# Documentation

# Airborne Wind Energy Trajectory Analysis

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Version 1.0

28 February, 2017

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The software was created in the frame of AWESCO, an Innovative Training Network (ITN) project funded by the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie Action (MSCA), grant agreement No. 642682.

# Contents

# 1 About this software

This software computes, for specified parameters of the kite flying in crosswind conditions, quantities such as the kite velocity, apparent wind, tether force and generated power. The computations are based on the assumption that the state of the kite moves in a constant force equilibrium where the forces exerted by the tether balance out the aerodynamic forces on the kite at every instant. The consequences of this assumption are that there is no acceleration of the kite, no tether drag and no gravitational forces, thus also the mass of the kite does not play a role. Note, that these are assumptions imposing strong simplifications on the dynamics of a kite flying in crosswind conditions. This applies, albeit in different ways, both for rigid wings as well as for soft kites. Nonetheless, since the forces regarded here are in standard operating conditions the dominating forces in crosswind flight, the analysis presented in this software provides a initial understanding of the variations and limits of the quantities of interest. The influence of the variation of parameters both relating to the kite as well as regarding the environmental conditions can be easily and quickly investigated.

The software was written in MATLAB (version R2016b) App Designer and is available open source under the GPL 3.0 license. See also the License Statement at the end of this documentation (Chapter 4).

The code for this software is available upon request. Please send code requests as well as bug reports or any questions to ahbee@control.ee.ethz.ch.

## 1.1 Installation

Double-click on the .mlappinstall file to install the software. After the successful installation an icon appears in the 'Apps' tab in MATLAB named 'AWE Trajectory Analysis v1.0'. The application can be started by clicking on the icon.

# 2 User Interface

## 2.1 Input

In the following, a list of the parameters is given which can be specified by the user in the interface of the software.

**Kite parameters:**

| Input field | Explanation | Units |
|---|---|---|
| Wing area | The area of the kite relevant for the generation of the aerodynamic forces | [$m^2$] |
| Lift coefficient | The constant (average) coefficient related to aerodynamic lift forces typical for the kite under consideration | [ ] |
| Drag coefficient | The constant (average) coefficient related to aerodynamic drag forces typical for the kite under consideration | [ ] |
| Line length | The length of the line (or tether) from the ground station to the kite | [m] |

**Flight Figure**

| Options | Parameterized equation (continuous) |
| --- | --- |
| Lemniscate | $$\theta(t) = \theta_c + m\ \mathsf{sin}(t)\,\mathsf{cos}(t)/[1 + \mathsf{sin}^2(t)]$$ $$\phi(t) = \phi_c + m\ \mathsf{cos}(t)/[1 + \mathsf{sin}^2(t)]$$ where $\theta_c$ and $\phi_c$ are the coordinates of the midpoint of the figure, $m$ is the distance from the midpoint to the leftmost/rightmost point of the figure. |
| Circle | $$\theta(t) = \theta_c + r\ \mathsf{sin}(t)$$ $$\phi(t) = \phi_c + r\ \mathsf{cos}(t)$$ where $\theta_c$ and $\phi_c$ are the coordinates of the midpoint of the figure, $r$ is the radius. |
| Ellipse | $$\theta(t) = \theta_c + a\ \mathsf{sin}(t)$$ $$\phi(t) = \phi_c + b\ \mathsf{cos}(t)$$ where $\theta_c$ and $\phi_c$ are the coordinates of the midpoint of the figure, $a$ is the distance from the focal points to the topmost point and $b$ is the vertical distance from the midpoint to the topmost point. Here, $b$ is fixed to $a/3$. |
| Superellipse | $$\theta(t) = \theta_c + a\,\lvert\mathsf{sin}(t)\rvert^{2/n}\,sign(\mathsf{sin}(t))$$ $$\phi(t) = \phi_c + b\,\lvert\mathsf{cos}(t)\rvert^{2/n}\,sign(\mathsf{cos}(t))$$ where $\theta_c$ and $\phi_c$ are the coordinates of the midpoint of the figure, $a$ is the distance from the focal points to the topmost point and $b$ is the vertical distance from the midpoint to the topmost point. $b$ is fixed to $a/3$. $n$ is the superellipse parameter, the larger $n$ the more the ellipse will be approximating a rectangle. Here, $n$ is set to 3 ($n =$ 2 corresponds to the standard ellipse). |

Note: Flight figures can be interesting when compared in power output and kite velocity, but also in terms of maximum curvature.

**Figure specifics:**

| Multioption | Explanation |
|---|---|
| Flight direction | Upwards - upwards turn of the flight direction of the kite at the outer edges of the lemniscate<br><br>Downwards - downwards turn of the flight direction of the kite at the outer edges of the lemniscate<br><br>Clockwise - flight direction is in clockwise direction around the figure (for circle, ellipse and superellipse)<br><br>Anti-clockwise - flight direction is in anti-clockwise direction around the figure (for circle, ellipse and superellipse) |

| Input field | Explanation | Units |
|---|---|---|
| Center point elevation | The center point coordinate of $\phi$ , $\phi_c$, is fixed to 0, while the center point coordinate of $\theta$, $\theta_c$, which corresponds to the elevation of the figure above the horizon, can be set individually in the feasible range, i.e. the whole figure has to fit into wind window (wind window reaches from $\theta$ = [0°,90°] and $\phi$ = [-180°, 180°]). | [°] |
| Lemniscate/ellipse/superellipse maximum distance, circle radius | Set $m$, $a$ and $r$, respectively, for figure shape defined in Table 'Flight figures'. The minimum value is set to 5° and the maximum value is adjusted such that the whole figure fits into the wind window (wind window reaches from $\theta$ = [0°,90°] and $\phi$ = [-180°, 180°]). | [°] |

**Environment Setup:**

| Input field | Explanation | Units |
|---|---|---|
| Wind velocity | Magnitude of wind velocity. The bound is adjusted such that feasible apparent winds are guaranteed. | [m/s] |
| Wind direction | The wind is assumed to have only horizontal components. If this field is set 0, the wind direction is aligned with the vector to the $\phi$=0 and $\theta$=0 coordinate. The bounds are adjusted such that feasible apparent winds are guaranteed. | [°] |
| Reel-out velocity | Tether reel-out velocity. Note that over one cycle the tether is kept constant at the previously defined tether length and the reel-out is assumed to apply starting from that length at each instant. The bounds are adjusted such that feasible apparent winds are guaranteed. Note also that the upper bounds will always be integer numbers. | [m/s] |

**Recommended changes lamp:** This lamp turns yellow when parameters in the input field of the panels 'Figure specifics' and 'Environment setup' are changed in a way that the total set of parameters no longer results in feasible flight conditions. The reel-out velocity is then automatically reduced to the maximum feasible value, the light turns on and the warning appears in order to signal the change to the user. The warning stays off and the light stays green when the specified flight conditions are feasible.

**Plot setup:**

| Input field | Explanation |
|---|---|
| Discretization points | The parametric equations in the Table 'Flight Figure' are discretized by the the number specified in this input field. |
| Plotted quantity | Choice of plotted quantity between: magnitude of kite (tangential) velocity in [m/s], magnitude of the apparent wind in [m/s], tether force in [N], generated power [W] (this quantity is only non-zero if the reel-out velocity is set to zero, see Chapter 3). The kite (tangential) velocity equals the kite velocity when reel-out is set to zero. Since the tangential velocity of the kite is perpendicular to the reel-out velocity, the total kite velocity can be computed simply by the root of the sum of the squares of both components (see Section 3.2). |

## 2.2 Output

**Flight feasibility lamp:** This lamp turns yellow after any change of input in the interface. Upon pressing the run button, the lamp turns green and the plot appears if the parameters specified result in feasible flight conditions. The lamp turns red and no plot appears if the parameters do not result in feasible flight conditions (e.g. the lift coefficient is too low for the minimum bounds to be met. Note that in the panels 'Flight specifics' and 'Environment setup' the bounds are adjusted such that only feasible values can appear in the associated input fields, see also the comment on the 'recommended changes lamp').

**Power output:**

| Output field | Explanation | Units |
|---|---|---|
| Average generated power per cycle | The average power generated over the whole cycle and its standard deviation. The average and standard deviation are computed by weighting the instantaneous power at the regarded and plotted instants by the individual amount of time relating to the each instant. This time is obtained by computing the path length between the individual instants and dividing this quantity by the kite velocity corresponding to each instant (see Chapter 3 for more details and formula). This quantity is zero if the reel-out is zero (see Chapter 3). | [W] |
| Total generated energy per cycle | The sum of the instantaneous power outputs multiplied by the time corresponding to each instant (see 'Average generated power per cycle') over the whole cycle. This quantity is zero if the reel-out is zero (see Chapter 3). | [Wh] |

**Data output file:** Upon pressing the run bottom and conditional on the feasibility of the specified parameters, a data file named 'dataset_AWETrajectoryAnalysis.mat' is generated and saved in your local MATLAB folder. The data file contains the values of the following input fields and output fields that were specified at the time of pressing the run button, and the following computed quantities:

'Version' - current version of the software used to produce the output file.

'Date' - date in the format dd/mm/yyyy.

'Time' - time in the format hh:mm:ss.

'A' - Wing area in [$m^2$].

'Cl' - Lift coefficient in [1].

'Cd' - Drag coefficient in [1].

'Linelength' - Line length in [m].

'windMag' - Wind velocity in [m/s].

'windAngle' - Wind direction in [$^\circ$].

'Figure' - a string showing the selected figure among the options 'Lemniscate', 'Circle', 'Ellipse' and 'Superellipse.

'flightDirection' - a string showing the selected direction of flight among the options 'Upwards', 'Downwards' for the lemniscate and 'Clockwise', 'Anti-clockwise' for the other figure choices.

'centerElevation' - Center point elevation in [$^\circ$].

'maxDistance' - Lemniscate/Ellipse/Superellipse max. Distance or Circle radius, in [$^\circ$].

'reeloutVel' - Reel-out velocity in [m/s].

'theta' - vector of length N, where N s the number of the discretization steps specified, containing the $\theta$ coordinates of each instance over the whole cycle, in [$^\circ$].

'phi' - vector of length N, where N is the number of the discretization steps specified, containing the $\phi$ coordinates of each instance over the whole cycle, in [$^\circ$].

'heading' - vector of length N, where N is the number of the discretization steps specified, containing the heading angle of the kite at each instance over the whole cycle, in [$^\circ$].

'kiteTanVel' - vector of length N, where N is the number of the discretization steps specified, containing the magnitude of the tangential velocity of the kite at each instance over the whole cycle, in [m/s].

'appVelVec' - Nx3-matrix, where N is the number of the discretization steps specified, containing the 3-D vector of the apparent wind velocity at the kite at each instance over the whole cycle, in [m/s].

'timePath' - vector of length N, where N is the number of the discretization steps specified, containing the flight time related to each instant. Since the kite velocity is not constant over the cycle and the path pieces are not equally long, the time associated with each instant varies. Units are [s].

‘tetherForce’ - vector of length N, where N is the number of the discretization steps specified, containing the tether force at each instance over the whole cycle, in [N].

‘genPower’ - vector of length N, where N is the number of the discretization steps specified, containing the generated power at each instant, in [W] .

‘meanPower’ - power generated on average over the whole cycle, in [W].

‘stdPower’ - standard deviation of the average power generated over the whole cycle, in [W].

‘totalEnergy’ - total energy generated over the whole cycle, in [Wh]

# 3 Background physics

This chapter states the physical laws underlying the computations performed within the software. Vectors and matrices are denoted by bold letters. Most of the equations and derivations presented here can be found in [1] and [3].

Note, as mentioned in Chapter 1 already, that the model derivations presented here are based on simplifications of the dynamics underlying the crosswind flight of power kite systems. In crosswind conditions, the dominating forces are the aerodynamic lift and drag forces on the kite. For light kites, such as flexible wings, these forces can be assumed to form an equilibrium with the tether forces at all times, meaning that the state of the kite changes in instantaneous transitions from one aerodynamic equilibrium to the other. The underlying assumptions for these dynamics are that gravitational forces are negligible, there are no inertial forces acting on the kite, as well as no acceleration of the kite (mass doesn't play a role), and further that the tether is a straight and massless line not subject to aerodynamic drag.

## 3.1 Coordinate system transformations

There are two coordinate systems relevant for the analysis applied in the software. The first one is the Cartesian coordinate system $(x, y, z)$ with the origin located at the ground station. Variables expressed in this coordinate system are denoted by the superscript 'c'.

The second coordinate system is the spherical coordinate system $(r, \theta, \phi)$ with its origin located at the ground station. If a variable has no superscript it is expressed in this coordinate system.

Both coordinate systems are sketched in Figure 3.1.

The transformation matrix from the Cartesian to the spherical coordinate system is denoted by $\mathbf{A}_{sc}$, where

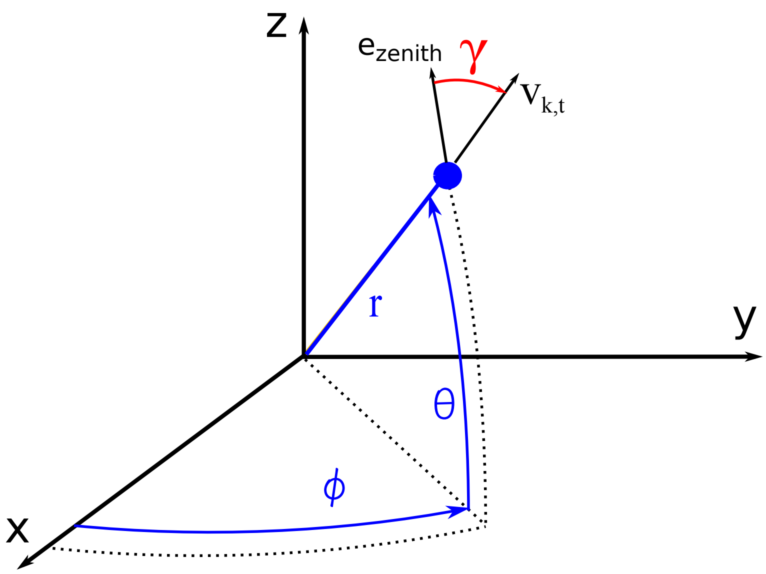


**Figure 3.1:** Cartesian $(x, y, z)$ and spherical $(r, \theta, \phi)$ coordinate system. The ground station is located at the origin of the coordinate system and the kite is marked by the blue dot. The vector $e_{zenith}$ is pointing towards the $z$-axis and lies in the tangential plane to the sphere of radius r located at the position of the kite. The angle $\gamma$ between the vector pointing towards the zenith and the velocity vector of the kite defines the heading angle of the kite, thus suggests that $\gamma$ is the direction of flight.

$$\mathbf{A}_{sc} = \begin{bmatrix} \cos\theta\ \cos\phi & \cos\theta\ \sin\phi & \sin\theta \\ -\sin\theta\ \cos\phi & -\sin\theta\ \sin\phi & \cos\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix}. \tag{3.1}$$

## 3.2 Wind

In the Cartesian coordinate system, the wind vector $\mathbf{v}_w$ lies in the x-y plane, where the angle $\phi_w$ between the vector and the x-axis is specified by the input field 'Wind direction'.

$$\mathbf{v}_w^c = \begin{bmatrix} \cos\phi_w \\ \sin\phi_w \\ 0 \end{bmatrix} v_w \tag{3.2}$$

$v_w$ is the magnitude of the wind velocity, specified in the input field 'Wind velocity'. The wind is assumed to be constant as a function of the angles $\theta$ and $\phi$

The apparent wind at the kite is defined by

$$\mathbf{v}_a = \mathbf{v}_w - \mathbf{v}_k = \mathbf{A}_{sc}\mathbf{v}_w^c - \mathbf{v}_k \tag{3.3}$$

where $\mathbf{v}_k$ is the kite velocity vector,

$$\mathbf{v}_k = \mathbf{v}_{k,r} + \mathbf{v}_{k,t} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} v_{k,r} + \begin{bmatrix} 0 \\ \cos\gamma \\ \sin\gamma \end{bmatrix} v_{k,t} \tag{3.4}$$

$\gamma$ is the heading angle (see also Figure 3.1), defined as in [2] by

$$\gamma = \arctan\left(\frac{\dot{\phi}\cos\theta}{\dot{\theta}}\right) \tag{3.5}$$

Equation (3.5) is approximated in the discrete computations (see also Section 3.5) by

$$\gamma(k) = \arctan\left(\frac{\Delta\phi(k)\cos\theta(k)}{\Delta\theta(k)}\right) \tag{3.6}$$

where $\Delta$ stands for the change of the coordinate over the interval spanned by the two midpoints enclosing the discretization point $k$, where $k = 1, ...., N$ and $N$ is specified in the input field 'Discretization points'.

The heading angle of the kite is zero when the kite points towards the zenith and increases when the kite traverses the trajectory in anti-clockwise direction when looked at the kite from the ground station. $v_{k,r}$ is the reel-out velocity specified in the input field 'Reel-out velocity' and $v_{k,t}$ is the kite tangential velocity, which is one of the output quantities of the software. The kite tangential velocity is the component of the kite velocity vector which lies in the plane tangential to the wind window at the location of the kite. In the following, the derivation of the tangential velocity is presented.

## 3.3 Kite velocity derivation

While the radial component of the kite velocity is simply given by the reel-out velocity $v_{k,r}$ which is specified by the user in the input field 'Reel-out velocity', the tangential component of the kite velocity results from the conditions of a prevailing force balance.

In an aerodynamic equilibrium, as it is assumed here to prevail at all times, the force balance is written as

$$\mathbf{F}_T = \mathbf{F}_L + \mathbf{F}_D \tag{3.7}$$

where $\mathbf{F}_T$ is the force exerted by the tether, $\mathbf{F}_L$ is the lift force and $\mathbf{F}_D$ is the drag force acting on the kite.

The two aerodynamic forces $\mathbf{F}_L$ and $\mathbf{F}_D$ are found by:

$$\mathbf{F}_L = 0.5\,\rho\,A\,C_L\,||\mathbf{v}_a||^2 \tag{3.8}$$
$$\mathbf{F}_D = 0.5\,\rho\,A\,C_D\,||\mathbf{v}_a||^2 \tag{3.9}$$

where $\rho$ is the density of the air, $A$ is the wing area specified in the input field 'Wing area', $C_L$ is the aerodynamic lift coefficient specified in the input field 'Lift coefficient' and $C_D$ is the aerodynamic drag coefficient specified in the input field 'Drag coefficient'.

Like the kite velocity vector, the apparent wind vector at the kite can be split into a vector containing the radial component $\mathbf{v}_{a,r}$ and a vector containing the tangential component $\mathbf{v}_{a,t}$.

From the following points:

- assume the tether is straight at all times: $\mathbf{F}_T \,||\, \mathbf{v}_{a,r}$
- the definition of the drag force: $\mathbf{F}_D \,||\, \mathbf{v}_a$
- the inherent definitions: $\mathbf{F}_D \perp \mathbf{F}_L$ and $\mathbf{v}_{a,t} \perp \mathbf{v}_{a,r}$

a geometric similarity for the angle $\beta_1$, defined as the angle between $\mathbf{v}_{a,r}$ and $\mathbf{v}_a$ and the angle $\beta_2$, defined as the angle between $\mathbf{F}_T$ and $\mathbf{F}_D$ can be found, such that:

$$\beta_1 = \beta_2, \quad \text{where} \;\; \beta_1 = \arctan\left(\frac{v_{a,t}}{v_{a,r}}\right), \beta_2 = \arctan\left(\frac{F_L}{F_D}\right) \tag{3.10}$$

From this follows

$$\frac{v_{a,t}}{v_{a,r}} = \frac{F_L}{F_D} = G \tag{3.11}$$

where $G$ is a constant which is also called the 'glide ratio' of the wing. The last equality in equation (3.11) results from inserting equations (3.8) into the second term in equation (3.11).

Finding the magnitude of the radial component $v_{a,r}$ as the first component of the apparent wind vector defined in equation (3.3) and writing the terms out:

$$v_{a,r} = [v_w \cos\theta \,(\cos\phi \, \cos\phi_w + \sin\phi \, \sin\phi_w) - v_{k,r}]. \tag{3.12}$$

From equation (3.11) follows

$$v_{a,t} = [v_w \cos\theta \,(\cos\phi \, \cos\phi_w + \sin\phi \, \sin\phi_w) - v_{k,r}]G \tag{3.13}$$

and

$$v_a = (v_{a,r}^2 + v_{a,t}^2)^{1/2} = [v_w \cos\theta \,(\cos\phi \, \cos\phi_w + \sin\phi \, \sin\phi_w) - v_{k,r}](1+G^2)^{1/2}. \tag{3.14}$$

For the tangential component of $\mathbf{v}_a$ we can find, next to equation (3.13), an additional expression directly from equation (3.3).

$$\mathbf{v}_{a,t} = \begin{bmatrix} -(\cos\phi \, \cos\phi_w - \sin\phi \, \sin\phi_w) \, \sin\theta \, v_w - \cos\gamma \, v_{k,t} \\ (\cos\phi \, \sin\phi_w - \cos\phi_w \, \sin\phi) \, v_w - \sin\gamma \, v_{k,t} \end{bmatrix}. \tag{3.15}$$

Equation (3.13) and taking the magnitude of equation (3.15) are both expressions for the magnitude of the tangential component of the apparent wind. Taking the square of both expressions and equaling them with each other allows us to solve the resulting expression for the kite tangential velocity squared. Since the kite tangential velocity has to be positive, the positive solution of the two resulting solutions is the one we are looking for. (If none of the two solutions is positive it means that the specified parameters do not result in feasible flight conditions.)

## 3.4 Tether force and generated power

The tether force results from the equation (3.7). Since $\mathbf{F}_L$ and $\mathbf{F}_D$ are perpendicular to each other, we find for the magnitude of the tether force, $F_T$,

$$F_T = (||\mathbf{F}_D||^2 + ||\mathbf{F}_L||^2)^{1/2} \tag{3.16}$$

Inserting equations (3.8) into equation (3.16) results in

$$F_T = 0.5\,\rho\,A\,(C_l^2 + C_d^2)^{1/2}\,||\mathbf{v}_a||^2. \tag{3.17}$$

The power $P$ generated by the kite can then be computed by

$$P = F_T \cdot v_{k,r} \tag{3.18}$$

## 3.5 Path length, flight time and quantities regarding the whole cycle

The parametric equations for the flight figures (see Section 2.1) are discretized by taking equal steps of the parameter $t$. The above derived quantities are then computed for each discrete point, named instances. Additionally, the coordinates for the midpoints between the discretization points are computed in order to achieve better accuracy when computing the path pieces (see equation (3.19)) and the heading (see equation (3.6)). In order to compute averages over the whole cycle or total quantities for the whole cycle, like the generated energy, the path piece relating to each instance has to be evaluated to weigh the quantities computed at the instances.

We use an approximation for the path piece $s(k)$ associated with each instance:

$$s(k) = r((\Delta\theta(k))^2 + \cos^2\theta(k)(\Delta\phi(k))^2)^{1/2}. \tag{3.19}$$

where $r$ is the line length specified in the 'Line length' input field. $\Delta$ stands for the change of the coordinate over the interval spanned by the two midpoints enclosing the discretization point $k$ where $k = 1, ...., N$ and $N$ is the total amount of steps defined by the user in the input field 'Discretization points'.

The flight time $t_s(k)$ related to every path piece $s(k)$ is then found by

$$t_s(k) = s(k)/v_{k,t}(k) \tag{3.20}$$

We then obtain for the generated energy $E(k)$ over each path piece:

$$E(k) = P(k) \cdot t_s(k) \tag{3.21}$$

The total energy generated over the whole cycle is computed by

$$E_{tot} = \sum_{k=1}^{N} E(k)^1 \tag{3.22}$$

For the average power $\bar{P}$ generated over one cycle we obtain

$$\bar{P} = \frac{P(k) \cdot t_s(k)}{\sum_{k=1}^{N} t_s(k)} \tag{3.23}$$

[1]In order to obtain energy in the units of [Wh] the resulting value for $E_{tot}$ is divided by 3600.

# 4 Copyright and License

# Bibliography


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[3] Loyd, M. L.: Crosswind kite power. Journal of Energy, vol. 4, 1980.