

**1 Functions of a Random Variable**

Consider a continuous random variable  $X$  with sample space  $\Omega = [0, 1]$  and cumulative distribution function:

$$F_X(x) = P[X \in [0, x]].$$

Let  $y = 2x$  be a function of  $X$ . If  $Y = y(x)$  is another random variable, what is the sample space of  $Y$ ? What is the cumulative distribution function  $F_Y(y) = P[y \in [0, y]]$  in terms of  $F_X$  and  $y$ ?

What would be the version of the above argument for  $y = -x$ ?

**Theorem 1** *Let  $X$  be a continuous random variable, and suppose that  $g(x)$  is a strictly increasing function on the range of  $X$ . Define  $Y = g(x)$ . If  $X$  and  $Y$  have cumulative distribution functions  $F_X$  and  $F_Y$  respectively. Then these functions are related by:*

$$F_Y(y) = F_X(g^{-1}(y)).$$

*If  $g(x)$  is strictly decreasing, then:*

$$F_Y(y) = 1 - F_X(g^{-1}(y)).$$