1 (a) $T(n) = 10T(n/3) + n^2$

Master Method:

$f(n) = n^2$

$g(n) = n^{\log_3 10 - \varepsilon}$ $\quad \varepsilon > 0$

$f(n)$ $g(n)$ relationship:

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^2}{n^{\log_3 10 - \varepsilon}}$

$\log_3 10 = 2.096$.

when $\varepsilon = 0.096$.

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 1$ constant

$f(n)$ is $O(g(n))$

$\Rightarrow$ Case 1 : $T(n)$ is $\Theta(n^{\log_3 10})$

(b) $T(n) = 4T(n/2) + n^3$

$f(n) = n^3$

$g(n) = n^{\log_2 4} = n^2$

$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \frac{n^2}{n^3} = 0$ constant

$\Rightarrow g(n)$ is $O(f(n))$

$f(n)$ is $\Omega(g(n))$

$4f(n/2) \leq cf(n)$ for $c > 1$

$\Rightarrow T(n)$ is $\Theta(n^3)$

(c) $T(n) = 9T(n/3) + n\log n$

$f(n) = n\log n$

$g(n) = n^{\log_3 9} = n^3$

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n\log n}{n^3} = 0$

$f(n)$ is $O(g(n))$ is also $o(g(n))$

Case 1:

$T(n)$ is $\Theta(n^3)$

(d) $T(n) = T(n-1) + n^7$

$= (T(n-2) + (n-1)^7) + n^7$

$= (T(n-3) + (n-2)^7) + (n-1)^7 + n^7$

$= \sum_{i=0}^{n} n^7$

$= \Theta(n^8)$

(e) $T(n) = 8T(n/2) + 5n^3$

$f(n) = 5n^3$

$g(n) = n^{\log_2 8} = n^3$

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{5n^3}{n^3} = 5$. $\quad \lim_{n\to\infty} \frac{g(n)}{f(n)} = \frac{n^3}{5n^3} = \frac{1}{5}$

$f(n)$ is $\Theta(g(n))$

Case 2: $T(n)$ is $\Theta(n^3 \log n)$

(f) $T(n) = n(T(n/2)^2)$

$T(n) < (n\,T(n/2))^2$

Transform domain : $n = 2^i$

$T(n) = T(2^i) = 2^i(T(2^{i-1}))$

$S(i) = T(2^i) = 2^i(S(i-1))$

$S(i) = 2^i(S(i-1))$

$= 2^i(2^{i-1} \cdot S(i-2))$

$= 2^i \cdot 2^{i-1} \cdot 2^{i-2} \cdot S(i-3)$

$\vdots$

$= 2^{\sum_{k=0}^{i} k} = 2^{\frac{i(i+1)}{2}}$

$S(i)$ is $\Theta(2^{i^2})$

$n = 2^i \Rightarrow i = \log_2 n$

$S(i) = T(n)$

is $\Theta(2^{(\log_2 n)^2})$

$T(n)$ is $\Theta(2^{(\log n)^2})$

2 (a) $T(n) = C + n + (n-1) + (n-2) + \cdots + 1$ ← last j loop execute only once.

1st j loop 2nd j loop 3rd j loop ....

$$= \frac{1}{2} n(n+1) + C.$$

$T(n)$ is $\Theta(n^2)$

(b) $T(n) = T(n-1) + C$

$$= (T(n-2) + C) + C$$

$$= \sum_{k=0}^{n} c$$

$$= nc.$$

$T(n)$ is $\Theta(n)$

(c) Assume return value is r

r follows this series

$$r = n_0 \cdot \left(\frac{1}{2}\right)^{k-1} + 5$$

program will terminate soon when

$r \leq 10.$

$$n_0\left(\frac{1}{2}\right)^{k-1} + 5 \leq 10$$

$$n_0\left(\frac{1}{2}\right)^{k-1} \leq 5$$

$$\left(\frac{1}{2}\right)^{k-1} \leq \frac{5}{n}$$

$$k = \log_{1/2}\frac{5}{n} + 1$$

$$= \log_2 \frac{n}{5} + 1$$

$$T(n) = \Theta(\log n)$$

(d) ii follow Geometric series:

$1, 2, 4 \cdots\cdots$

$$ii = 2^{k-1} = n$$

$$k-1 = \log_2 n$$

$$k = \log_2 n + 1$$

$$T(n) = n \cdot (\log_2 n + 1) + C$$

$$T(n) = n \log n$$