1. ________ (choose from Gilbert Strang, Stephen Boyd, Sheldon Axler) became famous for writing the paper “Down with Determinants “, in which it is postulated that Determinant is not required in Linear Algebra and hence no need of teaching it at any level.

a) Sheldon Axler

b) Gilbert Strang

c) Stephen Boyd

d) None of them

2 . ________ (choose from S.G .Johnson, Sheldon Axler, Boyd) became famous for writing the paper “A useful basis for defective matrices: Generalized eigenvectors and the Jordan form”.

a) Sheldon Axler

b) Gilbert Strang

c) Stephen Boyd

d) S.G. Johnson

3. In a 3 by 3 matrix if the third column is sum of first two columns which are independent , one eigenvalue is ________ and one right null space vector is ________

a) 0 , [1; 1 ;-1]

b) 1 , [1; 1 ;-1]

c) -1 , [1; 1 ;-1]

d) 1 , [-1; -1 ;2]

4. The vector matrix product $\begin{pmatrix} x & y & z \end{pmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ 3 & 5 & 6 \end{bmatrix}$

can expressed as a linear combination of row vectors as ________

a) $x(1\ 2\ \ 2) + y(2\ 1\ 1) + z(3\ 5\ 6)$

b) $x(1\ 2\ \ 3) + y(2\ 1\ 5) + z(2\ 1\ 6)$

c) $y(1\ 2\ \ 2) + z(2\ 1\ 1) + x(3\ 5\ 6)$

d) $z(1\ 2\ \ 2) + x(2\ 1\ 1) + y(3\ 5\ 6)$

5. If all the column sum of a 3 by 3 square matrix is 5, then one eigen value is

______ and one left eigen vector is _____

a) 5, [1 1 1];

b) 5, [-1 1 1];

c) -5, [1 -1 1];

d) 5, [1 1 -1];

6. A stochastic matrix is a square matrix with all elements as ________ numbers with all the row/column as _______

a) probability, vectors with sum as 1

b) real, vectors with sum as 0

c) negative, vectors with sum as -1.

d) integers, vectors with sum as 1

7. The common eigenvalue of any stochastic(also called markov) matrix is________

a) 1,

b) 0

c) -1

d) none of these

8. ______ and ______ subspaces of A and $\begin{bmatrix} A \\ A \end{bmatrix}$ is same

a) Rowspace and Right null space

b) Column space and Left null space

c) Rowspace and Left null space

d) cannot compare row space of these 2 matrices

9. If $A = uv^T + wz^T$ then ______ and ______ vectors span the column space and ______ and ______ vector span row space

a) u and w   v and z

b) v and z   u and w

c) v and w   u and z

d) u and z   v and w

10 Suppose S is spanned by vectors (1,5,1) and (2,2,2) . Then $S^{\perp}$ is the right null space of the matrix A= _______

a) $A = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 2 & 2 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 2 & 2 \end{bmatrix}^T$

c) $A = \begin{bmatrix} 1 & 2 \\ 5 & 2 \\ 1 & 2 \end{bmatrix}$

d) None of the above

11 A rank-one matrix can be expressed as $A = u\mathrm{v}^T$ where u and v are vectors. Then u , v for rank-one matrix $A = \begin{bmatrix} 2 & -2 \\ 6 & -6 \end{bmatrix}$ is ________ and ________

a) u=[2; 6] v = [1; -1]

b) u=[2, 6] v = [1; -1]

c) u=[-2; 6] v = [-1; -1]

d) u=[2; 6] v = [-1, -1]

12. A is an m by n matrix. If all columns are independent , row space dimension is ________ and right null space dimension is _______

a) n , 0

b) m, m-n

c) n, n-m

d) n, n

13. If the columns of A are linearly independent (A is m by n), then the rank is _____ , and the dimensions of the right null space is _____, the dimension of row space is _____ and there exists a _____ inverse (left/right).

a) n, 0 , n , left

b) m, 0 , n , right

c) n, 0 , m , left

d) n, 0 , n , right

Hint $(A^T A)^{-1}(A^T A) = I_{n\times n}$

14. If all columns of matrix A are independent ______ inverse (left/right) is given by ______

a) left, $(A^T A)^{-1}(A^T)$

b) right, $(A^T A)^{-1}(A)$

c) left, $(AA^T)^{-1}(A^T)$

d) right, $(A^TA)^{-1}(A^T)$

15 If all rows of matrix A are independent _______ inverse (left/right) is given by ______

a) right $A^T(AA^T)$

b) left $A^T(A^TA)$

c) right $A^T(AA^T)$

d) left $A^T(AA^T)$

hint $(\mathrm{AA}^T)(AA^T) = I_{m\times m}$

16. Ax=b can be considered as mapping from ______ space to ______ space

(Hint A takes 'x' and output 'b')

a) rowspace to column space

b) column space to row space

c) column space to left null space

d) row space to right null space

('x' is mapped to a 'b')

17. Let A be m by n matrix. Then any $x \in R^n$ can be expressed using the combined bases of ______ space and ______space

a) Row space and Right null space

b) Column space and Right null space

c) Row space and Left null space

d) Row space and Column space

18. Let A be m by n matrix. Then any $y \in R^m$ can be expressed using the combined bases of ______ space and ______ space

a) Column space and Left nullspace

b) Column space and Right nullspace

c) Row space and Right nullspace

d) Column space and Row nullspace

19. Let A be m by n matrix. Then row space and ______ space are subspaces of ______

a) Right nullspace , R^n

b) Left nullspace , R^m

c) Column space , R^n

d) Right nullspace , R^m

20 Let A be m by n matrix. Then column space and ______ space are subspaces of ______

a) Left nullspace , R^m

b) Right nullspace , R^n

c) Row space , R^m

d) Right nullspace , R^n

21 Row space and ______ of any matrix A are orthogonal

a) right null space

b) column space

c) left null space

22 ______ and left null space of any matrix A are orthogonal

a) column space

b) row space

c) right null space

23 Let A be m by n matrix. Then the sum of dimension of column space and dimension of ______ space is equal to ______

a) Left null space, m

b) Row space, n

c) Right null space, m

d) Column space, m

24 Let A be m by n matrix. Then the sum of dimension of row space and dimension of ______ space is equal to ______

a) Left null space, m

b) Row space, n

c) Right null space, n

d) Column space, m

25 Vector x is projected on to row space and obtained vector y. So upon projecting x onto right null space , we will obtain ______ (answer in terms of x and y)

a) $x - y$

b) $x + y$

c) $-x - y$

d) $y - x$

26 Expression for Projection of b vector on to **column space** of A is ______

a) $A(A^TA)^{-1}A^Tb$

b) $A^T(A^TA)^{-1}Ab$

c) $A(AA^T)^{-1}A^Tb$

d) $A(A^TA)^{-1}A^Tb$

27 Expression for Projection of c vector on to **row space** of A is ______

a) $A^T(AA^T)^{-1}Ac$

b) $A^T(A^TA)^{-1}Ac$

c) $A^T(AA^T)^{-1}A^Tc$

d) $A(AA^T)^{-1}Ac$

28 Expression for Projection of d vector on to left null space of A is ______

a) $\left(I_{m\times m} - A(A^TA)^{-1}A^T\right)b$

b) $\left(I_{n\times n} - A(A^TA)^{-1}A^T\right)b$

c) $\left(I_{m\times m} - A(A^TA)^{-1}A\right)b$

d) $\left(I_{n\times n} - A(A^TA)^{-1}A^T\right)b$

29 Eigenvalues of real symmetric matrix are ______

a) real

b) complex

c) always positive

d) always negative

30 Eigenvectors corresponding to different Eigenvalues of real symmetric matrix are ________

a) orthogonal

b) independent

c) dependent

d) non-orthogonal

32 If A is a hermitian matrix then , then $x^H Ax$ is _____

a) real

b) complex

c) always positive

d) always negative

33 2D-Rotation matrix that rotate (counter clock wise) a vector by 45 degree is given by _____

a) $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$

b) $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$

c) $\frac{1}{\sqrt{2}}\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$

d) $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

34 Absolute value of all eigenvalues of orthonormal matrix is ______

a) 1

b) 0

c) -1

d) depends on the values of the matrix elements

35 Eigenvalues of rotation matrix can never be _____

a) real

b) imaginary

c) positive

d) negative

36. A is an orthonormal matrix and let $Ax = y$ and $||x|| = 5$ then $||y|| =$ _______

a) 5

b) > 5

c) < 5

d) depends on the values of the elements of the matrix A

Write **a** if True else **b** for False

1. A 5×7 matrix never has linearly independent columns
2. Row elimination preserves right null space
3. Row elimination always destroy column space of a matrix of size $m \times n$ whose rank is < m
4. Row elimination preserve rank of the matrix
5. Row elimination preserve eigenvalues

6. Gaussian row elimination can be used to test whether given set of vectors are dependent or not.
7. (positive orthant) is a vector space
8. Set of points (x,y) on the line x+y =1 form a vector space
9. Set of points (x,y) on the line x+y =0 form a vector space
10. If the columns of a matrix are dependent, so are the rows
11. The column space of a 2 by 2 matrix is the same as its rows
12. The column space of a 2 by 2 matrix has the same dimension as its row space
13. The columns of a matrix are a basis for the column space
14. Rotation matrices are orthogonal matrices