10/6 Worksheet **Math 20 Fall 2014, Dartmouth College**

1 Functions of a Random Variable

Consider a continuous random variable X with sample space $\Omega = [0, 1]$ and cumulative distribution function:

$$F_X(x) = P[X \in [0, x]].$$

Let $y = 2x$ be a function of X. If $Y = 2X$ is another random variable, what is the sample space of Y? What is the cumulative distribution function $F_Y(y) = P[Y \in [0, y]]$ in terms of F_X and y?

In general, for a strictly increasing function $Y = g(X)$:

$$F_Y(y) =$$

What would be the version of the above argument for $Y = -X$?

In general, for a strictly decreasing function $Y = g(X)$:

$$F_Y(y) =$$

If $g(x)$ is strictly increasing and $Y = g(x)$, then:

$$f_Y(y) = f_X(g^{-1}(y))\frac{d}{dy}g^{-1}(y)$$

If $g(x)$ is strictly decreasing and $Y = g(x)$, then:

$$f_Y(y) = -f_X(g^{-1}(y))\frac{d}{dy}g^{-1}(y)$$

2 Choose a number U from the interval $[0, 1]$ with uniform density. Find the density functions $f_Y(y)$ for the random variables:

a. $Y = U + 2$

b. $Y = U^3$

c. $Y = 1/(U + 1)$

d. $Y = \log(U + 1)$

What about $U = |X - 1/2|$?

3 Exponential density

The exponential density function is:

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } 0 \leq x < \infty \\ 0, & \text{otherwise} \end{cases}$$

for some positive constant λ.

Plot a graph of the exponential densities for $\lambda = 2$ and $\lambda = 1$.

What is the cumulative distribution function $F(x)$ in this case?

Show that for any positive constants r and s, $P(X > r + s | X > r) = P(X > s)$. what does that mean, in words?