
## United.Kingdom_gdp.l1  -9.973e-02  6.152e-02  -1.621 0.108348
## United.Kingdom_unem.l1  3.864e-01  1.032e-01   3.745 0.000313 ***
## United.Kingdom_CPI.l2  -2.449e-03  1.378e-01  -0.018 0.985859
## United.Kingdom_gdp.l2  -1.902e-02  6.595e-02  -0.288 0.773622
## United.Kingdom_unem.l2  1.454e-01  1.084e-01   1.341 0.183057
## United.Kingdom_CPI.l3   3.996e-01  1.409e-01   2.837 0.005596 **
## United.Kingdom_gdp.l3  -2.699e-03  6.624e-02  -0.041 0.967589
## United.Kingdom_unem.l3  1.079e-01  1.097e-01   0.984 0.327706
## United.Kingdom_CPI.l4  -2.966e-01  1.265e-01  -2.345 0.021159 *
## United.Kingdom_gdp.l4   4.451e-02  6.152e-02   0.723 0.471237
## United.Kingdom_unem.l4 -8.516e-02  9.921e-02  -0.858 0.392851
## const                  -1.088e-02  1.471e-02  -0.740 0.461457
## trend                  -6.583e-05  1.118e-04  -0.589 0.557470
## sd1                     6.304e-02  2.177e-02   2.896 0.004716 **
## sd2                     8.642e-03  1.316e-02   0.657 0.512918
## sd3                     4.920e-02  2.200e-02   2.236 0.027738 *
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.03483 on 93 degrees of freedom
## Multiple R-Squared: 0.5148,  Adjusted R-squared: 0.4313
## F-statistic: 6.167 on 16 and 93 DF,  p-value: 4.547e-09
##
##
##
## Covariance matrix of residuals:
##                      United.Kingdom_CPI United.Kingdom_gdp United.Kingdom_unem
## United.Kingdom_CPI           0.0007028         -0.0002160           0.0001043
## United.Kingdom_gdp          -0.0002160          0.0036293          -0.0005354
## United.Kingdom_unem          0.0001043         -0.0005354           0.0012129
##
## Correlation matrix of residuals:
##                      United.Kingdom_CPI United.Kingdom_gdp United.Kingdom_unem
## United.Kingdom_CPI              1.0000            -0.1353              0.1130
## United.Kingdom_gdp             -0.1353             1.0000             -0.2552
## United.Kingdom_unem             0.1130            -0.2552              1.0000
```

Serial Test

```
serial.test(fit4, lags.pt = 8, type = "PT.asymptotic")
```

```
##
##  Portmanteau Test (asymptotic)
##
## data:  Residuals of VAR object fit4
## Chi-squared = 30.983, df = 36, p-value = 0.7059
```

由於serial test的結果不顯著，可知無法拒絕服從white noise process的假設。

Coefficients

```
Acoef(fit4)  #estimated AR coeff matrix
```

```
## [[1]]
##                      United.Kingdom_CPI.l1 United.Kingdom_gdp.l1
## United.Kingdom_CPI              0.2961087            0.04044376
## United.Kingdom_gdp             -0.4245142            0.41485629
## United.Kingdom_unem             0.2234930           -0.09973400
##                      United.Kingdom_unem.l1
## United.Kingdom_CPI              0.04570134
## United.Kingdom_gdp             -0.34559590
## United.Kingdom_unem             0.38636363
##
## [[2]]
##                      United.Kingdom_CPI.l2 United.Kingdom_gdp.l2
## United.Kingdom_CPI           -0.044216670            0.02866171
## United.Kingdom_gdp           -0.232582756            0.18572352
## United.Kingdom_unem          -0.002449422           -0.01902442
##                      United.Kingdom_unem.l2
## United.Kingdom_CPI             -0.06383873
## United.Kingdom_gdp              0.03901469
## United.Kingdom_unem             0.14537445
##
## [[3]]
##                      United.Kingdom_CPI.l3 United.Kingdom_gdp.l3
## United.Kingdom_CPI              0.1344138           -0.01021385
## United.Kingdom_gdp             -0.1747921           -0.21350973
## United.Kingdom_unem             0.3996444           -0.00269866
##                      United.Kingdom_unem.l3
## United.Kingdom_CPI              0.06107519
## United.Kingdom_gdp             -0.17019326
## United.Kingdom_unem             0.10789095
##
## [[4]]
##                      United.Kingdom_CPI.l4 United.Kingdom_gdp.l4
## United.Kingdom_CPI             0.16602575            0.04175325
## United.Kingdom_gdp             0.09320006           -0.01386687
## United.Kingdom_unem           -0.29659539            0.04450655
##                      United.Kingdom_unem.l4
## United.Kingdom_CPI              0.09137970
## United.Kingdom_gdp              0.29275881
## United.Kingdom_unem            -0.08516346
```

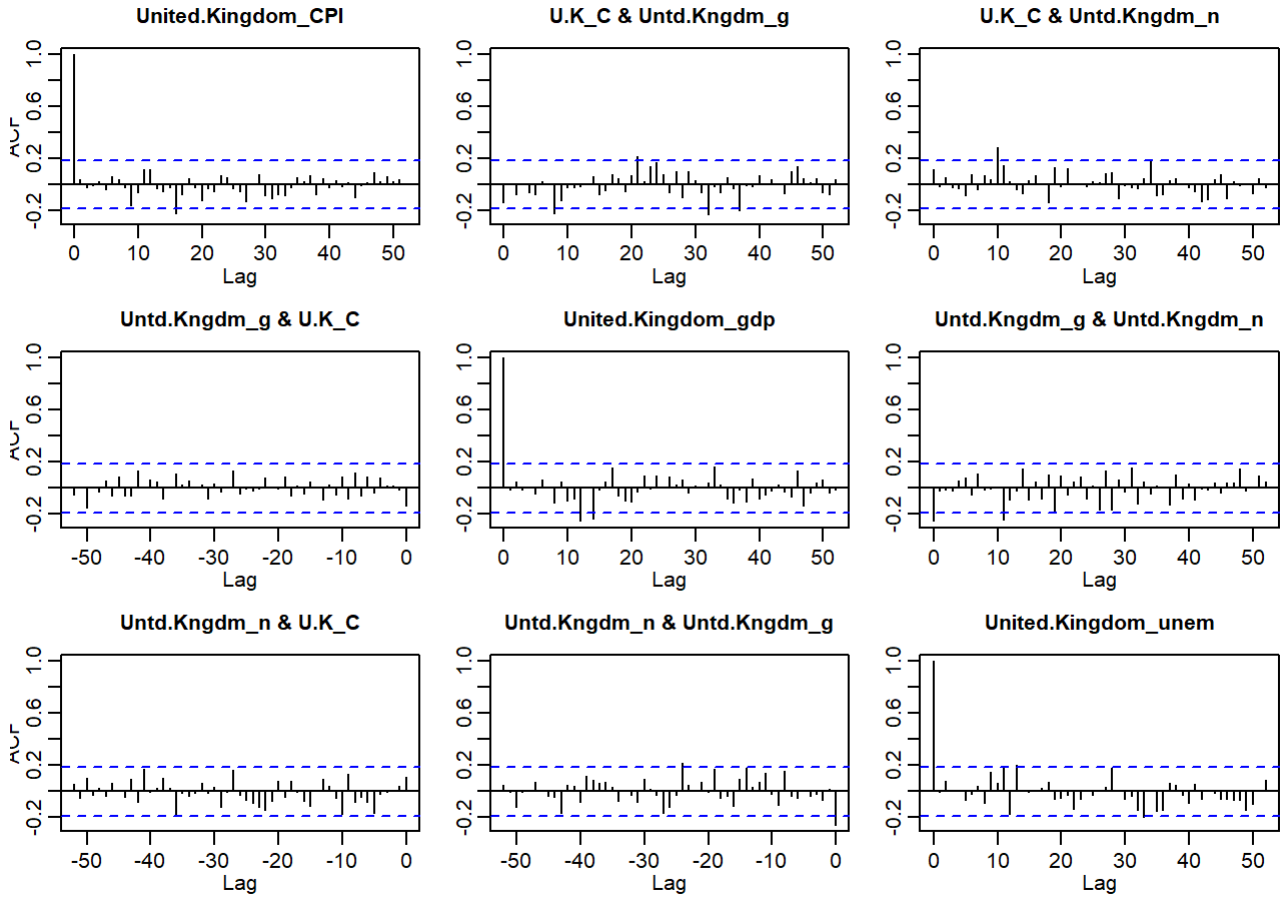
```
round(Bcoef(fit4),5)  #all estimated coeff
```

```
##                      United.Kingdom_CPI.l1 United.Kingdom_gdp.l1
## United.Kingdom_CPI                 0.29611                0.04044
## United.Kingdom_gdp                -0.42451                0.41486
## United.Kingdom_unem                0.22349               -0.09973
##                      United.Kingdom_unem.l1 United.Kingdom_CPI.l2
## United.Kingdom_CPI                  0.04570                -0.04422
## United.Kingdom_gdp                 -0.34560                -0.23258
## United.Kingdom_unem                 0.38636                -0.00245
##                      United.Kingdom_gdp.l2 United.Kingdom_unem.l2
## United.Kingdom_CPI                 0.02866                -0.06384
## United.Kingdom_gdp                 0.18572                 0.03901
## United.Kingdom_unem               -0.01902                 0.14537
##                      United.Kingdom_CPI.l3 United.Kingdom_gdp.l3
## United.Kingdom_CPI                 0.13441                -0.01021
## United.Kingdom_gdp                -0.17479                -0.21351
## United.Kingdom_unem                0.39964                -0.00270
##                      United.Kingdom_unem.l3 United.Kingdom_CPI.l4
## United.Kingdom_CPI                  0.06108                 0.16603
## United.Kingdom_gdp                 -0.17019                 0.09320
## United.Kingdom_unem                 0.10789                -0.29660
##                      United.Kingdom_gdp.l4 United.Kingdom_unem.l4    const
## United.Kingdom_CPI                 0.04175                 0.09138  0.00829
## United.Kingdom_gdp                -0.01387                 0.29276  0.09362
## United.Kingdom_unem                0.04451                -0.08516 -0.01088
##                         trend      sd1      sd2      sd3
## United.Kingdom_CPI    0.00011  0.08924 -0.00117  0.05040
## United.Kingdom_gdp   -0.00023 -0.04589  0.01456 -0.02925
## United.Kingdom_unem -0.00007  0.06304  0.00864  0.04920
```

上述結果即為參數估計的結果。

Residuals

```
fit4$resid = resid(fit4)
acf(fit4$resid,52)
```



由上圖可知，residuals 是 white noise process.

Model plots

```
plot(fit4) #plot fitted values and residuals w/ ACF and PACF
```

Diagram of fit and residuals for United.Kingdom_CPI

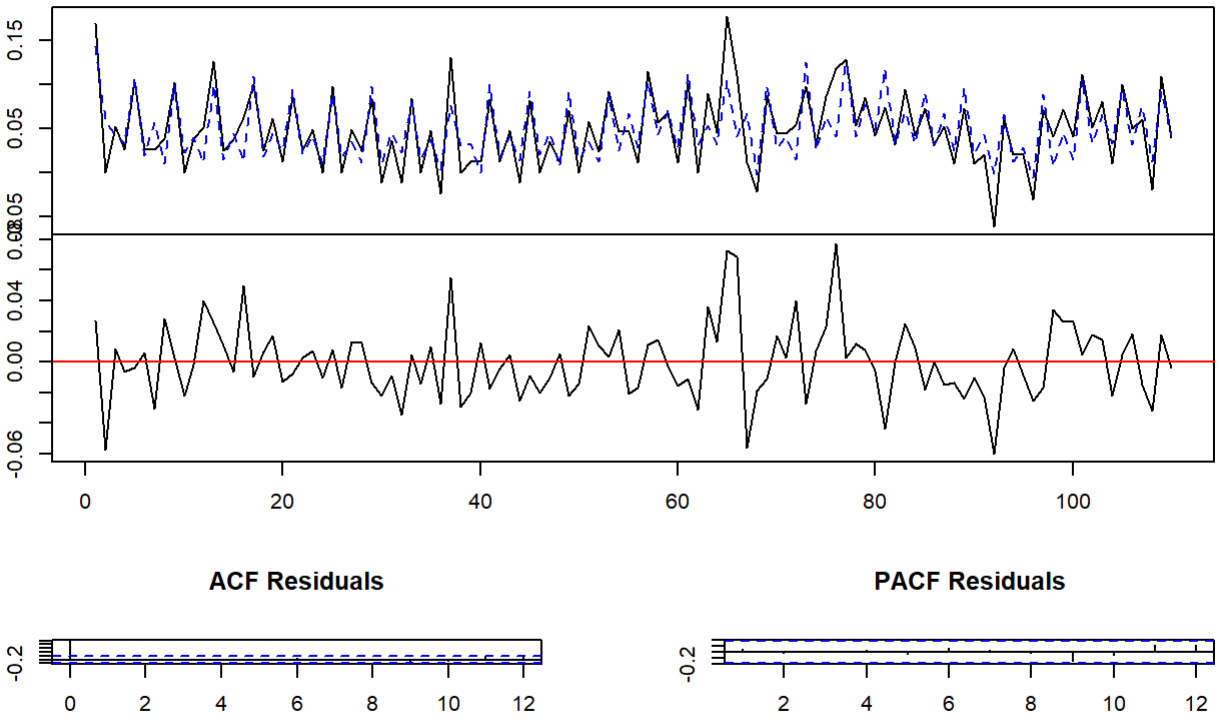


Diagram of fit and residuals for United.Kingdom_gdp

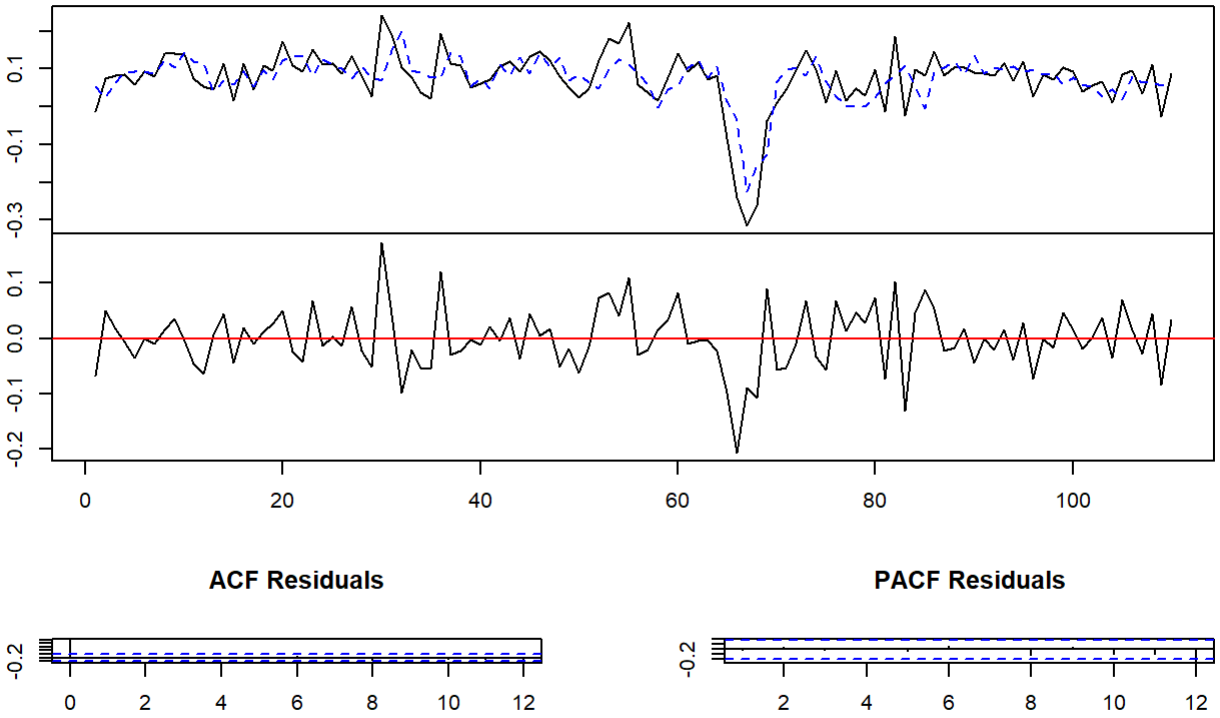
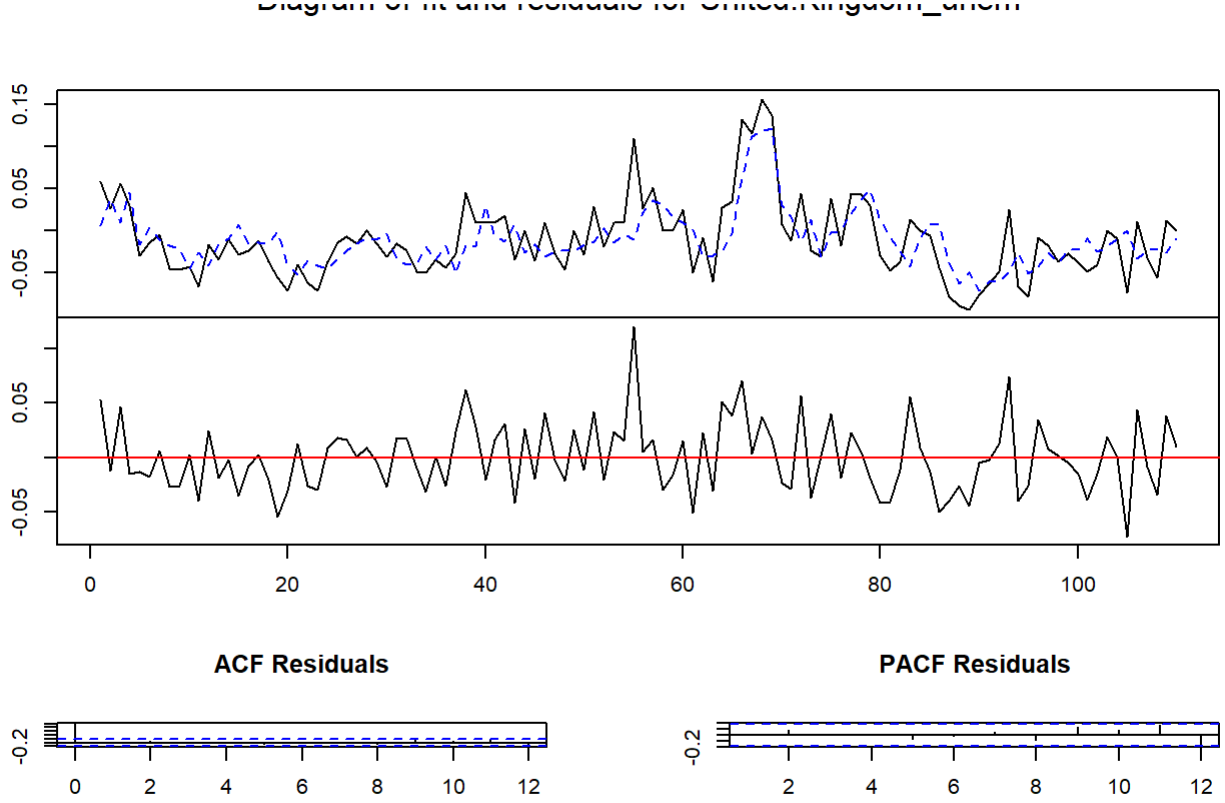


Diagram of fit and residuals for United.Kingdom_unem



結果為正相關。

Causality

```
causality(fit4, cause= "United.Kingdom_CPI")
```

```
## $Granger
##
##  Granger causality H0: United.Kingdom_CPI do not Granger-cause
##  United.Kingdom_gdp United.Kingdom_unem
##
## data:  VAR object fit4
## F-Test = 2.3, df1 = 8, df2 = 279, p-value = 0.02116
##
##
## $Instant
##
##  H0: No instantaneous causality between: United.Kingdom_CPI and
##  United.Kingdom_gdp United.Kingdom_unem
##
## data:  VAR object fit4
## Chi-squared = 2.6701, df = 2, p-value = 0.2631
```

```
causality(fit4, cause= "United.Kingdom_gdp")
```

```
## $Granger
##
##  Granger causality H0: United.Kingdom_gdp do not Granger-cause
##  United.Kingdom_CPI United.Kingdom_unem
##
## data:  VAR object fit4
## F-Test = 1.0008, df1 = 8, df2 = 279, p-value = 0.4356
##
##
## $Instant
##
##  H0: No instantaneous causality between: United.Kingdom_gdp and
##  United.Kingdom_CPI United.Kingdom_unem
##
## data:  VAR object fit4
## Chi-squared = 7.8267, df = 2, p-value = 0.01997
```

```
causality(fit4, cause= "United.Kingdom_unem")
```

```
## $Granger
##
##  Granger causality H0: United.Kingdom_unem do not Granger-cause
##  United.Kingdom_CPI United.Kingdom_gdp
##
## data:  VAR object fit4
## F-Test = 1.3763, df1 = 8, df2 = 279, p-value = 0.2066
##
##
## $Instant
##
##  H0: No instantaneous causality between: United.Kingdom_unem and
##  United.Kingdom_CPI United.Kingdom_gdp
##
## data:  VAR object fit4
## Chi-squared = 7.33, df = 2, p-value = 0.0256
```

檢查序列間是否存在granger causality 和 instantaneous causality並觀察United.Kingdom_CPI、United.Kingdom_gdp、United.Kingdom_unem分別對其他兩者的因果關係。

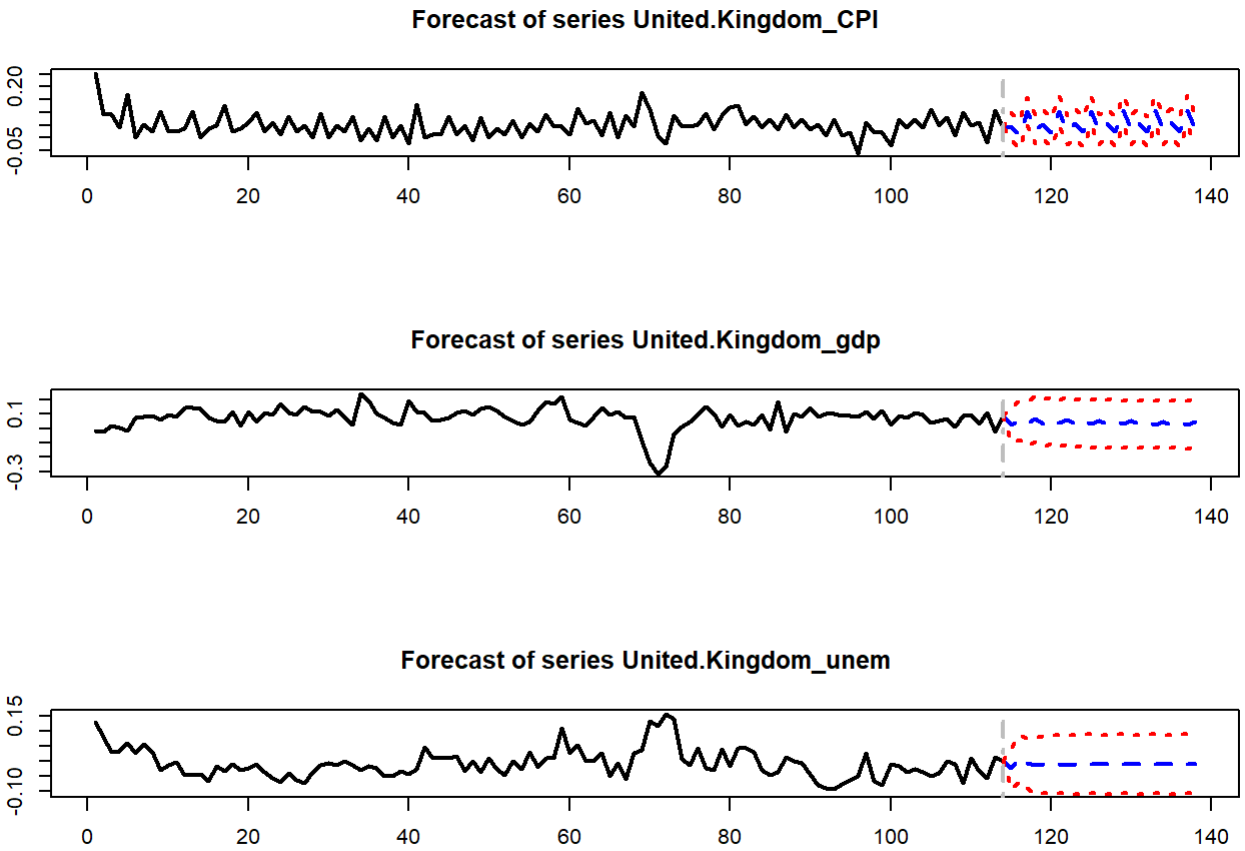
- 由第一個causality的結果可知可以拒絕H0，United.Kingdom_CPI 對於United.Kingdom_gdp和United.Kingdom_unem有相關性，並且也有當期的相關性。
- 由第二個causality的結果可知不能拒絕H0，United.Kingdom_gdp 對於United.Kingdom_CPI和United.Kingdom_unem無顯著相關性，然而他們有當期的相關性。

- 由第一個causality的結果可知不能拒絕H0，United.Kingdom_unem 對於United.Kingdom_gdp和United.Kingdom_CPI無顯著相關性，然而他們有當期的相關性。

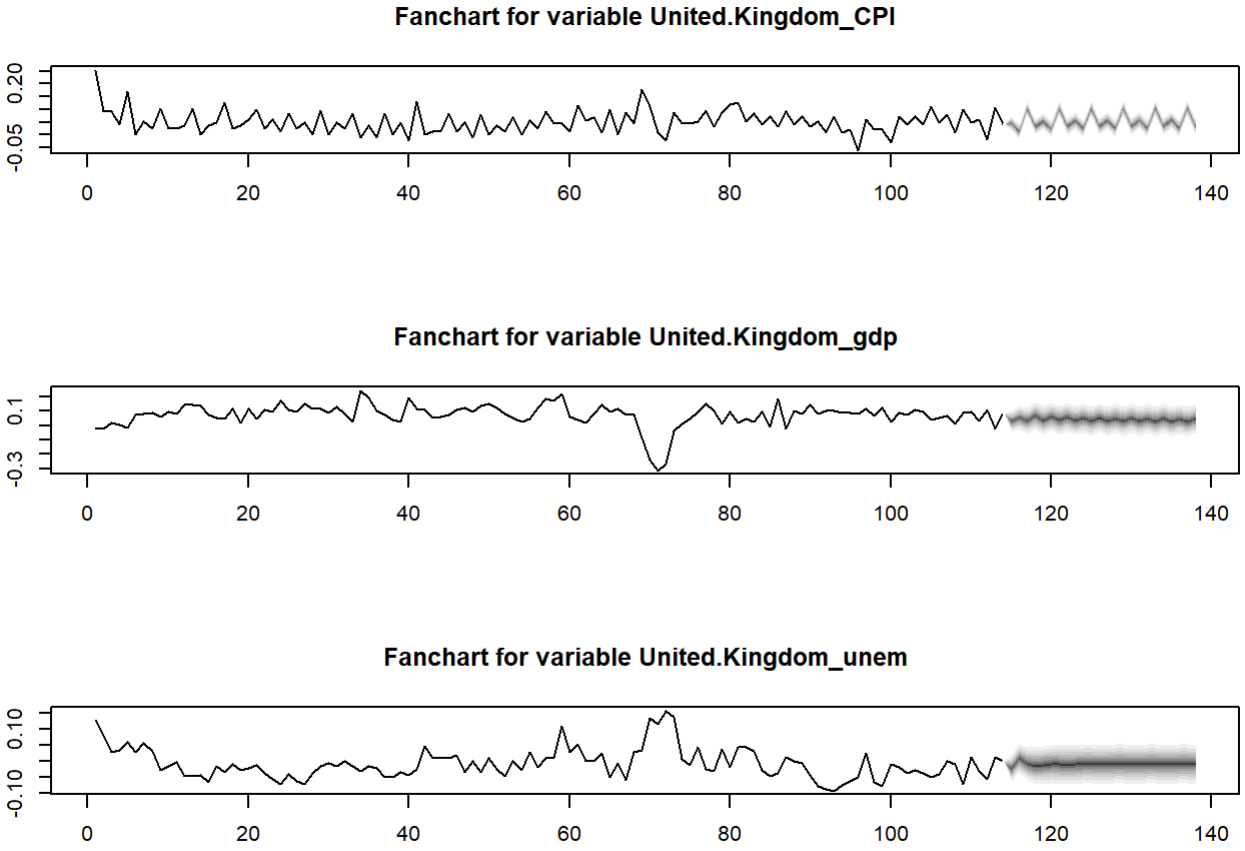
就結果可知，United.Kingdom_gdp及United.Kingdom_unem對其餘時間序列均無granger causality，顯示兩序列的過去值對其餘序列無顯著解釋能力。然而，United.Kingdom_CPI則對其餘序列有顯著解釋能力，即United.Kingdom_gdp這筆序列對其餘序列具有預測能力。

Prediction

```
fit4$pred = predict(fit4, n.ahead = 24, ci = 0.95)
plot(fit4$pred, lwd=2)
```




```
fanchart(fit4$pred)
```



AIC

```
AIC1<-2*9-2*638.761
AIC1
```

```
## [1] -1259.522
```

```
AIC4<-2*17-2*642.203
AIC4
```

```
## [1] -1250.406
```

由於AIC1(-1259.522) < AIC4(-1250.406)，可得知fit1模型的預測能力較好。