3.A Documentation of our package Puiseux

We have not found any implementation of rational Puiseux series as a package to any common software (Maple, Mathematica, Matlab), so we have implemented it in Mathematica. The package computes (rational) Puiseux series of a given polynomial. As the auxiliary procedures, it can also compute and plot the Newton polygon of the polynomial and the field of coefficients of the Puiseux series. The package is attached to this thesis and it is also downloadable from `www.karlin.mff.cuni.cz/~eblazkova/Puiseux`. The documentation follows.

Puiseux series

The command `PuiseuxExpansion` generates Puiseux series of a given polynomial at a given point.

`PuiseuxExpansion[f]`	generates list of Puiseux series of branches of f at the origin.
`PuiseuxExpansion[f,{x,y}]`	generates list of Puiseux series of branches of f at the point $[x, y]$.

The option `Length` influences the number of terms of Puiseux series. The default setting is 6.

In[1]:= `PuiseuxExpansion[`$2x^5 - x^3y + 2x^2y^2 - xy^3 + 2y^5 - x^5y^2 + 3x^2y^3$`]`

Out[1]= $$\Big\{\{x, 2x^2 + 8x^3 + 56x^4 + 504x^5 + 5052x^6 + 54080x^7\},$$
$$\Big\{\frac{x^2}{7}, \frac{x^2}{7} + \frac{x^3}{7} + \frac{6x^4}{49} + \frac{155x^5}{686} + \frac{123x^6}{343} + \frac{93519x^7}{134456}\Big\},$$
$$\Big\{2x^2, x - 2x^2 - 7x^3 - 28x^4 - \frac{313x^5}{2} - 1016x^6\Big\}\Big\}$$

In[2]:= `PuiseuxExpansion[`$2x^5 - x^3y + 2x^2y^2 - xy^3 + 2y^5 - x^5y^2 + 3x^2y^3$`,`
`Length -> 3]`

Out[2]= $$\Big\{\{x, 2x^2 + 8x^3 + 56x^4\}, \Big\{\frac{x^2}{7}, \frac{x^2}{7} + \frac{x^3}{7} + \frac{6x^4}{49}\Big\}, \{2x^2, x - 2x^2 - 7x^3\}\Big\}$$

In[3]:= `PuiseuxExpansion[`$-1 - 12x - 10x^2 - 4x^3 - x^4 + 8xy + 4x^2y - 2xy^2 - x^2y^2$`,`
`{-1, 2}, Length -> 3]`

Out[3]= $$\Big\{\Big\{-1 + x, 2 + x^2 + \frac{x^4}{2} + \frac{3x^6}{8}\Big\}, \Big\{-1 + x, 2 - x^2 - \frac{x^4}{2} - \frac{3x^6}{8}\Big\}\Big\}$$

Puiseux extension field

The field of coefficients of the Puiseux series is field extension of $\mathbb{Q}$. The adjoined elements are searched by `PuiseuxExtension` command. If we are interested only whether the field of coefficients is subfield of $\mathbb{Q}, \mathbb{R}$ or $\mathbb{C}$, we may use command `PuiseuxExtensionField`.

`PuiseuxExtension[f]`	generates elements of the field of coefficients of Puiseux series of f.
`PuiseuxExtensionField[f]`	returns the smallest field (from `Rationals`, `Reals` and `Complex`) which contains the field of coefficients of Puiseux series of f as subfield.

In[4]:= `PuiseuxExtension[`$x^2+y^2+x^3$`]`

Out[4]= `{{`$\mathbb{i}$`},{-`$\mathbb{i}$`}}`

In[5]:= `PuiseuxExtensionField[`$x^2+y^2+x^3$`]`

Out[5]= `{Complex, Complex}`

In[6]:= `PuiseuxExtension[`$2x^5-x^3y+2x^2y^2-xy^3+2y^5-x^5y^2+3x^2y^3$`]`

Out[6]= `{{}, {}, {}}`

In[7]:= `PuiseuxExtensionField[`$2x^5-x^3y+2x^2y^2-xy^3+2y^5-x^5y^2+3x^2y^3$`]`

Out[7]= `{Rationals, Rationals, Rationals}`

The possible options for both commands are `Series, Length`. For the command `PuiseuxExtensionField` is also possible to set `Extension`.

Option name	Default value	
`Series`	`False`	set whether the function returns also the Puiseux series of a given branch
`Length`	`3`	set the number of terms of the Puiseux series
`Extension`	`False`	set whether the function returns also the list of nonrational coefficients of Puiseux series

In[8]:= `PuiseuxExtension[`$x^2+y^2+x^3$`, Series -> True, Length -> 6]`

Out[8]= $$\left\{\left\{\{\mathbb{i}\},\left\{\mathbb{i}x,\ x+\frac{\mathbb{i}x^2}{2}+\frac{x^3}{8}-\frac{\mathbb{i}x^4}{16}-\frac{5x^5}{128}+\frac{7\mathbb{i}x^6}{256}\right\}\right\},\right.$$
$$\left.\left\{\{\mathbb{i}\},\left\{-\mathbb{i}x,\ x-\frac{\mathbb{i}x^2}{2}+\frac{x^3}{8}+\frac{\mathbb{i}x^4}{16}-\frac{5x^5}{128}-\frac{7\mathbb{i}x^6}{256}\right\}\right\}\right\}$$

In[9]:= `PuiseuxExtensionField[`$x^2+y^2+x^3$`, Extension -> True]`

Out[9]= `{{Complex, {`$\mathbb{i}$`}}, {Complex, {-`$\mathbb{i}$`}}}`

In[10]:= `PuiseuxExtensionField[`$x^2+y^2+x^3$`, Extension -> True,`
`Polynomial -> True, Length -> 3]`

Out[10]= $$\left\{\left\{\texttt{Complex},\ \{\mathbb{i}\},\left\{\mathbb{i}x,\ x+\frac{\mathbb{i}x^2}{2}+\frac{x^3}{8}\right\}\right\},\right.$$
$$\left.\left\{\texttt{Complex},\ \{-\mathbb{i}\},\left\{-\mathbb{i}x,\ x-\frac{\mathbb{i}x^2}{2}+\frac{x^3}{8}\right\}\right\}\right\}$$

Newton Polygon

The package Puiseux can generate the Newton polygon of a given two variables polynomial (`PolygonNewton`), its edges (`EdgesNewton`) and plot it (`PlotNewton`).

`PolygonNewton[f, z]`	generates the list of edges of Newton polygon of f. `{`$\mathtt{e_1, e_2, e_3, e_4}$`}` represents an edge with the normal vector $(e_1, e_2, -e_3)$ and the characteristic polynomial e_4 in the given variable z.
`EdgesNewton[f]`	generates the list of edges of the Newton polygon of f given by the points,
`PlotNewton[f]`	plots the Newton graph and polygon of f

In[11]:= `PolygonNewton[`$2x^5 - x^3y + 2x^2y^2 - xy^3 + 2y^5 - x^5y^2 + 3x^2y^3$`, z]`

Out[11]= `{{1, 2, 5, 2 - z}, {1, 1, 4,` $-1 + 2z - z^2$`}, {2, 1, 5, -1 + 2z}}`

In[12]:= `Factor[%]`

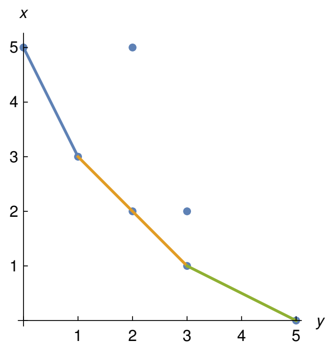
Out[12]= `{{1, 2, 5, 2 - z}, {1, 1, 4,` $-(-1+z)^2$`}, {2, 1, 5, -1 - 2z}}`

In[13]:= `EdgesNewton[`$2x^5 - x^3y + 2x^2y^2 - xy^3 + 2y^5 - x^5y^2 + 3x^2y^3$`]`

Out[13]= `{{{0, 5}, {1, 3}}, {{1, 3}, {3, 1}}, {{3, 1}, {5, 0}}}`

In[14]:= `PlotNewton[`$2x^5 - x^3y + 2x^2y^2 - xy^3 + 2y^5 - x^5y^2 + 3x^2y^3$`]`

Out[14]=



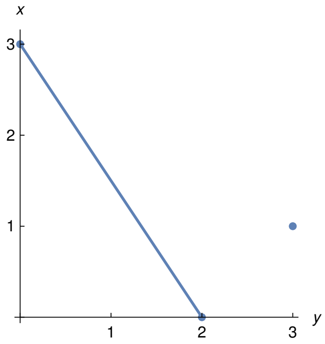
The following options for the Newton polygon methods are accepted:

Option name	Default	Commands
`PositiveSlope`	`True`	`PolygonNewton, EdgesNewton, PlotNewton`
`All`	`True`	`EdgesNewton, PlotNewton`

`PositiveSlope`

When this option is `True` the Newton polygon will contain all edges including the edges with the positive slope. The value `False` will disable the edges of the Newton polygon with positive slope. The default value is `False`.

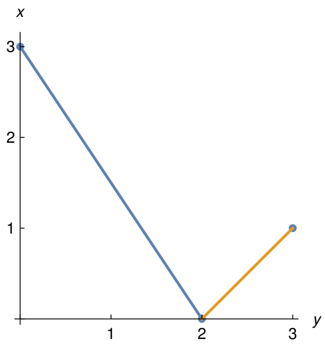
In[15]:= `PlotNewton[`$y^2 - 2x^3 + xy^3$`]`

Out[15]= 

In[16]:= PolygonNewton[$y^2 - 2x^3 + xy^3$, z]

Out[16]= {{2, 3, 6, -2 + z}}

In[17]:= PlotNewton[$y^2 - 2x^3 + xy^3$, PositiveSlope -> True]

Out[17]= 

In[18]:= PolygonNewton[$y^2 - 2x^3 + xy^3$, z PositiveSlope -> True]

Out[18]= {{2, 3, 6, -2 + z}, {1, -1, -2, 1 + z}}

`All`

When this option is set to `True` the edges of the Newton polygon will contain all points. Otherwise, when the option value is `False`, every edge will contain only its endpoints. The default value is `False`.

In[19]:= EdgesNewton[$- x^3y + 2x^2y^2 - xy^3 + y^4 + x^4 + x^5$]

Out[19]= {{{0, 4}, {4, 0}}}

In[20]:= EdgesNewton[$- x^3y + 2x^2y^2 - xy^3 + y^4 + x^4 + x^5$, All -> True]

Out[20]= {{{0, 4}, {1, 3}, {2, 2}, {3, 1}, {4, 0}}}