

# **Circuit Theory and Electronics Fundamentals**

## **Lab 1 - Circuit Analysis Methods**

### **Aerospace Engineering**

Laboratory Report

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# 1 Introduction

This report is being made for the subject of Circuit Theory and Electronics Fundamentals and is related to the second laboratory being its objective to study an RC circuit containing seven resistors (from  $R_1$  to  $R_7$ ), one sinusoidal voltage source ( $v_s$ ), one capacitor ( $C$ ), one current controlled voltage source ( $V_d$ ) and one voltage controlled current source ( $I_b$ ). The four elementary meshes are named after the current to which they are attributed, and the nodes are named after the numbers attributed to them, being  $V_0$  the ground node.

The current controlled voltage source  $V_d$  is calculated by multiplying  $K_d$  with the current  $I_d$ , whereas the voltage controlled current source  $I_b$  can be determined by multiplying  $K_b$  with the voltage source  $V_b$ .

The display of this circuit, as well as the equations used to determine the value of  $v_s$ , can be seen in Figure 1.

In Section 2 the circuit will be analysed theoretically with the aid of Octave, analysing firstly the circuit for  $t < 0$  using the nodal method, calculating the equivalent resistance  $R_{eq}$  as seen from the capacitor terminals, determining the natural and forced solution for  $V_6$  with the previous results, and finishing with the calculation of the frequency response for  $V_c$ ,  $v_s$  and  $V_6$  and the study of these results.

Secondly, in Section 3 it will be simulated the circuit using ngspice, with the aim of validating the results previously obtained by doing operating point, transient and frequency analysis.

Following with both results from Section 2 and Section 3 being compared and commented in Section 4

The conclusions of this study are outlined in Section 5.

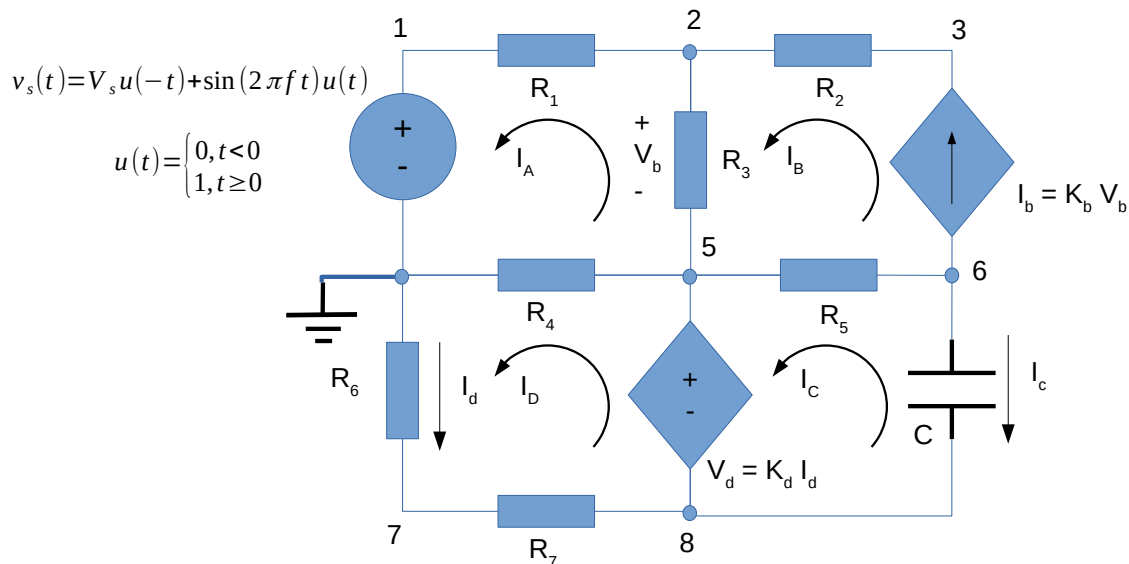


Figure 1: Circuit in analysis

Where:

Name	Value [A or V]
R1	1.034315078330000e+03
R2	2.028530907310000e+03
R3	3.146205063300000e+03
R4	4.034385474550000e+03
R5	3.121700422140000e+03
R6	2.071163796460000e+03
R7	1.015977530930000e+03
Vs	5.156959346000000e+00
C	1.014556835690000e-06
Kb	7.149794119600000e-03
Kd	8.125936425850000e+03

Table 1: Results obtained with t2 datagen python script

The units of the elements whose name starts with  $R$  (the resistors) are  $\Omega$  (ohm),  $V_s$  is expressed in  $V$  (volts) and  $C$  is given in  $F$  (farad). While  $K_b$  is given in  $S$  (siemens),  $K_d$  is also given in  $\Omega$ .

These values were obtained using the Python script using the lowest student number on our group - 95785.

## 2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, with the Nodal Analysis Method, which uses node voltages as the circuit variables.

### 2.1 Node analysis for $t < 0$

The aim of using this method is to determine every node voltage, therefore we considered the node 0 a reference node. However, due to the existence of the independent voltage source  $v_s$  and the current controlled voltage source  $V_d$ , it is useless to analyse nodes 0 and 1 (connected to  $v_s$ ) and also nodes 5 and 8 (connected to  $V_d$ ) since nodes connected to voltage sources can't be analysed. So, this means that we can only analyse nodes 2, 3, 6 and 7.

In order to determine all the unknown node voltage values, it is necessary to have eight linearly independent equations. Before  $t = 0s$ ,  $v_s$  is constant, which means that the capacitor is assumed to also be constant and fully charged, behaving like an open-circuit, therefore  $I_c = 0$ .

Four of the needed equations are given by the nodal analysis:

Node 2

$$(V_3 - V_2)G_2 - (V_2 - V_5)G_3 - (V_2 - V_1)G_1 = 0 \quad (1)$$

Node 3

$$(V_2 - V_5)K_b - (V_3 - V_2)G_2 = 0 \quad (2)$$

Node 6

$$(V_2 - V_5)K_b - (V_6 - V_5)G_5 = 0 \quad (3)$$

Node 7

$$-V_7G_6 - (V_7 - V_8)G_8 = 0 \quad (4)$$

We still need another four equations. For this reason, we can use these two trivial equations

$$V_0 = 0 \quad (5)$$

$$V_1 = V_s \quad (6)$$

We can also use the fact that  $V_5 - V_8 = V_d = K_d I_d$  and  $I_d$ , according the Ohm's Law, is equal to  $G_7(V_0 - V_7)$  which means

$$-V_7G_6K_d = V_5 - V_8 \quad (7)$$

At last, since there was still missing an equation, we considered a super node containing the branch that includes  $v_s$

$$(V_2 - V_1)G_1 + V_5G_4 + V_7G_6 = 0 \quad (8)$$

The system of equations that will be solved in form of matrix and with the assistance of Octave is:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_1 & -G_1 - G_2 - G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & 0 & K_b + G_2 & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & -G_1 & G_1 & 0 & G_4 & 0 & G_6 & 0 \\ 0 & 0 & K_b & 0 & -K_b - G_5 & G_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & G_6 - G_7 & G_7 \\ 0 & 0 & 0 & 0 & 1 & 0 & K_d G_6 & -1 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (9)$$

Name	Value [V]
V0i	0.000000000000000e+00
V1i	5.156959346000000e+00
V2i	4.904491610977624e+00
V3i	4.386308256622717e+00
V5i	4.940219576984384e+00
V6i	5.737650461673599e+00
V7i	-2.030644972553098e+00
V8i	-3.026746617887047e+00

Table 2: Nodal voltage values ( $t < 0$ )

By using Ohm's Law, we calculate the values of the currents passing in the resistors. The value of the current in  $v_s$  is symmetrical to  $I_{R1}$  and the value of the current in  $V_d$  is symmetrical to  $I_{R6}$ .

Name	Value [A or V]
Ib	-2.554476012603980e-04
IR1i	-2.440917089113781e-04
IR2i	-2.554476012603928e-04
IR3i	-1.135589234901462e-05
IR4i	-1.224528396740627e-03
IR5i	-2.554476012603919e-04
IR6i	9.804366878292505e-04
Ivsi	-2.440917089113781e-04
IVdi	-9.804366878292496e-04

Table 3: Branch and voltage sources current values ( $t < 0$ )

## 2.2 Determining $R_{eq}$

Analysing the circuit now for  $t = 0$ , we notice that  $v_s$  equals 0 (short circuit), thus  $V_1$  is also null. In order to determine the equivalent resistance ( $R_{eq}$ ) of the circuit seen from the capacitors terminals and the time constant (much needed for the following subsections), we replace the capacitor with a voltage source  $V_x = V_6 - V_8$  and do another nodal analysis to determine the current supplied by  $V_x$ , which will be called  $I_x$ . These two values determine  $R_{eq}$  with the equation:

$$R_{eq} = V_x / I_x \quad (10)$$

We use the values  $V_6$  and  $V_8$  from the previous subsection since the voltage drop at the terminals of the capacitor needs to be a continuous function, this means that there cannot be a sudden energy discontinuity in the capacitor. This is the most efficient procedure to determine the equivalent resistance in such a complex circuit, with this reasoning being based on the usage of the *Thevenin* and *Norton* theorems, where  $V_x$  is equivalent to *Thevenin's* voltage and  $I_x$  is *Norton's* current.

For this nodal analysis, we used almost the same equations that were used in the previous subsection, except, of course, the trivial one for  $V_1$ , since now  $V_1 = 0$ , and we replaced the node 6 equation with the new equation  $V_x = V_6 - V_8$ .

The system of equations that will be solved in form of matrix and with the assistance of Octave is:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_1 & -G_1 - G_2 - G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & 0 & K_b + G_2 & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & -G_1 & G_1 & 0 & G_4 & 0 & G_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & G_6 - G_7 & G_7 \\ 0 & 0 & 0 & 0 & 1 & 0 & K_d G_6 & -1 \end{bmatrix} \begin{bmatrix} V_0 t_0 \\ V_1 t_0 \\ V_2 t_0 \\ V_3 t_0 \\ V_5 t_0 \\ V_6 t_0 \\ V_7 t_0 \\ V_8 t_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_x \\ 0 \\ 0 \end{bmatrix} \quad (11)$$

Name	Value [V]
V0t0	0.0000000000000000e+00
V1t0	0.0000000000000000e+00
V2t0	0.0000000000000000e+00
V3t0	0.0000000000000000e+00
V5t0	0.0000000000000000e+00
V6t0	8.764397079560647e+00
V7t0	-0.0000000000000000e+00
V8t0	0.0000000000000000e+00

Table 4: Nodal voltage values ( $t < 0$ )

By using Ohm's Law, we calculate the values of the currents passing in the resistors.

Name	Value [A]
Ib	0.0000000000000000e+00
IR1t0	0.0000000000000000e+00
IR2t0	0.0000000000000000e+00
IR3t0	0.0000000000000000e+00
IR4t0	-0.0000000000000000e+00
IR5t0	-2.807571481683833e-03
IR6t0	0.0000000000000000e+00
IR7t0	-0.0000000000000000e+00
Ivst0	0.0000000000000000e+00
IVdt0	2.807571481683833e-03

Table 5: Branch and voltage sources current values ( $t=0$ )

Now that we have all values, we can use the Kirchhoff Current Law (KCL) in node 6 in order to compute the value of  $I_x$

$$I_x = -K_b(V_2 - V_5) - (V_6 - V_5)G_5 \quad (12)$$

With the values of  $V_x$  and  $I_x$  determined, we can calculate  $R_{eq}$  with 10 and the time constant value with the equation

$$\tau = R_{eq}C \quad (13)$$

Name	Value
Ixt0	-2.807571481683833e-03
Reqt0	3.121700422140000e+03
tau	3.167142502258496e-03
Vx	8.764397079560647e+00

Table 6:  $I_x$  (in A),  $R_{eq}$  (in Ohm),  $\tau$  (adimensional) and  $V_x$  (in V) values

### 2.3 Natural Solution with node analysis for $t \geq 0$

The aim of this section is to calculate the natural solution of  $v_{6n}(t)$ . The natural response is what the circuit does including the initial conditions (initial voltage of the capacitor) but with the input suppressed. Knowing the general solution ( $v_{6n}(t) = V_6(+\infty) + (V_6(0) - V_6(+\infty))e^{\frac{-t}{\tau}}$ ), and the fact that the capacitor begins charged but it discharges as the time passes, since it consumes energy, meaning that  $V_6(+\infty) = 0$ , we can write the following equation:

$$v_{6n}(t) = V_6(0)e^{\frac{-t}{\tau}} \quad (14)$$

We also know that  $V_x = V_6 - V_8$ , and that the value of  $V_8(0)$  is approximately 0, which means that

$$v_{6n}(t) = V_x e^{\frac{-t}{\tau}} \quad (15)$$

Hence, the graph of  $V_{6n}$  in function of the time, in the interval  $[0;20]$  ms is represent in 2. The result is no suprise, as it shows below, being a negative exponential graph.

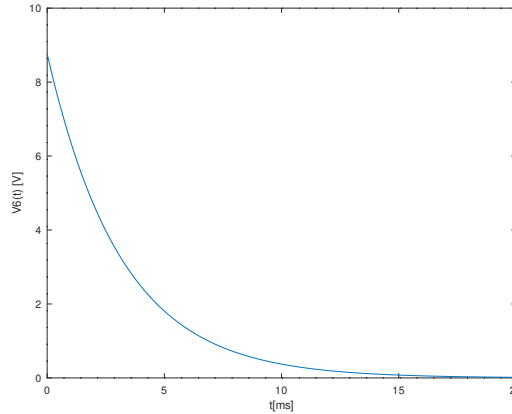


Figure 2: Natural solution  $v_{6n}(t)$

### 2.4 Forced Solution with node analysis for $t \geq 0$

In this Section, we have used a phasor voltage source  $V_s$ , which was replaced with its complex amplitude 1.

After analysing the circuit we notice that  $v_1$  equals  $v_s$  we do not have to include it in our next matricial system. Also, if we look at the equations on it, the only voltages that depend on the frequency are  $v_6$  and  $v_8$ .



Considering all the informations above, we are able to solve the system with only 4 equations. Therefore, using nodal method, we have:

$$\begin{bmatrix} -K_b - \frac{1}{R_2} & \frac{1}{R_2} & K_b & 0 \\ \frac{1}{R_3} - K_b & 0 & K_b - \frac{1}{R_3} - \frac{1}{R_4} & -\frac{1}{R_6} \\ K_b - \frac{1}{R_1} - \frac{1}{R_3} & 0 & \frac{1}{R_3} - K_b & 0 \\ 0 & 0 & 1 & \frac{K_d - R_7}{R_6} - 1 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \\ V_5 \\ V_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ phasorv_s \\ 0 \end{bmatrix} \quad (16)$$

For the matrix above, we considered that:

$$phasorv_s = e^{-i\pi/2} \quad (17)$$

Also considering that,

$$Z_c = \frac{1}{j\omega c} \quad (18)$$

Where  $\omega = 2\pi f$  and  $f = 1000Hz$ .

Then, replacing C with impedance  $Z_c$  and running the nodal analysis to determine the phasor voltages in all nodes we have the following equations for the determination of the remaining variables:

$$v_8 = R_7 V_7 \left( \frac{1}{R_1} + \frac{1}{R_6} \right); \quad (19)$$

$$v_6 = \frac{V_5 \left( K_b + \frac{1}{R_5} \right) - K_b V_2 + \frac{v_8}{Z_c}}{\frac{1}{R_5} + \frac{1}{Z_c}} \quad (20)$$

$$v_c = v_6 - v_8 \quad (21)$$

We have calculated the module of  $V_6$  with "abs" Octave's function. Then, the phase is calculated with the "imag", the "real" and the "atan" functions. The obtained results are the ones that follow next.

Name	Value [Rad]
Phase1	0.000000000000000e+00
Phase2	-5.224782481821870e-17
Phase3	1.727520704050529e-17
Phase5	-5.650387200837146e-17
Phase6	1.451830770421718e-01
Phase7	-5.650387200837146e-17
Phase8	-5.650387200837146e-17

Table 7: Phase Results

Name	Value [V]
V1	1.000000000000000e+00
V2	9.510432954608804e-01
V3	8.505609531370399e-01
V5	9.579714024343183e-01
V6	5.888386465685786e-01
V7	3.937678845826408e-01
V8	5.869246613772022e-01

Table 8: Amplitudes Results

## 2.5 Natural and Forced Superimposed

In this subsection, we determine the final total solution for the value of  $v_6$  for the given frequency of 1000 Hz (= 1kHz) by superimposing the natural and forced solutions (determined in the third and forth subsection, respectively), giving us the following equation:

$$v_6(t) = v_{6n}(t) + v_{6f}(t) \quad (22)$$

In Figure 3 we plotted the graphs of  $v_6(t)$  and  $v_s(t)$  in the interval [-5;20] ms. We can clearly divide the solutions in three parts:

- $t = [-5; 0]ms$ ,  $v_s = V_s$  and  $v_6 = V_6i$  (the value we calculated in the first subsection);
- $t = 0ms$ ,  $v_s = 0$  and  $v_6 = V_6t_0$  (the value we calculated in the second subsection);
- $t = ]0; 20]ms$ ,  $v_s = \sin(2\pi ft)$  and  $v_6$  is given by the equation 22.

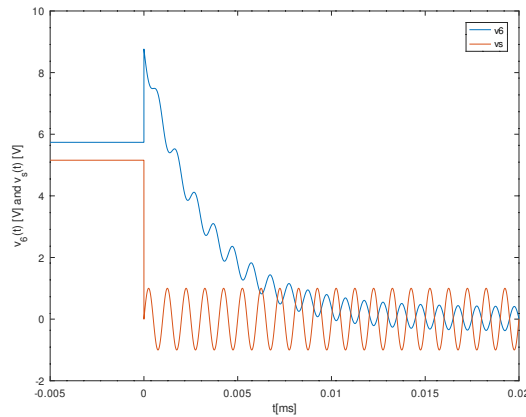


Figure 3:  $v_s(t)$  and the final solution of  $v_6(t)$  in the interval [-5;20]ms for the frequency of 1000Hz

## 2.6 Frequency Responses

In this subsection, we analysed how independent (or not) are the complex voltages  $v_c$ ,  $v_s$  and  $v_6$  from the frequency values, comparing them in the same graphs, with the frequency range being from 0.1Hz to 1MHz.

Just like before, we used the nodal method in order to obtain the values of the voltages that don't depend on the frequency. By knowing these voltages, we can now compute the values  $v_6$ ,  $v_8$  and, therefore,  $v_c$  and also  $v_s$ , knowing that the value of this voltage is constant due to its independence from the frequency ( $v_s = e^{-i\pi/2}$ ), so there is no need to do any calculation for the phase.

The values had their angles (that correspond to their phases) converted from radians to degrees, were restricted to the interval  $[-90;90]^\circ$  in the computation and the frequencies were in a logarithmic scale. We can see in Figure 4 that, as predicted,  $v_6$  and  $v_c$  have their angles decreasing with the increase of the frequency.

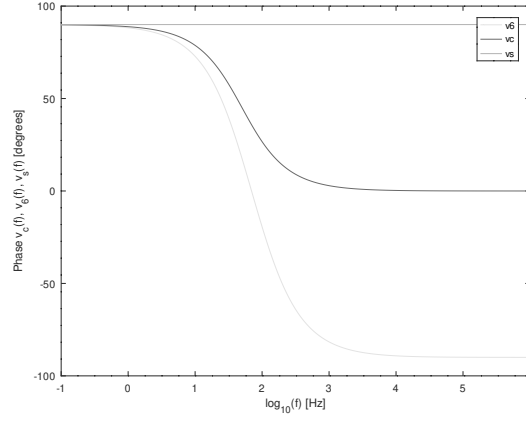


Figure 4: Phase of  $v_6$ ,  $v_c$  and  $v_s$  (in degrees)

In order to represent the results, we used the magnitude values in dB (decibels) calculated with the following equation:

$$magnitude = 20\log_{10}(|x|) \quad (23)$$

With  $x$  being the phasor voltage value. We also put the frequencies here in a logarithmic scale. The amplitude attributed to  $v_s$  is 1, which means that the magnitude is 0 ( $20\log_{10}(1) = 0$ ), and is, as expected, constant (proving that this voltage is independent from the frequency), while the other two voltages decrease when the frequency goes up.

This should be no surprise for  $v_c$  to decrease because of the impedance of the capacitor.

$$Z = -i/(wC) \quad (24)$$

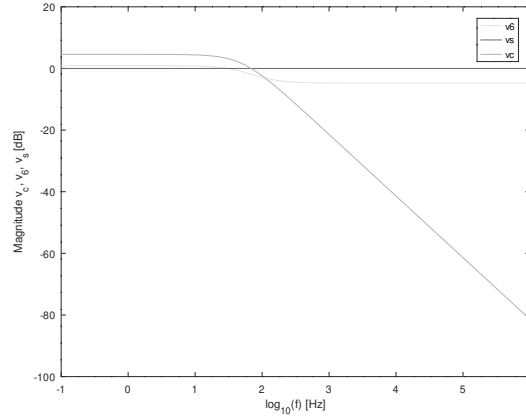


Figure 5: Magnitude of  $v_6$ ,  $v_c$  and  $v_s$  (in decibels)

### 3 Simulation Analysis

First of all, in this simulation is important to explain the creation of an auxiliary voltage  $V_{aux}$  (with a the same voltage of  $V_7$ ) that was put between N7 and R7 as shown in Figure 6. Consequently, this led to the appearance of a node that we designated by N9 that has the same voltage as N7 (the drop voltage is 0).

This was necessary because of Ngspice software requirements. After doing that ngspice was able to compute and determine all node voltages and current branches.

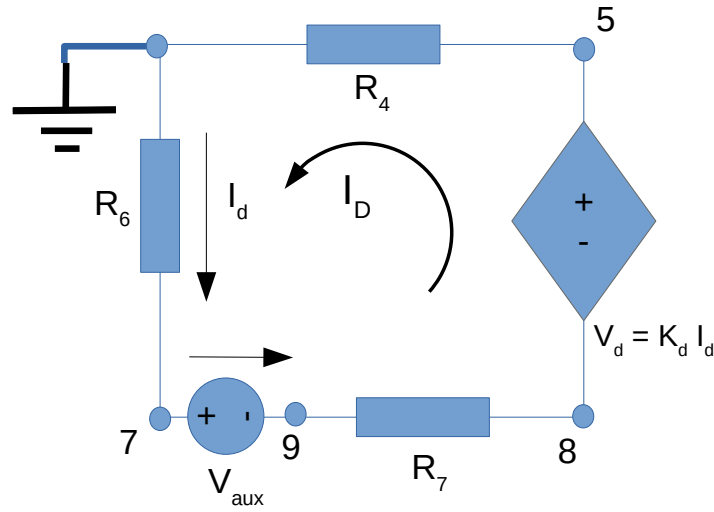


Figure 6: D Mesh with an additional voltage source

#### 3.1 Operating Point Analysis for $t < 0$

The Table 9 shows the simulated operating point results for the circuit described in Figure 1, considering  $t < 0$ , which means  $V_s(t) = V_s$ .

#### 3.2 Operating Point Analysis for $t = 0$

This second part covers the simulation of the circuit for  $t = 0$ . To do that the capacitor is replaced with a voltage source  $V_x = V_6 - V_8$  using the values obtained in the previous section. This is necessary because for  $t \leq 0$  the voltage in the capacitor is the same. So to maintain the boundary conditions  $V_6$  and  $V_8$  the capacitor is replaced with the initial voltage source. The results are presented on Table 10.

Name	Value [A or V]
@hd[i]	-9.80437e-04
@vs[i]	-2.44092e-04
@c[i]	0.000000e+00
@gb[i]	-2.55448e-04
@r1[i]	-2.44092e-04
@r2[i]	-2.55448e-04
@r3[i]	-1.13559e-05
@r4[i]	-1.22453e-03
@r5[i]	-2.55448e-04
@r6[i]	9.804367e-04
@r7[i]	9.804367e-04
n1	5.156959e+00
n2	4.904492e+00
n3	4.386308e+00
n5	4.940220e+00
n6	5.737650e+00
n7	-2.03064e+00
n8	-3.02675e+00
n9	-2.03064e+00

Table 9: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

Name	Value [A or V]
@hd[i]	2.807571e-03
@vs[i]	0.000000e+00
@gb[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	-2.80757e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00
n1	0.000000e+00
n2	0.000000e+00
n3	0.000000e+00
n5	0.000000e+00
n6	8.764397e+00
n7	0.000000e+00
n8	0.000000e+00
n9	0.000000e+00

Table 10: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

### 3.3 Natural Solution

In order to study the natural solution response of the circuit in the interval [0;20]ms using the boundary conditions ( $V_6$  and  $V_8$ ) calculated before, a transient analysis was realized. Fig. 7 shows the plot of the required results.

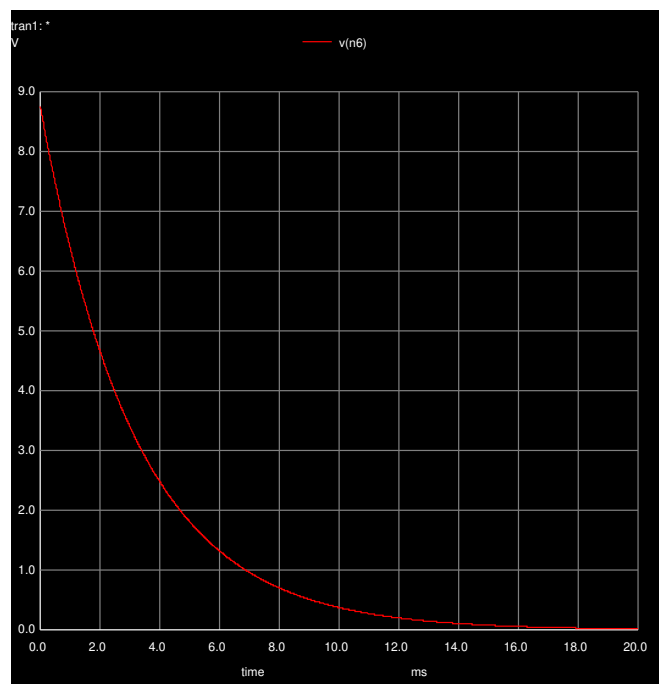


Figure 7: Natural Response of  $V_6$

### 3.4 Total Solution

In the fourth section a total response of node 6 was performed, using the same procedure and interval of 3.3 with a initial sinusoidal voltage source  $V_s(t)$  that has a frequency of 1000Hz. Fig. 8 shows the plot of the required results.

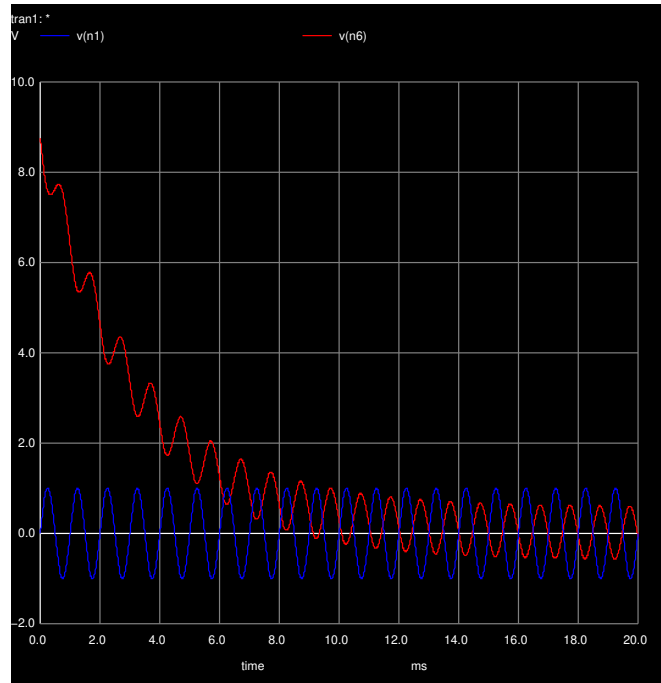


Figure 8: Total Response of  $V_6$  and  $V_s$

It is important to note that during the time interval considered the voltage in the capacitor diminish until its phase differs  $\pi$  from the voltage source.

### 3.5 Frequency Responses

In this part of the chapter a small signal analysis was realized. The frequency response is simulated on node 6 for the frequency between 0.1Hz to 1MHz. Since  $V_s$  is the source of the frequency and  $V_6$  is an output voltage is expected to  $V_s$  to remain constant and  $V_6$  to decrease. This can be seen in the following figures 9 and 10.

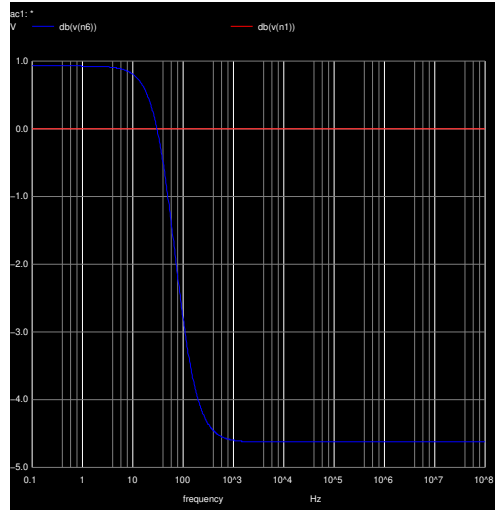


Figure 9: Magnitude response for  $V_6$  and  $V_s$  (in dB)

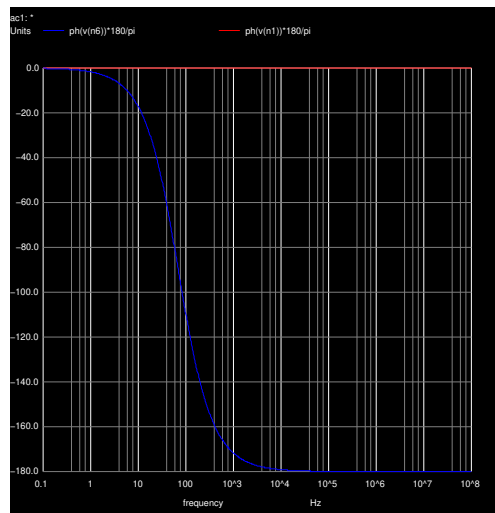


Figure 10: Phase response for  $V_6$  and  $V_s$  (in degrees)



## 4 Side by Side Comparison

After ending both simulation and theoretical analysis processes, the results were presented on their own sections. However, for presenting a prudent interpretation of the result both tables were put side by side.

In the simulation process is important to refer again the creation of an auxiliary voltage that was put between N7 and R7 as shown in Figure 6. Consequently, this resulted on the appearance of a node that we designated by N9 that has the same voltage as N7.

Name	Value [A or V]
@hd[i]	-9.80437e-04
@vs[i]	-2.44092e-04
@c[i]	0.000000e+00
@gb[i]	-2.55448e-04
@r1[i]	-2.44092e-04
@r2[i]	-2.55448e-04
@r3[i]	-1.13559e-05
@r4[i]	-1.22453e-03
@r5[i]	-2.55448e-04
@r6[i]	9.804367e-04
@r7[i]	9.804367e-04
n1	5.156959e+00
n2	4.904492e+00
n3	4.386308e+00
n5	4.940220e+00
n6	5.737650e+00
n7	-2.03064e+00
n8	-3.02675e+00
n9	-2.03064e+00

Table 11: Simulation nodal voltage results. All variables are expressed in Volt or Ampere. (Ngspice)

Name	Value [A or V]
IVdi	-9.804366878292496e-04
Ivsi	-2.440917089113781e-04
Ib	-2.554476012603980e-04
IR1i	-2.440917089113781e-04
IR2i	-2.554476012603928e-04
IR3i	-1.135589234901462e-05
IR4i	-1.224528396740627e-03
IR5i	-2.554476012603919e-04
IR6i	9.804366878292505e-04
V1i	5.156959346000000e+00
V2i	4.904491610977624e+00
V3i	4.386308256622717e+00
V5i	4.940219576984384e+00
V6i	5.737650461673599e+00
V7i	-2.030644972553098e+00
V8i	-3.026746617887047e+00

Table 12: Theoretical nodal voltage results. All variables are expressed in Volt.(Octave)

It shows that simulated operating point results from NGSpice and the nodal method results from Octave for the circuit that is being studied are the same.

However, was also calculated the relative errors made in order to understand the accuracy of the results.

Related to that calculations, it was noticed that in an experimental procedure, the calculation of relative errors is made by comparing experimental values and theoretical values, meaning that the decimal places used in the theoretical value to be considered will be in accordance with the experimentally obtained places.

Considering that, in our case, the experimental values are obtained through NGSpice, we must use theoretical values with the same number of decimal places returned by the simulation for calculating the errors, as it has a number of decimal places lower than that of the octave. By doing this calculating, all error values absolute and relative are equal to zero.

Therefore, we can see that the order of magnitude of the errors will be residual.

Related to Point 2, it was also required the analysis side by side of the results as shown on Tables below.

Name	Value [A or V]
@hd[i]	2.807571e-03
@vs[i]	0.000000e+00
@gb[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	-2.80757e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00
n1	0.000000e+00
n2	0.000000e+00
n3	0.000000e+00
n5	0.000000e+00
n6	8.764397e+00
n7	0.000000e+00
n8	0.000000e+00
n9	0.000000e+00

Table 13: Simulation nodal voltage results. All variables are expressed in Volt or Ampere. (Ngspice)

Name	Value [A or V]
IVdt0	2.807571481683833e-03
Ivst0	0.000000000000000e+00
Ib	0.000000000000000e+00
IR1t0	0.000000000000000e+00
IR2t0	0.000000000000000e+00
IR3t0	0.000000000000000e+00
IR4t0	-0.000000000000000e+00
IR5t0	-2.807571481683833e-03
IR6t0	0.000000000000000e+00
IR7t0	-0.000000000000000e+00
V1t0	0.000000000000000e+00
V2t0	0.000000000000000e+00
V3t0	0.000000000000000e+00
V5t0	0.000000000000000e+00
V6t0	8.764397079560647e+00
V7t0	-0.000000000000000e+00
V8t0	0.000000000000000e+00

Table 14: Theoretical nodal voltage results. All variables are expressed in Volt.(Octave)

The conclusions obtained were the same that on the first side by side comparison with errors that can be considered zero.

Although, differences in the order of  $10^{-15}$  (or lower), are very likely connected to the way the computer programs deal with mathematical operations and calculations (seen that  $10^{-15}$  is approximately the precision of a double's mantissa). It's important to notice that the format of the data presented in the Ngspice tables are automatically chosen but the ones from Octave were used with a bigger precision.

Therefore, we can see that the order of magnitude of the errors will always be residual.

## 5 Conclusion

The objective of this laboratory assignment is to analyse the circuit and solve it. After discussing with all members of the group we can conclude that this goal was achieved.

As presented the results obtained by the Octave math tool and Ngspice simulation tool are the same. This perfect match was achieved in all the analysis done (operationg, transient and frequency) as presented in Section 4.

Also, all the components used in this circuit (resistors, branches, nodes,...) are perfect this means they don't dissipate energy by heating. This is one of the advantages of simulating rather than doing it on the laboratory, the other one being the elimination of "humam error". It's known that this type of error can influence the experimental results causing considerable relative errors, which in our case weren't made.

Finally, this similarity proves the efficiency and precision of the methods that were used.