TÉCNICO LISBOA

Circuit Theory and Electronics Fundamentals

Lab 1 - Circuit Analysis Methods

Aerospace Engineering

Laboratory Report

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Eva Claro, 95785

Miguel Isidoro, 95834

Pedro Braz, 95837

Contents

1 Introduction

This report is being made for the subject of Circuit Theory and Electronics Fundamentals and is related to the 2^{st} laboratory being its objective to study an RC circuit containing seven resistors, one sinusoidal voltage source, one capacitor, one current controlled voltage source and one voltage controlled current source. The four elementary meshes are named after the current to which they are attributed.

The current controlled voltage source V_d is calculated by multiplying K_d with the current I_d, whereas the voltage controlled current source I_b can be determined by multiplying K_b with the voltage source V_b.

The display of this circuit can be seen in Figure 1.

In Section 2 the circuit will be analysed theoretically ATENÇÃO ending with the presentation of the results obtained by Octave.

Secondly, in Section 3 it will be simulated the circuit using ngspice, the results obtained will be presented and

Following with both results from Section 2 and Section 3 being compared and commented.

The conclusions of this study are outlined in Section 4.

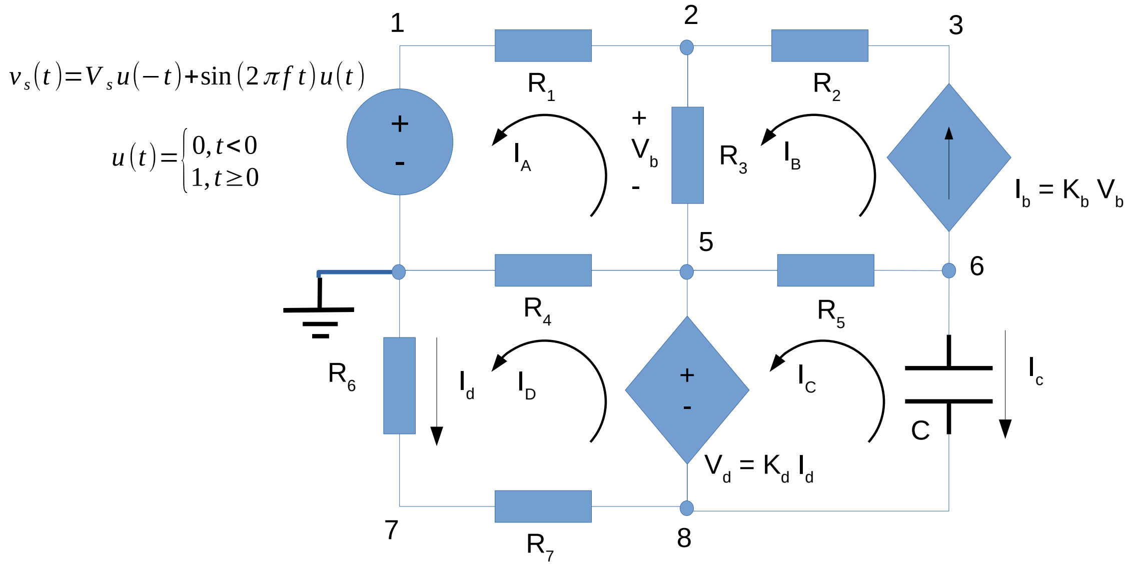

Figure 1: Circuit in analysis

Where:

Name	Value [A or V]
R1	1.034315078330000e+03
R2	2.028530907310000e+03
R3	3.146205063300000e+03
R4	4.034385474550000e+03
R5	3.121700422140000e+03
R6	2.071163796460000e+03
R7	1.015977530930000e+03
Vs	5.156959346000000e+00
C	1.014556835690000e-06
Kb	7.149794119600000e-03
Kd	8.125936425850000e+03

Table 1: Results obtained by mesh analysis method with octave

MUDAR The units of the elements whose name starts with R (the resistors) are $k\Omega$ (kilo-ohm), the ones that start with I are expressed in mA (miliampere) and the ones starting with V are expressed in V (volts). While Kb is given in mS (milisiemens), Kc is also given in $k\Omega$.

These values where obtained using the Python script using the lowest student number on our group - 95785.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, in terms of its node voltages and mesh currents.

For this mesh method, circular currents are defined in the counter-clockwise direction and then the circuit is evaluated considering those new currents.

Starting by number the nodes arbitrarily, assigning current names and directions to all branches also arbitrarily and defining one node as ground (GND).

Being mesh A the one with the resistors R_1, R_3 and R_4, and the voltage source V_a, the mesh B with the resistors R_2, R_3 and R_5 and the voltage controlled current source I_b, the mesh C with the resistors R_4, R_6 and R_7, and the current controlled voltage source V_c, and, at last, the mesh D with the resistor R_5, the current source I_d and the current controlled voltage source V_c.

A system of equations obtained applying the Kirchhoff Current Law (KCL)to each mesh can be written as

It was used 3 equations (4 meshes - 1 = 3 linearly independent equations): Mesh A, Mesh C and an addicional equation which is

$$I_b = K_b V_b, \text{ where } V_b = (I_B - I_A)R_3 \tag{1}$$

It's important to notice that D loop is independent of the remaining ones so it isn't don't need to determine the current I_D as it is given on the initial data.

The Nodal Analysis Method is another general procedure for analysing circuits using node voltages as the circuit variables.

To find the nodal voltages we chose 7 equations (8 nodes - 1 = 7 linearly independent equations) that comprise:

- KCL in nodes not connected to voltage sources;
- Additional equations for nodes related by voltage sources.

It was used the equations regarding the nodes 0, 2, 5, 6 therefore it was necessary three additional equations.

We chose to put the ground zero between three branches corresponding to the ones with R_1, R_2 and R_3 because it will facilitate the system of equations.

The next equation was used for node 1 because node 1 and node 4 are connected to an independent voltage source.

$$V_1 - V_4 = V_a \tag{2}$$

Secondly knowing that $V_c = K_c I_c$ and $V_c = V_3 - V_7$ it was concluded that for node 7 the equation obtained was

$$V_3 - V_7 = K_c(V_3 - V_6)G_6 \tag{3}$$

To find the third equation (Equation 4) it was used the continuity of the current to create a "super-knot" (nodes 3 and 7)

$$(V_4 - V_3)G_4 + (V_0 - V_3)G_3 + (V_5 - V_3)G_5 - I_d + (V_6 - V_7)G_7 = 0 \tag{4}$$

2.1 Node analysis for t<0

The system of equations that will be solved is:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_1 & -G_1-G_2-G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & 0 & K_b+G_2 & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & -G_1 & G_1 & 0 & G_4 & 0 & G_6 & 0 \\ 0 & 0 & K_b & 0 & -K_b-G_5 & G_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & G_6-G_7 & G_7 \\ 0 & 0 & 0 & 0 & 1 & 0 & K_dG_6 & -1 \end{bmatrix} \begin{bmatrix} V_0i \\ V_1i \\ V_2i \\ V_3i \\ V_5i \\ V_6i \\ V_7i \\ V_8i \end{bmatrix} = \begin{bmatrix} 0 \\ V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

Name	Value [A or V]
Ib	-2.554476012603980e-04
IR1i	-2.440917089113781e-04
IR2i	-2.554476012603928e-04
IR3i	-1.135589234901462e-05
IR4i	-1.224528396740627e-03
IR5i	-2.554476012603919e-04
IR6i	9.804366878292505e-04
Ivsi	-2.440917089113781e-04
IVdi	-9.804366878292496e-04

Table 2: LEGENDA

Name	Value [A or V]
V0i	0.000000000000000e+00
V1i	5.156959346000000e+00
V2i	4.904491610977624e+00
V3i	4.386308256622717e+00
V5i	4.940219576984384e+00
V6i	5.737650461673599e+00
V7i	-2.030644972553098e+00
V8i	-3.026746617887047e+00

Table 3: LEGENDA

2.2 Determining R_{eq}

The system of equations that will be solved is:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_1 & -G_1-G_2-G_3 & G_2 & G_3 & 0 & 0 & 0 \\ 0 & 0 & K_b+G_2 & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & -G_1 & G_1 & 0 & G_4 & 0 & G_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & G_6-G_7 & G_7 \\ 0 & 0 & 0 & 0 & 1 & 0 & K_d G_6 & -1 \end{bmatrix} \begin{bmatrix} V_0 t0 \\ V_1 t0 \\ V_2 t0 \\ V_3 t0 \\ V_5 t0 \\ V_6 t0 \\ V_7 t0 \\ V_8 t0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_x \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

Name	**Value [A or V]**
V0t0	0.000000000000000e+00
V1t0	0.000000000000000e+00
V2t0	0.000000000000000e+00
V3t0	0.000000000000000e+00
V5t0	0.000000000000000e+00
V6t0	8.764397079560647e+00
V7t0	-0.000000000000000e+00
V8t0	0.000000000000000e+00

Table 4: LEGENDA

Name	**Value [A or V]**
Ib	0.000000000000000e+00
IR1t0	0.000000000000000e+00
IR2t0	0.000000000000000e+00
IR3t0	0.000000000000000e+00
IR4t0	-0.000000000000000e+00
IR5t0	-2.807571481683833e-03
IR6t0	0.000000000000000e+00
IR7t0	-0.000000000000000e+00
Ivst0	0.000000000000000e+00
IVdt0	2.807571481683833e-03

Table 5: LEGENDA

Name	**Value [A or V]**
Ixt0	-2.807571481683833e-03
Reqt0	3.121700422140000e+03
tau	3.167142502258496e-03

Table 6: LEGENDA

2.3 Natural Solution with node analysis for t≥0

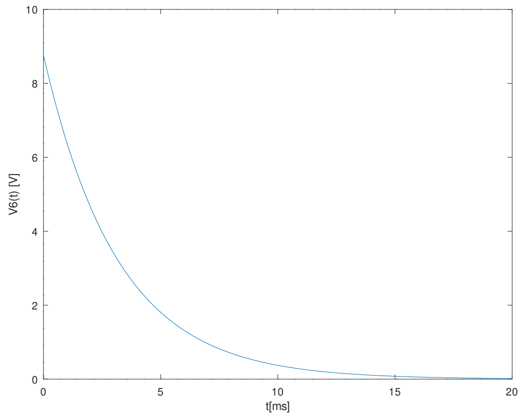

Figure 2: LEGENDA

2.4 Forced Solution with node analysis for t≥0

$$\begin{bmatrix} -G_1 & G_1+G_2+G_3 & -G_2 & -G_3 & 0 & 0 & 0 & \\ 0 & -G_2-K_b & G_2 & K_b & 0 & 0 & 0 & \\ 0 & K_b & 0 & K_b-G_5 & G_5+1/Z_c & 0 & -1/Z_c & \\ 0 & 0 & 0 & 0 & 0 & G_6+G_7 & -G_7 & \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & K_dG_6 & -1 & \\ G_1 & -G_1 & 0 & -G_4 & 0 & -G_6 & 0 & \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

Name	Value [A or V]
Phase1	0.000000000000000e+00
Phase2	-5.224782481821870e-17
Phase3	1.727520704050529e-17
Phase5	-5.650387200837146e-17
Phase6	1.451830770421718e-01
Phase7	-5.650387200837146e-17
Phase8	-5.650387200837146e-17

Table 7: LEGENDA

Name	Value [A or V]
V1	1.000000000000000e+00
V2	9.510432954608804e-01
V3	8.505609531370399e-01
V5	9.579714024343183e-01
V6	5.888386465685786e-01
V7	3.937678845826408e-01
V8	5.869246613772022e-01

Table 8: LEGENDA

2.5 Ponto 5

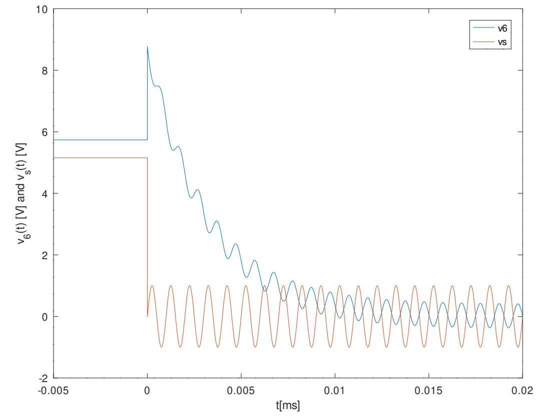

Figure 3: LEGENDA

2.6 Frequency Responses

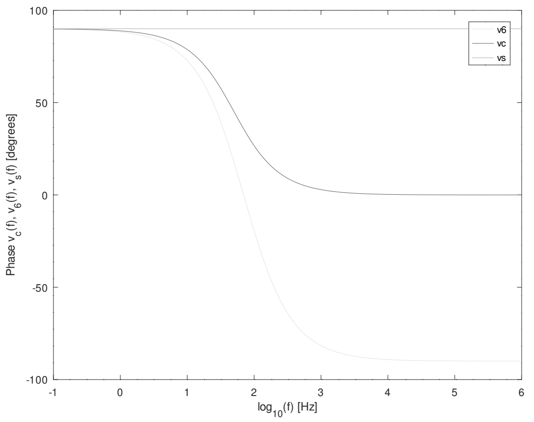

Figure 4: LEGENDA

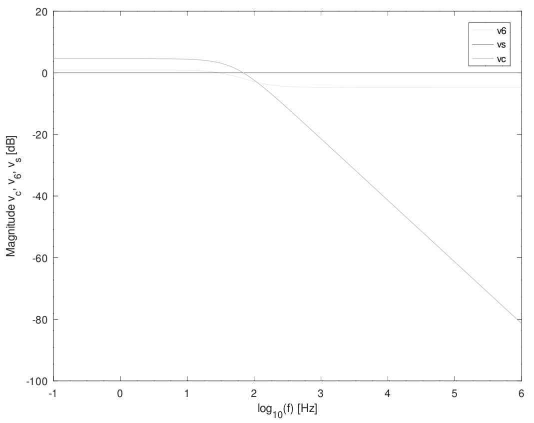

Figure 5: LEGENDA

After solving the system with Octave tools we get the Table results.In Table voltage values are identified with V and their measure is V(Volts), the remaining ones are current values so their units are A(Amperes).

3 Simulation Analysis

The Table 9 shows the simulated operating point results for the circuit described in Figure 1.

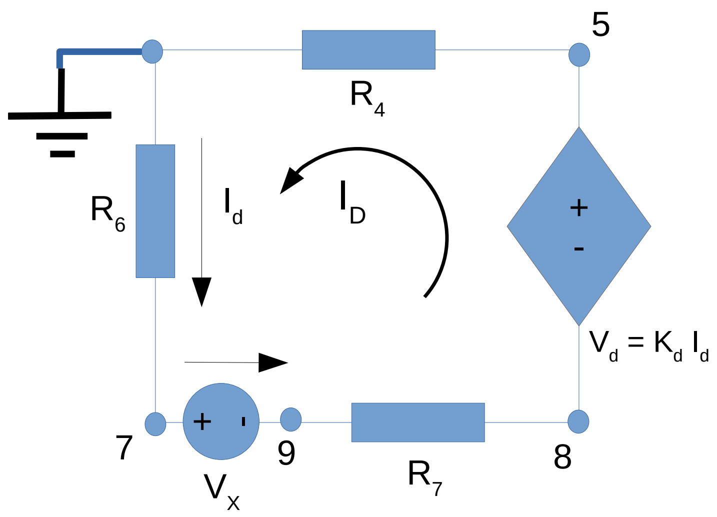

Figure 6: D Mesh with an adicional voltage source

3.1 Operating Point Analysis for t<0

Name	**Value [A or V]**
@vaux[i]	9.804367e-04
@hd[i]	-9.80437e-04
@vs[i]	-2.44092e-04
@c[i]	0.000000e+00
@gb[i]	-2.55448e-04
@r1[i]	-2.44092e-04
@r2[i]	-2.55448e-04
@r3[i]	-1.13559e-05
@r4[i]	-1.22453e-03
@r5[i]	-2.55448e-04
@r6[i]	9.804367e-04
@r7[i]	9.804367e-04
n1	5.156959e+00
n2	4.904492e+00
n3	4.386308e+00
n5	4.940220e+00
n6	5.737650e+00
n7	-2.03064e+00
n8	-3.02675e+00
n9	-2.03064e+00

Table 9: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3.2 Determining R_{eq}

Name	Value [A or V]
@vaux[i]	0.000000e+00
@hd[i]	2.807571e-03
@vs[i]	0.000000e+00
@gb[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	-2.80757e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00
n1	0.000000e+00
n2	0.000000e+00
n3	0.000000e+00
n5	0.000000e+00
n6	8.764397e+00
n7	0.000000e+00
n8	0.000000e+00
n9	0.000000e+00

Table 10: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3.3 Natural Solution

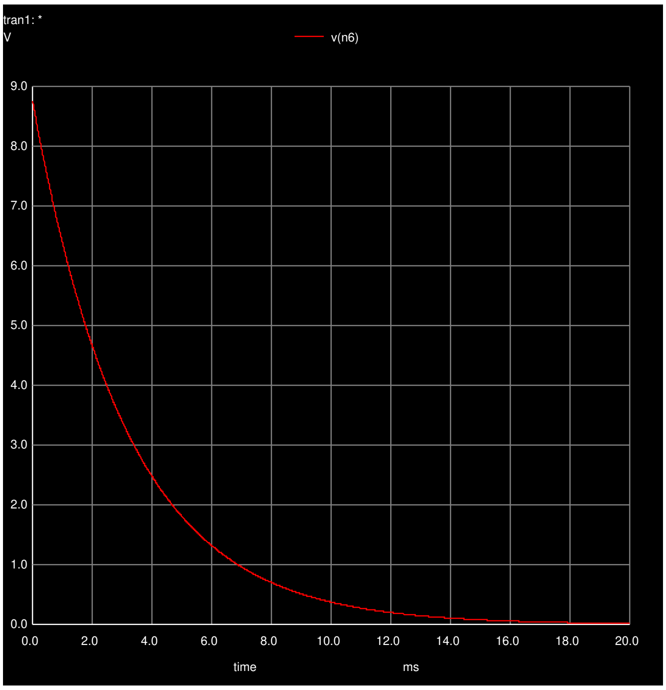

Figure 7: LEGENDA

3.4 Total Solution

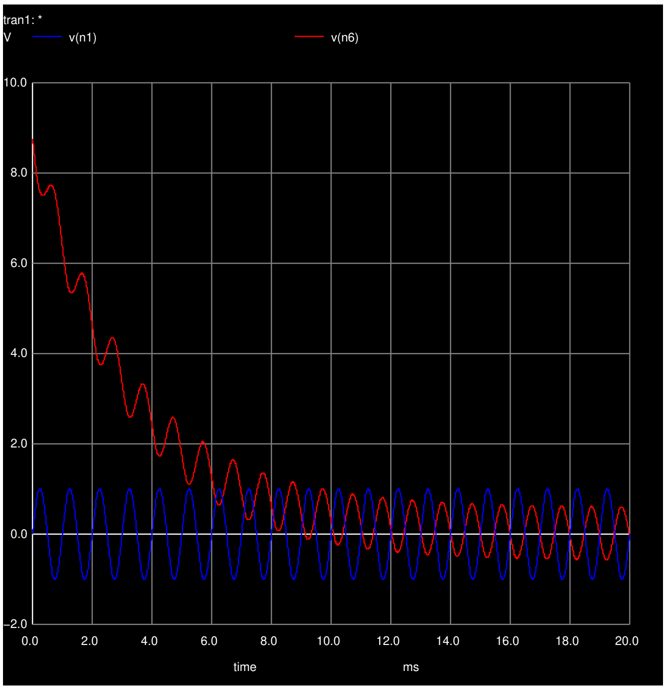

Figure 8: LEGENDA

3.5 Frequency Responses

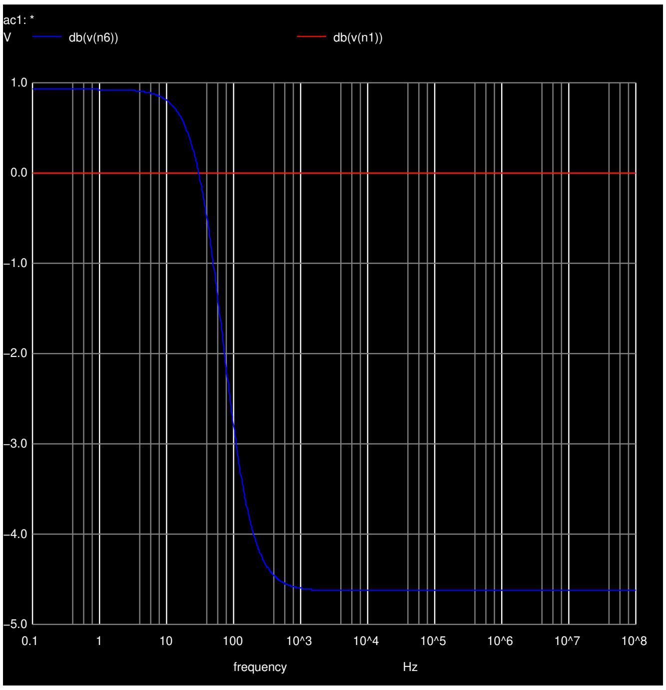

Figure 9: LEGENDA

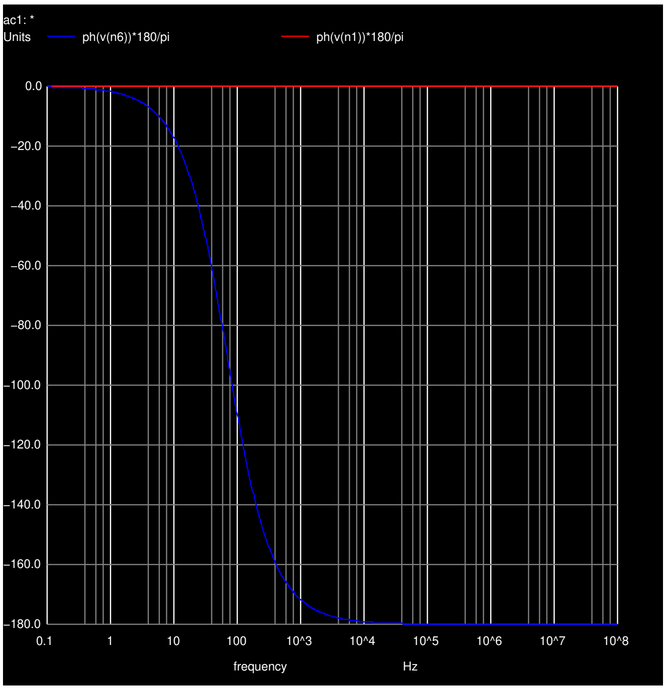

Figure 10: LEGENDA

In this simulation is important to explain the creation of an auxiliary voltage V_b (with a voltage equal to $0V$) that was put between N6 and R7 as shown in Figure 6. Consequently, this led to the appearance of a node that we designated by N8 that has the same voltage as N6.

This was necessary because of Ngspice software requirements.

By observing the Table 9,Table 2 and Table 4 values it's possible to conclude that the simulated results are the same as the theoretical results.

However, was also calculated the relative errors made in order to understand the accuracy of the results.

Related to that calculations, it was noticed that in an experimental procedure, the calculation of relative errors is made by comparing experimental values and theorical values, meaning that the decimal places used in the theoretical value to be considered will be in accordance with the experimentally obtained places.

Considering that, in our case, the experimental values are obtained through NGSice, we must use theoretical values with the same number of decimal places returned by the simulation for calculating the errors, as it has a number of decimal places lower than that of the octave. By doing this calculating, all error values absolute and relative are equal to zero.

Therefore, we can see that the order of magnitude of the errors will be residual.

4 Conclusion

The objective of this laboratory assignment is to analyse the circuit and solve it. After discussing with all members of the group we can conclude that this goal was achieved.

As presented the results obtained by the Octave math tool and Ngspice simulation tool are the same. This perfect match was achieved because the circuit is not very complex, being only composed of linear components so both models (Ngspice and Octave) used the same methods to solve the circuit and therefore the results can not differ.

Also, all the components used in this circuit (resistors, branches, nodes,...) are perfect this means they don't dissipate energy by heating.This is one of the advantages of simulating rather than doing it on the laboratory, the other one being the elimination of ”humam error”. It's known that this type of error can influence the experimental results causing considerable relative errors, which in our case weren't made.

Finally, this similarity proves the efficiency and importance of the nodal and mesh methods.