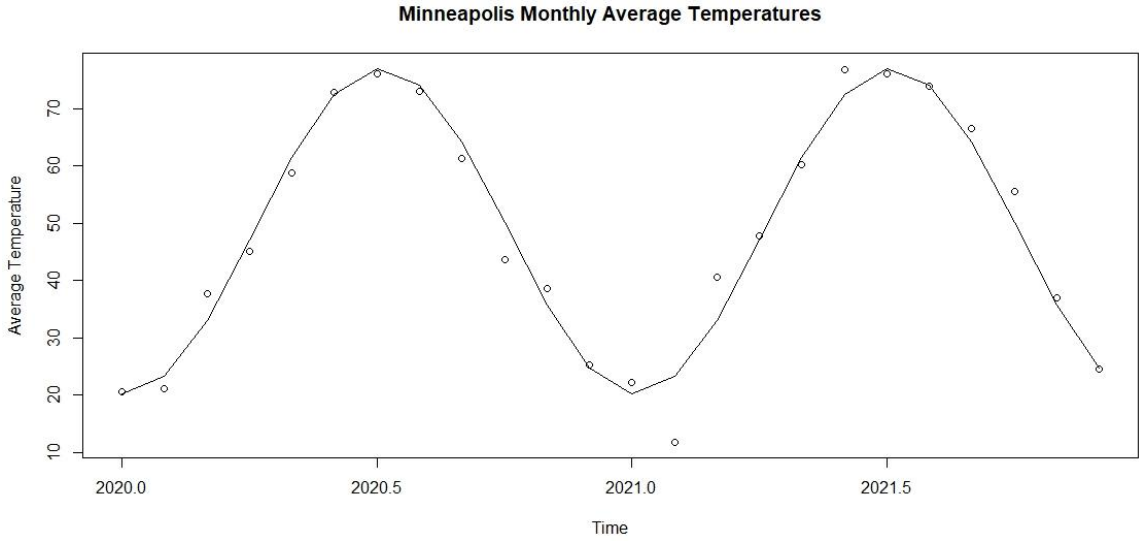
Minneapolis and Houston Temperature Average Trends from January 2020 to December 2021

Minneapolis (44.9778°N, 93.2650°W) and Houston (29.7604°N, 95.3698°W), two cities of similar longitude with approximately 1000 miles between the locations, exhibit greatly different weather trends. This theory is examined through the analysis of two years of collections of data by the National Weather Service by considering the monthly averages for a total of 24 observations in Minneapolis and 24 observations in Houston. The trend line equation used will be $y = \beta0 + \beta1 * \cos(2 * \pi * t) + \beta2 * \sin(2 * \pi * t)$.

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Residuals:
     Min       1Q   Median       3Q      Max
-11.4865  -1.4084   0.1743   2.0202   7.4899

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)     48.6042     0.8333  58.326   <2e-16 ***
harcos(2*pi*t) -28.4305     1.1785 -24.125   <2e-16 ***
harsin(2*pi*t)  -1.5922     1.1785  -1.351    0.191
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.082 on 21 degrees of freedom
Multiple R-squared:  0.9653,    Adjusted R-squared:  0.962
F-statistic: 291.9 on 2 and 21 DF,  p-value: 4.745e-16
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Estimated Trend:

$y = 48.6 - 28.43 * \cos(2 * \pi * t) - 1.59 * \sin(2 * \pi * t)$

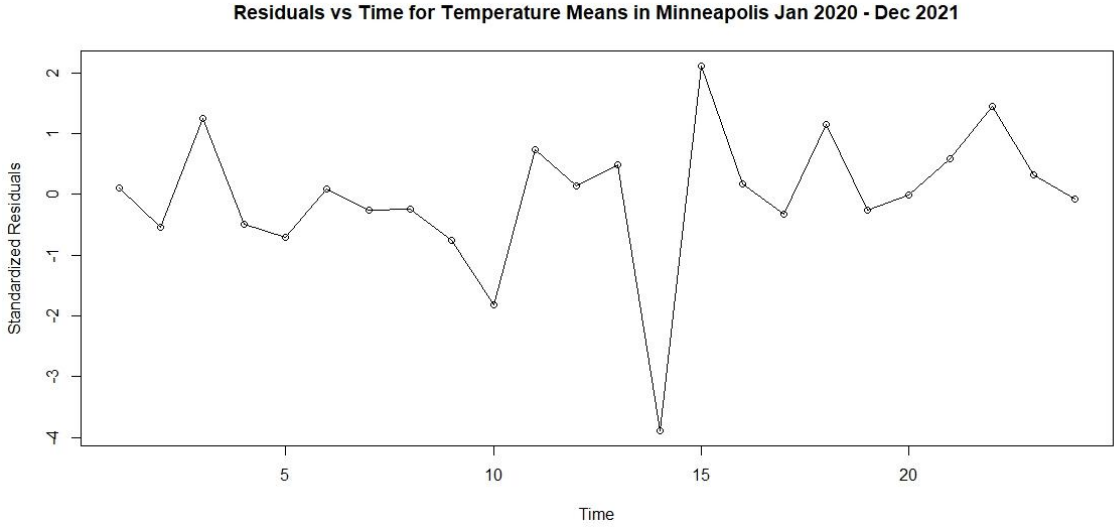
Where y is degrees Fahrenheit. T = day of the year/365.

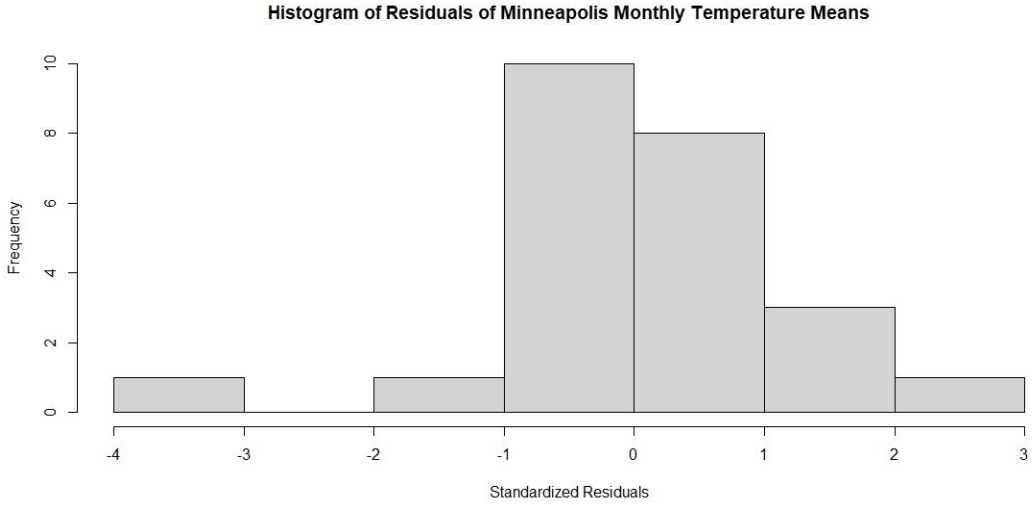
95% Confidence Interval:

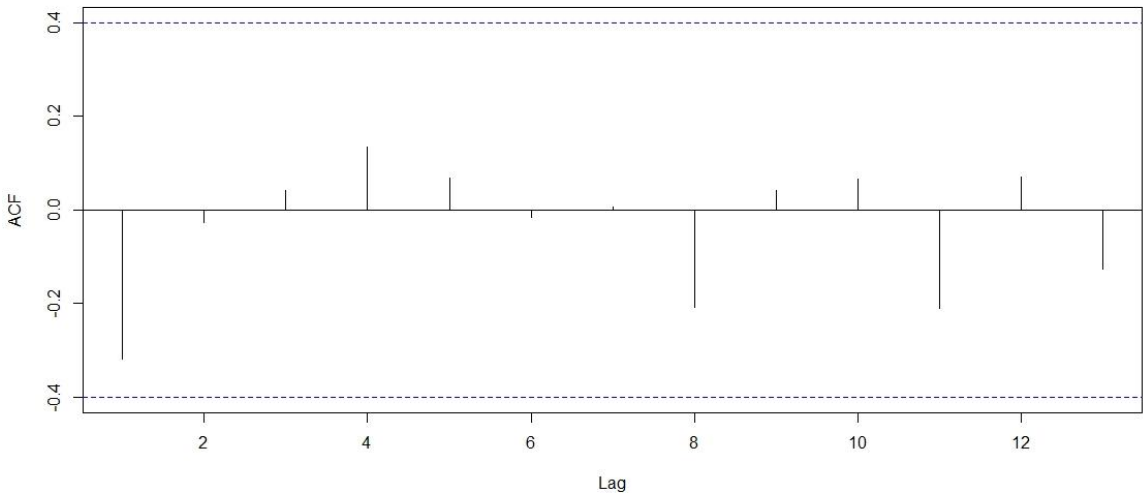
```
                   2.5 %      97.5 %
(Intercept)     46.871186  50.3371475
harcos(2*pi*t) -30.881308 -25.9796983
harsin(2*pi*t)  -4.042968   0.8586417
```

Plotting the average monthly weather in Minneapolis a cosine wave trend is present, where then a harmonic time series line was generated to fit to the data. The summary of this data shows that the cosine wave is significant at an $\alpha = 0.05$ level, however the sine wave is not as significant as a predictor to the response variable. The best line of fit would be $y = 48.6 - 28.43 * \cos(2 * \pi * t) - 1.59 * \sin(2 * \pi * t)$, which would account for 96.53% of the variation in the estimated average weather seasonal trend in

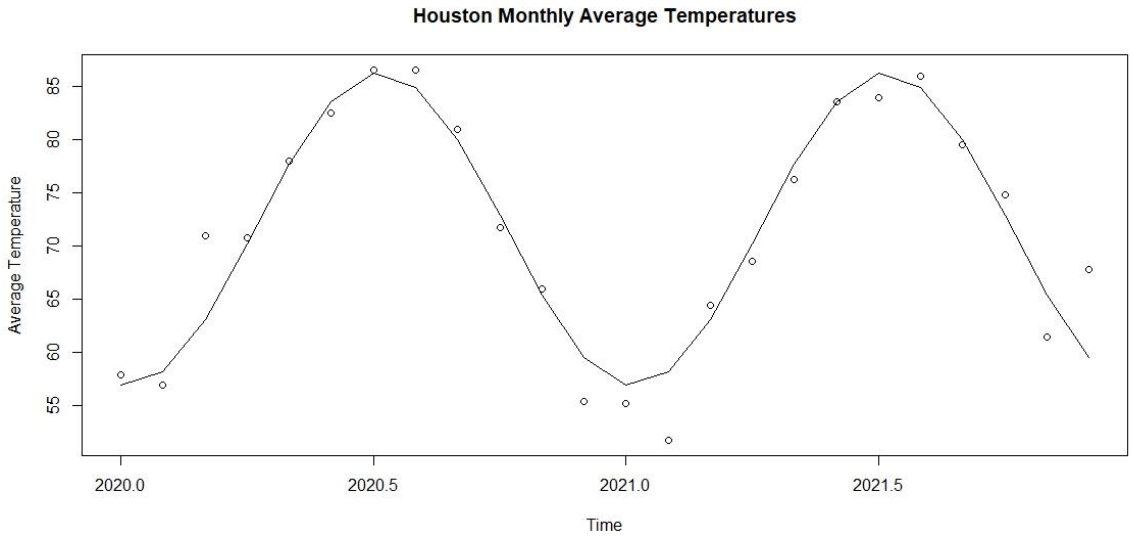
Minneapolis. The amplitude of the cosine wave on the Minneapolis weather data has an estimated value of 28.43°, with a 95% Confidence Interval of the amplitude being between 25.98° and 30.88°.







The above three figures show graphs from performing a residual analysis on the 24 observations of the Minneapolis monthly temperature averages collected. To determine if the estimated trend can be used to determine past and future cyclical trends for the area this analysis must prove that the residuals are white noise, and a trend is not present. In the graph comparing the residual vs. time no apparent trend is present and there appears to be an outlier in data from February 2021. The histogram has a bell curve, and the ACF shows white noise. Based on these findings, we can conclude that the estimated trend found for the Minneapolis monthly averages may be a source of estimating past and future trends for that region.




```
Residuals:
    Min      1Q  Median      3Q     Max
-6.4930 -1.4921  0.1231  1.0146  8.2660

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)    71.5708     0.6838 104.670  < 2e-16 ***
harcos(2*pi*t) -14.6732     0.9670 -15.174 8.61e-13 ***
harsin(2*pi*t)  -1.3409     0.9670  -1.387     0.18
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3.35 on 21 degrees of freedom
Multiple R-squared:  0.9171,    Adjusted R-squared:  0.9092
F-statistic: 116.1 on 2 and 21 DF,  p-value: 4.442e-12
```

Estimated Trend:

$y = 71.57 – 14.67 * \cos(2* \pi *t) – 1.34 * \sin(2* \pi *t)$

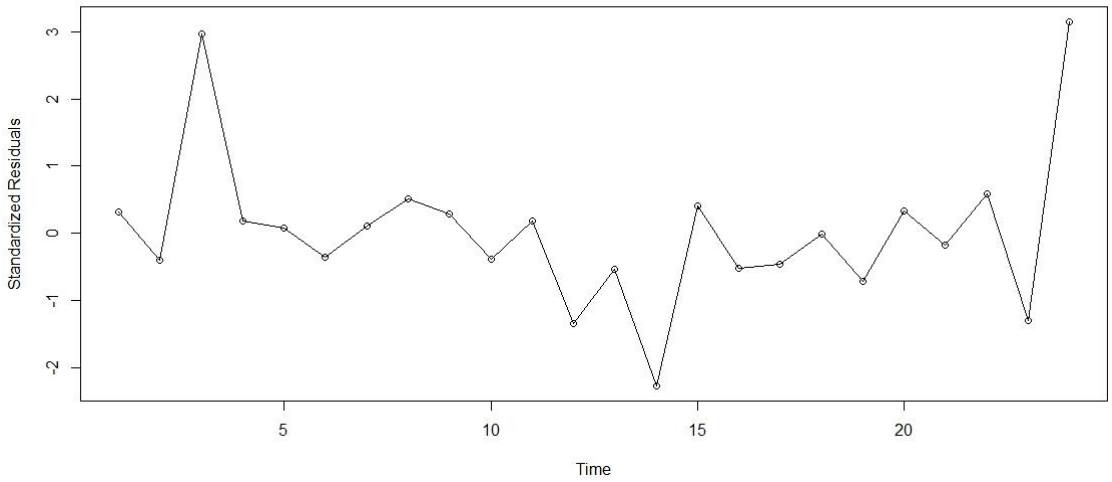
Where y is degrees Fahrenheit. T = day of the year/365.

95% Confidence Interval:

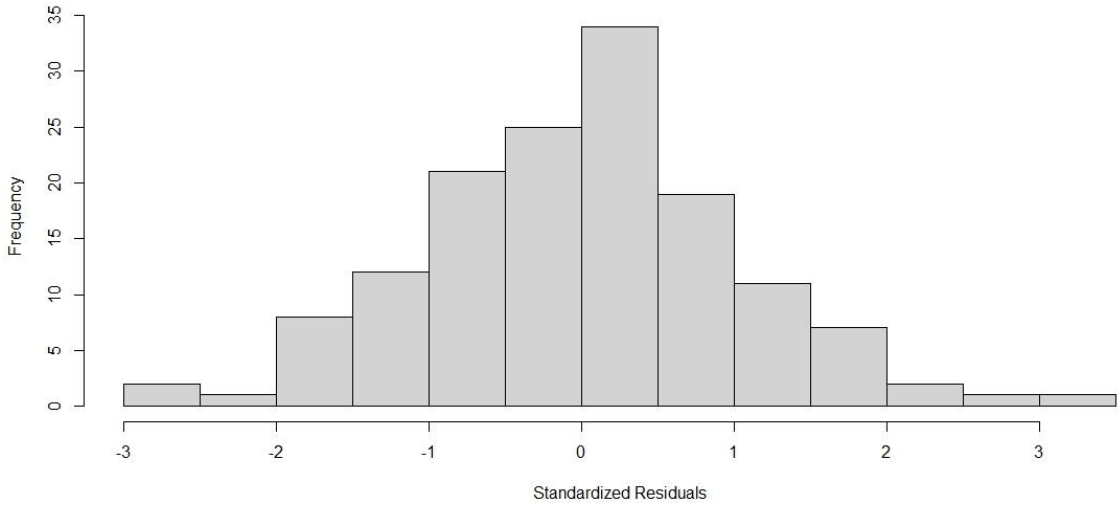
```
                    2.5 %      97.5 %
(Intercept)      70.148837  72.9928301
har2cos(2*pi*t) -16.684183 -12.6621692
har2sin(2*pi*t)  -3.351937   0.6700776
```

Like the Minneapolis data, Houston has a present cosine wave trend, and based on the analysis of the data the cosine wave is significant when testing at the $\alpha = 0.05$ level while the sine wave is not significant. This creates the cosine trend $y = 71.57 – 14.67 * \cos(2* \pi *t) – 1.34 * \sin(2* \pi *t)$ where 91.71% of the variation of data in monthly weather averages is accounted for in the equation of the estimated trend. The amplitude of the cosine wave on the Houston weather data has an estimated value of 14.67°, with a 95% Confidence Interval of the amplitude being between 12.66° and 16.68°. Comparatively the amplitude of the estimated harmonic time series model on the Minneapolis monthly averages of temperature is twice that of the estimated harmonic time series model of the Houston monthly averages of temperature.

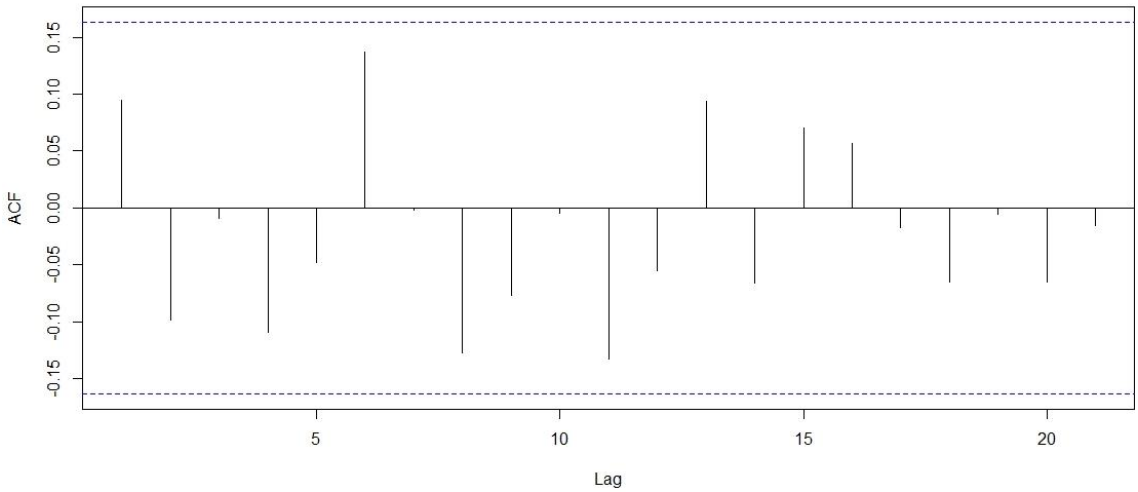
Residuals vs Time for Temperature Means in Houston Jan 2020 - Dec 2021



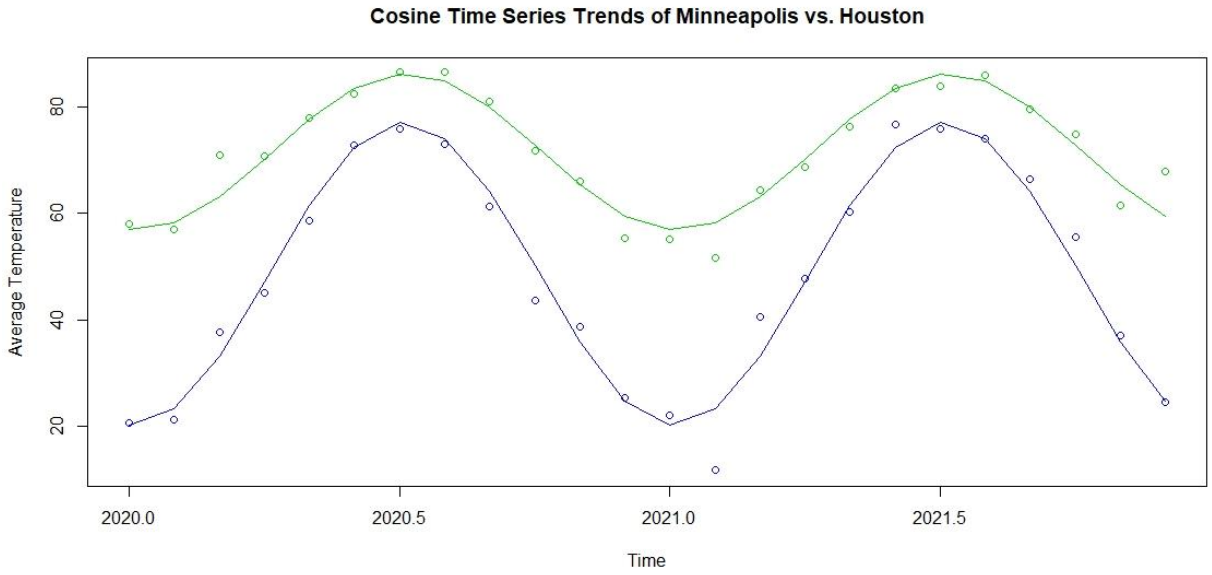
Histogram of Residuals of Houston Monthly Temperature Means



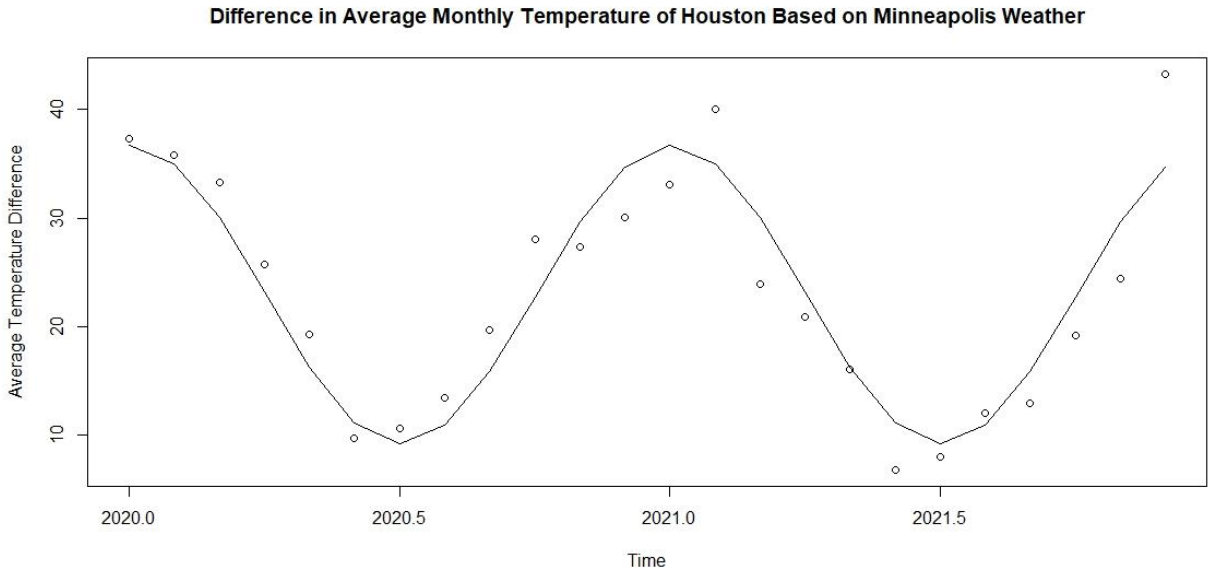
Autocorrelation of Residuals of Houston Monthly Temperature Means



When looking at the residual analysis of the 24 observations of the Houston monthly averages, it can be determined that that the estimated trend found from the data may be used for determining previous and future monthly averages of temperature for that region. Similar to the residual analysis of the Minneapolis weather data, the graph shows no significant trend, with an even bell-shaped histogram of the distribution of the residuals, and the ACF reflects white noise.



In the above figure the Minneapolis data is fitted in the blue trend line and the Houston data is fitted in the green trend line. While both exhibit a cosine wave trend, the trend of weather monthly averages in Minneapolis show a much greated amplitude than that in Houston.



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Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)      22.9667     0.8114  28.306  < 2e-16 ***
har3cos(2*pi*t)  13.7573     1.1474  11.990  7.4e-11 ***
har3sin(2*pi*t)   0.2512     1.1474   0.219    0.829
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3.975 on 21 degrees of freedom
Multiple R-squared:  0.8726,    Adjusted R-squared:  0.8604
F-statistic:  71.9 on 2 and 21 DF,  p-value: 4.03e-10
```

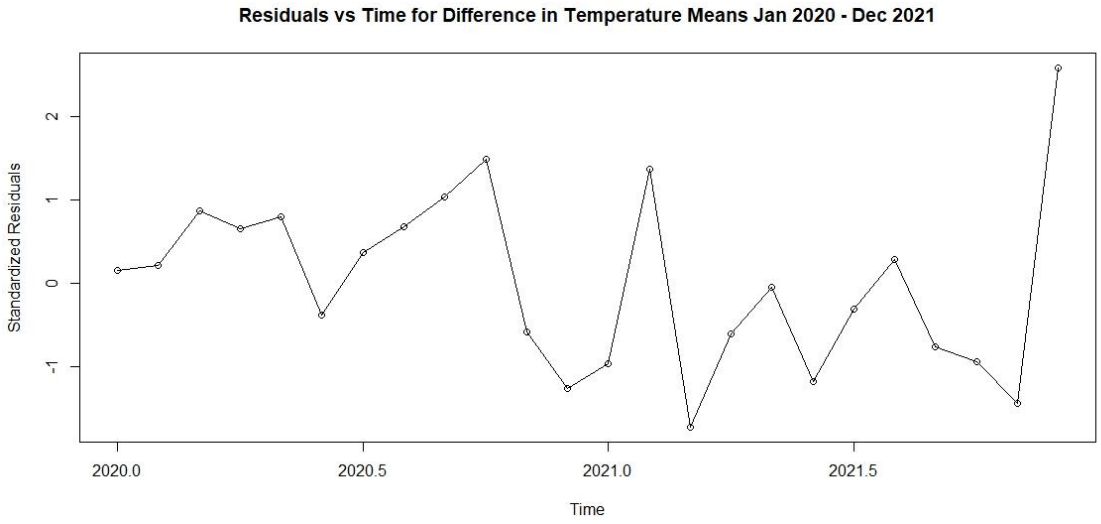
Estimated Trend:

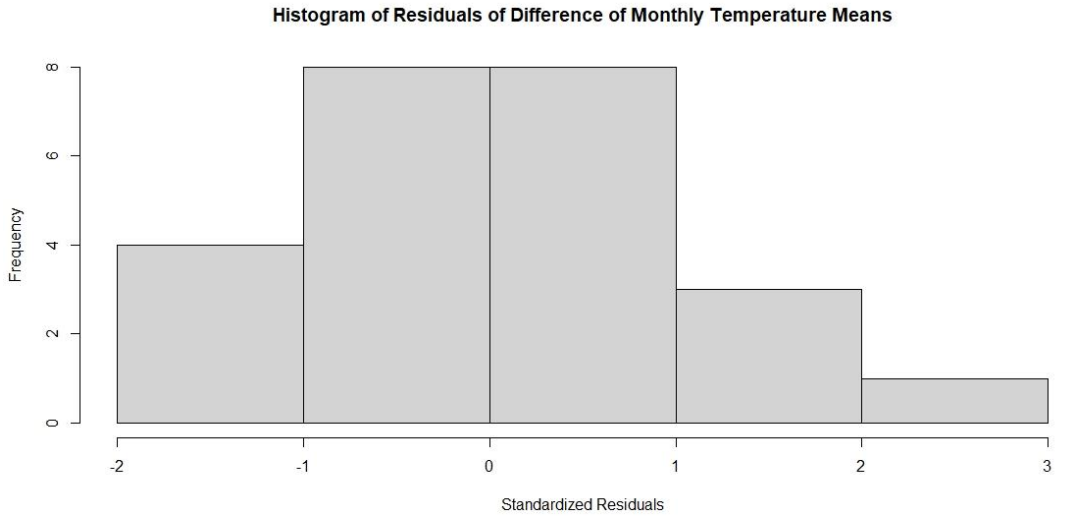
y_diff = 22.97 + 13.75 * cos (2* π *t) + 0.25 * sin (2* π *t)

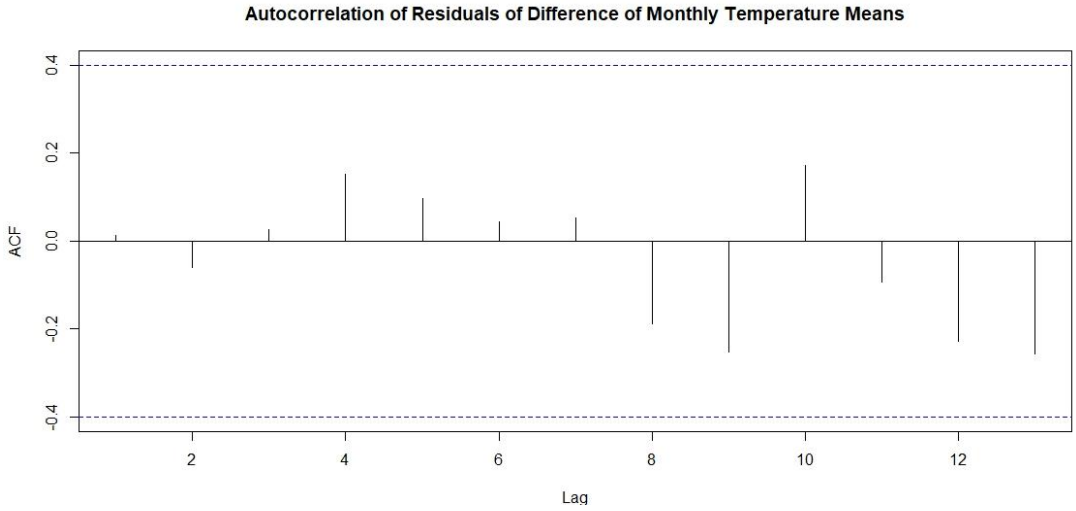
Where y is degrees Fahrenheit. T = day of the year/365.

```
                    2.5 %    97.5 %
(Intercept)     21.279347 24.653986
har3cos(2*pi*t) 11.371097 16.143557
har3sin(2*pi*t) -2.134997  2.637464
```

The estimated trends created from the Minneapolis and Houston data both displayed cosine trends on the same cycle, which suggests the change between the monthly temperature averages in the two locations may have a trend of its own. This is displayed with the 24 data points of the difference in the monthly temperature averages, where a cosine wave is fitted to the data when looking at the increase of weather in Houston compared to Minneapolis. The estimated trend y_diff = 22.97 + 13.75 * cos (2* π *t) + 0.25 * sin (2* π *t) accounts for 87.26% of the variation in data from the two locations. A 95% confidence interval shows that the amplitude, though estimated to be 13.75°, may be on the interval of 11.37° to 16.14°.







By looking at graphs above of the residual vs time graph showing no trend, the bell-shaped histogram, and an ACF showing white noise, it is apparent based on this residual analysis that the expected trend line fitted for the difference in the weather data may be considered for predicting future values.

In conclusion, Minneapolis, and Houston both exhibit a cosine trend over the same period from year to year. The estimated amplitude of Minneapolis of 28.43° compared to the estimated amplitude of Houston, which is 14.67°, is almost twice that of Houston. Geographically both cities reside near a similar longitude, however different factors are present in the 1000-mile distance which cause the climates of the cities to exhibit significantly different cosine trends with different amplitudes. Due to the cities experiencing weather temperature changes during the same seasons, a trend was apparent in the analysis of the difference in temperature from Minneapolis to Houston, therefore an estimated trend may be suggested to predict the weather in Houston given the weather in Minneapolis.

Sources:

National Weather Service. "NOW Data – NOAA Online Weather Data" *Monthly Mean Avg Temperature for Minneapolis-St Paul Area.* Years: 2020-2021 http://www.weather.gov/wrh/climate

National Weather Service. "NOW Data – NOAA Online Weather Data" *Monthly Mean Avg Temperature for Houston Area.* Years: 2020-2021 http://www.weather.gov/wrh/climate