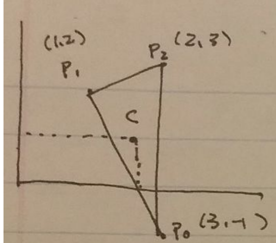
Eva (Fan-Yi) Kung
998137573, g3ekungg

CSC418 Assignment #2: Part 1

Q1)

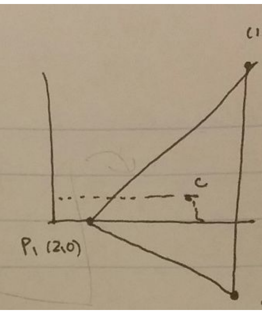
a)



centroid (c)

$$= \left[\frac{1+2+3}{3} , \frac{2+3-1}{3} \right]$$

$$= [2, 4/3]$$



C =

$$\left[\frac{2+8+10}{3} , \frac{0-4+8}{3} \right]$$

$$= [20/3, 4/3]$$

Centralize on origin.

P_0 $\begin{bmatrix} 3-2 \\ -1-\frac{4}{3} \end{bmatrix} = \begin{bmatrix} 1 \\ -\frac{7}{3} \end{bmatrix}$

① P_1 $\begin{bmatrix} 1-2 \\ 2-4/3 \end{bmatrix} = \begin{bmatrix} -1 \\ \frac{2}{3} \end{bmatrix}$

P_2 $\begin{bmatrix} 2-2 \\ 3-\frac{4}{3} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{5}{3} \end{bmatrix}$

Flip on y axis. $\therefore x = 0$

② $[P_0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{7}{3} \end{bmatrix}$. $[P2] \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{5}{3} \end{bmatrix}$

$[P_1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -\frac{2}{3} \end{bmatrix}$

③ Shift up by $\frac{2}{3}$ (translate)

$[P_0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $[P_1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ $[P_2] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

④ Translate right along x by 3

$[P_0] \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ $[P_1] \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $[P_2] \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

⑤ Non uniform scale each.

$[P_0] \begin{bmatrix} 5/2 & 0 & 0 \\ 0 & 8/3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \end{bmatrix}$ $[P_1] = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $[P_2] \begin{bmatrix} 8/3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$

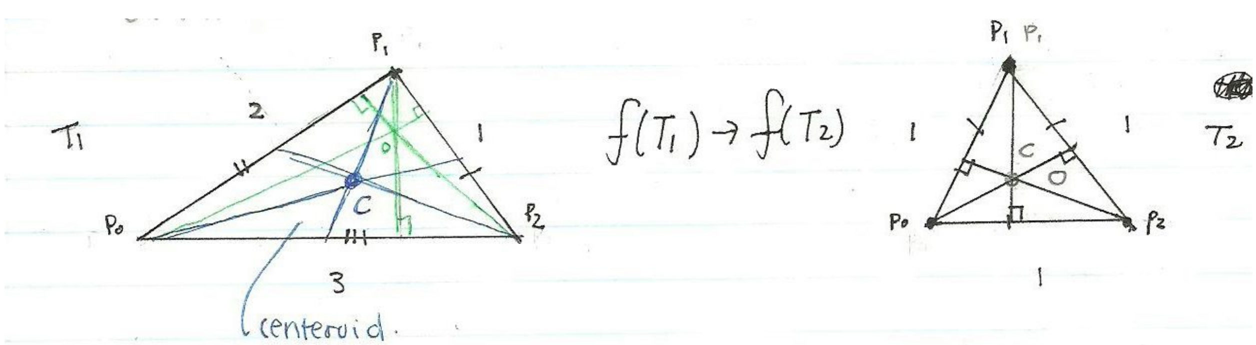
$\therefore$ The Final Transformation Matrix :

$$
\begin{array}{ccccccccc}
 & & ⑤ & ④ & ③ & ② & ① & & \\
(10,8) & \Leftarrow & \begin{bmatrix} 5/2 & 0 & 0 \\ 0 & 8/3 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 6 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2/3 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -4/3 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} [P_0] \\ 1 \end{bmatrix} & \Leftarrow P_0 \\
(2,0) & \Leftarrow & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2/3 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -4/3 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} [P_1] \\ 1 \end{bmatrix} & \Leftarrow P_1 \\
(8,-4) & \Leftarrow & \begin{bmatrix} 8/3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2/3 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -4/3 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} [P_2] \\ 1 \end{bmatrix} & \Leftarrow P_2
\end{array}
$$

b)
A 2D homography requires at 3 point mappings, meaning (x1,x2, x3) that lying on the same line such that h(x1), h(x2), h(x3) is on the same line still. (3x3 matrix)

Since 2D rigid transformations means that both the before and after images are congruent, only one pair of point mappings need to be specified. As we are only performing translations or rotations on the point. For instance moving a point (x,y) on a triangle up by 10 units : (x+10, y+10).

c)
The centroid and orthocenter of a triangle are not preserved under an affine projection.
Consider this counter example:



Let F(T1) be an affine transformation of T1, to T2 , it is transformed into an equilateral triangle, then this goes against the rule. You can see that through some transformation from T1 to T2, in this specific example (where T2 is an equilateral triangle), the orthocenter and centroid merges and becomes the same point where as it was not prior to the transformation. Therefore, it is not preserved under an affine projection.

Q2)

a)

In a real camera, the lens contains a variable focal length, it serves to create images of objects seen through it.

The focal length denotes the length of the lens, it is used to determine how near or far your image is seen through, which in other words, is called a ‘zoom’ function.

The aperture of a camera is the diameter of opening of adjustable depth, that controls the amount of light that goes through the lens affecting the amount of time it takes to snap a photo. A lower aperture creates a sharper focus whereas a higher aperture creates a more blurrier image

b)

Camera position : $(2,1,3)$

View direction $= (-1-2,\ 2-1,\ (-3)) = (-3,1,-2)$

up direction $= (0,1,0)$

→ Let A be unit vector : $\dfrac{(-3,1,-2)}{\sqrt{14}}$, $B = \dfrac{(3,-1,2)}{\sqrt{14}}$, $C = (0,1,0)$

$$u = \frac{C \times B}{\|C \times B\|} \qquad C \times B = (0,1,0) \times \frac{(3,-1,2)}{\sqrt{14}} = \frac{(2,0,-3)}{\sqrt{14}} \qquad \|C \times B\| = \frac{1}{\sqrt{14}}\frac{1}{\sqrt{4+9}} = \frac{1}{\sqrt{14}}\frac{1}{\sqrt{13}}$$

$$\boxed{\therefore u = \frac{1}{\sqrt{13}}(2,0,-3)}$$

$$V = \frac{B \times u}{\|B \times u\|} \qquad B \times u = \frac{(3,-1,2)}{\sqrt{14}} \times \frac{(2,0,-3)}{\sqrt{13}} = \frac{1}{\sqrt{14}\sqrt{13}}(3,13,2)$$

$$\|B \times u\| = \frac{1}{\sqrt{14}\sqrt{13}} \cdot \frac{1}{\sqrt{182}} = \frac{1}{\sqrt{182}\sqrt{182}} = \frac{1}{182} \qquad \boxed{\therefore V = \frac{1}{\sqrt{182}}(3,13,2)}$$

$$\begin{bmatrix} u_x & v_x & B_x & e_x \\ u_y & v_y & B_y & e_y \\ u_z & v_z & B_z & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{13}} & \frac{3}{\sqrt{182}} & \frac{3}{\sqrt{14}} & 2 \\ 0 & \frac{13}{\sqrt{182}} & \frac{-1}{\sqrt{14}} & 1 \\ \frac{-3}{\sqrt{13}} & \frac{2}{\sqrt{182}} & \frac{2}{\sqrt{14}} & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} , \quad \begin{bmatrix} \frac{2}{\sqrt{13}} & 0 & \frac{-3}{\sqrt{13}} \\ \frac{3}{\sqrt{182}} & \frac{13}{\sqrt{182}} & \frac{2}{\sqrt{182}} \\ \frac{3}{\sqrt{14}} & \frac{-1}{\sqrt{14}} & \frac{2}{\sqrt{14}} \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{4-9}{\sqrt{13}} \\ \frac{6+13+6}{\sqrt{182}} \\ \frac{6-1+6}{\sqrt{14}} \end{bmatrix} = \begin{bmatrix} \frac{-5}{\sqrt{13}} \\ \frac{25}{\sqrt{182}} \\ \frac{11}{\sqrt{14}} \end{bmatrix}$$

Therefore, the Camera to world transformation is:

$$\begin{bmatrix} \frac{2}{\sqrt{13}} & 0 & \frac{-3}{\sqrt{13}} & \frac{-5}{\sqrt{13}} \\ \frac{3}{\sqrt{182}} & \frac{13}{\sqrt{182}} & \frac{2}{\sqrt{182}} & \frac{25}{\sqrt{182}} \\ \frac{3}{\sqrt{14}} & \frac{-1}{\sqrt{14}} & \frac{2}{\sqrt{14}} & \frac{11}{\sqrt{14}} \end{bmatrix}$$

c)
The solution to this problem is done through Perspective/Planar projection.

d)
A family of parallel lines with the direction vector b do not remain parallel after perspective projection

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Q3)
a)

$$P = (x,y,z)$$
$$f(x,y,z) = (R - \sqrt{x^2+y^2}\,)^2 + z^2 - r^2$$
$$= (R - \sqrt{x^2+y^2})(R - \sqrt{x^2+y^2}) + z^2 - r^2$$
$$= R^2 - 2R\sqrt{x^2+y^2} + x^2 + y^2 - r^2$$

$$Gradient(f) = f'(x,y,z) = \langle f_x, f_y, f_z \rangle$$
$$= \langle -\frac{2Rx}{\sqrt{x^2+y^2}} + 2x\,,\ -\frac{2Ry}{\sqrt{x^2+y^2}} + 2y,\ 2z \rangle$$

b)

$$Implicit\ equation\ of\ the\ surface: \langle f_x,\ f_y,\ f_z \rangle \cdot \langle x-x0,\ y-y0,\ z-z0 \rangle = 0$$
$$Therefore,\ \langle -\frac{2Rx}{\sqrt{x^2+y^2}} + 2x,\ -\frac{2Ry}{\sqrt{x^2+y^2}} + 2y,\ 2z \rangle \cdot \langle x-x0,\ y-y0,\ z-z0 \rangle = 0$$

c)

$q(\lambda) = (Rcos\lambda,\ Rsin\lambda,\ r)$, point q lies on the surface if f(q(λ))=0

$$(R - \sqrt{(Rcos\lambda)^2 + (Rsin\lambda\,)^2 + r - r^2}$$
$$= R - \sqrt{R^2cos^2\lambda + R^2sin^2\lambda}$$
$$= R - \sqrt{R^2(cos^2\lambda + sin^2\lambda)}$$
$$= R - R = 0$$

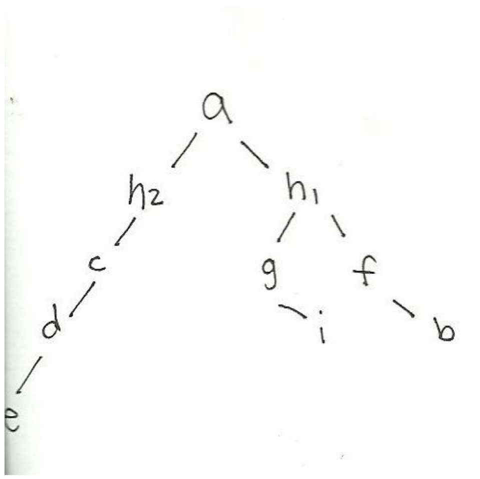
d)
Let T(λ) be the tangent directional vector for q(λ)
Since $q'(\lambda) = (-Rsin\lambda,\ Rcos\lambda,\ 0) \Rightarrow$ Then T(λ) = $-Rsin\lambda + Rcos\lambda$

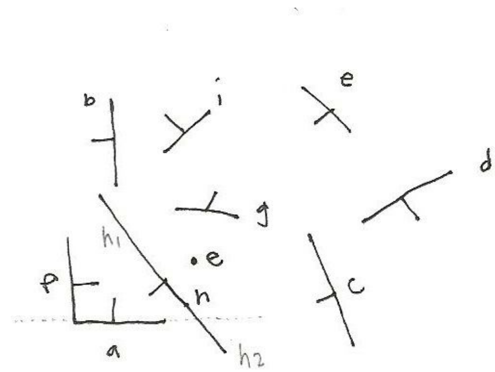
Q4)
a)

Yes, it is possible. By using perspective projection, some of the segments may not be included.

Consider the following in figure 2, since we want to render the furthest wall first, segments h f and a will not be seen since h has a outward normal facing OPPOSITE direction from the eye. Therefore, segments f and a will not be seen as it is going to be blocked from wall h. The Eye/Camera can only see the back of wall h, but nothing beyond it.

b)





c)
Render order: h2, c, d, e, a, b, f , h1, g, i.
Rendering order above goes from the furthest wall to the nearest.